Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2019-2020 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

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Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

The independence of the Maxwell equations

Time domain - Differential form

$$
\begin{cases}
\nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\
\nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) + \vec{\mathbf{j}}_0(\vec{\mathbf{r}},t) \\
\nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) + \rho_0(\vec{\mathbf{r}},t) \\
\nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0
\end{cases}
$$

Even assuming knowledge of the impressed sources $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t); \rho_0(\vec{\mathbf{r}},t)$

Maxwell equations involve a number of unknowns larger than the number of equations!

The additional missing equations are provided by the **constitutive relationships**, which describe interaction of fields and matter from a macroscopic point of view

Constitutive relationships

Inductions and currents represented in terms of fields

$$\vec{\mathbf{d}} = \mathscr{D}\left[\vec{\mathbf{e}},\vec{\mathbf{h}}\right]$$

$$\vec{\mathbf{b}} = \mathscr{B}\left[\vec{\mathbf{e}},\vec{\mathbf{h}}\right]$$

$$\vec{\mathbf{j}} = \mathscr{J}\left[\vec{\mathbf{e}},\vec{\mathbf{h}}\right]$$

$$\underbrace{\vec{\mathbf{e}},\vec{\mathbf{h}}}_{\textbf{Inputs}} \longrightarrow \boxed{\mathscr{D}[\cdot]} \longrightarrow \underbrace{\vec{\mathbf{d}}}_{\textbf{Output}}$$

$\mathscr{D}[\cdot],\mathscr{B}[\cdot]$ and $\mathscr{J}[\cdot]$ are functionals which depend upon the medium in which the electromagnetic field is considered and upon the fields themselves.

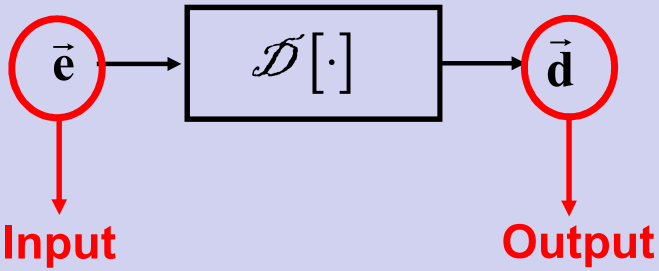
Constitutive relationships

Linear & Anisotropic media

$$\vec{\mathbf{d}} = \mathscr{D}\left[\vec{\mathbf{e}}\right]$$

$$\vec{\mathbf{b}} = \mathscr{B}\left[\vec{\mathbf{h}}\right]$$

$$\vec{\mathbf{j}} = \mathscr{J}\left[\vec{\mathbf{e}}\right]$$



Constitutive relationships

Linear media

- **Local (non-dispersive) media**
- **Anisotropic media**

$$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\boldsymbol{\varepsilon}(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$$
$$\vec{\mathbf{b}}(\vec{\mathbf{r}},t)=\boldsymbol{\mu}(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$
$$\vec{\mathbf{j}}(\vec{\mathbf{r}},t)=\boldsymbol{\sigma}(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$$

$\boldsymbol{\varepsilon}(\vec{\mathbf{r}},t);\boldsymbol{\mu}(\vec{\mathbf{r}},t);\boldsymbol{\sigma}(\vec{\mathbf{r}},t):$ 3x3 matrices

- **Local (non-dispersive) media**
- **Isotropic media**

$$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon(\vec{\mathbf{r}},t)\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$$
$$\vec{\mathbf{b}}(\vec{\mathbf{r}},t)=\mu(\vec{\mathbf{r}},t)\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$
$$\vec{\mathbf{j}}(\vec{\mathbf{r}},t)=\sigma(\vec{\mathbf{r}},t)\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$$

$\varepsilon(\vec{\mathbf{r}},t);\mu(\vec{\mathbf{r}},t);\sigma(\vec{\mathbf{r}},t):$ scalar functions

Constitutive relationships

Linear media

Class	Property	Effect on the I-O relation

Constitutive relationships

Linear media

Class	Property	Effect on the I-O relation
Time-dispersive		
Time-nondispersive		
Time-variant		
Time-invariant		

Constitutive relationships

Linear media

Class	Property	Effect on the I-O relation
Space-dispersive		
Space-nondispersive		
Space-variant		
Space-invariant		

Constitutive relationships

Linear media

Class

Time-dispersive

Time-nondispersive

Time-variant

Time-invariant

Property

The output **at time t depends only** on the value of the input **at the same time t**

Constitutive relationships

Linear media

Class

Time-dispersive

Time-nondispersive

Time-variant

Time-invariant

Property

The output **at time t depends** on the values of the input **throughout a time-interval**.

Constitutive relationships

Linear media

Class

Time-dispersive

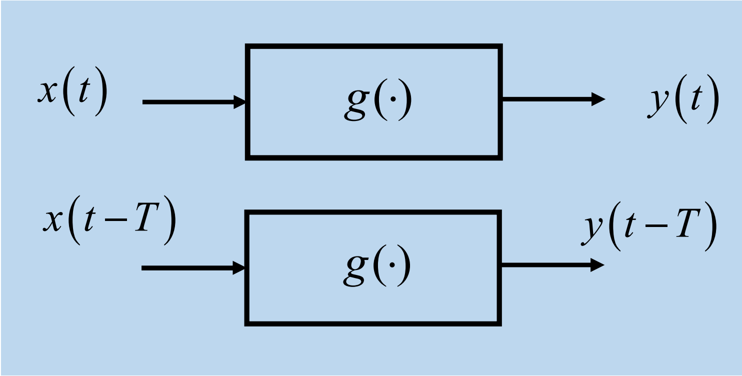
Time-nondispersive

Time-variant

Time-invariant

Property

A **time translation** of the input implies **the same translation** of the output



Constitutive relationships

Linear media

Class	Property
Time-dispersive **Time-nondispersive** **Time-variant** **Time-invariant**	A **time translation** of the input **does not** imply **the same translation** of the output

Constitutive relationships

Linear media

Class	Property
Space-dispersive	
Space-nondispersive	The output **at space $\vec{\mathbf{r}}$ depends only** on the value of the input **at the same space $\vec{\mathbf{r}}$**
Space-variant	
Space-invariant	

Constitutive relationships

Linear media

Class	Property
Space-dispersive	The output **at space $\vec{\mathbf{r}}$ depends** on the values of the input **throughout a space-interval**
Space-nondispersive	
Space-variant	
Space-invariant	

Constitutive relationships

Linear media

Class

Space-dispersive

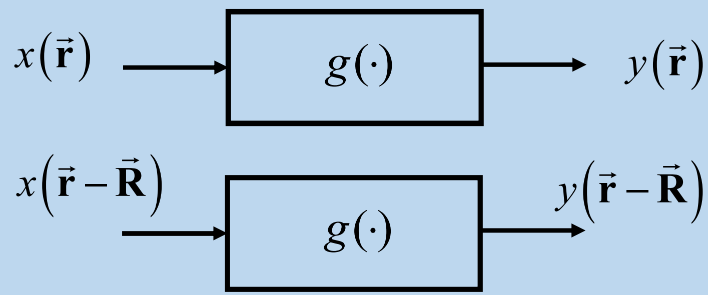
Space-nondispersive

Space-variant

Space-invariant

Property

A **space translation** of the input implies **the same translation** of the output



Constitutive relationships

Linear media

Class

Space-dispersive

Space-nondispersive

Space-variant

Space-invariant

Property

A **space translation** of the input **does not** imply **the same translation** of the output

Constitutive relationships

Linear media

Class	Property	Effect on the I-O relation

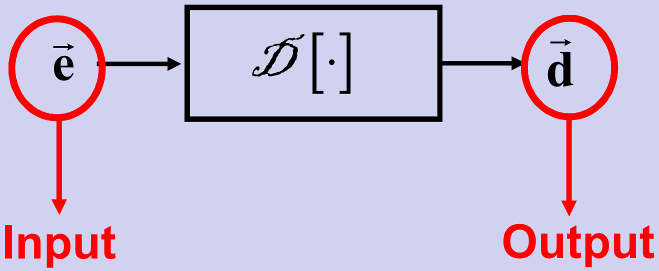
Constitutive relationships

Linear & Anisotropic media

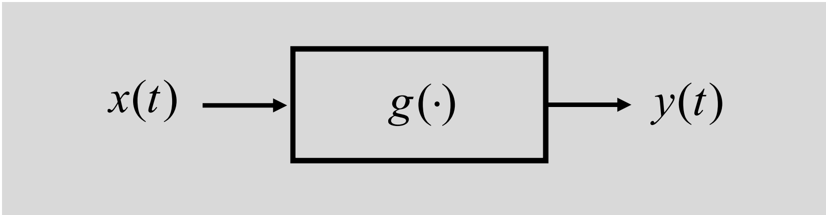
$$\vec{\mathbf{d}} = \mathscr{D}\left[\vec{\mathbf{e}}\right]$$

$$\vec{\mathbf{b}} = \mathscr{B}\left[\vec{\mathbf{h}}\right]$$

$$\vec{\mathbf{j}} = \mathscr{J}\left[\vec{\mathbf{e}}\right]$$



Memo: time-dispersive (TD) linear systems



Effect on the I-O relation

$$y(t) = \int dt'\, g(t,t')\, x(t')$$

The output **at time t depends** on the values of the input **throughout a time-interval**.

In the most general case, these systems possess an heredity: they are called **dispersive**

Linear systems

$$x(t) \longrightarrow \boxed{\;g(\cdot)\;} \longrightarrow y(t)$$

$x(t)$ and $y(t)$ are scalar

	Time-dispersive (TD)	**Time-nondispersive (TND)**
Time-variant (TV)	$y(t)=\int dt'\, g(t,t')\, x(t')$	$y(t)=\tilde{g}(t)\, x(t)$
Time-invariant (TI)	$y(t)=\int dt'\, g(t-t')\, x(t')$	$y(t)=\tilde{g}\, x(t)$

Linear systems

$$\vec{\mathbf{x}}(t) \longrightarrow \boxed{\mathbf{g}(\cdot)} \longrightarrow \vec{\mathbf{y}}(t)$$

$\vec{\mathbf{x}}(t)$ and $\vec{\mathbf{y}}(t)$ are vectors

	Time-dispersive (TD)	**Time-nondispersive (TND)**
Time-variant (TV)	$\vec{\mathbf{y}}(t) = \int dt' \, \mathbf{g}(t,t') \cdot \vec{\mathbf{x}}(t')$	$\vec{\mathbf{y}}(t) = \tilde{\mathbf{g}}(t) \cdot \vec{\mathbf{x}}(t)$
Time-invariant (TI)	$\vec{\mathbf{y}}(t) = \int dt' \, \mathbf{g}(t-t') \cdot \vec{\mathbf{x}}(t')$	$\vec{\mathbf{y}}(t) = \tilde{\mathbf{g}} \cdot \vec{\mathbf{x}}(t)$

Linear systems

$$\vec{\mathbf{x}}(\vec{\mathbf{r}}) \longrightarrow \boxed{\mathbf{g}(\cdot)} \longrightarrow \vec{\mathbf{y}}(\vec{\mathbf{r}})$$

$\vec{\mathbf{x}}(\vec{\mathbf{r}})$ and $\vec{\mathbf{y}}(\vec{\mathbf{r}})$ are vectors

	Space-dispersive (SD)	**Space-nondispersive (SND)**
Space-variant (SV)	$\vec{\mathbf{y}}(\vec{\mathbf{r}})=\int d\vec{\mathbf{r}}'\,\mathbf{g}(\vec{\mathbf{r}},\vec{\mathbf{r}}')\cdot\vec{\mathbf{x}}(\vec{\mathbf{r}}')$	$\vec{\mathbf{y}}(\vec{\mathbf{r}})=\tilde{\mathbf{g}}(\vec{\mathbf{r}})\cdot\vec{\mathbf{x}}(\vec{\mathbf{r}})$
Space-invariant (SI)	$\vec{\mathbf{y}}(\vec{\mathbf{r}})=\int d\vec{\mathbf{r}}'\,\mathbf{g}(\vec{\mathbf{r}}-\vec{\mathbf{r}}')\cdot\vec{\mathbf{x}}(\vec{\mathbf{r}}')$	$\vec{\mathbf{y}}(\vec{\mathbf{r}})=\tilde{\mathbf{g}}\cdot\vec{\mathbf{x}}(\vec{\mathbf{r}})$

	Time-dispersive (TD)	Time-nondispersive (TND)
Time-variant (TV)	$\boxed{\vec{\mathbf{y}}(t)=\int dt'\,\mathbf{g}(t,t')\cdot\vec{\mathbf{x}}(t')}$	$\vec{\mathbf{y}}(t)=\tilde{\mathbf{g}}(t)\cdot\vec{\mathbf{x}}(t)$
Time-invariant (TI)	$\vec{\mathbf{y}}(t)=\int dt'\,\mathbf{g}(t-t')\cdot\vec{\mathbf{x}}(t')$	$\vec{\mathbf{y}}(t)=\tilde{\mathbf{g}}\cdot\vec{\mathbf{x}}(t)$

	Space-dispersive (SD)	Space-nondispersive (SND)
Space-variant (SV)	$\boxed{\vec{\mathbf{y}}(\vec{\mathbf{r}})=\int d\vec{\mathbf{r}}'\,\mathbf{g}(\vec{\mathbf{r}},\vec{\mathbf{r}}')\cdot\vec{\mathbf{x}}(\vec{\mathbf{r}}')}$	$\vec{\mathbf{y}}(\vec{\mathbf{r}})=\tilde{\mathbf{g}}(\vec{\mathbf{r}})\cdot\vec{\mathbf{x}}(\vec{\mathbf{r}})$
Space-invariant (SI)	$\vec{\mathbf{y}}(\vec{\mathbf{r}})=\int d\vec{\mathbf{r}}'\,\mathbf{g}(\vec{\mathbf{r}}-\vec{\mathbf{r}}')\cdot\vec{\mathbf{x}}(\vec{\mathbf{r}}')$	$\vec{\mathbf{y}}(\vec{\mathbf{r}})=\tilde{\mathbf{g}}\cdot\vec{\mathbf{x}}(\vec{\mathbf{r}})$

Constitutive relationships

Linear & Anisotropic & Dispersive media

$$\vec{\mathbf{d}} = \mathscr{D}\left[\vec{\mathbf{e}}\right] \qquad \vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\,\mathbf{g}_d\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t,t'\right)\cdot\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$$

$$\vec{\mathbf{b}} = \mathscr{B}\left[\vec{\mathbf{h}}\right] \qquad \vec{\mathbf{b}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\,\mathbf{g}_b\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t,t'\right)\cdot\vec{\mathbf{h}}\left(\vec{\mathbf{r}}',t'\right)$$

$$\vec{\mathbf{j}} = \mathscr{J}\left[\vec{\mathbf{e}}\right] \qquad \vec{\mathbf{j}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\,\mathbf{g}_j\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t,t'\right)\cdot\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$$

Linear & Isotropic & Dispersive media

$$\vec{\mathbf{d}} = \mathscr{D}\left[\vec{\mathbf{e}}\right] \qquad \vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\,g_d\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t,t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$$

$$\vec{\mathbf{b}} = \mathscr{B}\left[\vec{\mathbf{h}}\right] \qquad \vec{\mathbf{b}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\,g_b\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t,t'\right)\vec{\mathbf{h}}\left(\vec{\mathbf{r}}',t'\right)$$

$$\vec{\mathbf{j}} = \mathscr{J}\left[\vec{\mathbf{e}}\right] \qquad \vec{\mathbf{j}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\,g_j\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t,t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$$

Constitutive relationships

In the following, just for the sake of simplicity, we will consider isotropic media

Time: dispersive
Space: dispersive

	Space-Dispersive (SD) Time-Dispersive (TD)
SV-TV	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t,t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$
SV-TI	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$
SI-TV	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g\left(\vec{\mathbf{r}}-\vec{\mathbf{r}}',t,t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$
SI-TI	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g\left(\vec{\mathbf{r}}-\vec{\mathbf{r}}',t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$

Time: nondispersive
Space: dispersive

	Space-Dispersive (SD) Time-Dispersive (TD)	Space-Dispersive (SD) Time-Nondispersive (TND)
SV-TV	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g(\vec{\mathbf{r}},\vec{\mathbf{r}}',t,t')\vec{\mathbf{e}}(\vec{\mathbf{r}}',t')$	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\, g(\vec{\mathbf{r}},\vec{\mathbf{r}}',t)\vec{\mathbf{e}}(\vec{\mathbf{r}}',t)$
SV-TI	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g(\vec{\mathbf{r}},\vec{\mathbf{r}}',t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}}',t')$	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\, g(\vec{\mathbf{r}},\vec{\mathbf{r}}')\vec{\mathbf{e}}(\vec{\mathbf{r}}',t)$
SI-TV	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g(\vec{\mathbf{r}}-\vec{\mathbf{r}}',t,t')\vec{\mathbf{e}}(\vec{\mathbf{r}}',t')$	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\, g(\vec{\mathbf{r}}-\vec{\mathbf{r}}',t)\vec{\mathbf{e}}(\vec{\mathbf{r}}',t)$
SI-TI	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g(\vec{\mathbf{r}}-\vec{\mathbf{r}}',t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}}',t')$	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\, g(\vec{\mathbf{r}}-\vec{\mathbf{r}}')\vec{\mathbf{e}}(\vec{\mathbf{r}}',t)$

Time: dispersive
Space: nondispersive

	Space-Dispersive (SD) Time-Dispersive (TD)	Space-Nondispersive (SND) Time-Dispersive (TD)
SV-TV	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\,g(\vec{\mathbf{r}},\vec{\mathbf{r}}',t,t')\vec{\mathbf{e}}(\vec{\mathbf{r}}',t')$	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\,g(\vec{\mathbf{r}},t,t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$
SV-TI	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\,g(\vec{\mathbf{r}},\vec{\mathbf{r}}',t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}}',t')$	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\,g(\vec{\mathbf{r}},t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$
SI-TV	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\,g(\vec{\mathbf{r}}-\vec{\mathbf{r}}',t,t')\vec{\mathbf{e}}(\vec{\mathbf{r}}',t')$	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\,g(t,t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$
SI-TI	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\,g(\vec{\mathbf{r}}-\vec{\mathbf{r}}',t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}}',t')$	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\,g(t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$

Dispersive (time & space) vs. nondispersive (time & space)

	Space-Dispersive (SD) Time-Dispersive (TD)	Space-Nondispersive (SND) Time-Nondispersive (TND)
SV-TV	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\,g\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t,t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon\left(\vec{\mathbf{r}},t\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t\right)$
SV-TI	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\,g\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon\left(\vec{\mathbf{r}}\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t\right)$
SI-TV	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\,g\left(\vec{\mathbf{r}}-\vec{\mathbf{r}}',t,t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon\left(t\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t\right)$
SI-TI	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\,g\left(\vec{\mathbf{r}}-\vec{\mathbf{r}}',t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon\,\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t\right)$

Permittivity

	SND+TND: Local
SV-TV	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon(\vec{\mathbf{r}},t)\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$
SV-TI	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$
SI-TV	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon(t)\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$
SI-TI	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon\,\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$

$$[\varepsilon]=?$$

$$[\varepsilon]=\frac{Coulomb}{m^2}\frac{m}{Volt}=\frac{Coulomb}{Volt}\frac{1}{m}=\frac{Farad}{m}$$

$$\left[\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right]=\frac{Volt}{m} \qquad \left[\vec{\mathbf{d}}(\vec{\mathbf{r}},t)\right]=\frac{Coulomb}{m^2}$$

$$C=\frac{q}{\Delta \mathrm{v}} \qquad Farad=\frac{Coulomb}{Volt}$$

Permeability

	SND+TND: Local
SV-TV	$\vec{\mathbf{b}}(\vec{\mathbf{r}},t)=\mu(\vec{\mathbf{r}},t)\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$
SV-TI	$\vec{\mathbf{b}}(\vec{\mathbf{r}},t)=\mu(\vec{\mathbf{r}})\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$
SI-TV	$\vec{\mathbf{b}}(\vec{\mathbf{r}},t)=\mu(t)\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$
SI-TI	$\vec{\mathbf{b}}(\vec{\mathbf{r}},t)=\mu\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$

$$[\mu]=?$$

$$[\mu]=\frac{Weber}{m^2}\frac{m}{Ampere}=\frac{Weber}{Ampere}\frac{1}{m}=\frac{Henry}{m}$$

$$\left[\vec{\mathbf{b}}(\vec{\mathbf{r}},t)\right]=\frac{Weber}{m^2}\qquad\left[\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right]=\frac{Ampere}{m}$$

$$L=\frac{\Phi_{\vec{\mathbf{b}}}}{i}\qquad Henry=\frac{Weber}{Ampere}$$

Conductivity

	SND+TND: Local
SV-TV	$\vec{\mathbf{j}}(\vec{\mathbf{r}},t)=\sigma\left(\vec{\mathbf{r}},t\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t\right)$
SV-TI	$\vec{\mathbf{j}}(\vec{\mathbf{r}},t)=\sigma\left(\vec{\mathbf{r}}\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t\right)$
SI-TV	$\vec{\mathbf{j}}(\vec{\mathbf{r}},t)=\sigma\left(t\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t\right)$
SI-TI	$\vec{\mathbf{j}}(\vec{\mathbf{r}},t)=\sigma\,\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t\right)$

$$\left[\sigma\right]=?$$

$$\left[\sigma\right]=\frac{Ampere}{m^2}\frac{m}{Volt}=\frac{Ampere}{Volt}\frac{1}{m}=\frac{1}{\Omega\, m}=\frac{Siemens}{m}$$

$$\left[\vec{\mathbf{j}}(\vec{\mathbf{r}},t)\right]=\frac{Ampere}{m^2} \qquad \left[\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t\right)\right]=\frac{Volt}{m}$$

$$\Delta\mathrm{v}=R\,i \qquad \Omega=\frac{Volt}{Ampere}=\frac{1}{Siemens}$$

Exercise n.1

Linear+ Isotropic + Space-Variant (SV) + Time-Variant (TV)

Space-dispersive (SD) +Time-dispersive (TD) $\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g_d\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t,t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$	$\left[g_d\right]=\dfrac{Farad}{s\cdot m^4}$
Space-dispersive (SD) +Time-nondispersive (TD) $\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}' g_d\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t\right)$	$\left[g_d\right]=\dfrac{Farad}{m^4}$
Space-nondispersive (SND) +Time-dispersive (TD) $\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d\left(\vec{\mathbf{r}},t,t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t'\right)$	$\left[g_d\right]=\dfrac{Farad}{s\cdot m}$
Space-nondispersive (SND) +Time-nondispersive (TD) $\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon\left(\vec{\mathbf{r}},t\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t\right)$	$\left[\varepsilon\right]=\dfrac{Farad}{m}$

Exercise n.2

Linear+ Isotropic + Space-Variant (SV) + Time-Invariant (TI)

Space-Dispersive (SD) +Time-Dispersive (TD) $\vec{\mathbf{b}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g_b\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t-t'\right)\vec{\mathbf{h}}\left(\vec{\mathbf{r}}',t'\right)$	$\left[g_b\right]=\frac{Henry}{s\cdot m^4}$
Space-Dispersive (SD) +Time-Nondispersive (TD) $\vec{\mathbf{b}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}' g_b\left(\vec{\mathbf{r}},\vec{\mathbf{r}}'\right)\vec{\mathbf{h}}\left(\vec{\mathbf{r}}',t\right)$	$\left[g_b\right]=\frac{Henry}{m^4}$
Space-Nondispersive (SND) +Time-Dispersive (TD) $\vec{\mathbf{b}}(\vec{\mathbf{r}},t)=\int dt'\, g_b\left(\vec{\mathbf{r}},t-t'\right)\vec{\mathbf{h}}\left(\vec{\mathbf{r}},t'\right)$	$\left[g_b\right]=\frac{Henry}{s\cdot m}$
Space-Nondispersive (SND) +Time-Nondispersive (TD) $\vec{\mathbf{b}}(\vec{\mathbf{r}},t)=\mu\left(\vec{\mathbf{r}}\right)\vec{\mathbf{h}}\left(\vec{\mathbf{r}},t\right)$	$\left[\mu\right]=\frac{Henry}{m}$

Conductors

$$\vec{\mathbf{j}}(\vec{\mathbf{r}},t)=\sigma\,\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$$

Metal	Conductivity σ [siemens/m]
Silver, 99.98% pure	6.14×10^7
Copper, annealed	5.80×10^7
Copper, hard drawn	5.65×10^7
Gold, pure drawn	4.10×10^7
Aluminum, commercial hard drawn	3.54×10^7
Magnesium	2.17×10^7
Tungsten	1.81×10^7
Zinc	1.74×10^7
Nickel	1.28×10^7
Iron, 99.98% pure	1.00×10^7
Steel	$1.00–0.5 \times 10^7$
Lead	0.48×10^7
Mercury	0.10×10^7

From G.Franceschetti, ‘Electromagnetics, Theory, Techniques, and Engineering paradigms’, Plenum Press

Perfect Electric Conductors (PEC)

$$\sigma \to \infty \quad \Rightarrow \quad \vec{\mathbf{e}} = 0 \quad \Rightarrow \quad \vec{\mathbf{h}} = 0$$

Vacuo

$$\vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \varepsilon_0 \vec{\mathbf{e}}(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{b}}(\vec{\mathbf{r}},t) = \mu_0 \vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

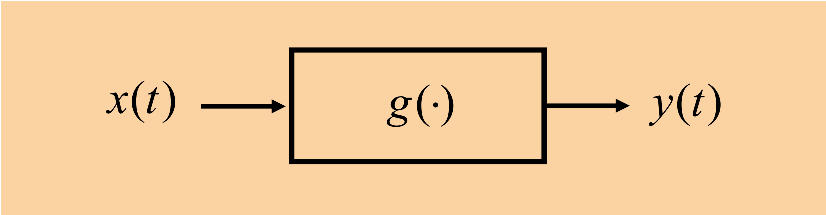
$$\sigma = 0$$

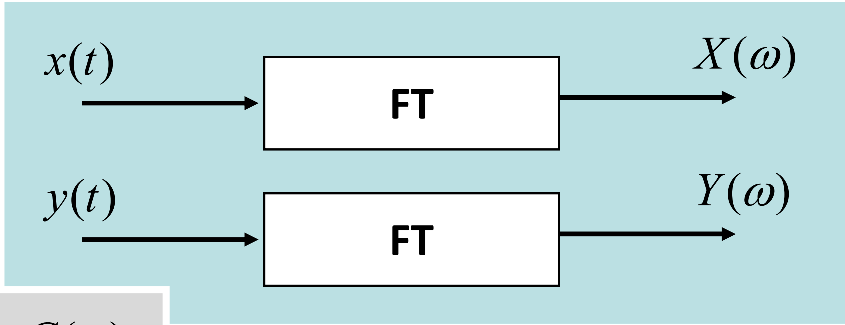
$$\mu_0 = 4\pi \times 10^{-7} \quad Henry\,/\,m$$

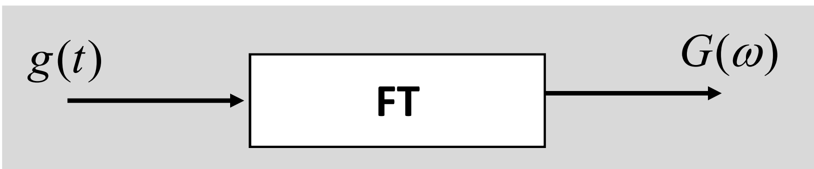
$$\varepsilon_0 = 8.8 \times 10^{-12} \quad Farad\,/\,m$$

Fourier and Phasor domains

Memo: linear time-invariant (LTI) systems

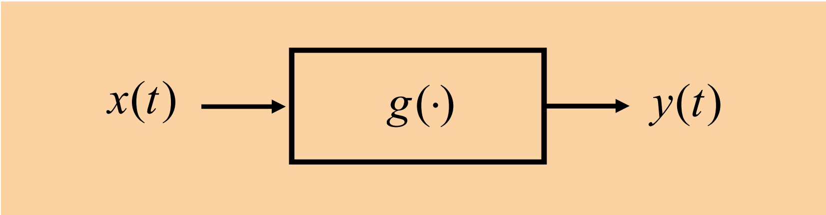


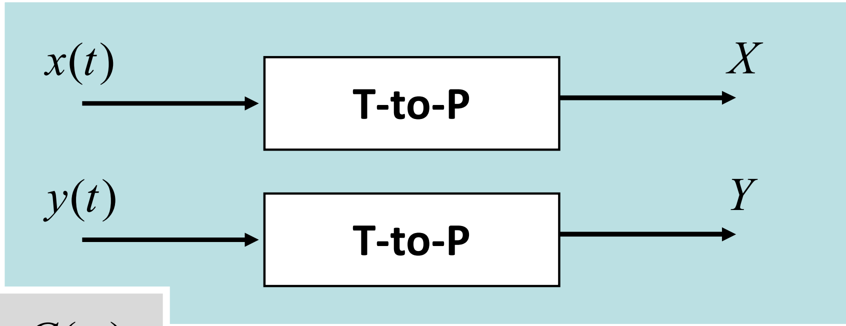


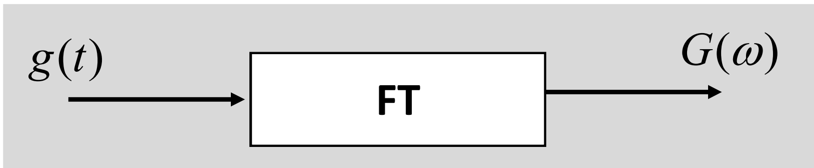


	Time domain	Frequency domain
Time-dispersive	$y(t)=\int dt'\, g(t-t')\,x(t')$	$Y(\omega)=G(\omega)X(\omega)$
Time-nondispersive	$y(t)=\tilde{g}\,x(t)$	$Y(\omega)=\tilde{g}X(\omega)$

Memo: linear time-invariant (LTI) systems





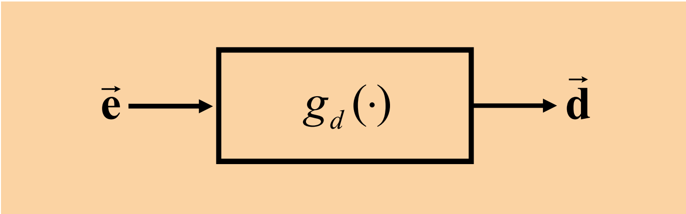


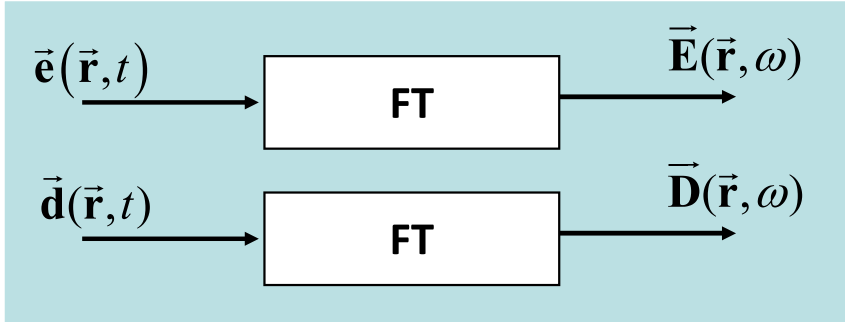
	Time domain	Frequency domain	Phasor domain
Time-dispersive	$y(t)=\int dt'\, g(t-t')\,x(t')$	$Y(\omega)=G(\omega)X(\omega)$	$Y=G(\omega_0)X$
Time-nondispersive	$y(t)=\tilde{g}\,x(t)$	$Y(\omega)=\tilde{g}X(\omega)$	$Y=\tilde{g}X$

Fourier and Phasor domains

	Time domain	Frequency domain	Phasor domain
Time-nondispersive Time-invariant Space-nondispersive Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$		
Time-nondispersive Time-invariant Space-nondispersive Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$		
Time-dispersive Time-invariant Space-nondispersive Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\,g_d(t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$		
Time-dispersive Time-invariant Space-nondispersive Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\,g_d(\vec{\mathbf{r}},t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$		

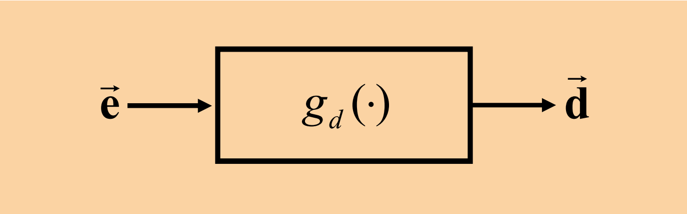
Time: nondispersive & invariant
Space: nondispersive

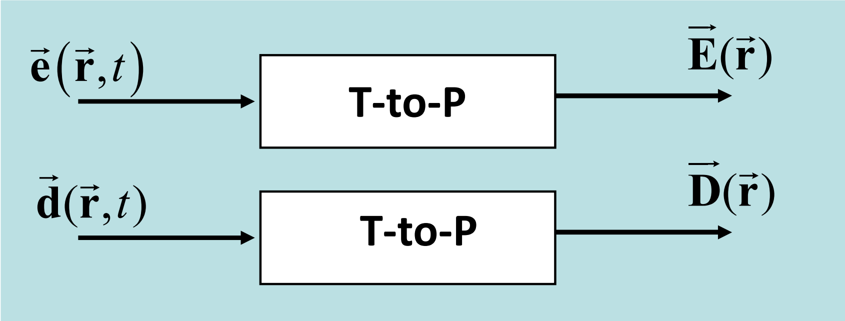




	Time domain	Frequency domain	Phasor domain
Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	
Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	

Time: nondispersive & invariant
Space: nondispersive





	Time domain	Frequency domain	Phasor domain
Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}})$

Time: nondispersive & invariant
Space: nondispersive

$$\vec{\mathbf{e}} \longrightarrow \boxed{g_d(\cdot)} \longrightarrow \vec{\mathbf{d}}$$

Real quantities, all equal each other

	Time domain	Frequency domain	Phasor domain
Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \varepsilon \vec{\mathbf{e}}(\vec{\mathbf{r}},t)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) = \varepsilon \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}}) = \varepsilon \vec{\mathbf{E}}(\vec{\mathbf{r}})$
Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \varepsilon(\vec{\mathbf{r}}) \vec{\mathbf{e}}(\vec{\mathbf{r}},t)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) = \varepsilon(\vec{\mathbf{r}}) \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}}) = \varepsilon(\vec{\mathbf{r}}) \vec{\mathbf{E}}(\vec{\mathbf{r}})$

Time: nondispersive & invariant
Space: nondispersive

$$\vec{\mathbf{e}} \longrightarrow \boxed{g_d(\cdot)} \longrightarrow \vec{\mathbf{d}}$$

Real quantities, all equal each other

	Time domain	Frequency domain	Phasor domain
Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \varepsilon \vec{\mathbf{e}}(\vec{\mathbf{r}},t)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) = \varepsilon \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}}) = \varepsilon \vec{\mathbf{E}}(\vec{\mathbf{r}})$
Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \varepsilon(\vec{\mathbf{r}}) \vec{\mathbf{e}}(\vec{\mathbf{r}},t)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) = \varepsilon(\vec{\mathbf{r}}) \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}}) = \varepsilon(\vec{\mathbf{r}}) \vec{\mathbf{E}}(\vec{\mathbf{r}})$

Time: dispersive & invariant
Space: nondispersive

$$\vec{\mathbf{e}} \longrightarrow \boxed{g_d(\cdot)} \longrightarrow \vec{\mathbf{d}}$$

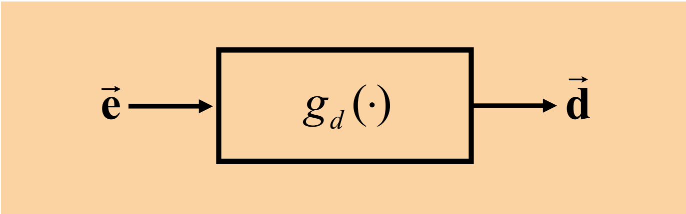
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) \longrightarrow \boxed{\text{FT}} \longrightarrow \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$$

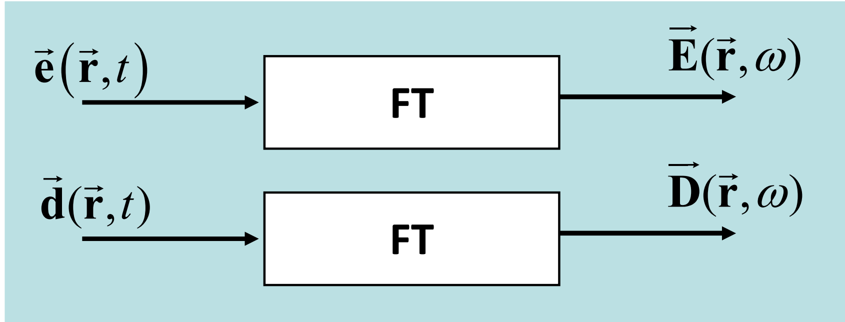
$$\vec{\mathbf{d}}(\vec{\mathbf{r}},t) \longrightarrow \boxed{\text{FT}} \longrightarrow \vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)$$

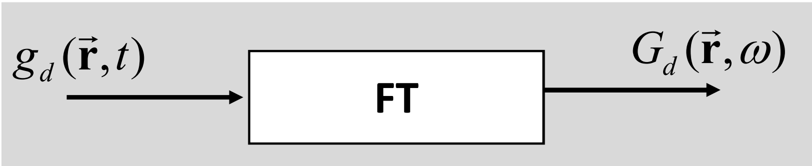
$$g_d(t) \longrightarrow \boxed{\text{FT}} \longrightarrow G_d(\omega)$$

	Time domain	Frequency domain	Phasor domain
Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(t-t')\,\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=G_d(\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	
Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(\vec{\mathbf{r}},t-t')\,\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$		

Time: dispersive & invariant
Space: nondispersive







	Time domain	Frequency domain	Phasor domain
Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(t-t')\,\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=G_d(\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	
Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(\vec{\mathbf{r}},t-t')\,\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=G_d(\vec{\mathbf{r}},\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	

Time: dispersive & invariant
Space: nondispersive

$$\vec{\mathbf{e}} \longrightarrow \boxed{g_d(\cdot)} \longrightarrow \vec{\mathbf{d}}$$

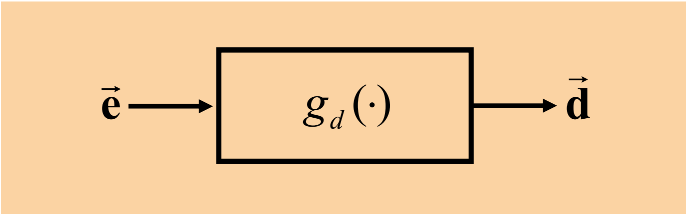
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) \longrightarrow \boxed{\textbf{T-to-P}} \longrightarrow \vec{\mathbf{E}}(\vec{\mathbf{r}})$$

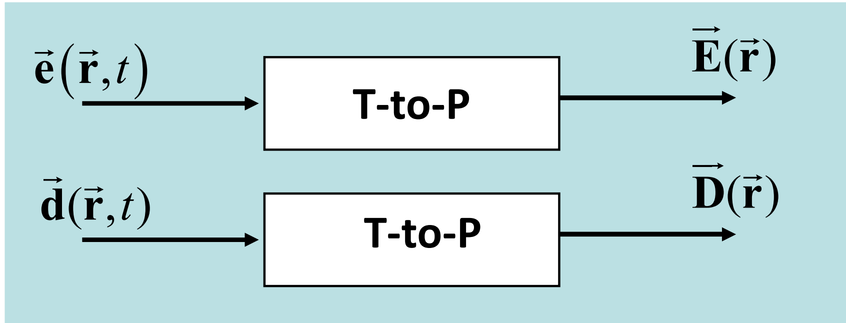
$$\vec{\mathbf{d}}(\vec{\mathbf{r}},t) \longrightarrow \boxed{\textbf{T-to-P}} \longrightarrow \vec{\mathbf{D}}(\vec{\mathbf{r}})$$

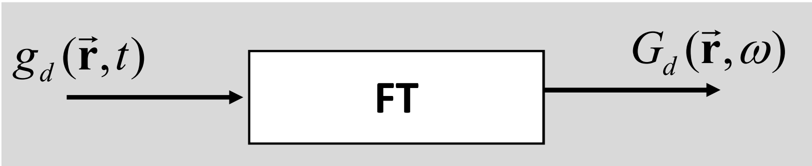
$$g_d(t) \longrightarrow \boxed{\textbf{FT}} \longrightarrow G_d(\omega)$$

	Time domain	Frequency domain	Phasor domain
Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=G_d(\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=G_d(\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(\vec{\mathbf{r}},t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=G_d(\vec{\mathbf{r}},\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	

Time: dispersive & invariant
Space: nondispersive







	Time domain	Frequency domain	Phasor domain
Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=G_d(\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=G_d(\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(\vec{\mathbf{r}},t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=G_d(\vec{\mathbf{r}},\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=G_d(\vec{\mathbf{r}},\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$

Time: dispersive & invariant
Space: nondispersive

$$\vec{\mathbf{e}} \longrightarrow \boxed{g_d(\cdot)} \longrightarrow \vec{\mathbf{d}}$$

Real | Complex | Complex

	Time domain	Frequency domain	Phasor domain
Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(t-t')\,\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=G_d(\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=G_d(\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(\vec{\mathbf{r}},t-t')\,\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=G_d(\vec{\mathbf{r}},\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=G_d(\vec{\mathbf{r}},\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$

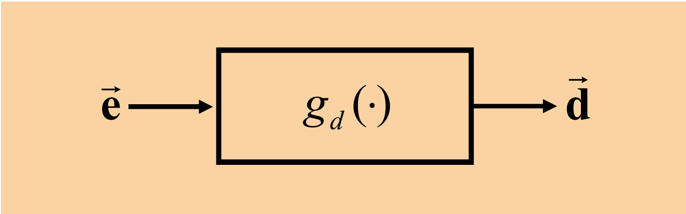
Time: dispersive & invariant
Space: nondispersive

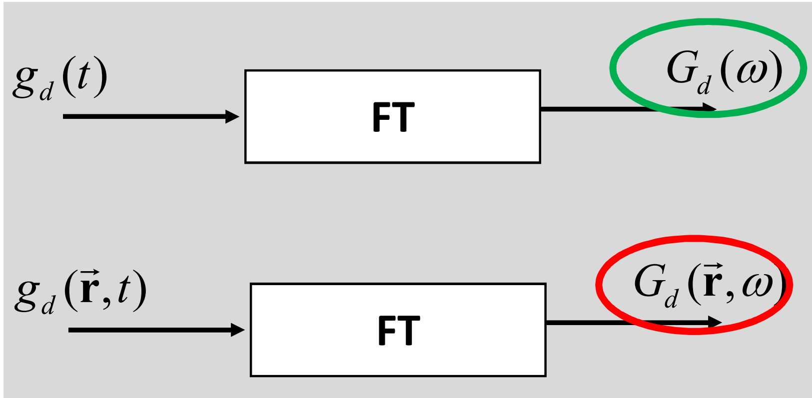
$$\vec{\mathbf{e}} \longrightarrow \boxed{g_d(\cdot)} \longrightarrow \vec{\mathbf{d}}$$

Real | **Complex** | **Complex**

	Time domain	Frequency domain	Phasor domain
Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(t-t')\,\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=G_d(\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=G_d(\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(\vec{\mathbf{r}},t-t')\,\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=G_d(\vec{\mathbf{r}},\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=G_d(\vec{\mathbf{r}},\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$

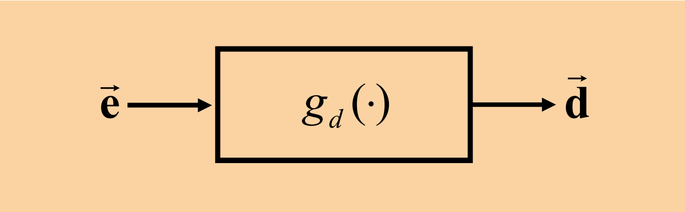
Time: dispersive & invariant
Space: nondispersive

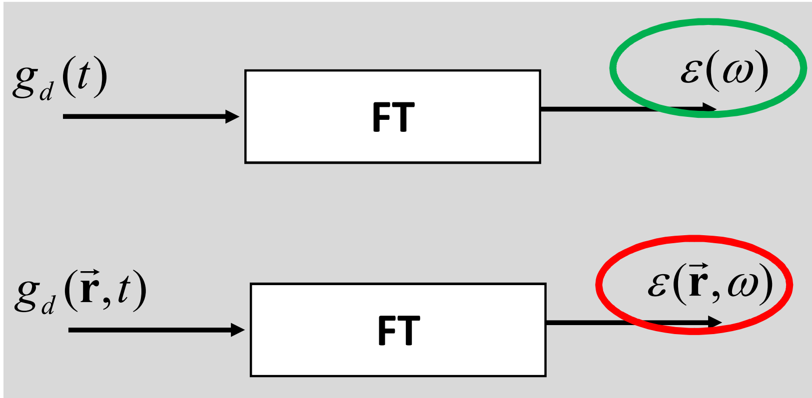




	Time domain	Frequency domain	Phasor domain
Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d\left(t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t'\right)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=G_d(\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=G_d(\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d\left(\vec{\mathbf{r}},t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t'\right)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=G_d(\vec{\mathbf{r}},\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=G_d(\vec{\mathbf{r}},\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$

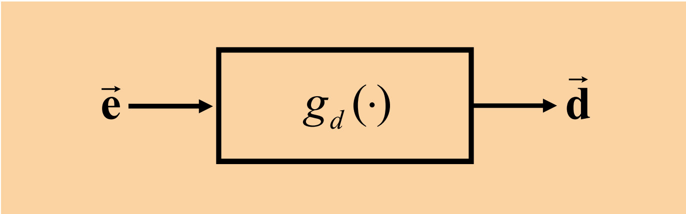
Time: dispersive & invariant
Space: nondispersive

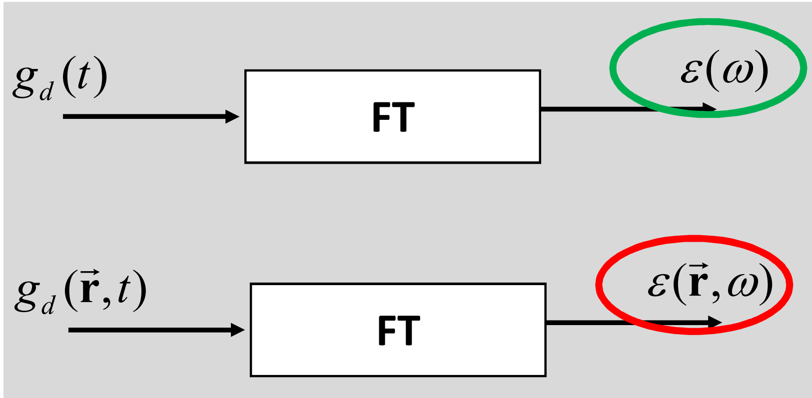




	Time domain	Frequency domain	Phasor domain
Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d\left(t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t'\right)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=G_d(\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=G_d(\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d\left(\vec{\mathbf{r}},t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t'\right)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=G_d(\vec{\mathbf{r}},\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=G_d(\vec{\mathbf{r}},\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$

Time: dispersive & invariant
Space: nondispersive





	Time domain	Frequency domain	Phasor domain
Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon(\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon(\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(\vec{\mathbf{r}},t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon(\vec{\mathbf{r}},\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon(\vec{\mathbf{r}},\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$

Fourier and Phasor domains

	Time domain	Frequency domain	Phasor domain
Time-nondispersive Time-invariant Space-nondispersive Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Time-nondispersive Time-invariant Space-nondispersive Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Time-dispersive Time-invariant Space-nondispersive Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon(\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon(\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Time-dispersive Time-invariant Space-nondispersive Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(\vec{\mathbf{r}},t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon(\vec{\mathbf{r}},\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon(\vec{\mathbf{r}},\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$

Fourier and Phasor domains

	Time domain	Frequency domain	Phasor domain
Time-nondispersive Time-invariant Space-nondispersive Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Time-nondispersive Time-invariant Space-nondispersive Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Time-dispersive Time-invariant Space-nondispersive Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon(\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon(\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Time-dispersive Time-invariant Space-nondispersive Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(\vec{\mathbf{r}},t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon(\vec{\mathbf{r}},\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon(\vec{\mathbf{r}},\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$

Fourier and Phasor domains

	Time domain	Frequency domain	Phasor domain
Time-nondispersive Time-invariant Space-nondispersive Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Time-nondispersive Time-invariant Space-nondispersive Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Time-dispersive Time-invariant Space-nondispersive Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon(\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon(\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Normal media	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(\vec{\mathbf{r}},t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon(\vec{\mathbf{r}},\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon(\vec{\mathbf{r}},\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$

Fourier and Phasor domains

	Time domain	Frequency domain	Phasor domain
Normal media	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d\left(\vec{\mathbf{r}},t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t'\right)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon(\vec{\mathbf{r}},\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon(\vec{\mathbf{r}},\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$
	$\vec{\mathbf{b}}(\vec{\mathbf{r}},t)=\int dt'\, g_b\left(\vec{\mathbf{r}},t-t'\right)\vec{\mathbf{h}}\left(\vec{\mathbf{r}},t'\right)$	$\vec{\mathbf{B}}(\vec{\mathbf{r}},\omega)=\mu(\vec{\mathbf{r}},\omega)\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{B}}(\vec{\mathbf{r}})=\mu(\vec{\mathbf{r}},\omega_0)\vec{\mathbf{H}}(\vec{\mathbf{r}})$
Conductors	$\vec{\mathbf{j}}(\vec{\mathbf{r}},t)=\sigma\,\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$	$\vec{\mathbf{J}}(\vec{\mathbf{r}},\omega)=\sigma\,\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{J}}(\vec{\mathbf{r}})=\sigma\,\vec{\mathbf{E}}(\vec{\mathbf{r}})$

Fourier and Phasor domains

	Time domain	Frequency domain	Phasor domain
Time-nondispersive Time-invariant Space-nondispersive Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Time-nondispersive Time-invariant Space-nondispersive Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Time-dispersive Time-invariant Space-nondispersive Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\,g_d(t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon(\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon(\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Normal media	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\,g_d(\vec{\mathbf{r}},t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon(\vec{\mathbf{r}},\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon(\vec{\mathbf{r}},\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$

Fourier and Phasor domains

	Time domain	Frequency domain / Phasor domain
Time-nondispersive Time-invariant Space-nondispersive Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$	$\vec{\mathbf{D}}=\varepsilon\vec{\mathbf{E}}$ $\vec{\mathbf{B}}=\mu\vec{\mathbf{H}}$ $\vec{\mathbf{J}}=\sigma\vec{\mathbf{E}}$
Time-nondispersive Time-invariant Space-nondispersive Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$	
Time-dispersive Time-invariant Space-nondispersive Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	
Normal media	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(\vec{\mathbf{r}},t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	

Fourier and Phasor domains

	Time domain
Time-nondispersive Time-invariant Space-nondispersive Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$
Time-nondispersive Time-invariant Space-nondispersive Space-variant	
Time-dispersive Time-invariant Space-nondispersive Space-invariant	
Normal media	

$$\begin{cases} \nabla\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=-\dfrac{\partial\vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\dfrac{\partial\vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t}+\vec{\mathbf{j}}(\vec{\mathbf{r}},t)+\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t) \\ \nabla\cdot\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\rho(\vec{\mathbf{r}},t)+\rho_0(\vec{\mathbf{r}},t) \\ \nabla\cdot\vec{\mathbf{b}}(\vec{\mathbf{r}},t)=0 \end{cases}$$

$$\Downarrow$$

$$\begin{cases} \nabla\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=-\mu\dfrac{\partial\vec{\mathbf{h}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\varepsilon\dfrac{\partial\vec{\mathbf{e}}(\vec{\mathbf{r}},t)}{\partial t}+\sigma\vec{\mathbf{e}}(\vec{\mathbf{r}},t)+\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t) \\ \varepsilon\nabla\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\rho(\vec{\mathbf{r}},t)+\rho_0(\vec{\mathbf{r}},t) \\ \nabla\cdot\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=0 \end{cases}$$

Fourier and Phasor domains

	Time domain
Time-nondispersive Time-invariant Space-nondispersive Space-invariant	
Time-nondispersive Time-invariant Space-nondispersive Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$
Time-dispersive Time-invariant Space-nondispersive Space-invariant	
Normal media	

$$\begin{cases}\nabla\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=-\dfrac{\partial\vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t}\\ \nabla\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\dfrac{\partial\vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t}+\vec{\mathbf{j}}(\vec{\mathbf{r}},t)+\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)\\ \nabla\cdot\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\rho(\vec{\mathbf{r}},t)+\rho_0(\vec{\mathbf{r}},t)\\ \nabla\cdot\vec{\mathbf{b}}(\vec{\mathbf{r}},t)=0\end{cases}$$

$$\Downarrow$$

$$\begin{cases}\nabla\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=-\mu(\vec{\mathbf{r}})\dfrac{\partial\vec{\mathbf{h}}(\vec{\mathbf{r}},t)}{\partial t}\\ \nabla\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\varepsilon(\vec{\mathbf{r}})\dfrac{\partial\vec{\mathbf{e}}(\vec{\mathbf{r}},t)}{\partial t}+\sigma\vec{\mathbf{e}}(\vec{\mathbf{r}},t)+\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)\\ \nabla\cdot\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\rho(\vec{\mathbf{r}},t)+\rho_0(\vec{\mathbf{r}},t)\\ \nabla\cdot\mu(\vec{\mathbf{r}})\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=0\end{cases}$$

Fourier and Phasor domains

	Time domain
Time-nondispersive **Time-invariant** **Space-nondispersive** **Space-invariant**	
Time-nondispersive **Time-invariant** **Space-nondispersive** **Space-variant**	
Time-dispersive **Time-invariant** **Space-nondispersive** **Space-invariant**	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d\left(t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t'\right)$
Normal media	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d\left(\vec{\mathbf{r}},t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t'\right)$

$$\begin{cases} \nabla\times\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t\right)=-\dfrac{\partial\vec{\mathbf{b}}\left(\vec{\mathbf{r}},t\right)}{\partial t} \\ \nabla\times\vec{\mathbf{h}}\left(\vec{\mathbf{r}},t\right)=\dfrac{\partial\vec{\mathbf{d}}\left(\vec{\mathbf{r}},t\right)}{\partial t}+\vec{\mathbf{j}}\left(\vec{\mathbf{r}},t\right)+\vec{\mathbf{j}}_0\left(\vec{\mathbf{r}},t\right) \\ \nabla\cdot\vec{\mathbf{d}}\left(\vec{\mathbf{r}},t\right)=\rho\left(\vec{\mathbf{r}},t\right) \\ \nabla\cdot\vec{\mathbf{b}}\left(\vec{\mathbf{r}},t\right)=0 \end{cases}$$

Much more convenient to work in the frequency/phasor domains!

Fourier and Phasor domains

Time-nondispersive
Time-invariant
Space-nondispersive
Space-invariant

Time-nondispersive
Time-invariant
Space-nondispersive
Space-variant

Time-dispersive
Time-invariant
Space-nondispersive
Space-invariant

Normal media

Frequency domain

$$\begin{cases} \nabla\times\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega\vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) \\ \nabla\times\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = \boxed{j\omega\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)+\vec{\mathbf{J}}(\vec{\mathbf{r}},\omega)}+\vec{\mathbf{J}}_0(\vec{\mathbf{r}},\omega) \\ \nabla\cdot\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) = \rho(\vec{\mathbf{r}},\omega)+\rho_0(\vec{\mathbf{r}},\omega) \\ \nabla\cdot\vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) = 0 \end{cases}$$

Phasor domain

$$\begin{cases} \nabla\times\vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0\vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla\times\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \boxed{j\omega_0\vec{\mathbf{D}}(\vec{\mathbf{r}})+\vec{\mathbf{J}}(\vec{\mathbf{r}})}+\vec{\mathbf{J}}_0(\vec{\mathbf{r}}) \\ \nabla\cdot\vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}})+\rho_0(\vec{\mathbf{r}}) \\ \nabla\cdot\vec{\mathbf{B}}(\vec{\mathbf{r}}) = 0 \end{cases}$$

$$\vec{\mathbf{D}} = \varepsilon\vec{\mathbf{E}}; \quad \vec{\mathbf{B}} = \mu\vec{\mathbf{H}}; \quad \vec{\mathbf{J}} = \sigma\vec{\mathbf{E}};$$

$$\begin{cases} \nabla\times\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega\mu\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) \\ \nabla\times\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = \boxed{j\omega\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)+\sigma\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)}+\vec{\mathbf{J}}_0(\vec{\mathbf{r}},\omega) \\ \nabla\cdot\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = \rho(\vec{\mathbf{r}},\omega)+\rho_0(\vec{\mathbf{r}},\omega) \\ \nabla\cdot\mu\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = 0 \end{cases}$$

$$\begin{cases} \nabla\times\vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0\mu\vec{\mathbf{H}}(\vec{\mathbf{r}}) \\ \nabla\times\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = \boxed{j\omega_0\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}})+\sigma\vec{\mathbf{E}}(\vec{\mathbf{r}})}+\vec{\mathbf{J}}_0(\vec{\mathbf{r}}) \\ \nabla\cdot\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}})+\rho_0(\vec{\mathbf{r}}) \\ \nabla\cdot\mu\vec{\mathbf{H}}(\vec{\mathbf{r}}) = 0 \end{cases}$$

Fourier and Phasor domains

Time-nondispersive
Time-invariant
Space-nondispersive
Space-invariant

Time-nondispersive
Time-invariant
Space-nondispersive
Space-variant

Time-dispersive
Time-invariant
Space-nondispersive
Space-invariant

Normal media

Frequency domain

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega\mu \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = \boxed{j\omega\varepsilon \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) + \sigma \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)} + \vec{\mathbf{J}}_0(\vec{\mathbf{r}},\omega) \\ \nabla \cdot \varepsilon \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = \rho(\vec{\mathbf{r}},\omega) + \rho_0(\vec{\mathbf{r}},\omega) \\ \nabla \cdot \mu \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = 0 \end{cases}$$

Phasor domain

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0\mu \vec{\mathbf{H}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = \boxed{j\omega_0\varepsilon \vec{\mathbf{E}}(\vec{\mathbf{r}}) + \sigma \vec{\mathbf{E}}(\vec{\mathbf{r}})} + \vec{\mathbf{J}}_0(\vec{\mathbf{r}}) \\ \nabla \cdot \varepsilon \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) + \rho_0(\vec{\mathbf{r}}) \\ \nabla \cdot \mu \vec{\mathbf{H}}(\vec{\mathbf{r}}) = 0 \end{cases}$$

$$\vec{\mathbf{D}} = \varepsilon \vec{\mathbf{E}} \qquad \vec{\mathbf{J}} = \sigma \vec{\mathbf{E}}$$

$$j\omega\varepsilon \vec{\mathbf{E}} + \sigma \vec{\mathbf{E}} = j\omega\varepsilon \left[1 + \frac{\sigma}{j\omega\varepsilon} \right] \vec{\mathbf{E}} = j\omega\varepsilon \left[1 - j\frac{\sigma}{\omega\varepsilon} \right] \vec{\mathbf{E}} = j\omega\varepsilon_{eq} \vec{\mathbf{E}}$$

$$\varepsilon_{eq} = \varepsilon \left[1 - j\frac{\sigma}{\omega\varepsilon} \right]$$

One consideration

- In time-dispersive media, when working in the Fourier/Phasor domain, we have:

$$\varepsilon = \varepsilon_1 - j\varepsilon_2$$
$$\mu = \mu_1 - j\mu_2$$

It can be shown that the real and imaginary parts of these quantities are not independent each other: they are related by the Kramers- Kröning relations

…. two last considerations

- Note that causality and finite velocity of propagation must be enforced when writing the impulse response that describes the medium

- Note that, due to the finite velocity of propagation, space-dispersive media are time-dispersive too