Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2019-2020 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

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Phasors and vector functions

Time domain

$$\vec{\mathbf{f}}(x,y,z,t)=f_x(x,y,z,t)\hat{i}_x+f_y(x,y,z,t)\hat{i}_y+f_z(x,y,z,t)\,\hat{i}_z$$

Phasor domain

$$\vec{\mathbf{F}}(x,y,z)=F_x(x,y,z)\hat{i}_x+F_y(x,y,z)\hat{i}_y+F_z(x,y,z)\hat{i}_z$$

$$f_x(x,y,z,t)=A_x(x,y,z)\cos\left(\omega_0 t+\alpha_x(x,y,z)\right) \longrightarrow \boxed{\text{T-to-P}} \longrightarrow F_x(x,y,z)=A_x(x,y,z)e^{j\alpha_x(x,y,z)}$$

$$f_y(x,y,z,t)=A_y(x,y,z)\cos\left(\omega_0 t+\alpha_y(x,y,z)\right) \longrightarrow \boxed{\text{T-to-P}} \longrightarrow F_y(x,y,z)=A_y(x,y,z)e^{j\alpha_y(x,y,z)}$$

$$f_z(x,y,z,t)=A_z(x,y,z)\cos\left(\omega_0 t+\alpha_z(x,y,z)\right) \longrightarrow \boxed{\text{T-to-P}} \longrightarrow F_z(x,y,z)=A_z(x,y,z)e^{j\alpha_z(x,y,z)}$$

$$\vec{\mathbf{f}}(x,y,z,t) \longrightarrow \boxed{\text{T-to-P}} \longrightarrow \vec{\mathbf{F}}(x,y,z)$$

$$\vec{\mathbf{F}}(x,y,z) \longrightarrow \boxed{\text{P-to-T}} \longrightarrow \vec{\mathbf{f}}(x,y,z,t)=\mathrm{Re}\left\{\vec{\mathbf{F}}(x,y,z)e^{j\omega_0 t}\right\}$$

Phasors and vector functions

Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Phasor domain

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z$$

$f_x(t) = A_x\cos(\omega_0 t + \alpha_x)$ → **T-to-P** → $F_x = A_x e^{j\alpha_x} = R_x + jI_x$

$f_y(t) = A_y\cos(\omega_0 t + \alpha_y)$ → **T-to-P** → $F_y = A_y e^{j\alpha_y} = R_y + jI_y$

$f_z(t) = A_z\cos(\omega_0 t + \alpha_z)$ → **T-to-P** → $F_z = A_z e^{j\alpha_z} = R_z + jI_z$

$\vec{\mathbf{f}}(t)$ → **T-to-P** → $\vec{\mathbf{F}}$

$\vec{\mathbf{F}}$ → **P-to-T** → $\vec{\mathbf{f}}(t) = \mathrm{Re}\left\{\vec{\mathbf{F}}e^{j\omega_0 t}\right\}$

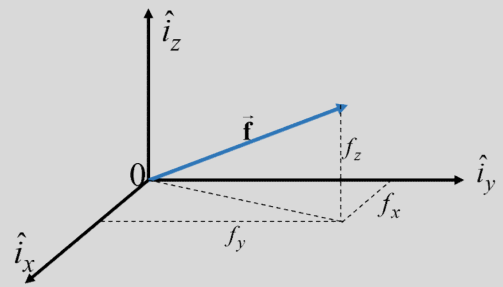
Complex vectors: graphical representation

Real numbers

$$f$$

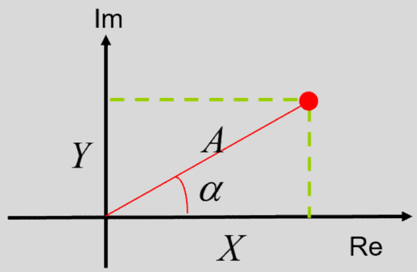
Real vectors (3 components)

$$\vec{\mathbf{f}} = f_x\hat{i}_x + f_y\hat{i}_y + f_z\hat{i}_z$$



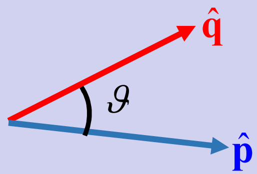
Complex numbers

$$F = Ae^{j\alpha} = X + jY$$



Complex vectors (3 components)

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z$$



Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

P1 $\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$

P2 $\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

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$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

P1 $\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$

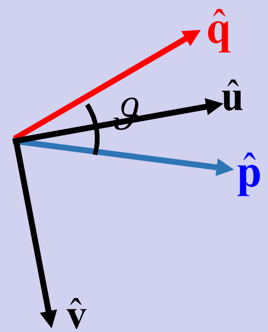
P2 $\vec{\mathbf{F}} = |\vec{\mathbf{F}}|e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + |\vec{\mathbf{F}}|e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$

$$\begin{cases} F_p = |\vec{\mathbf{F}}|\cos\phi \\ F_q = |\vec{\mathbf{F}}|\sin\phi \end{cases}$$

$$|\vec{\mathbf{F}}| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

Polarization plane



$$F_p + jF_q = |\vec{\mathbf{F}}|e^{j\phi}$$

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

P1 $\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$

T1 $\vec{\mathbf{f}}(t) = F_p\cos(\omega_0 t)\hat{\mathbf{p}} - F_q\sin(\omega_0 t)\hat{\mathbf{q}}$

P2 $\vec{\mathbf{F}} = |\vec{\mathbf{F}}|e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + |\vec{\mathbf{F}}|e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$

T2 $\vec{\mathbf{f}}(t) = |\vec{\mathbf{F}}|\cos\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t + \phi)\hat{\mathbf{u}} + |\vec{\mathbf{F}}|\sin\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t - \phi)\hat{\mathbf{v}}$

$\vec{\mathbf{f}}(t) \longrightarrow$ **T-to-P** $\longrightarrow \vec{\mathbf{F}}$

$\vec{\mathbf{F}} \longrightarrow$ **P-to-T** $\longrightarrow \vec{\mathbf{f}}(t) = \mathrm{Re}\left\{\vec{\mathbf{F}}e^{j\omega_0 t}\right\}$

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

P1 $\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$

T1 $\vec{\mathbf{f}}(t) = F_p\cos(\omega_0 t)\hat{\mathbf{p}} - F_q\sin(\omega_0 t)\hat{\mathbf{q}}$

P2 $\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$

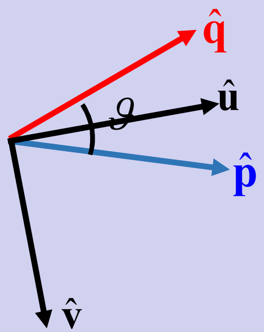
T2 $\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right|\cos\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t + \phi)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right|\sin\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t - \phi)\hat{\mathbf{v}}$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

Polarization plane



The vector $\vec{\mathbf{f}}(t)$ lies in the polarization plane $(\hat{\mathbf{p}},\hat{\mathbf{q}})$ (which is coincident with the plane $(\hat{\mathbf{u}},\hat{\mathbf{v}})$), and it changes in general both its amplitude and its direction as the time elapses.

Two cases are of particular interest:

- **Linear polarization**
- **Circular polarization**

Linear Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

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P1 $\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$

T1 $\vec{\mathbf{f}}(t) = F_p\cos(\omega_0 t)\hat{\mathbf{p}} - F_q\sin(\omega_0 t)\hat{\mathbf{q}}$

P2 $\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$

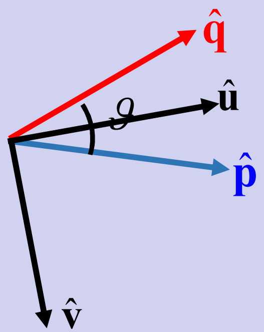
T2 $\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right|\cos\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t + \phi)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right|\sin\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t - \phi)\hat{\mathbf{v}}$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

Polarization plane

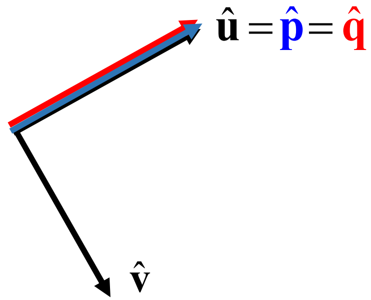


The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line:

- $\hat{\mathbf{p}} = \hat{\mathbf{q}}$ ⟹ $\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right|\cos(\omega_0 t + \phi)\hat{\mathbf{u}}$ $(\hat{\mathbf{u}} = \hat{\mathbf{p}} = \hat{\mathbf{q}})$

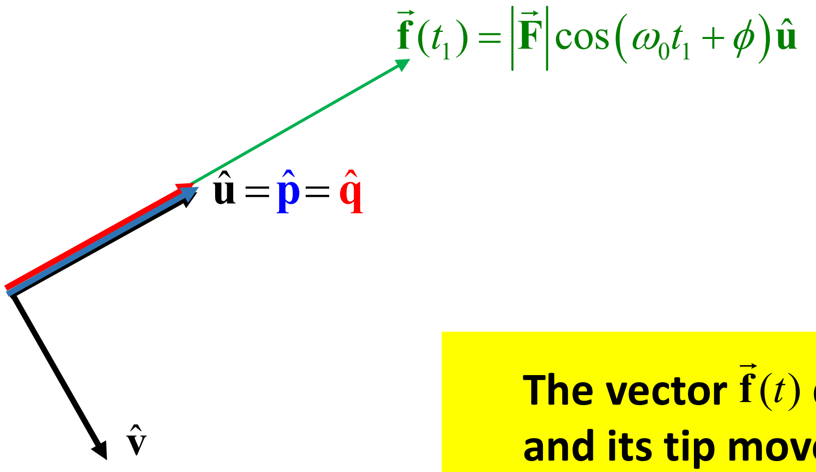
Linear Polarization

$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right| \cos\left(\omega_0 t + \phi\right) \hat{\mathbf{u}}$$



Linear Polarization

$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right| \cos\left(\omega_0 t + \phi\right) \hat{\mathbf{u}}$$



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Linear Polarization

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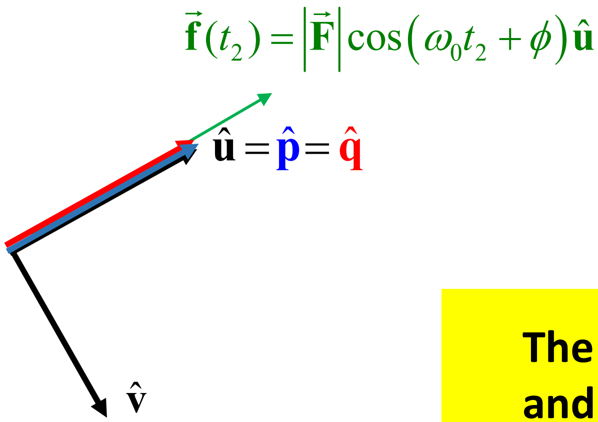

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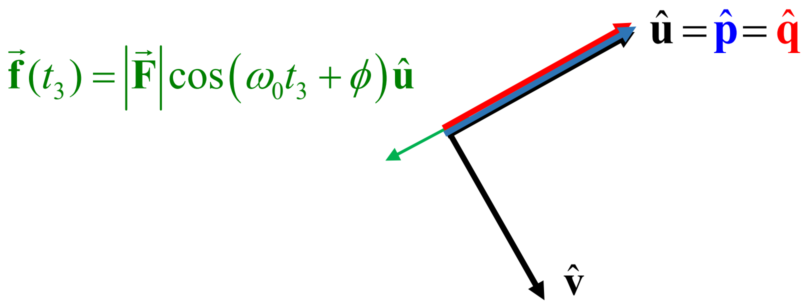
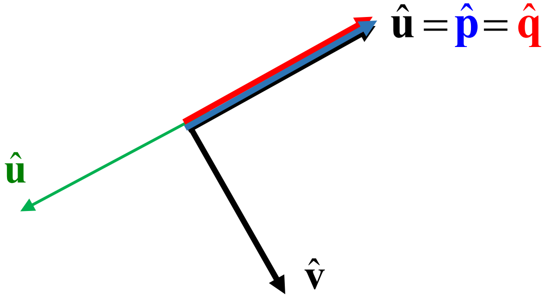


The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line

Linear Polarization

$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right| \cos\left(\omega_0 t + \phi\right) \hat{\mathbf{u}}$$

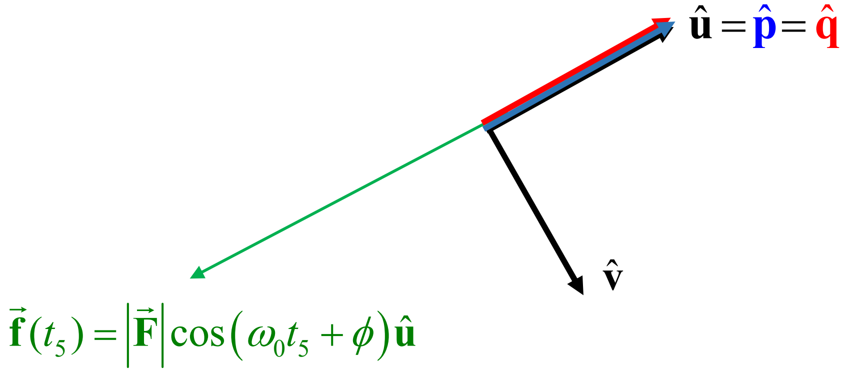
$$\vec{\mathbf{f}}(t_4) = \left|\vec{\mathbf{F}}\right| \cos\left(\omega_0 t_4 + \phi\right) \hat{\mathbf{u}}$$



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line

Linear Polarization

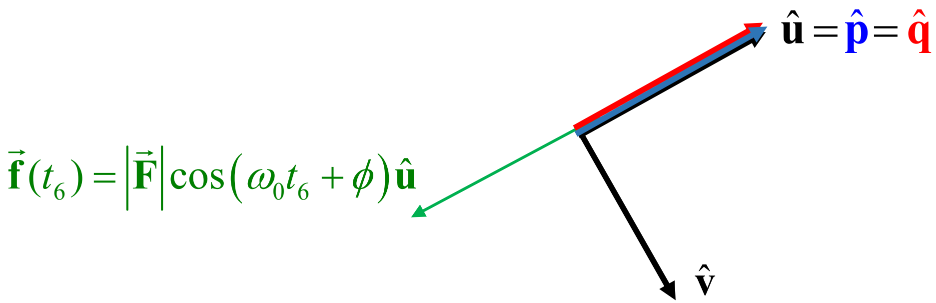
$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right| \cos\left(\omega_0 t + \phi\right)\hat{\mathbf{u}}$$



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line

Linear Polarization

$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right| \cos\left(\omega_0 t + \phi\right) \hat{\mathbf{u}}$$



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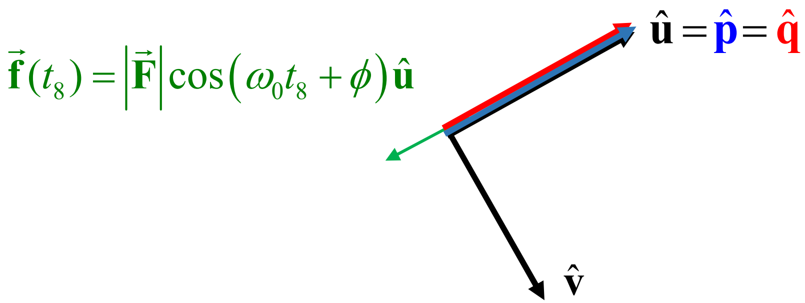

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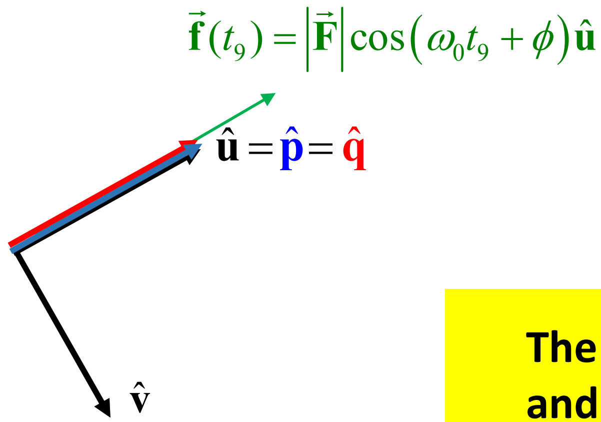

The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line

Linear Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

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$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

P1 $\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$

T1 $\vec{\mathbf{f}}(t) = F_p\cos(\omega_0 t)\hat{\mathbf{p}} - F_q\sin(\omega_0 t)\hat{\mathbf{q}}$

P2 $\vec{\mathbf{F}} = |\vec{\mathbf{F}}|e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + |\vec{\mathbf{F}}|e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$

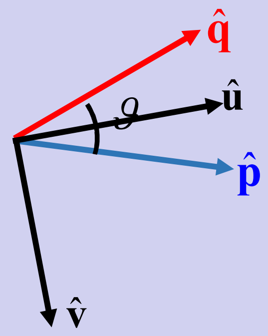
T2 $\vec{\mathbf{f}}(t) = |\vec{\mathbf{F}}|\cos\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t + \phi)\hat{\mathbf{u}} + |\vec{\mathbf{F}}|\sin\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t - \phi)\hat{\mathbf{v}}$

$$\begin{cases} F_p = |\vec{\mathbf{F}}|\cos\phi \\ F_q = |\vec{\mathbf{F}}|\sin\phi \end{cases}$$

$$|\vec{\mathbf{F}}| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

Polarization plane



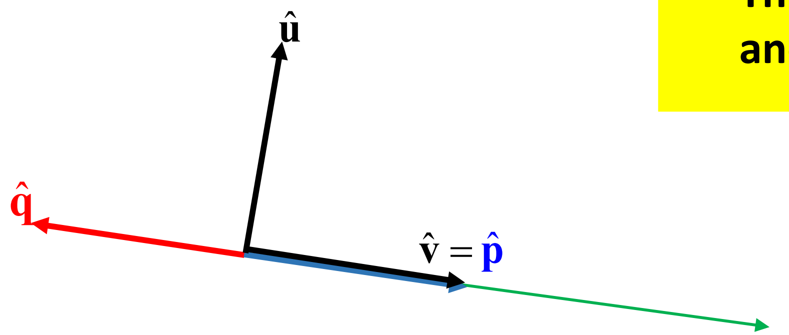
The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line:

- $\hat{\mathbf{p}} = \hat{\mathbf{q}}$ ➡ $\vec{\mathbf{f}}(t) = |\vec{\mathbf{F}}|\cos(\omega_0 t + \phi)\hat{\mathbf{u}}$ $(\hat{\mathbf{u}} = \hat{\mathbf{p}} = \hat{\mathbf{q}})$
- $\hat{\mathbf{p}} = -\hat{\mathbf{q}}$ ➡ $\vec{\mathbf{f}}(t) = |\vec{\mathbf{F}}|\cos(\omega_0 t - \phi)\hat{\mathbf{v}}$ $(\hat{\mathbf{v}} = \hat{\mathbf{p}} = -\hat{\mathbf{q}})$

Linear Polarization

$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right| \cos\left(\omega_0 t - \phi\right) \hat{\mathbf{v}}$$

The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line

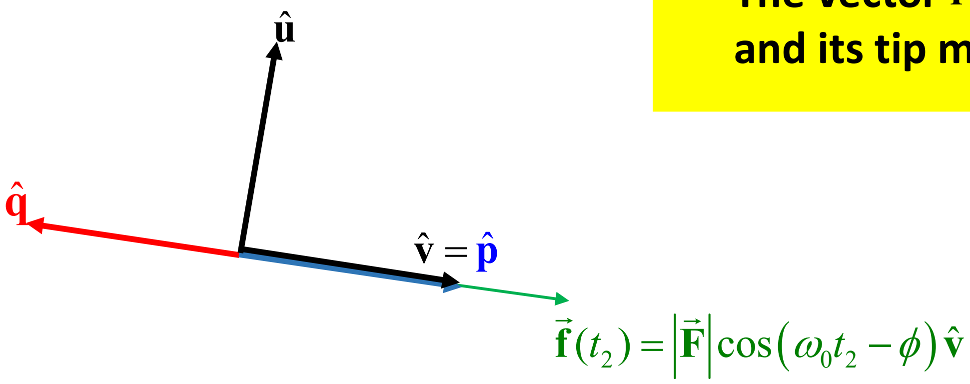


$$\vec{\mathbf{f}}(t_1) = \left|\vec{\mathbf{F}}\right| \cos\left(\omega_0 t_1 - \phi\right) \hat{\mathbf{v}}$$

Linear Polarization

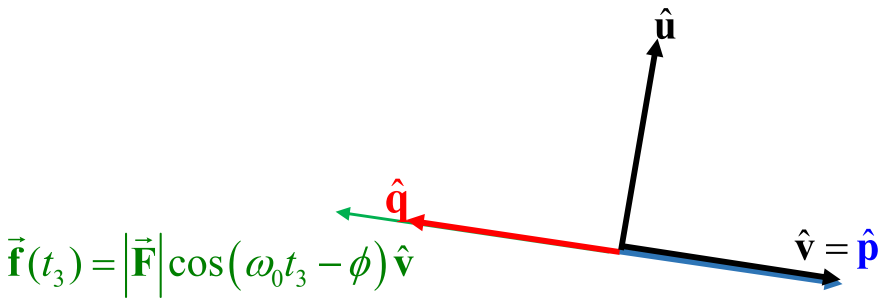
$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right| \cos\left(\omega_0 t - \phi\right) \hat{\mathbf{v}}$$

The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line



Linear Polarization

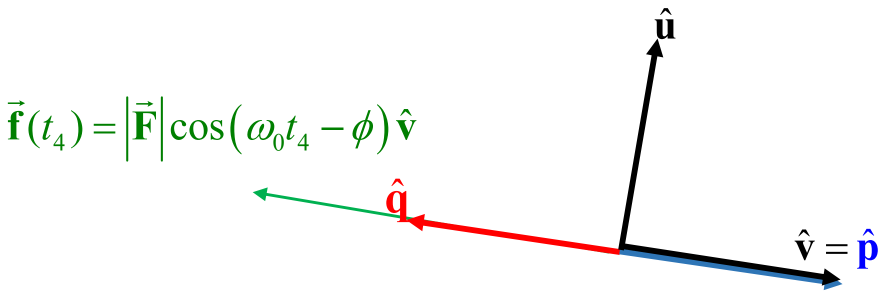
$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right| \cos\left(\omega_0 t - \phi\right) \hat{\mathbf{v}}$$



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line

Linear Polarization

$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right| \cos\left(\omega_0 t - \phi\right) \hat{\mathbf{v}}$$



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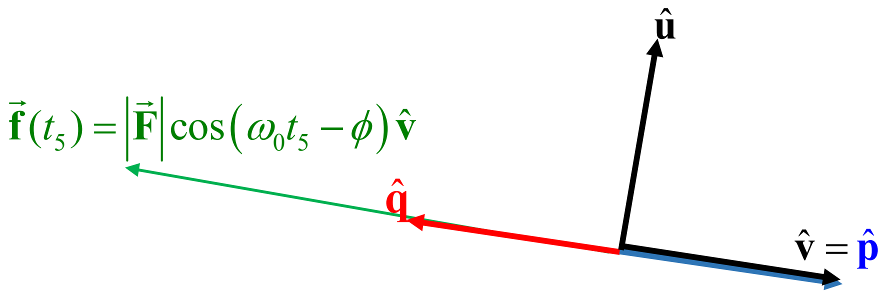

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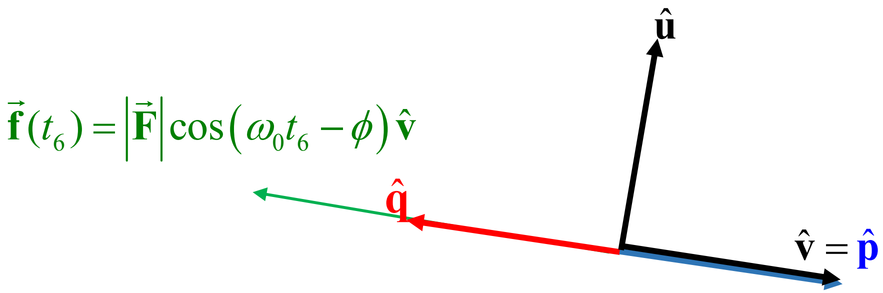

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Linear Polarization

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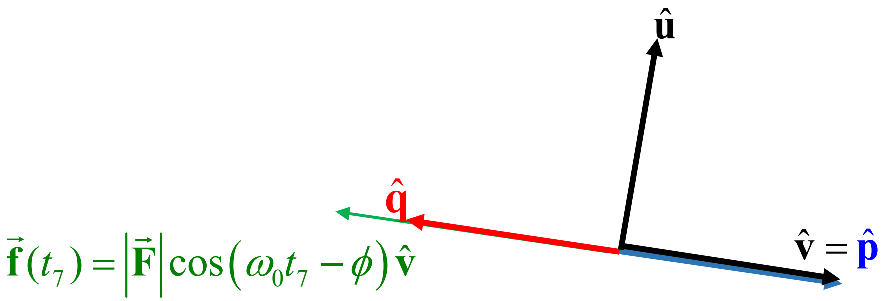

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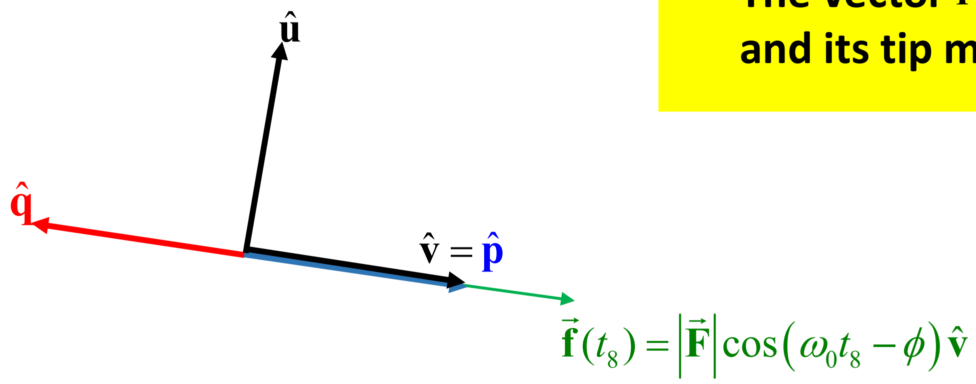

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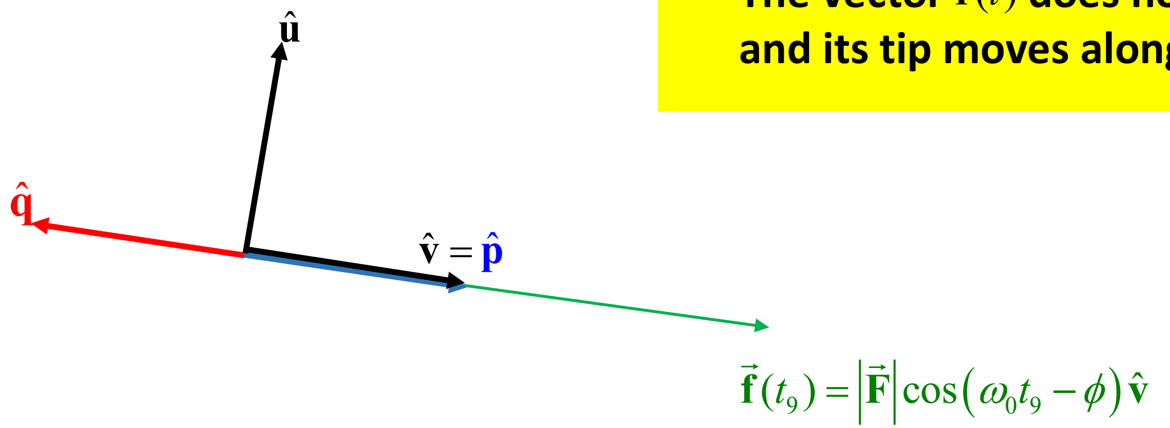

Linear Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

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P1 $\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$

T1 $\vec{\mathbf{f}}(t) = F_p\cos(\omega_0 t)\hat{\mathbf{p}} - F_q\sin(\omega_0 t)\hat{\mathbf{q}}$

P2 $\vec{\mathbf{F}} = |\vec{\mathbf{F}}|e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + |\vec{\mathbf{F}}|e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$

T2 $\vec{\mathbf{f}}(t) = |\vec{\mathbf{F}}|\cos\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t + \phi)\hat{\mathbf{u}} + |\vec{\mathbf{F}}|\sin\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t - \phi)\hat{\mathbf{v}}$

$$\begin{cases} F_p = |\vec{\mathbf{F}}|\cos\phi \\ F_q = |\vec{\mathbf{F}}|\sin\phi \end{cases}$$

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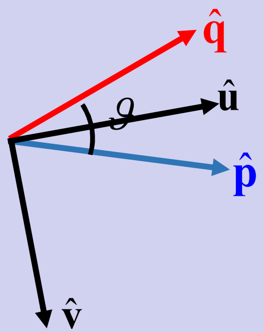
Polarization plane



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line:

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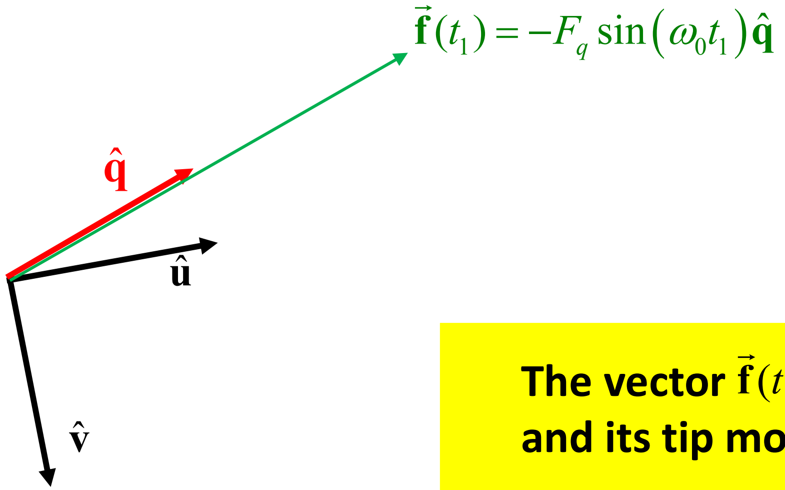
- $\hat{\mathbf{p}} = -\hat{\mathbf{q}}$ ➡ $\vec{\mathbf{f}}(t) = |\vec{\mathbf{F}}|\cos(\omega_0 t - \phi)\hat{\mathbf{v}}$ $(\hat{\mathbf{v}} = \hat{\mathbf{p}} = -\hat{\mathbf{q}})$
- $F_p = 0$ and $F_q \neq 0$ ➡ $\vec{\mathbf{f}}(t) = -F_q\sin(\omega_0 t)\hat{\mathbf{q}}$

Linear Polarization

$$\vec{\mathbf{f}}(t) = -F_q \sin\left(\omega_0 t\right)\hat{\mathbf{q}}$$



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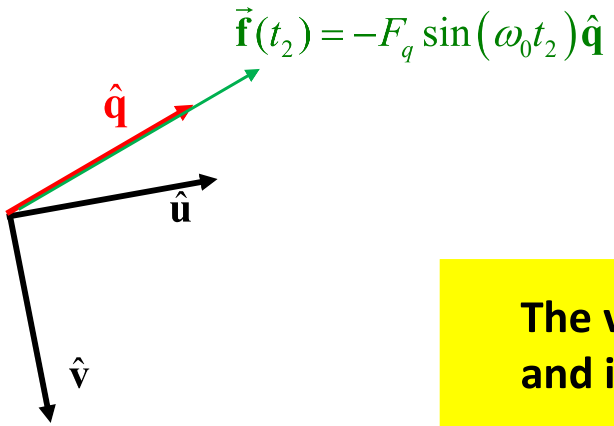

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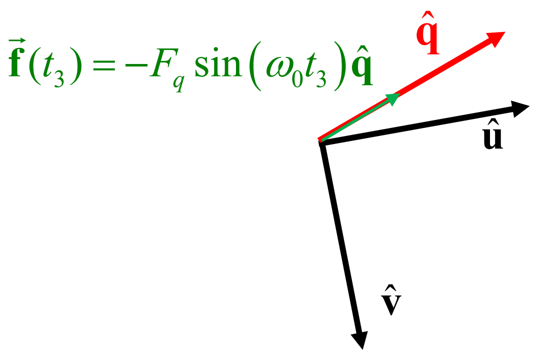

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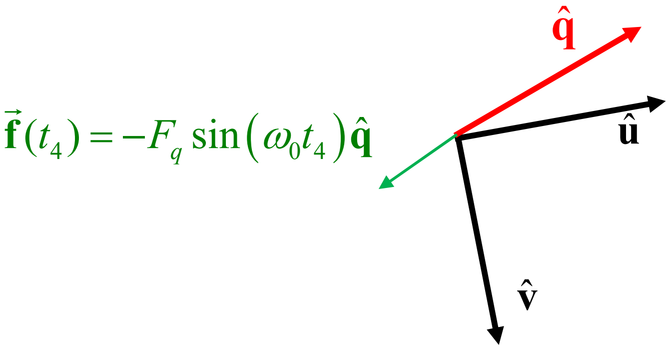

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Linear Polarization

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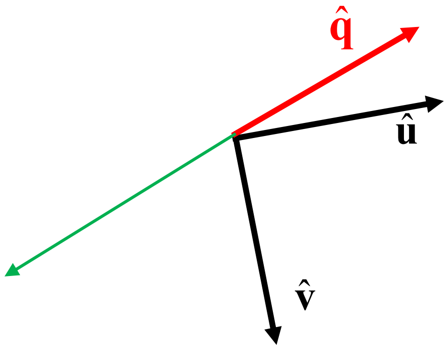
$$\vec{\mathbf{f}}(t_5) = -F_q \sin\left(\omega_0 t_5\right)\hat{\mathbf{q}}$$



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Linear Polarization

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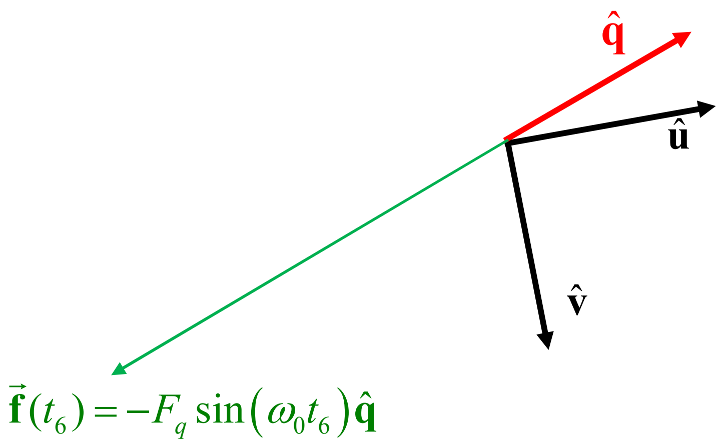

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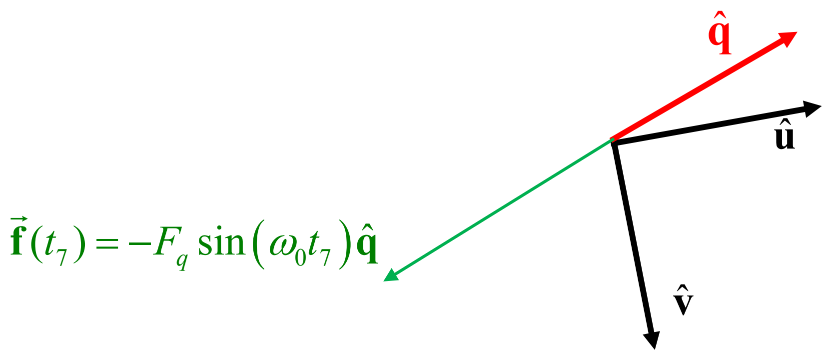

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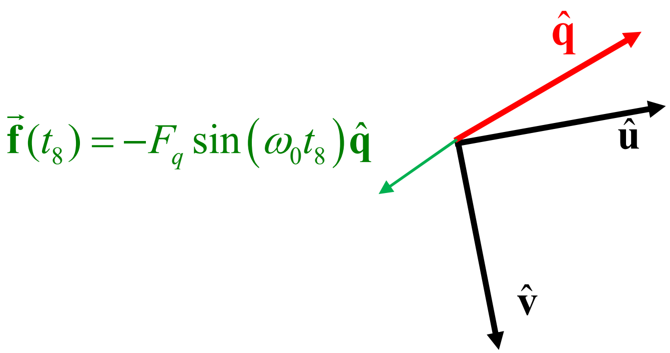

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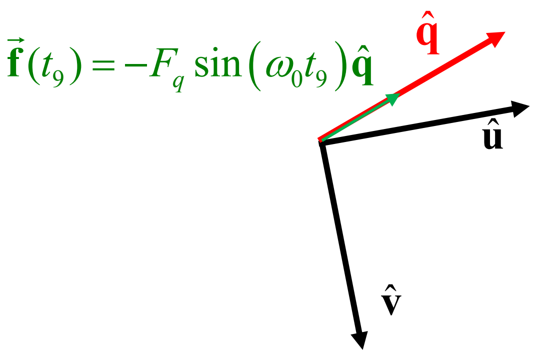

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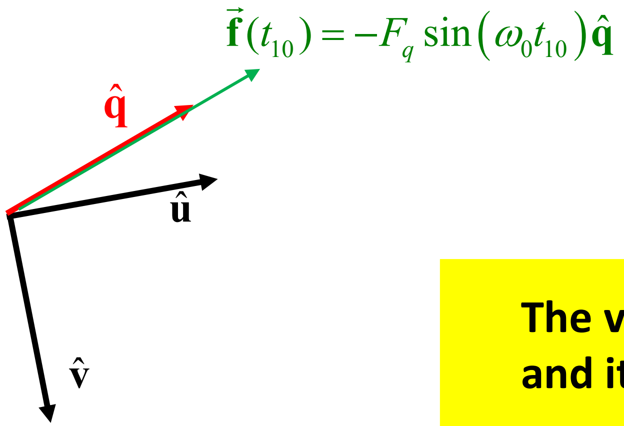

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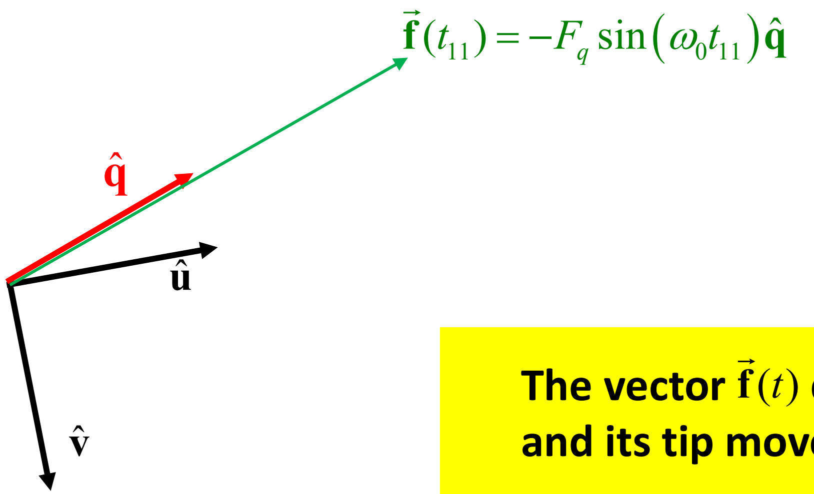

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T1 $\vec{\mathbf{f}}(t) = F_p\cos(\omega_0 t)\hat{\mathbf{p}} - F_q\sin(\omega_0 t)\hat{\mathbf{q}}$

P2 $\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$

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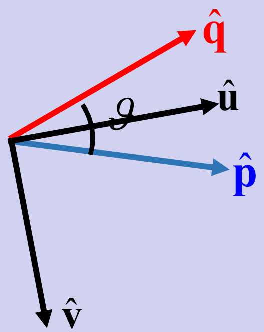
$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

Polarization plane



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- **C** $F_p = 0$ and $F_q \neq 0$ ➡ $\vec{\mathbf{f}}(t) = -F_q\sin(\omega_0 t)\hat{\mathbf{q}}$
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Linear Polarization

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P1 $\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$

T1 $\vec{\mathbf{f}}(t) = F_p\cos(\omega_0 t)\hat{\mathbf{p}} - F_q\sin(\omega_0 t)\hat{\mathbf{q}}$

P2 $\vec{\mathbf{F}} = |\vec{\mathbf{F}}|e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + |\vec{\mathbf{F}}|e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$

T2 $\vec{\mathbf{f}}(t) = |\vec{\mathbf{F}}|\cos\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t + \phi)\hat{\mathbf{u}} + |\vec{\mathbf{F}}|\sin\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t - \phi)\hat{\mathbf{v}}$

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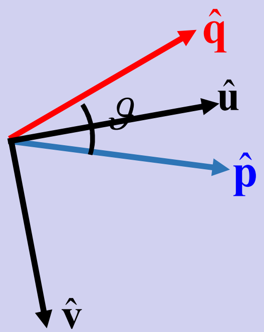
$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

Polarization plane



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- **B** $\hat{\mathbf{p}} = -\hat{\mathbf{q}} \Longrightarrow \vec{\mathbf{f}}(t) = |\vec{\mathbf{F}}|\cos(\omega_0 t - \phi)\hat{\mathbf{v}}$ $(\hat{\mathbf{v}} = \hat{\mathbf{p}} = -\hat{\mathbf{q}})$
- **C** $F_p = 0$ and $F_q \neq 0 \Longrightarrow \vec{\mathbf{f}}(t) = -F_q\sin(\omega_0 t)\hat{\mathbf{q}}$
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Linear Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

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$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

P1 $\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$

T1 $\vec{\mathbf{f}}(t) = F_p\cos(\omega_0 t)\hat{\mathbf{p}} - F_q\sin(\omega_0 t)\hat{\mathbf{q}}$

P2 $\vec{\mathbf{F}} = |\vec{\mathbf{F}}|e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + |\vec{\mathbf{F}}|e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$

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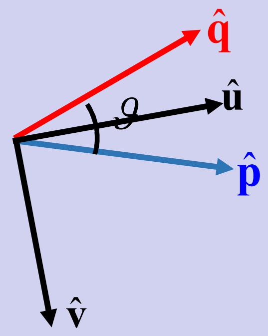
Polarization plane



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- **C** $F_p = 0$ and $F_q \neq 0$ ➡ $\vec{\mathbf{f}}(t) = -F_q\sin(\omega_0 t)\hat{\mathbf{q}}$
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Linear Polarization

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P2 $\vec{\mathbf{F}} = |\vec{\mathbf{F}}|e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + |\vec{\mathbf{F}}|e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$

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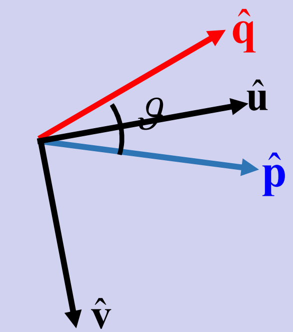

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- **B** $\hat{\mathbf{p}} = -\hat{\mathbf{q}}$ ➡ $\vec{\mathbf{f}}(t) = |\vec{\mathbf{F}}|\cos(\omega_0 t - \phi)\hat{\mathbf{v}}$ $(\hat{\mathbf{v}} = \hat{\mathbf{p}} = -\hat{\mathbf{q}})$
- **C** $F_p = 0$ and $F_q \neq 0$ ➡ $\vec{\mathbf{f}}(t) = -F_q\sin(\omega_0 t)\hat{\mathbf{q}}$
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Linear Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

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P1 $\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$

T1 $\vec{\mathbf{f}}(t) = F_p\cos(\omega_0 t)\hat{\mathbf{p}} - F_q\sin(\omega_0 t)\hat{\mathbf{q}}$

P2 $\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$

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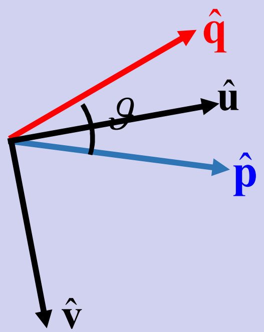
Polarization plane



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Linear Polarization

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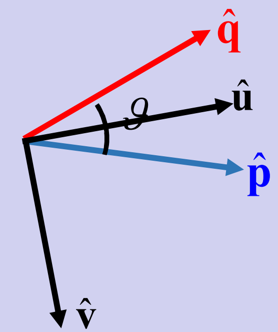
Polarization plane



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Linear Polarization

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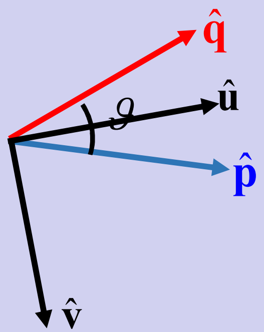
Polarization plane



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Linear Polarization

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$$F_p = \left|\vec{\mathbf{F}}\right|\cos\phi$$

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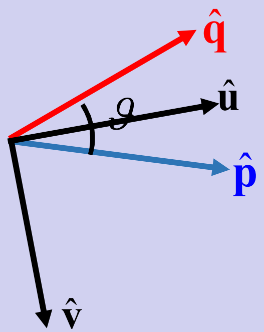
$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

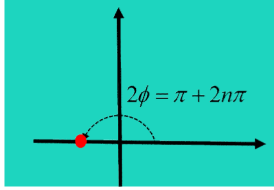
Polarization plane

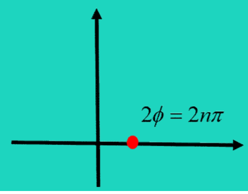


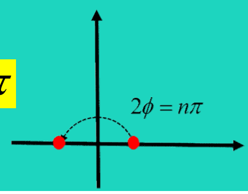
$F_p = 0$ and $F_q \neq 0$ ➡ $\cos\phi = 0$ ➡ $\phi = \frac{\pi}{2} + n\pi$ ➡ $2\phi = \pi + 2n\pi$

$F_p \neq 0$ and $F_q = 0$ ➡ $\sin\phi = 0$ ➡ $\phi = n\pi$ ➡ $2\phi = 2n\pi$

$2\phi = n\pi$







Linear Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

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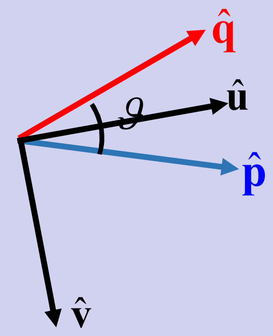
Polarization plane



$$(F_p = 0 \text{ and } F_q \neq 0) \text{ or } (F_p \neq 0 \text{ and } F_q = 0)$$

$$\Downarrow$$

$$2\phi = n\pi$$

$$\Downarrow$$

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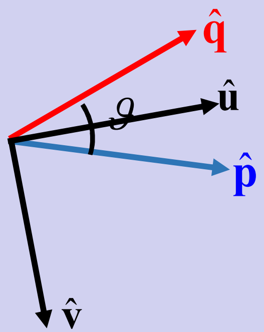

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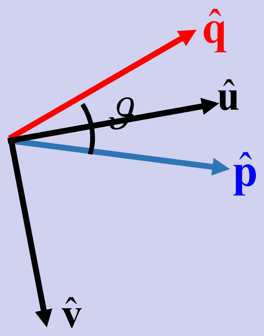

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C $F_p = 0$ and $F_q \neq 0$

or

D $F_p \neq 0$ and $F_q = 0$

➡ $\angle F_u - \angle F_v = n\pi$

Linear Polarization

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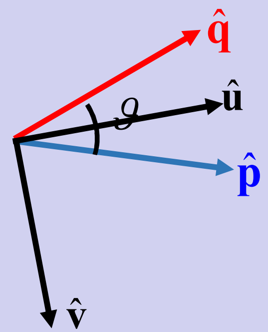


The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line:

- **A** $\hat{\mathbf{p}} = \hat{\mathbf{q}}$ $\Rightarrow$ $F_u \neq 0$ and $F_v = 0$
- **B** $\hat{\mathbf{p}} = -\hat{\mathbf{q}}$ $\Rightarrow$ $F_u = 0$ and $F_v \neq 0$
- **C** $F_p = 0$ and $F_q \neq 0$
- or
- **D** $F_p \neq 0$ and $F_q = 0$ $\Rightarrow$ $\angle F_u - \angle F_v = n\pi$

Linear Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

P1 $\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$

T1 $\vec{\mathbf{f}}(t) = F_p\cos(\omega_0 t)\hat{\mathbf{p}} - F_q\sin(\omega_0 t)\hat{\mathbf{q}}$

P2 $\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$

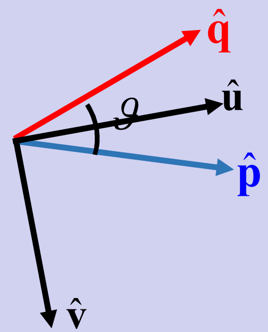
T2 $\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right|\cos\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t + \phi)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right|\sin\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t - \phi)\hat{\mathbf{v}}$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

Polarization plane



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line:

$(F_u \neq 0 \text{ and } F_v = 0)$ **or** $(F_u = 0 \text{ and } F_v \neq 0)$ **or** $(\angle F_u - \angle F_v = n\pi)$

Linear Polarization

P1 $\vec{\mathbf{F}} = F_p \hat{\mathbf{p}} + jF_q \hat{\mathbf{q}}$

T1 $\vec{\mathbf{f}}(t) = F_p \cos(\omega_0 t)\hat{\mathbf{p}} - F_q \sin(\omega_0 t)\hat{\mathbf{q}}$

P2 $\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi} \cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi} \sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}} = F_u \hat{\mathbf{u}} + F_v \hat{\mathbf{v}}$

T2 $\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right| \cos\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t + \phi)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| \sin\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t - \phi)\hat{\mathbf{v}}$

Linear polarization: the vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line.
To obtain linear polarization, one (**just one**) of the following three conditions must be enforced:

$$(F_u \neq 0 \text{ and } F_v = 0) \quad \text{or} \quad (F_u = 0 \text{ and } F_v \neq 0) \quad \text{or} \quad (\angle F_u - \angle F_v = n\pi)$$

Polarization plane

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

P1 $\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$

T1 $\vec{\mathbf{f}}(t) = F_p\cos(\omega_0 t)\hat{\mathbf{p}} - F_q\sin(\omega_0 t)\hat{\mathbf{q}}$

P2 $\vec{\mathbf{F}} = |\vec{\mathbf{F}}|e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + |\vec{\mathbf{F}}|e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$

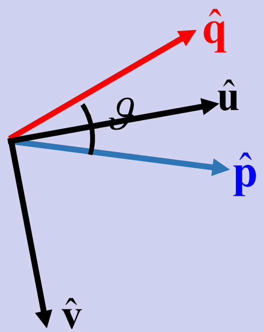
T2 $\vec{\mathbf{f}}(t) = |\vec{\mathbf{F}}|\cos\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t + \phi)\hat{\mathbf{u}} + |\vec{\mathbf{F}}|\sin\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t - \phi)\hat{\mathbf{v}}$

$$\begin{cases} F_p = |\vec{\mathbf{F}}|\cos\phi \\ F_q = |\vec{\mathbf{F}}|\sin\phi \end{cases}$$

$$|\vec{\mathbf{F}}| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

Polarization plane



The vector $\vec{\mathbf{f}}(t)$ lies in the polarization plane $(\hat{\mathbf{p}},\hat{\mathbf{q}})$ (which is coincident with the plane $(\hat{\mathbf{u}},\hat{\mathbf{v}})$), and it changes in general both its amplitude and its direction as the time elapses.

Two cases are of particular interest:

- Linear polarization
- **Circular polarization**

Circular Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

P1 $\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$

T1 $\vec{\mathbf{f}}(t) = F_p\cos(\omega_0 t)\hat{\mathbf{p}} - F_q\sin(\omega_0 t)\hat{\mathbf{q}}$

P2 $\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$

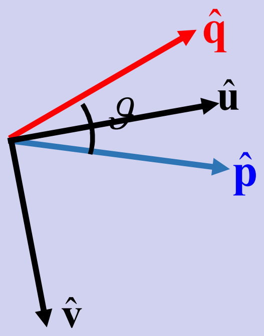
T2 $\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right|\cos\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t + \phi)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right|\sin\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t - \phi)\hat{\mathbf{v}}$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\hat{\mathbf{u}} \cdot \hat{\mathbf{v}} = 0$$

Polarization plane



$$\begin{cases} \left|F_u\right| = \left|F_v\right| \\ \angle F_u - \angle F_v = \frac{\pi}{2} + n\pi \end{cases} \Rightarrow \begin{cases} \vartheta = \frac{\pi}{2} \\ \phi = \pm\frac{\pi}{4} \end{cases}$$

Circular Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

P1 $\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$

T1 $\vec{\mathbf{f}}(t) = F_p\cos(\omega_0 t)\hat{\mathbf{p}} - F_q\sin(\omega_0 t)\hat{\mathbf{q}}$

P2 $\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$

T2 $\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right|\cos\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t + \phi)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right|\sin\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t - \phi)\hat{\mathbf{v}}$

$$\sin\left(\alpha + \frac{\pi}{2}\right) = \cos\alpha \;;\; \sin\left(\alpha - \frac{\pi}{2}\right) = -\cos\alpha$$

$$\sin\left(\omega_0 t + \frac{\pi}{4}\right) = \sin\left(\omega_0 t - \frac{\pi}{4} + \frac{\pi}{2}\right) = \cos\left(\omega_0 t - \frac{\pi}{4}\right)$$

$$\sin\left(\omega_0 t - \frac{\pi}{4}\right) = \sin\left(\omega_0 t + \frac{\pi}{4} - \frac{\pi}{2}\right) = -\cos\left(\omega_0 t + \frac{\pi}{4}\right)$$

$$\vartheta = \frac{\pi}{2};\; \phi = \pm\frac{\pi}{4}$$

$$\vec{\mathbf{f}}(t) = \frac{\left|\vec{\mathbf{F}}\right|}{\sqrt{2}}\cos\left(\omega_0 t \pm \frac{\pi}{4}\right)\hat{\mathbf{u}} + \frac{\left|\vec{\mathbf{F}}\right|}{\sqrt{2}}\cos\left(\omega_0 t \mp \frac{\pi}{4}\right)\hat{\mathbf{v}}$$

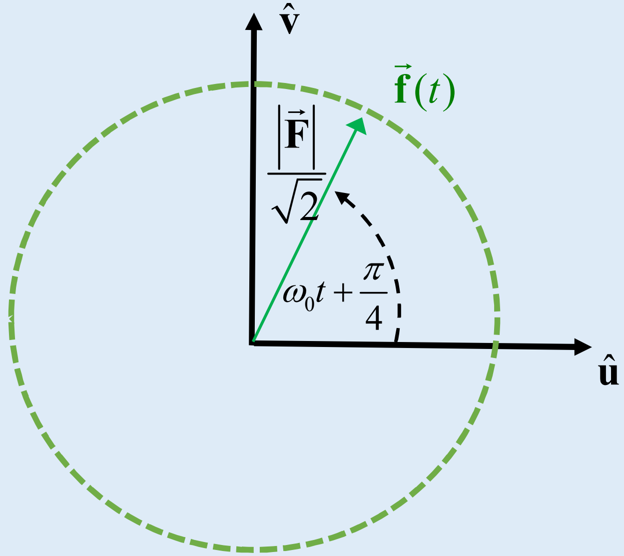
$$= \frac{\left|\vec{\mathbf{F}}\right|}{\sqrt{2}}\cos\left(\omega_0 t \pm \frac{\pi}{4}\right)\hat{\mathbf{u}} \pm \frac{\left|\vec{\mathbf{F}}\right|}{\sqrt{2}}\sin\left(\omega_0 t \pm \frac{\pi}{4}\right)\hat{\mathbf{v}}$$

Circular Polarization

$$\vartheta = \frac{\pi}{2};\ \phi = +\frac{\pi}{4}$$

$$\vec{\mathbf{f}}(t) = \frac{|\vec{\mathbf{F}}|}{\sqrt{2}}\cos\left(\omega_0 t + \frac{\pi}{4}\right)\hat{\mathbf{u}} + \frac{|\vec{\mathbf{F}}|}{\sqrt{2}}\sin\left(\omega_0 t + \frac{\pi}{4}\right)\hat{\mathbf{v}}$$

$$|\vec{\mathbf{f}}(t)|^2 = \frac{|\vec{\mathbf{F}}|^2}{2}\left[\cos\left(\omega_0 t + \frac{\pi}{4}\right)\right]^2 + \frac{|\vec{\mathbf{F}}|^2}{2}\left[\sin\left(\omega_0 t + \frac{\pi}{4}\right)\right]^2 = \frac{|\vec{\mathbf{F}}|^2}{2}$$



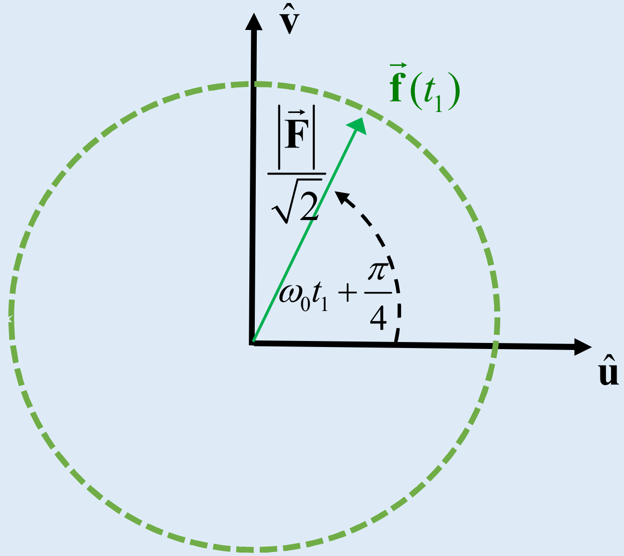
The vector $\vec{\mathbf{f}}(t)$ maintains a constant modulus

Circular Polarization

$$\vartheta = \frac{\pi}{2};\ \phi = +\frac{\pi}{4}$$

$$\vec{\mathbf{f}}(t) = \frac{|\vec{\mathbf{F}}|}{\sqrt{2}}\cos\left(\omega_0 t + \frac{\pi}{4}\right)\hat{\mathbf{u}} + \frac{|\vec{\mathbf{F}}|}{\sqrt{2}}\sin\left(\omega_0 t + \frac{\pi}{4}\right)\hat{\mathbf{v}}$$

$$|\vec{\mathbf{f}}(t)|^2 = \frac{|\vec{\mathbf{F}}|^2}{2}\left[\cos\left(\omega_0 t + \frac{\pi}{4}\right)\right]^2 + \frac{|\vec{\mathbf{F}}|^2}{2}\left[\sin\left(\omega_0 t + \frac{\pi}{4}\right)\right]^2 = \frac{|\vec{\mathbf{F}}|^2}{2}$$



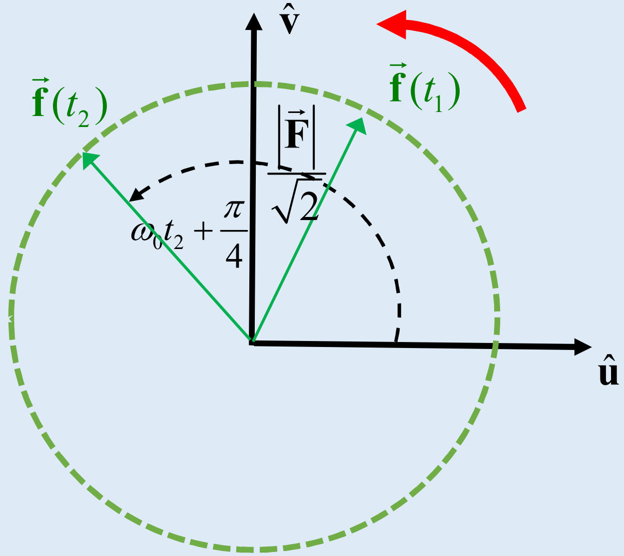
The vector $\vec{\mathbf{f}}(t)$ maintains a constant modulus

Circular Polarization

$$\vartheta = \frac{\pi}{2};\ \phi = +\frac{\pi}{4}$$

$$\vec{\mathbf{f}}(t) = \frac{|\vec{\mathbf{F}}|}{\sqrt{2}}\cos\left(\omega_0 t + \frac{\pi}{4}\right)\hat{\mathbf{u}} + \frac{|\vec{\mathbf{F}}|}{\sqrt{2}}\sin\left(\omega_0 t + \frac{\pi}{4}\right)\hat{\mathbf{v}}$$

$$|\vec{\mathbf{f}}(t)|^2 = \frac{|\vec{\mathbf{F}}|^2}{2}\left[\cos\left(\omega_0 t + \frac{\pi}{4}\right)\right]^2 + \frac{|\vec{\mathbf{F}}|^2}{2}\left[\sin\left(\omega_0 t + \frac{\pi}{4}\right)\right]^2 = \frac{|\vec{\mathbf{F}}|^2}{2}$$



The vector $\vec{\mathbf{f}}(t)$ maintains a constant modulus

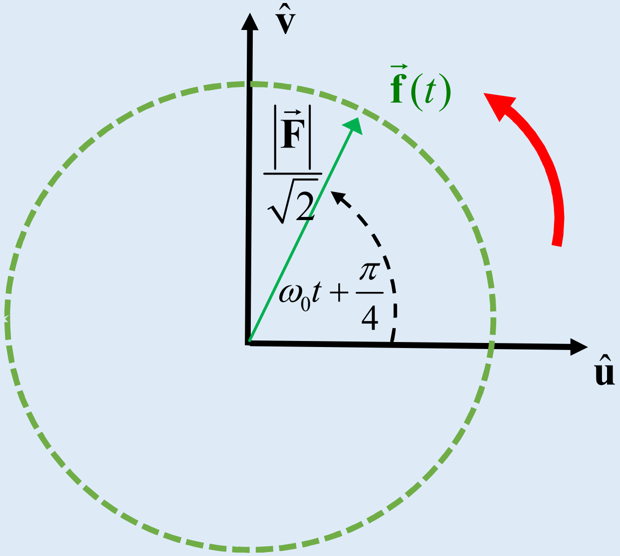
Its tip moves along a circle with angular velocity ω_0

Circular Polarization

$$\vartheta = \frac{\pi}{2};\ \phi = +\frac{\pi}{4}$$

$$\vec{\mathbf{f}}(t) = \frac{|\vec{\mathbf{F}}|}{\sqrt{2}}\cos\left(\omega_0 t + \frac{\pi}{4}\right)\hat{\mathbf{u}} + \frac{|\vec{\mathbf{F}}|}{\sqrt{2}}\sin\left(\omega_0 t + \frac{\pi}{4}\right)\hat{\mathbf{v}}$$

$$|\vec{\mathbf{f}}(t)|^2 = \frac{|\vec{\mathbf{F}}|^2}{2}\left[\cos\left(\omega_0 t + \frac{\pi}{4}\right)\right]^2 + \frac{|\vec{\mathbf{F}}|^2}{2}\left[\sin\left(\omega_0 t + \frac{\pi}{4}\right)\right]^2 = \frac{|\vec{\mathbf{F}}|^2}{2}$$



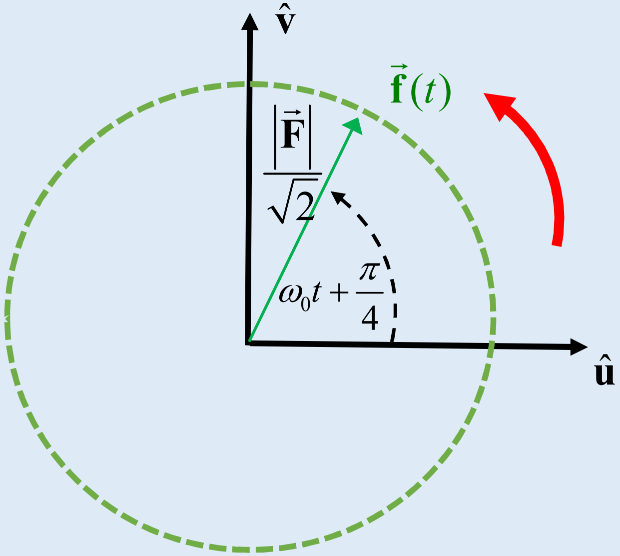
The vector $\vec{\mathbf{f}}(t)$ maintains a constant modulus and its tip moves along a circle with angular velocity ω_0

Circular Polarization

$$\vartheta = \frac{\pi}{2};\ \phi = +\frac{\pi}{4}$$

$$\vec{\mathbf{f}}(t) = \frac{|\vec{\mathbf{F}}|}{\sqrt{2}}\cos\left(\omega_0 t + \frac{\pi}{4}\right)\hat{\mathbf{u}} + \frac{|\vec{\mathbf{F}}|}{\sqrt{2}}\sin\left(\omega_0 t + \frac{\pi}{4}\right)\hat{\mathbf{v}}$$

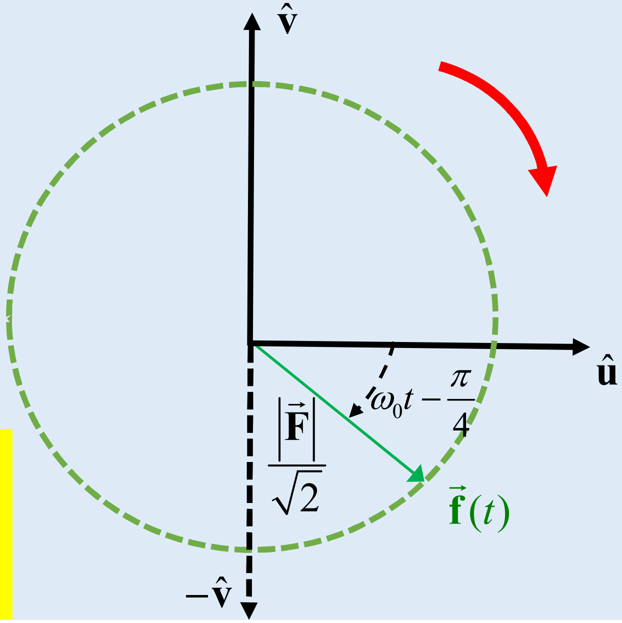
$$|\vec{\mathbf{f}}(t)|^2 = \frac{|\vec{\mathbf{F}}|^2}{2}\left[\cos\left(\omega_0 t + \frac{\pi}{4}\right)\right]^2 + \frac{|\vec{\mathbf{F}}|^2}{2}\left[\sin\left(\omega_0 t + \frac{\pi}{4}\right)\right]^2 = \frac{|\vec{\mathbf{F}}|^2}{2}$$



$$\vartheta = \frac{\pi}{2};\ \phi = -\frac{\pi}{4}$$

$$\vec{\mathbf{f}}(t) = \frac{|\vec{\mathbf{F}}|}{\sqrt{2}}\cos\left(\omega_0 t - \frac{\pi}{4}\right)\hat{\mathbf{u}} - \frac{|\vec{\mathbf{F}}|}{\sqrt{2}}\sin\left(\omega_0 t - \frac{\pi}{4}\right)\hat{\mathbf{v}}$$

$$|\vec{\mathbf{f}}(t)|^2 = \frac{|\vec{\mathbf{F}}|^2}{2}\left[\cos\left(\omega_0 t - \frac{\pi}{4}\right)\right]^2 + \frac{|\vec{\mathbf{F}}|^2}{2}\left[\sin\left(\omega_0 t - \frac{\pi}{4}\right)\right]^2 = \frac{|\vec{\mathbf{F}}|^2}{2}$$



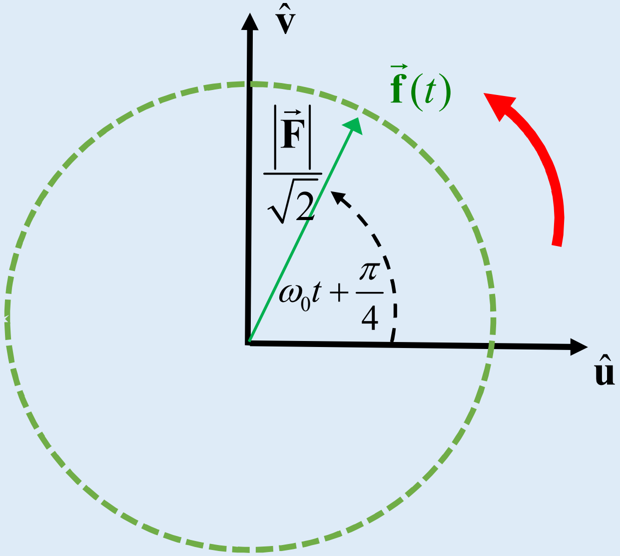
The vector $\vec{\mathbf{f}}(t)$ maintains a constant modulus and its tip moves along a circle with angular velocity ω_0

Circular Polarization

$$\vartheta = \frac{\pi}{2};\ \phi = +\frac{\pi}{4}$$

$$\vec{\mathbf{f}}(t) = \frac{|\vec{\mathbf{F}}|}{\sqrt{2}}\cos\left(\omega_0 t + \frac{\pi}{4}\right)\hat{\mathbf{u}} + \frac{|\vec{\mathbf{F}}|}{\sqrt{2}}\sin\left(\omega_0 t + \frac{\pi}{4}\right)\hat{\mathbf{v}}$$

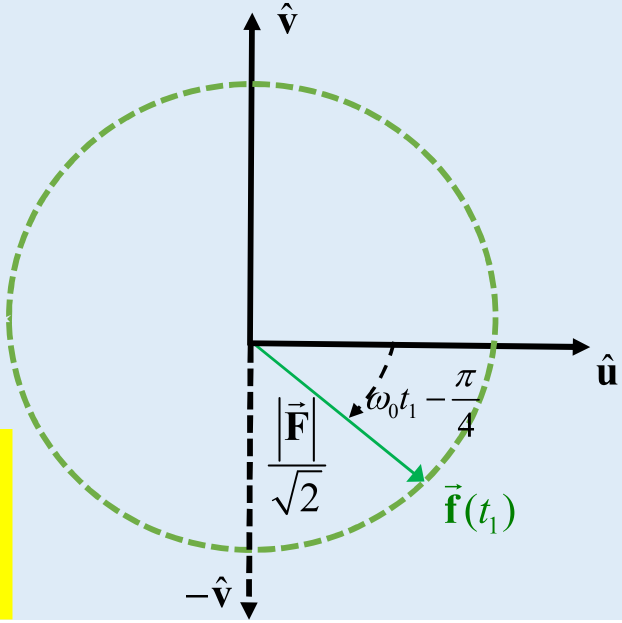
$$|\vec{\mathbf{f}}(t)|^2 = \frac{|\vec{\mathbf{F}}|^2}{2}\left[\cos\left(\omega_0 t + \frac{\pi}{4}\right)\right]^2 + \frac{|\vec{\mathbf{F}}|^2}{2}\left[\sin\left(\omega_0 t + \frac{\pi}{4}\right)\right]^2 = \frac{|\vec{\mathbf{F}}|^2}{2}$$



$$\vartheta = \frac{\pi}{2};\ \phi = -\frac{\pi}{4}$$

$$\vec{\mathbf{f}}(t) = \frac{|\vec{\mathbf{F}}|}{\sqrt{2}}\cos\left(\omega_0 t - \frac{\pi}{4}\right)\hat{\mathbf{u}} - \frac{|\vec{\mathbf{F}}|}{\sqrt{2}}\sin\left(\omega_0 t - \frac{\pi}{4}\right)\hat{\mathbf{v}}$$

$$|\vec{\mathbf{f}}(t)|^2 = \frac{|\vec{\mathbf{F}}|^2}{2}\left[\cos\left(\omega_0 t - \frac{\pi}{4}\right)\right]^2 + \frac{|\vec{\mathbf{F}}|^2}{2}\left[\sin\left(\omega_0 t - \frac{\pi}{4}\right)\right]^2 = \frac{|\vec{\mathbf{F}}|^2}{2}$$



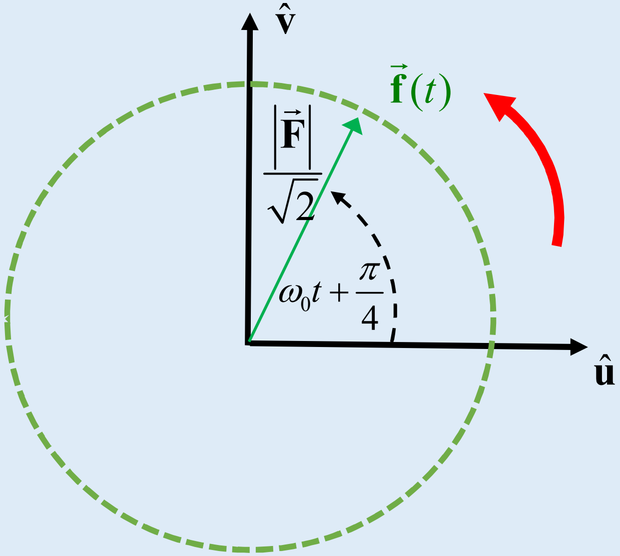
The vector $\vec{\mathbf{f}}(t)$ maintains a constant modulus and its tip moves along a circle with angular velocity ω_0

Circular Polarization

$$\vartheta = \frac{\pi}{2};\ \phi = +\frac{\pi}{4}$$

$$\vec{\mathbf{f}}(t) = \frac{\left|\vec{\mathbf{F}}\right|}{\sqrt{2}}\cos\left(\omega_0 t + \frac{\pi}{4}\right)\hat{\mathbf{u}} + \frac{\left|\vec{\mathbf{F}}\right|}{\sqrt{2}}\sin\left(\omega_0 t + \frac{\pi}{4}\right)\hat{\mathbf{v}}$$

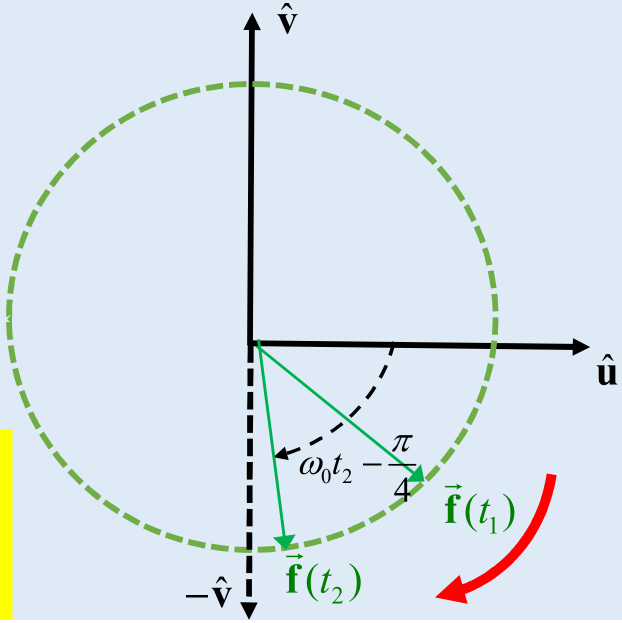
$$\left|\vec{\mathbf{f}}(t)\right|^2 = \frac{\left|\vec{\mathbf{F}}\right|^2}{2}\left[\cos\left(\omega_0 t + \frac{\pi}{4}\right)\right]^2 + \frac{\left|\vec{\mathbf{F}}\right|^2}{2}\left[\sin\left(\omega_0 t + \frac{\pi}{4}\right)\right]^2 = \frac{\left|\vec{\mathbf{F}}\right|^2}{2}$$



$$\vartheta = \frac{\pi}{2};\ \phi = -\frac{\pi}{4}$$

$$\vec{\mathbf{f}}(t) = \frac{\left|\vec{\mathbf{F}}\right|}{\sqrt{2}}\cos\left(\omega_0 t - \frac{\pi}{4}\right)\hat{\mathbf{u}} - \frac{\left|\vec{\mathbf{F}}\right|}{\sqrt{2}}\sin\left(\omega_0 t - \frac{\pi}{4}\right)\hat{\mathbf{v}}$$

$$\left|\vec{\mathbf{f}}(t)\right|^2 = \frac{\left|\vec{\mathbf{F}}\right|^2}{2}\left[\cos\left(\omega_0 t - \frac{\pi}{4}\right)\right]^2 + \frac{\left|\vec{\mathbf{F}}\right|^2}{2}\left[\sin\left(\omega_0 t - \frac{\pi}{4}\right)\right]^2 = \frac{\left|\vec{\mathbf{F}}\right|^2}{2}$$



The vector $\vec{\mathbf{f}}(t)$ maintains a constant modulus and its tip moves along a circle with angular velocity ω_0

Circular Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

P1 $\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$

T1 $\vec{\mathbf{f}}(t) = F_p\cos(\omega_0 t)\hat{\mathbf{p}} - F_q\sin(\omega_0 t)\hat{\mathbf{q}}$

P2 $\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$

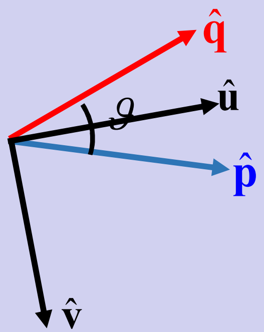
T2 $\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right|\cos\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t + \phi)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right|\sin\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t - \phi)\hat{\mathbf{v}}$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

Polarization plane



The vector $\vec{\mathbf{f}}(t)$ maintains a constant modulus and its tip moves along a circle with angular velocity ω_0

$$\left(\left|F_u\right| = \left|F_v\right|\right) \ \underline{\textbf{and}}\ \left(\angle F_u - \angle F_v = \frac{\pi}{2} + n\pi\right)$$

Linear Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

P1 $\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$

T1 $\vec{\mathbf{f}}(t) = F_p\cos(\omega_0 t)\hat{\mathbf{p}} - F_q\sin(\omega_0 t)\hat{\mathbf{q}}$

P2 $\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$

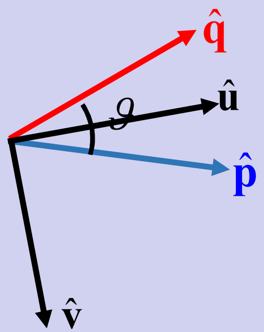
T2 $\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right|\cos\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t + \phi)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right|\sin\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t - \phi)\hat{\mathbf{v}}$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

Polarization plane



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line:

$(F_u \neq 0 \text{ and } F_v = 0)$ **or** $(F_u = 0 \text{ and } F_v \neq 0)$ **or** $(\angle F_u - \angle F_v = n\pi)$

Field Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

P1 $\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$

T1 $\vec{\mathbf{f}}(t) = F_p\cos(\omega_0 t)\hat{\mathbf{p}} - F_q\sin(\omega_0 t)\hat{\mathbf{q}}$

P2 $\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}} = F_u\hat{\mathbf{u}} + F_v\hat{\mathbf{v}}$

T2 $\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right|\cos\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t + \phi)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right|\sin\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t - \phi)\hat{\mathbf{v}}$

Field Polarization

P1 $\vec{\mathbf{F}} = F_p \hat{\mathbf{p}} + jF_q \hat{\mathbf{q}}$

T1 $\vec{\mathbf{f}}(t) = F_p \cos(\omega_0 t)\hat{\mathbf{p}} - F_q \sin(\omega_0 t)\hat{\mathbf{q}}$

P2 $\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi} \cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi} \sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}} = F_u \hat{\mathbf{u}} + F_v \hat{\mathbf{v}}$

T2 $\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right| \cos\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t + \phi)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| \sin\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t - \phi)\hat{\mathbf{v}}$

Linear polarization: the vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line.
To obtain linear polarization, one (**just one**) of the following three conditions must be enforced:

$$\left(F_u \neq 0 \text{ and } F_v = 0\right) \quad \underline{\textbf{or}} \quad \left(F_u = 0 \text{ and } F_v \neq 0\right) \quad \underline{\textbf{or}} \quad \left(\angle F_u - \angle F_v = n\pi\right)$$

Circular polarization: the vector $\vec{\mathbf{f}}(t)$ maintains a constant modulus and its tip moves with angular velocity ω_0 along a circle in the polarization plane.
To obtain circular polarization, the following two conditions must be **simultaneously** enforced:

$$\left|F_u\right| = \left|F_v\right| \quad \underline{\textbf{and}} \quad \angle F_u - \angle F_v = \frac{\pi}{2} + n\pi$$

In the more general case, the tip of the vector $\vec{\mathbf{f}}(t)$ moves along an ellipse in the polarization plane. This case is referred to as **elliptical polarization.**