Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2019-2020 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

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Phasors and vector functions

Time domain

$$\vec{\mathbf{f}}(x,y,z,t) = f_x(x,y,z,t)\hat{i}_x + f_y(x,y,z,t)\hat{i}_y + f_z(x,y,z,t)\,\hat{i}_z$$

Phasor domain

$$\vec{\mathbf{F}}(x,y,z) = F_x(x,y,z)\hat{i}_x + F_y(x,y,z)\hat{i}_y + F_z(x,y,z)\hat{i}_z$$

$$f_x(x,y,z,t) = A_x(x,y,z)\cos\left(\omega_0 t + \alpha_x(x,y,z)\right) \longrightarrow \boxed{\text{T-to-P}} \longrightarrow F_x(x,y,z) = A_x(x,y,z)e^{j\alpha_x(x,y,z)}$$

$$f_y(x,y,z,t) = A_y(x,y,z)\cos\left(\omega_0 t + \alpha_y(x,y,z)\right) \longrightarrow \boxed{\text{T-to-P}} \longrightarrow F_y(x,y,z) = A_y(x,y,z)e^{j\alpha_y(x,y,z)}$$

$$f_z(x,y,z,t) = A_z(x,y,z)\cos\left(\omega_0 t + \alpha_z(x,y,z)\right) \longrightarrow \boxed{\text{T-to-P}} \longrightarrow F_z(x,y,z) = A_z(x,y,z)e^{j\alpha_z(x,y,z)}$$

$$\vec{\mathbf{f}}(x,y,z,t) \longrightarrow \boxed{\text{T-to-P}} \longrightarrow \vec{\mathbf{F}}(x,y,z)$$

$$\vec{\mathbf{F}}(x,y,z) \longrightarrow \boxed{\text{P-to-T}} \longrightarrow \vec{\mathbf{f}}(x,y,z,t) = \mathrm{Re}\left\{\vec{\mathbf{F}}(x,y,z)e^{j\omega_0 t}\right\}$$

Phasors and vector functions

Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Phasor domain

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z$$

$f_x(t) = A_x \cos(\omega_0 t + \alpha_x)$ → **T-to-P** → $F_x = A_x e^{j\alpha_x} = R_x + jI_x$

$f_y(t) = A_y \cos(\omega_0 t + \alpha_y)$ → **T-to-P** → $F_y = A_y e^{j\alpha_y} = R_y + jI_y$

$f_z(t) = A_z \cos(\omega_0 t + \alpha_z)$ → **T-to-P** → $F_z = A_z e^{j\alpha_z} = R_z + jI_z$

$\vec{\mathbf{f}}(t)$ → **T-to-P** → $\vec{\mathbf{F}}$

$\vec{\mathbf{F}}$ → **P-to-T** → $\vec{\mathbf{f}}(t) = \mathrm{Re}\left\{\vec{\mathbf{F}} e^{j\omega_0 t}\right\}$

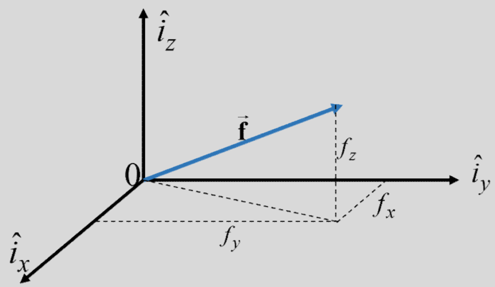
Complex vectors: graphical representation

Real numbers

f

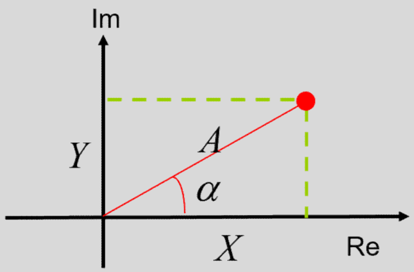
Real vectors (3 components)

$$\vec{\mathbf{f}} = f_x \hat{i}_x + f_y \hat{i}_y + f_z \hat{i}_z$$



Complex numbers

$$F = Ae^{j\alpha} = X + jY$$



Complex vectors (3 components)

$$\vec{\mathbf{F}} = F_x \hat{i}_x + F_y \hat{i}_y + F_z \hat{i}_z$$

?

Color legend

New formulas, important considerations,
important formulas, important concepts

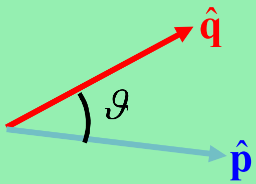
Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Mathematical tools that we will exploit today



$$\begin{cases} \hat{\mathbf{p}} \cdot \hat{\mathbf{p}} = 1 \\ \hat{\mathbf{q}} \cdot \hat{\mathbf{q}} = 1 \\ \hat{\mathbf{p}} \cdot \hat{\mathbf{q}} = \hat{\mathbf{q}} \cdot \hat{\mathbf{p}} = \cos\vartheta \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{\vec{\mathbf{F}} \cdot \vec{\mathbf{F}}^*}$$

$$\sin\left(\alpha + \frac{\pi}{2}\right) = \cos\alpha \;;\; \sin\left(\alpha - \frac{\pi}{2}\right) = -\cos\alpha$$

$$\left[\sin\left(\frac{\vartheta}{2}\right)\right]^2 = \frac{1-\cos(\vartheta)}{2};\left[\cos\left(\frac{\vartheta}{2}\right)\right]^2 = \frac{1+\cos(\vartheta)}{2}$$

$$\cos\phi = \frac{e^{j\phi} + e^{-j\phi}}{2}$$

$$\sin\phi = \frac{e^{j\phi} - e^{-j\phi}}{2j}$$

$$e^{j\phi} = \cos\phi + j\sin\phi$$

Phasors and vector functions

Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Phasor domain

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z$$

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$f_y(t) = A_y\cos(\omega_0 t + \alpha_y)$ → **T-to-P** → $F_y = A_y e^{j\alpha_y} = R_y + jI_y$

$f_z(t) = A_z\cos(\omega_0 t + \alpha_z)$ → **T-to-P** → $F_z = A_z e^{j\alpha_z} = R_z + jI_z$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = \left(A_x e^{j\alpha_x}\right)\hat{i}_x + \left(A_y e^{j\alpha_y}\right)\hat{i}_y + \left(A_z e^{j\alpha_z}\right)\hat{i}_z = (R_x + jI_x)\hat{i}_x + (R_y + jI_y)\hat{i}_y + (R_z + jI_z)\hat{i}_z$$

$$= \underbrace{\left[R_x\hat{i}_x + R_y\hat{i}_y + R_z\hat{i}_z\right]}_{F_p\hat{\mathbf{p}}} + j\underbrace{\left[I_x\hat{i}_x + I_y\hat{i}_y + I_z\hat{i}_z\right]}_{F_q\hat{\mathbf{q}}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

F_p and F_q are real!

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

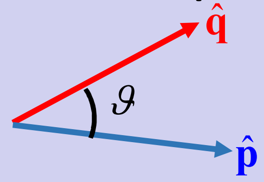
$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

Polarization plane



Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

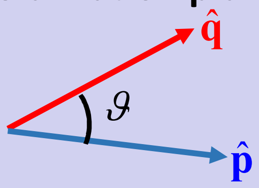
$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

Polarization plane



$$\begin{cases} \hat{\mathbf{p}}\cdot\hat{\mathbf{p}} = 1 \\ \hat{\mathbf{q}}\cdot\hat{\mathbf{q}} = 1 \\ \hat{\mathbf{p}}\cdot\hat{\mathbf{q}} = \hat{\mathbf{q}}\cdot\hat{\mathbf{p}} = \cos\vartheta \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{\vec{\mathbf{F}}\cdot\vec{\mathbf{F}}^*} = \sqrt{\left(F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}\right)\cdot\left(F_p\hat{\mathbf{p}} - jF_q\hat{\mathbf{q}}\right)} = \sqrt{\left(F_p^2\hat{\mathbf{p}}\cdot\hat{\mathbf{p}} - jF_pF_q\hat{\mathbf{p}}\cdot\hat{\mathbf{q}} + jF_qF_p\hat{\mathbf{q}}\cdot\hat{\mathbf{p}} + F_q^2\hat{\mathbf{q}}\cdot\hat{\mathbf{q}}\right)} = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{\vec{\mathbf{F}}\cdot\vec{\mathbf{F}}^*} = \sqrt{F_xF_x^* + F_yF_y^* + F_zF_z^*} = \sqrt{\left|F_x\right|^2 + \left|F_y\right|^2 + \left|F_z\right|^2}$$

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

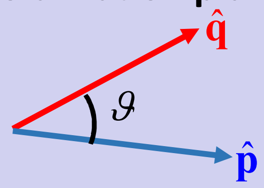
$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

Polarization plane



$$C_1 = X + jY = |C_1|e^{j\alpha}$$

$$|C_1| = \sqrt{X^2 + Y^2}$$

$$\begin{cases} \mathrm{Re}\{C_1\} = X = |C_1|\cos\alpha \\ \mathrm{Im}\{C_1\} = Y = |C_1|\sin\alpha \end{cases}$$

$$C_2 = F_p + jF_q = |C_2|e^{j\phi}$$

$$|C_2| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\begin{cases} \mathrm{Re}\{C_2\} = F_p = |C_2|\cos\phi \\ \mathrm{Im}\{C_2\} = F_q = |C_2|\sin\phi \end{cases}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

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$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

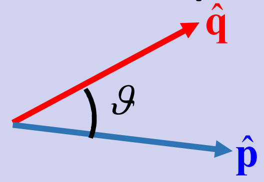
$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

Polarization plane



$$C_2 = F_p + jF_q = |C_2|e^{j\phi}$$

$$|C_2| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\begin{cases} \mathrm{Re}\{C_2\} = F_p = |C_2|\cos\phi \\ \mathrm{Im}\{C_2\} = F_q = |C_2|\sin\phi \end{cases}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

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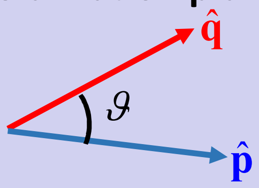
$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

Polarization plane



$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\left(\frac{\hat{\mathbf{p}} + \hat{\mathbf{q}}}{2}\right) + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\left(\frac{\hat{\mathbf{p}} - \hat{\mathbf{q}}}{2}\right)$$

$$\cos\phi = \frac{e^{j\phi} + e^{-j\phi}}{2}$$

$$\sin\phi = \frac{e^{j\phi} - e^{-j\phi}}{2j}$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}} = \left|\vec{\mathbf{F}}\right|\cos\phi\,\hat{\mathbf{p}} + j\left|\vec{\mathbf{F}}\right|\sin\phi\,\hat{\mathbf{q}}$$

$$= \left|\vec{\mathbf{F}}\right|\left[\frac{e^{j\phi} + e^{-j\phi}}{2}\right]\hat{\mathbf{p}} + j\left|\vec{\mathbf{F}}\right|\left[\frac{e^{j\phi} - e^{-j\phi}}{2j}\right]\hat{\mathbf{q}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\left(\frac{\hat{\mathbf{p}} + \hat{\mathbf{q}}}{2}\right) + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\left(\frac{\hat{\mathbf{p}} - \hat{\mathbf{q}}}{2}\right)$$

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

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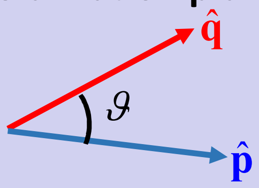
$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

Polarization plane



$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\left(\frac{\hat{\mathbf{p}} + \hat{\mathbf{q}}}{2}\right) + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\left(\frac{\hat{\mathbf{p}} - \hat{\mathbf{q}}}{2}\right)$$

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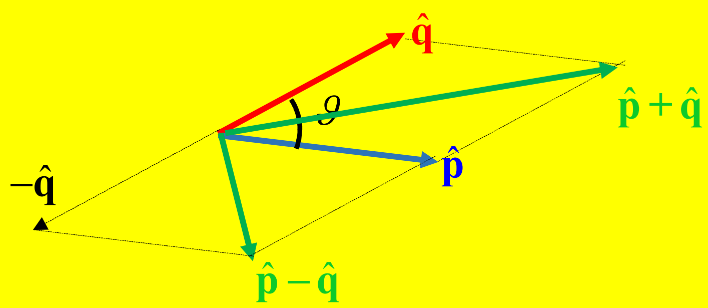
Polarization plane



$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\underbrace{\left(\frac{\hat{\mathbf{p}} + \hat{\mathbf{q}}}{2}\right)}_{\vec{\mathbf{U}}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\underbrace{\left(\frac{\hat{\mathbf{p}} - \hat{\mathbf{q}}}{2}\right)}_{\vec{\mathbf{V}}}$$

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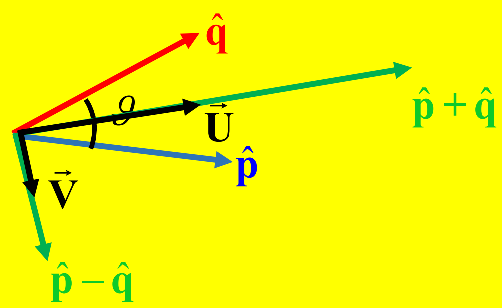
Polarization plane



$$\vec{\mathbf{F}} = |\vec{\mathbf{F}}|e^{j\phi}\underbrace{\left(\frac{\hat{\mathbf{p}}+\hat{\mathbf{q}}}{2}\right)}_{\vec{\mathbf{U}}} + |\vec{\mathbf{F}}|e^{-j\phi}\underbrace{\left(\frac{\hat{\mathbf{p}}-\hat{\mathbf{q}}}{2}\right)}_{\vec{\mathbf{V}}}$$

Phasors and vector functions

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$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

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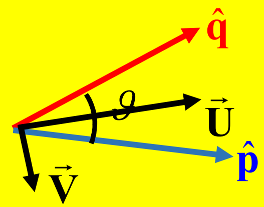
$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

Polarization plane



$$\vec{\mathbf{F}} = \underbrace{\left|\vec{\mathbf{F}}\right| e^{j\phi}\left(\frac{\hat{\mathbf{p}}+\hat{\mathbf{q}}}{2}\right)}_{\vec{\mathbf{U}}} + \underbrace{\left|\vec{\mathbf{F}}\right| e^{-j\phi}\left(\frac{\hat{\mathbf{p}}-\hat{\mathbf{q}}}{2}\right)}_{\vec{\mathbf{V}}}$$

$$\vec{\mathbf{U}}\cdot\vec{\mathbf{V}} = 0$$

$$\left|\vec{\mathbf{U}}\right| \neq 1$$

$$\left|\vec{\mathbf{V}}\right| \neq 1$$

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

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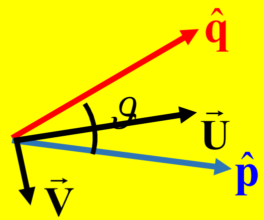
$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$|\vec{\mathbf{F}}| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\begin{cases} F_p = |\vec{\mathbf{F}}|\cos\phi \\ F_q = |\vec{\mathbf{F}}|\sin\phi \end{cases}$$

Polarization plane



$$\vec{\mathbf{F}} = |\vec{\mathbf{F}}|e^{j\phi}\underbrace{\left(\frac{\hat{\mathbf{p}} + \hat{\mathbf{q}}}{2}\right)}_{\vec{\mathbf{U}}} + |\vec{\mathbf{F}}|e^{-j\phi}\underbrace{\left(\frac{\hat{\mathbf{p}} - \hat{\mathbf{q}}}{2}\right)}_{\vec{\mathbf{V}}}$$

$$\vec{\mathbf{U}}\cdot\vec{\mathbf{V}} = 0 \qquad |\vec{\mathbf{U}}| \neq 1 \qquad |\vec{\mathbf{V}}| \neq 1$$

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos\left(\omega_0 t + \alpha_x\right) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos\left(\omega_0 t + \alpha_y\right) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

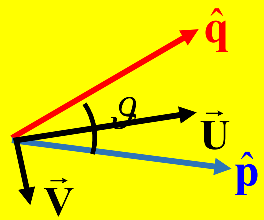
$$f_z(t) = A_z\cos\left(\omega_0 t + \alpha_z\right) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

Polarization plane



$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\underbrace{\left(\frac{\hat{\mathbf{p}}+\hat{\mathbf{q}}}{2}\right)}_{\vec{\mathbf{U}}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\underbrace{\left(\frac{\hat{\mathbf{p}}-\hat{\mathbf{q}}}{2}\right)}_{\vec{\mathbf{V}}}$$

$$\vec{\mathbf{U}}\cdot\vec{\mathbf{V}} = 0 \qquad \left|\vec{\mathbf{U}}\right| \neq 1 \qquad \left|\vec{\mathbf{V}}\right| \neq 1$$

$$\vec{\mathbf{U}} = \frac{\hat{\mathbf{p}}+\hat{\mathbf{q}}}{2}$$

$$\vec{\mathbf{V}} = \frac{\hat{\mathbf{p}}-\hat{\mathbf{q}}}{2}$$

$$\vec{\mathbf{U}}\cdot\vec{\mathbf{V}} = \left(\frac{\hat{\mathbf{p}}+\hat{\mathbf{q}}}{2}\right)\cdot\left(\frac{\hat{\mathbf{p}}-\hat{\mathbf{q}}}{2}\right) = \frac{\hat{\mathbf{p}}\cdot\hat{\mathbf{p}} - \hat{\mathbf{p}}\cdot\hat{\mathbf{q}} + \hat{\mathbf{q}}\cdot\hat{\mathbf{p}} - \hat{\mathbf{q}}\cdot\hat{\mathbf{q}}}{4} = 0$$

$\vec{\mathbf{U}}$ and $\vec{\mathbf{V}}$ are orthogonal

$$\begin{cases} \hat{\mathbf{p}}\cdot\hat{\mathbf{p}} = 1 \\ \hat{\mathbf{q}}\cdot\hat{\mathbf{q}} = 1 \end{cases}$$

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

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$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

$$\left[\cos\left(\frac{\vartheta}{2}\right)\right]^2 = \frac{1+\cos(\vartheta)}{2}$$

$$\left[\sin\left(\frac{\vartheta}{2}\right)\right]^2 = \frac{1-\cos(\vartheta)}{2}$$

$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right|e^{j\phi}\underbrace{\left(\frac{\hat{\mathbf{p}}+\hat{\mathbf{q}}}{2}\right)}_{\vec{\mathbf{U}}} + \left|\vec{\mathbf{F}}\right|e^{-j\phi}\underbrace{\left(\frac{\hat{\mathbf{p}}-\hat{\mathbf{q}}}{2}\right)}_{\vec{\mathbf{V}}}$$

$$\vec{\mathbf{U}}\cdot\vec{\mathbf{V}} = 0 \qquad \left|\vec{\mathbf{U}}\right| \neq 1 \qquad \left|\vec{\mathbf{V}}\right| \neq 1$$

$$\vec{\mathbf{U}} = \frac{\hat{\mathbf{p}}+\hat{\mathbf{q}}}{2} \qquad \left|\vec{\mathbf{U}}\right|^2 = \left(\frac{\hat{\mathbf{p}}+\hat{\mathbf{q}}}{2}\right)\cdot\left(\frac{\hat{\mathbf{p}}+\hat{\mathbf{q}}}{2}\right) = \frac{(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}+\hat{\mathbf{p}}\cdot\hat{\mathbf{q}}+\hat{\mathbf{q}}\cdot\hat{\mathbf{p}}+\hat{\mathbf{q}}\cdot\hat{\mathbf{q}})}{4} = \frac{2+2\cos\vartheta}{4} = \frac{1+\cos\vartheta}{2} = \left[\cos\left(\frac{\vartheta}{2}\right)\right]^2$$

$$\vec{\mathbf{V}} = \frac{\hat{\mathbf{p}}-\hat{\mathbf{q}}}{2} \qquad \left|\vec{\mathbf{V}}\right|^2 = \left(\frac{\hat{\mathbf{p}}-\hat{\mathbf{q}}}{2}\right)\cdot\left(\frac{\hat{\mathbf{p}}-\hat{\mathbf{q}}}{2}\right) = \frac{(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}-\hat{\mathbf{p}}\cdot\hat{\mathbf{q}}-\hat{\mathbf{q}}\cdot\hat{\mathbf{p}}+\hat{\mathbf{q}}\cdot\hat{\mathbf{q}})}{4} = \frac{2-2\cos\vartheta}{4} = \frac{1-\cos\vartheta}{2} = \left[\sin\left(\frac{\vartheta}{2}\right)\right]^2$$

$$\begin{cases} \hat{\mathbf{p}}\cdot\hat{\mathbf{p}} = 1 \\ \hat{\mathbf{q}}\cdot\hat{\mathbf{q}} = 1 \\ \hat{\mathbf{p}}\cdot\hat{\mathbf{q}} = \hat{\mathbf{q}}\cdot\hat{\mathbf{p}} = \cos\vartheta \end{cases}$$

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

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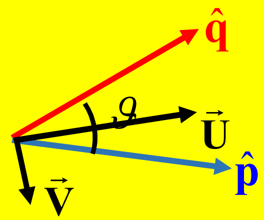
$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$|\vec{\mathbf{F}}| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\begin{cases} F_p = |\vec{\mathbf{F}}|\cos\phi \\ F_q = |\vec{\mathbf{F}}|\sin\phi \end{cases}$$

Polarization plane



$$\vec{\mathbf{F}} = |\vec{\mathbf{F}}|e^{j\phi}\left(\frac{\hat{\mathbf{p}}+\hat{\mathbf{q}}}{2}\right) + |\vec{\mathbf{F}}|e^{-j\phi}\left(\frac{\hat{\mathbf{p}}-\hat{\mathbf{q}}}{2}\right)$$

$$\frac{\hat{\mathbf{p}}+\hat{\mathbf{q}}}{2} = \vec{\mathbf{U}} = \cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} \qquad \frac{\hat{\mathbf{p}}-\hat{\mathbf{q}}}{2} = \vec{\mathbf{V}} = \sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}$$

$$\vec{\mathbf{U}}\cdot\vec{\mathbf{V}} = 0 \qquad |\vec{\mathbf{U}}| \neq 1 \qquad |\vec{\mathbf{V}}| \neq 1$$

$$\vec{\mathbf{U}} = \frac{\hat{\mathbf{p}}+\hat{\mathbf{q}}}{2} \qquad |\vec{\mathbf{U}}|^2 = \left[\cos\left(\frac{\vartheta}{2}\right)\right]^2$$

$$\vec{\mathbf{V}} = \frac{\hat{\mathbf{p}}-\hat{\mathbf{q}}}{2} \qquad |\vec{\mathbf{V}}|^2 = \left[\sin\left(\frac{\vartheta}{2}\right)\right]^2$$

$$\hat{\mathbf{u}} = \frac{\vec{\mathbf{U}}}{|\vec{\mathbf{U}}|} = \frac{\hat{\mathbf{p}}+\hat{\mathbf{q}}}{2}\frac{1}{\cos\left(\frac{\vartheta}{2}\right)}$$

$$\vec{\mathbf{U}} = \frac{\hat{\mathbf{p}}+\hat{\mathbf{q}}}{2} = \cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}}$$

$$\hat{\mathbf{v}} = \frac{\vec{\mathbf{V}}}{|\vec{\mathbf{V}}|} = \frac{\hat{\mathbf{p}}-\hat{\mathbf{q}}}{2}\frac{1}{\sin\left(\frac{\vartheta}{2}\right)}$$

$$\vec{\mathbf{V}} = \frac{\hat{\mathbf{p}}-\hat{\mathbf{q}}}{2} = \sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}$$

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x \cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y \cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

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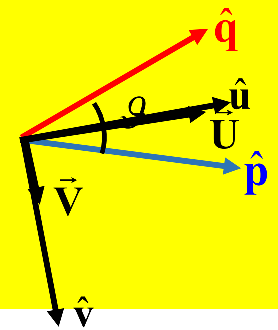
$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$|\vec{\mathbf{F}}| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\begin{cases} F_p = |\vec{\mathbf{F}}|\cos\phi \\ F_q = |\vec{\mathbf{F}}|\sin\phi \end{cases}$$

Polarization plane



$$\hat{\mathbf{u}} \cdot \hat{\mathbf{v}} = 0 \qquad |\hat{\mathbf{u}}| = 1 \qquad |\hat{\mathbf{v}}| = 1$$

$$\vec{\mathbf{F}} = |\vec{\mathbf{F}}| e^{j\phi}\underbrace{\left(\frac{\hat{\mathbf{p}} + \hat{\mathbf{q}}}{2}\right)}_{\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}}} + |\vec{\mathbf{F}}| e^{-j\phi}\underbrace{\left(\frac{\hat{\mathbf{p}} - \hat{\mathbf{q}}}{2}\right)}_{\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}}$$

$$\vec{\mathbf{U}} \cdot \vec{\mathbf{V}} = 0 \qquad |\vec{\mathbf{U}}| \neq 1 \qquad |\vec{\mathbf{V}}| \neq 1$$

$$\vec{\mathbf{U}} = \frac{\hat{\mathbf{p}} + \hat{\mathbf{q}}}{2} \qquad |\vec{\mathbf{U}}|^2 = \left[\cos\left(\frac{\vartheta}{2}\right)\right]^2$$

$$\vec{\mathbf{V}} = \frac{\hat{\mathbf{p}} - \hat{\mathbf{q}}}{2} \qquad |\vec{\mathbf{V}}|^2 = \left[\sin\left(\frac{\vartheta}{2}\right)\right]^2$$

$$\hat{\mathbf{u}} = \frac{\vec{\mathbf{U}}}{|\vec{\mathbf{U}}|} = \frac{\hat{\mathbf{p}} + \hat{\mathbf{q}}}{2}\,\frac{1}{\cos\left(\frac{\vartheta}{2}\right)}$$

$$\vec{\mathbf{U}} = \frac{\hat{\mathbf{p}} + \hat{\mathbf{q}}}{2} = \cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}}$$

$$\hat{\mathbf{v}} = \frac{\vec{\mathbf{V}}}{|\vec{\mathbf{V}}|} = \frac{\hat{\mathbf{p}} - \hat{\mathbf{q}}}{2}\,\frac{1}{\sin\left(\frac{\vartheta}{2}\right)}$$

$$\vec{\mathbf{V}} = \frac{\hat{\mathbf{p}} - \hat{\mathbf{q}}}{2} = \sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}$$

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

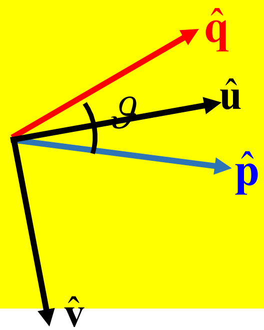
$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\boxed{\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

Polarization plane



$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

$$\left|\hat{\mathbf{u}}\right| = 1$$

$$\left|\hat{\mathbf{v}}\right| = 1$$

$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\underbrace{\left(\frac{\hat{\mathbf{p}} + \hat{\mathbf{q}}}{2}\right)}_{\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\underbrace{\left(\frac{\hat{\mathbf{p}} - \hat{\mathbf{q}}}{2}\right)}_{\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}}$$

$$\vec{\mathbf{U}}\cdot\vec{\mathbf{V}} = 0 \qquad \left|\vec{\mathbf{U}}\right| \neq 1 \qquad \left|\vec{\mathbf{V}}\right| \neq 1$$

$$\vec{\mathbf{U}} = \frac{\hat{\mathbf{p}} + \hat{\mathbf{q}}}{2} \qquad \left|\vec{\mathbf{U}}\right|^2 = \left[\cos\left(\frac{\vartheta}{2}\right)\right]^2$$

$$\vec{\mathbf{V}} = \frac{\hat{\mathbf{p}} - \hat{\mathbf{q}}}{2} \qquad \left|\vec{\mathbf{V}}\right|^2 = \left[\sin\left(\frac{\vartheta}{2}\right)\right]^2$$

$$\boxed{\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\vec{\mathbf{v}}}$$

Phasors and vector functions

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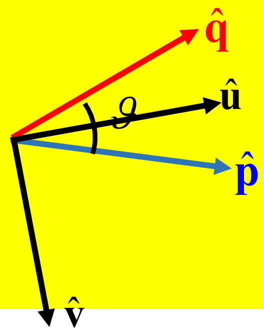
$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

Polarization plane



$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

$$\left|\hat{\mathbf{u}}\right| = 1$$

$$\left|\hat{\mathbf{v}}\right| = 1$$

$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\vec{\mathbf{v}}$$

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

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$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

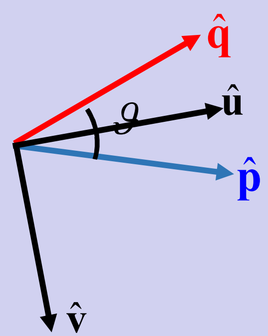
$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

Polarization plane



Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\vec{\mathbf{f}}(t) = F_p\cos(\omega_0 t)\hat{\mathbf{p}} - F_q\sin(\omega_0 t)\hat{\mathbf{q}}$$

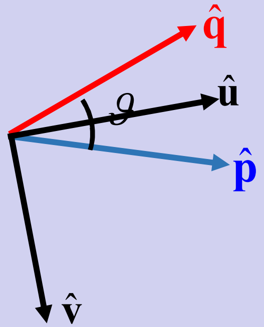
$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

Polarization plane



$$\vec{\mathbf{f}}(t) = \mathrm{Re}\left\{\vec{\mathbf{F}}e^{j\omega_0 t}\right\} = \mathrm{Re}\left\{\left[F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}\right]e^{j\omega_0 t}\right\}$$

$$= \mathrm{Re}\left\{\left[F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}\right]\left[\cos(\omega_0 t) + j\sin(\omega_0 t)\right]\right\}$$

$$= \mathrm{Re}\left\{F_p\cos(\omega_0 t)\hat{\mathbf{p}} - F_q\sin(\omega_0 t)\hat{\mathbf{q}} + jF_p\sin(\omega_0 t)\hat{\mathbf{p}} + jF_q\cos(\omega_0 t)\hat{\mathbf{q}}\right\}$$

$$= F_p\cos(\omega_0 t)\hat{\mathbf{p}} - F_q\sin(\omega_0 t)\hat{\mathbf{q}}$$

$$e^{j\omega_0 t} = \cos(\omega_0 t) + j\sin(\omega_0 t)$$

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\vec{\mathbf{f}}(t) = F_p\cos(\omega_0 t)\hat{\mathbf{p}} - F_q\sin(\omega_0 t)\hat{\mathbf{q}}$$

$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}$$

$$e^{j\omega_0 t}e^{j\phi} = \cos(\omega_0 t + \phi) + j\sin(\omega_0 t + \phi)$$

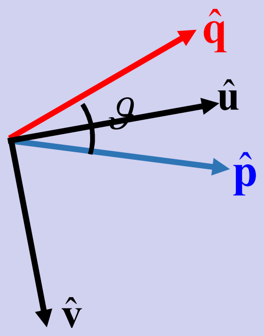
$$e^{j\omega_0 t}e^{-j\phi} = \cos(\omega_0 t - \phi) + j\sin(\omega_0 t - \phi)$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

Polarization plane



$$\vec{\mathbf{f}}(t) = \mathrm{Re}\left\{\vec{\mathbf{F}}e^{j\omega_0 t}\right\} = \mathrm{Re}\left\{\left[\left|\vec{\mathbf{F}}\right| e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}\right]e^{j\omega_0 t}\right\}$$

$$= \mathrm{Re}\left\{\left|\vec{\mathbf{F}}\right|\cos\left(\frac{\vartheta}{2}\right)e^{j\omega_0 t}e^{j\phi}\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right|\sin\left(\frac{\vartheta}{2}\right)e^{j\omega_0 t}e^{-j\phi}\hat{\mathbf{v}}\right\}$$

$$= \left|\vec{\mathbf{F}}\right|\cos\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t + \phi)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right|\sin\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t - \phi)\hat{\mathbf{v}}$$

Polarization plane

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\vec{\mathbf{f}}(t) = F_p\cos(\omega_0 t)\hat{\mathbf{p}} - F_q\sin(\omega_0 t)\hat{\mathbf{q}}$$

$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}$$

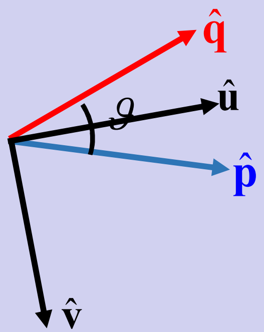
$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right|\cos\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t + \phi)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right|\sin\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t - \phi)\hat{\mathbf{v}}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\hat{\mathbf{u}} \cdot \hat{\mathbf{v}} = 0$$

Polarization plane



The vector $\vec{\mathbf{f}}(t)$ lies in the polarization plane $(\hat{\mathbf{p}}, \hat{\mathbf{q}})$ (which is coincident with the plane $(\hat{\mathbf{u}}, \hat{\mathbf{v}})$), and it changes in general both its amplitude and its direction as the time elapses.

Two cases are of particular interest:

- **Linear polarization**
- **Circular polarization**

Polarization plane

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\vec{\mathbf{f}}(t) = F_p\cos(\omega_0 t)\hat{\mathbf{p}} - F_q\sin(\omega_0 t)\hat{\mathbf{q}}$$

$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}$$

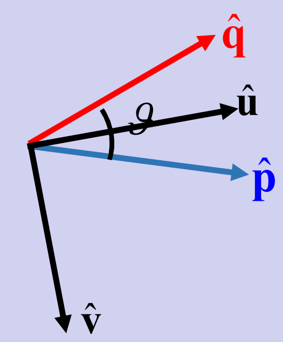
$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right|\cos\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t + \phi)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right|\sin\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t - \phi)\hat{\mathbf{v}}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

Polarization plane



The vector $\vec{\mathbf{f}}(t)$ lies in the polarization plane $(\hat{\mathbf{p}},\hat{\mathbf{q}})$ (which is coincident with the plane $(\hat{\mathbf{u}},\hat{\mathbf{v}})$), and it changes in general both its amplitude and its direction as the time elapses.

Two cases are of particular interest:

- **Linear polarization**
- Circular polarization

Linear Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x \cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y \cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z \cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\vec{\mathbf{f}}(t) = F_p\cos(\omega_0 t)\hat{\mathbf{p}} - F_q\sin(\omega_0 t)\hat{\mathbf{q}}$$

$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}$$

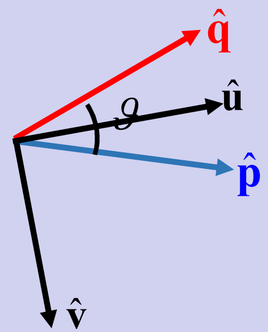
$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right|\cos\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t + \phi)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right|\sin\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t - \phi)\hat{\mathbf{v}}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

Polarization plane



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line:

Linear Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\vec{\mathbf{f}}(t) = F_p\cos(\omega_0 t)\hat{\mathbf{p}} - F_q\sin(\omega_0 t)\hat{\mathbf{q}}$$

$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}$$

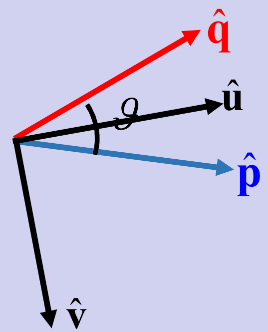
$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right|\cos\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t + \phi)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right|\sin\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t - \phi)\hat{\mathbf{v}}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

Polarization plane



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line:

- $\hat{\mathbf{p}} = \hat{\mathbf{q}}$

Linear Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\vec{\mathbf{f}}(t) = F_p\cos(\omega_0 t)\hat{\mathbf{p}} - F_q\sin(\omega_0 t)\hat{\mathbf{q}}$$

$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}$$

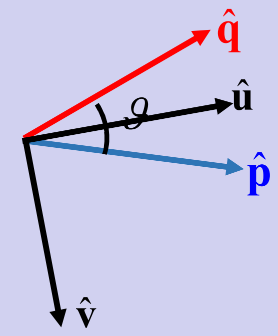
$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right|\cos\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t + \phi)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right|\sin\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t - \phi)\hat{\mathbf{v}}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\ 2} + F_q^{\ 2}}$$

$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

Polarization plane



$$\hat{\mathbf{p}} = \hat{\mathbf{q}}$$

$$\vartheta = 0 \Rightarrow \cos\left(\frac{\vartheta}{2}\right) = 1 \quad \sin\left(\frac{\vartheta}{2}\right) = 0$$

$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right|\cos(\omega_0 t + \phi)\hat{\mathbf{u}}$$

$$\frac{\hat{\mathbf{p}} + \hat{\mathbf{q}}}{2} = \cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}}$$

$$\frac{\hat{\mathbf{p}} - \hat{\mathbf{q}}}{2} = \sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}$$

$$\hat{\mathbf{u}} = \hat{\mathbf{p}} = \hat{\mathbf{q}}$$

Linear Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\vec{\mathbf{f}}(t) = F_p\cos(\omega_0 t)\hat{\mathbf{p}} - F_q\sin(\omega_0 t)\hat{\mathbf{q}}$$

$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}$$

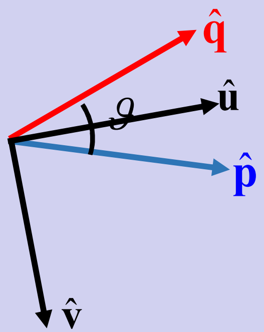
$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right|\cos\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t + \phi)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right|\sin\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t - \phi)\hat{\mathbf{v}}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

Polarization plane



$$\hat{\mathbf{p}} = \hat{\mathbf{q}}$$

$$\vartheta = 0 \Rightarrow \cos\left(\frac{\vartheta}{2}\right) = 1 \qquad \sin\left(\frac{\vartheta}{2}\right) = 0$$

$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right|\cos(\omega_0 t + \phi)\hat{\mathbf{u}}$$

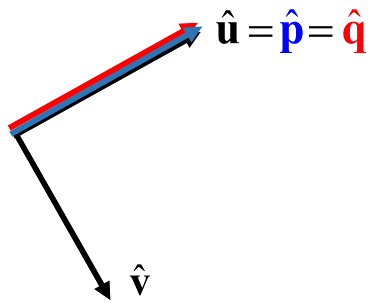
$$\frac{\hat{\mathbf{p}} + \hat{\mathbf{q}}}{2} = \cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}}$$

$$\frac{\hat{\mathbf{p}} - \hat{\mathbf{q}}}{2} = \sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}$$

$$\hat{\mathbf{u}} = \hat{\mathbf{p}} = \hat{\mathbf{q}}$$

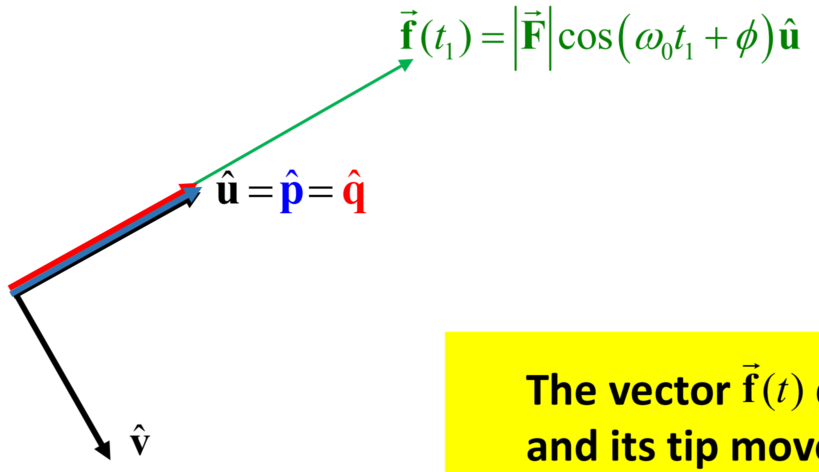
Linear Polarization

$$\vec{\mathbf{f}}(t)=\left|\vec{\mathbf{F}}\right|\cos\left(\omega_0 t+\phi\right)\hat{\mathbf{u}}$$



Linear Polarization

$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right| \cos\left(\omega_0 t + \phi\right)\hat{\mathbf{u}}$$

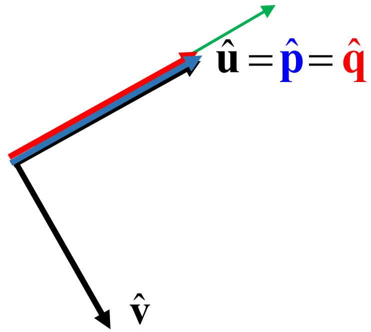


The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line

Linear Polarization

$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right| \cos\left(\omega_0 t + \phi\right)\hat{\mathbf{u}}$$

$$\vec{\mathbf{f}}(t_2) = \left|\vec{\mathbf{F}}\right| \cos\left(\omega_0 t_2 + \phi\right)\hat{\mathbf{u}}$$

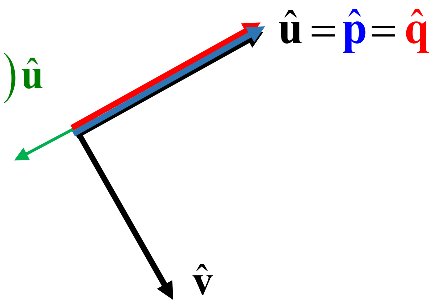


The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line

Linear Polarization

$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right| \cos\left(\omega_0 t + \phi\right)\hat{\mathbf{u}}$$

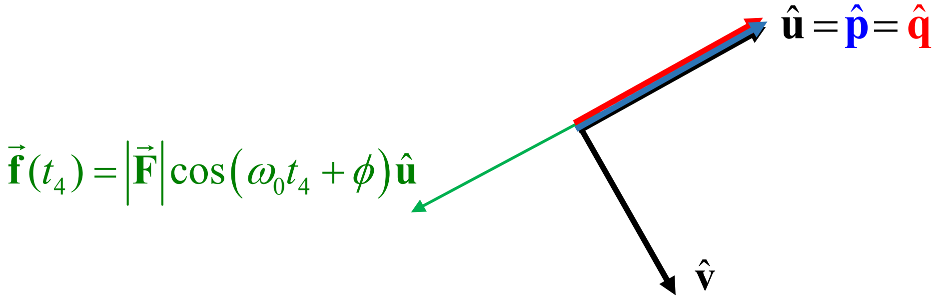
$$\vec{\mathbf{f}}(t_3) = \left|\vec{\mathbf{F}}\right| \cos\left(\omega_0 t_3 + \phi\right)\hat{\mathbf{u}}$$



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line

Linear Polarization

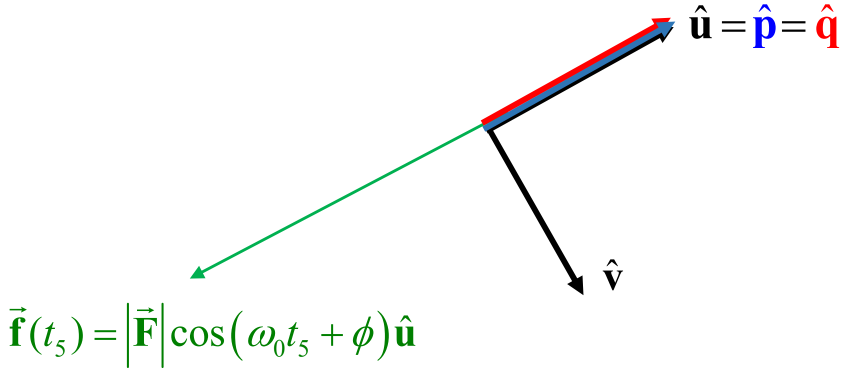
$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right| \cos\left(\omega_0 t + \phi\right) \hat{\mathbf{u}}$$



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line

Linear Polarization

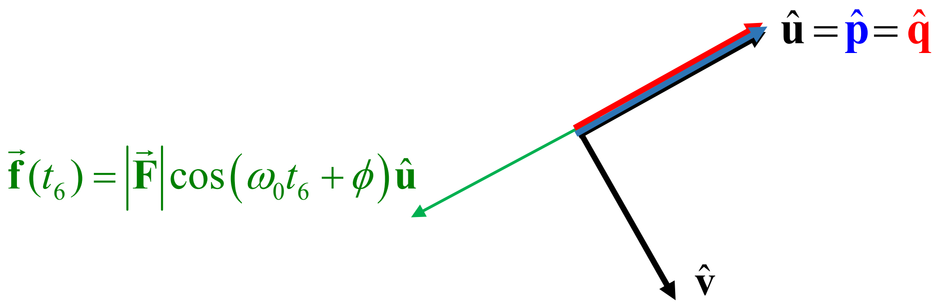
$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right| \cos\left(\omega_0 t + \phi\right) \hat{\mathbf{u}}$$



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line

Linear Polarization

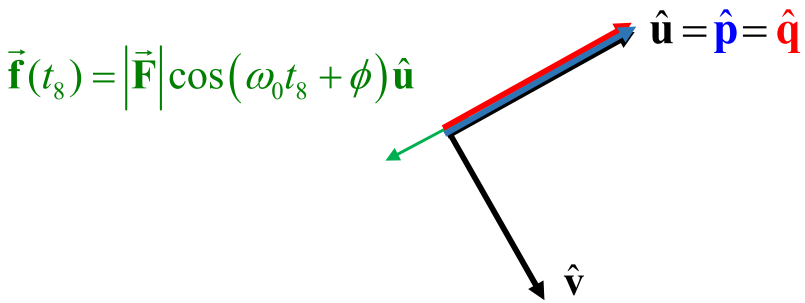
$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right| \cos\left(\omega_0 t + \phi\right) \hat{\mathbf{u}}$$



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line

Linear Polarization

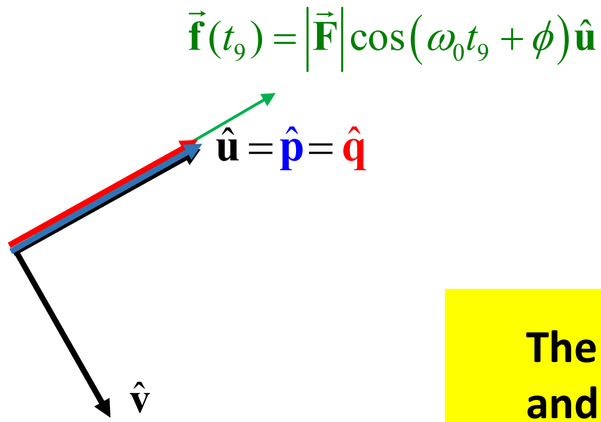
$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right| \cos\left(\omega_0 t + \phi\right) \hat{\mathbf{u}}$$



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line

Linear Polarization

$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right| \cos\left(\omega_0 t + \phi\right) \hat{\mathbf{u}}$$



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line

Linear Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\vec{\mathbf{f}}(t) = F_p\cos(\omega_0 t)\hat{\mathbf{p}} - F_q\sin(\omega_0 t)\hat{\mathbf{q}}$$

$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}$$

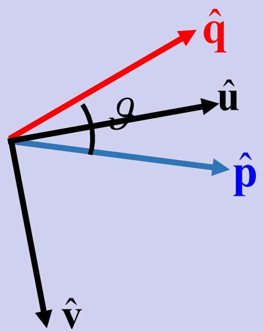
$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right|\cos\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t + \phi)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right|\sin\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t - \phi)\hat{\mathbf{v}}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

Polarization plane



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line:

- $\hat{\mathbf{p}} = \hat{\mathbf{q}}$ $\quad\Longrightarrow\quad$ $\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right|\cos(\omega_0 t + \phi)\hat{\mathbf{u}}$
- $\hat{\mathbf{p}} = -\hat{\mathbf{q}}$

Linear Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

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$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\vec{\mathbf{f}}(t) = F_p\cos(\omega_0 t)\hat{\mathbf{p}} - F_q\sin(\omega_0 t)\hat{\mathbf{q}}$$

$$\vec{\mathbf{F}} = |\vec{\mathbf{F}}|e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + |\vec{\mathbf{F}}|e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}$$

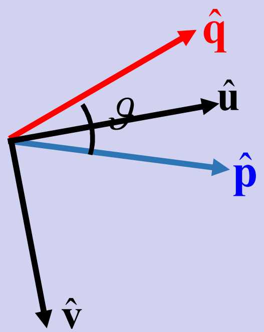
$$\vec{\mathbf{f}}(t) = |\vec{\mathbf{F}}|\cos\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t + \phi)\hat{\mathbf{u}} + |\vec{\mathbf{F}}|\sin\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t - \phi)\hat{\mathbf{v}}$$

$$\begin{cases} F_p = |\vec{\mathbf{F}}|\cos\phi \\ F_q = |\vec{\mathbf{F}}|\sin\phi \end{cases}$$

$$|\vec{\mathbf{F}}| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

Polarization plane



$$\hat{\mathbf{p}} = -\hat{\mathbf{q}}$$

$$\vartheta = \pi \Rightarrow \cos\left(\frac{\vartheta}{2}\right) = 0 \quad \sin\left(\frac{\vartheta}{2}\right) = 1$$

$$\vec{\mathbf{f}}(t) = |\vec{\mathbf{F}}|\cos(\omega_0 t - \phi)\hat{\mathbf{v}}$$

$$\frac{\hat{\mathbf{p}} + \hat{\mathbf{q}}}{2} = \cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}}$$

$$\frac{\hat{\mathbf{p}} - \hat{\mathbf{q}}}{2} = \sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}$$

$$\hat{\mathbf{v}} = \hat{\mathbf{p}} = -\hat{\mathbf{q}}$$

Linear Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x \cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y \cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z \cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\vec{\mathbf{f}}(t) = F_p\cos(\omega_0 t)\hat{\mathbf{p}} - F_q\sin(\omega_0 t)\hat{\mathbf{q}}$$

$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}$$

$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right|\cos\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t + \phi)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right|\sin\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t - \phi)\hat{\mathbf{v}}$$

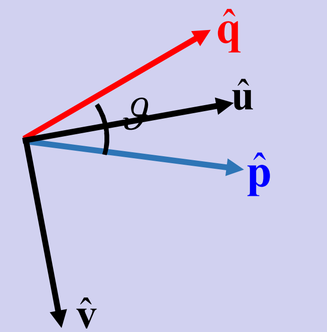
$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$\hat{\mathbf{q}}$

$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

Polarization plane



$$\hat{\mathbf{p}} = -\hat{\mathbf{q}}$$

$$\Downarrow$$

$$\vartheta = \pi \Rightarrow \cos\left(\frac{\vartheta}{2}\right) = 0 \qquad \sin\left(\frac{\vartheta}{2}\right) = 1$$

$$\Downarrow$$

$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right|\cos(\omega_0 t - \phi)\hat{\mathbf{v}}$$

$$\frac{\hat{\mathbf{p}} + \hat{\mathbf{q}}}{2} = \cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}}$$

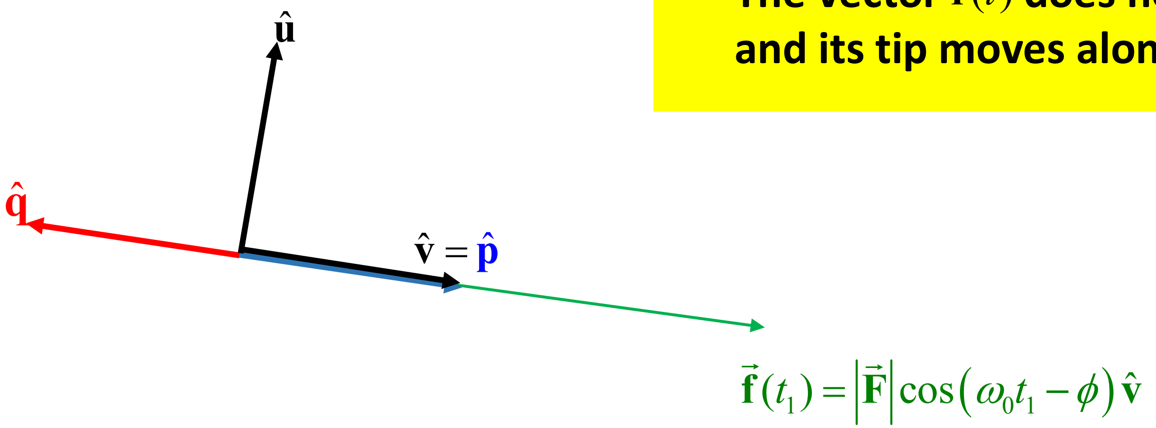
$$\frac{\hat{\mathbf{p}} - \hat{\mathbf{q}}}{2} = \sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}$$

$$\hat{\mathbf{v}} = \hat{\mathbf{p}} = -\hat{\mathbf{q}}$$

Linear Polarization

$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right| \cos\left(\omega_0 t - \phi\right) \hat{\mathbf{v}}$$

The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line



Linear Polarization

$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right| \cos\left(\omega_0 t - \phi\right) \hat{\mathbf{v}}$$

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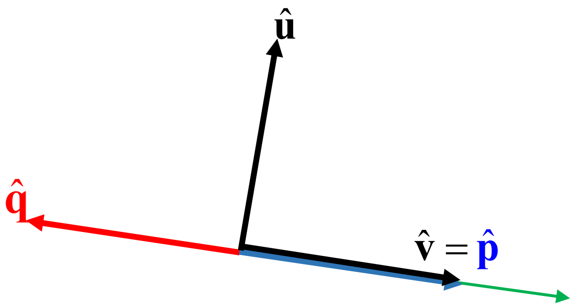
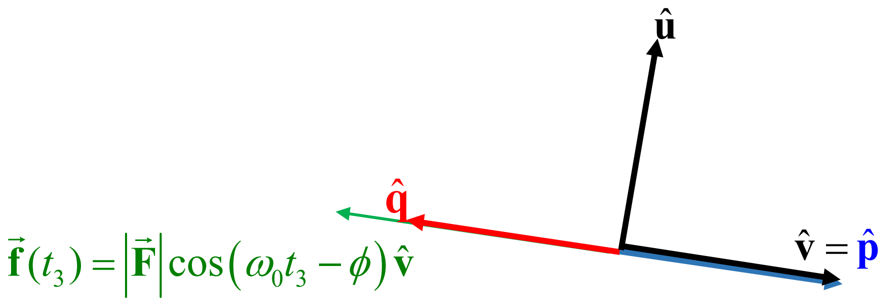


$$\vec{\mathbf{f}}(t_2) = \left|\vec{\mathbf{F}}\right| \cos\left(\omega_0 t_2 - \phi\right) \hat{\mathbf{v}}$$

Linear Polarization

$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right| \cos\left(\omega_0 t - \phi\right) \hat{\mathbf{v}}$$



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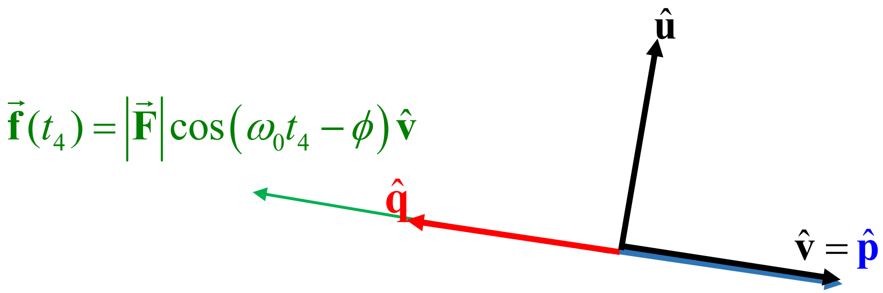

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Linear Polarization

$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right| \cos\left(\omega_0 t - \phi\right) \hat{\mathbf{v}}$$

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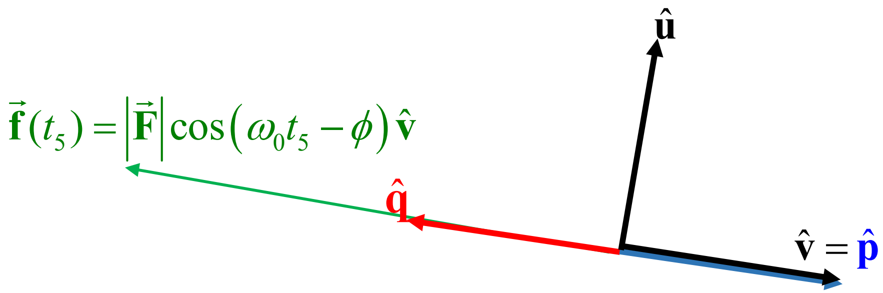
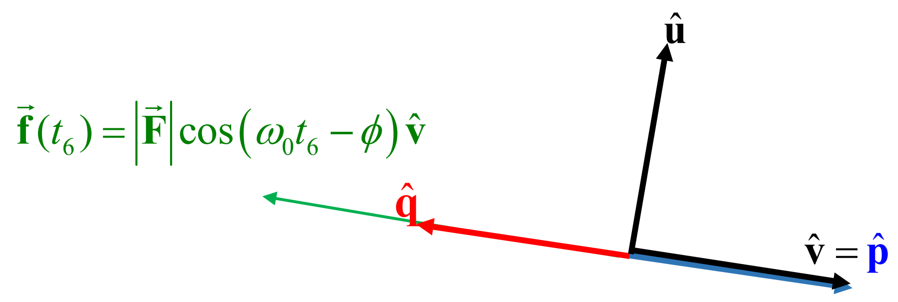


Linear Polarization

$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right| \cos\left(\omega_0 t - \phi\right) \hat{\mathbf{v}}$$



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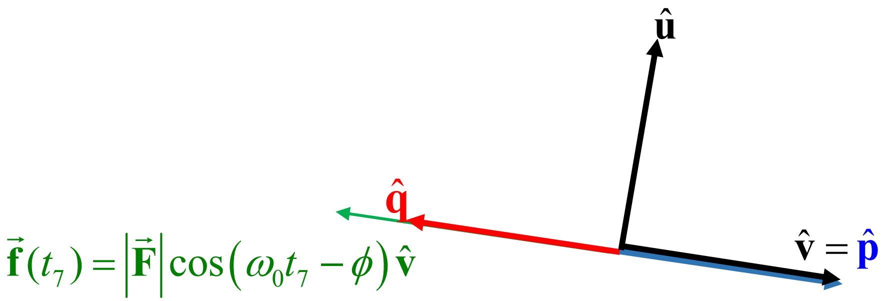

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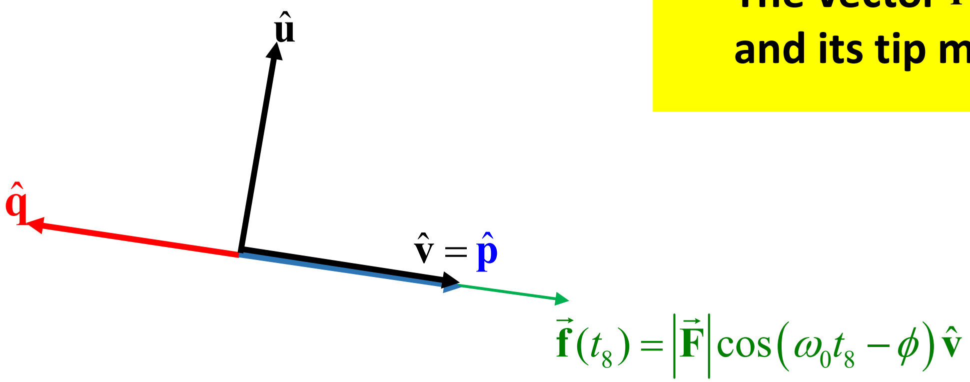

Linear Polarization

$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right| \cos\left(\omega_0 t - \phi\right) \hat{\mathbf{v}}$$

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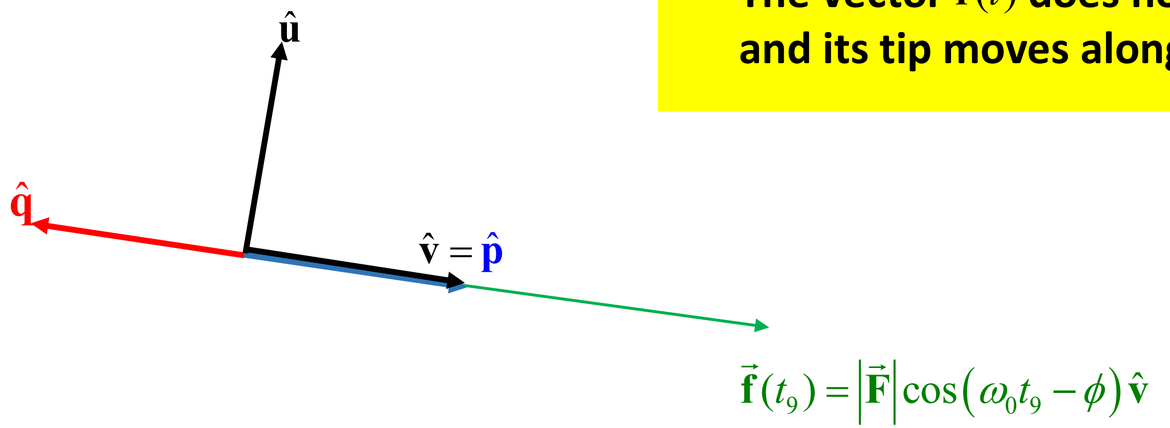

Linear Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

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$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\vec{\mathbf{f}}(t) = F_p\cos(\omega_0 t)\hat{\mathbf{p}} - F_q\sin(\omega_0 t)\hat{\mathbf{q}}$$

$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}$$

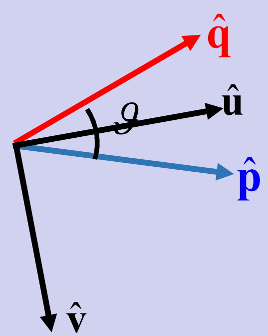
$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right|\cos\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t + \phi)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right|\sin\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t - \phi)\hat{\mathbf{v}}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

Polarization plane



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line:

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- $\hat{\mathbf{p}} = -\hat{\mathbf{q}}$ ➡ $\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right|\cos(\omega_0 t - \phi)\hat{\mathbf{v}}$
- $F_p = 0$ and $F_q \neq 0$

Linear Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

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$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\vec{\mathbf{f}}(t) = F_p\cos(\omega_0 t)\hat{\mathbf{p}} - F_q\sin(\omega_0 t)\hat{\mathbf{q}}$$

$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}$$

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$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

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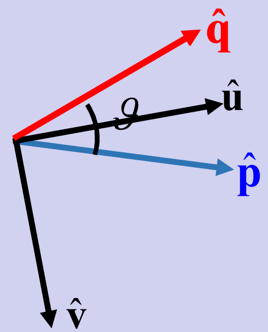
$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

Polarization plane



$F_p = 0$ and $F_q \neq 0$ ➡ $\vec{\mathbf{f}}(t) = -F_q\sin(\omega_0 t)\hat{\mathbf{q}}$

Linear Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

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$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\vec{\mathbf{f}}(t) = F_p\cos(\omega_0 t)\hat{\mathbf{p}} - F_q\sin(\omega_0 t)\hat{\mathbf{q}}$$

$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}$$

$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right|\cos\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t + \phi)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right|\sin\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t - \phi)\hat{\mathbf{v}}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

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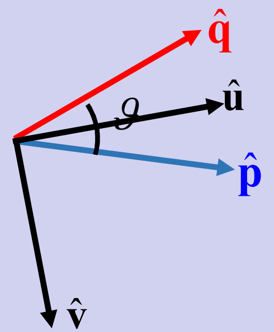
Polarization plane



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- $\hat{\mathbf{p}} = -\hat{\mathbf{q}}$ ➡ $\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right|\cos(\omega_0 t - \phi)\hat{\mathbf{v}}$
- $F_p = 0$ and $F_q \neq 0$ ➡ $\vec{\mathbf{f}}(t) = -F_q\sin(\omega_0 t)\hat{\mathbf{q}}$
- $F_p \neq 0$ and $F_q = 0$ ➡ $\vec{\mathbf{f}}(t) = F_p\cos(\omega_0 t)\hat{\mathbf{p}}$