Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2020-2021 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

Stefano Perna

Phasors and vector functions

$$\text{Time domain} \longrightarrow \text{Phasor domain}$$

Time domain

$$\vec{\mathbf{f}}(x,y,z,t) = f_x(x,y,z,t)\hat{i}_x + f_y(x,y,z,t)\hat{i}_y + f_z(x,y,z,t)\,\hat{i}_z$$

Phasor domain

$$\vec{\mathbf{F}}(x,y,z) = F_x(x,y,z)\hat{i}_x + F_y(x,y,z)\hat{i}_y + F_z(x,y,z)\hat{i}_z$$

$$f_x(x,y,z,t) = A_x(x,y,z)\cos\left(\omega_0 t + \alpha_x(x,y,z)\right) \longrightarrow \boxed{\text{T-to-P}} \longrightarrow F_x(x,y,z) = A_x(x,y,z)e^{j\alpha_x(x,y,z)}$$

$$f_y(x,y,z,t) = A_y(x,y,z)\cos\left(\omega_0 t + \alpha_y(x,y,z)\right) \longrightarrow \boxed{\text{T-to-P}} \longrightarrow F_y(x,y,z) = A_y(x,y,z)e^{j\alpha_y(x,y,z)}$$

$$f_z(x,y,z,t) = A_z(x,y,z)\cos\left(\omega_0 t + \alpha_z(x,y,z)\right) \longrightarrow \boxed{\text{T-to-P}} \longrightarrow F_z(x,y,z) = A_z(x,y,z)e^{j\alpha_z(x,y,z)}$$

$$\vec{\mathbf{f}}(x,y,z,t) \longrightarrow \boxed{\text{T-to-P}} \longrightarrow \vec{\mathbf{F}}(x,y,z)$$

$$\vec{\mathbf{F}}(x,y,z) \longrightarrow \boxed{\text{P-to-T}} \longrightarrow \vec{\mathbf{f}}(x,y,z,t) = \mathrm{Re}\left\{\vec{\mathbf{F}}(x,y,z)e^{j\omega_0 t}\right\}$$

Phasors and vector functions

Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Phasor domain

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z$$

$$f_x(t) = A_x\cos\left(\omega_0 t + \alpha_x\right) \longrightarrow \boxed{\text{T-to-P}} \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos\left(\omega_0 t + \alpha_y\right) \longrightarrow \boxed{\text{T-to-P}} \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos\left(\omega_0 t + \alpha_z\right) \longrightarrow \boxed{\text{T-to-P}} \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{f}}(t) \longrightarrow \boxed{\text{T-to-P}} \longrightarrow \vec{\mathbf{F}}$$

$$\vec{\mathbf{F}} \longrightarrow \boxed{\text{P-to-T}} \longrightarrow \vec{\mathbf{f}}(t) = \mathrm{Re}\left\{\vec{\mathbf{F}}e^{j\omega_0 t}\right\}$$

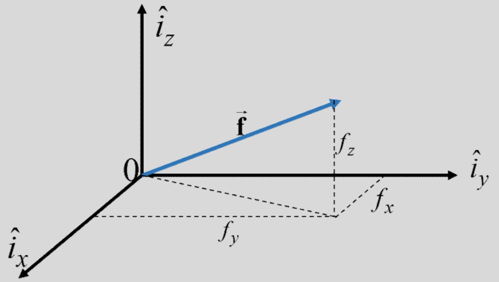
Complex vectors: graphical representation

Real numbers

f

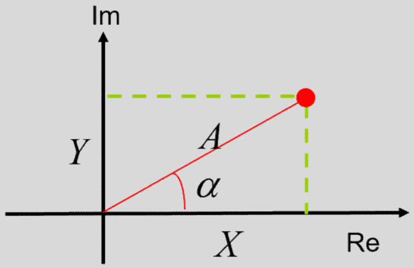
Real vectors (3 components)

$$\vec{\mathbf{f}} = f_x \hat{i}_x + f_y \hat{i}_y + f_z \hat{i}_z$$



Complex numbers

$$F = Ae^{j\alpha} = X + jY$$



Complex vectors (3 components)

$$\vec{\mathbf{F}} = F_x \hat{i}_x + F_y \hat{i}_y + F_z \hat{i}_z$$

?

Color legend

New formulas, important considerations, important formulas, important concepts

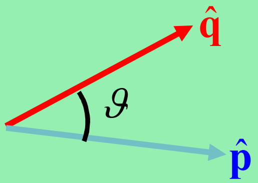
Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Mathematical tools that we will exploit today



$$\begin{cases} \hat{\mathbf{p}}\cdot\hat{\mathbf{p}}=1 \\ \hat{\mathbf{q}}\cdot\hat{\mathbf{q}}=1 \\ \hat{\mathbf{p}}\cdot\hat{\mathbf{q}}=\hat{\mathbf{q}}\cdot\hat{\mathbf{p}}=\cos\vartheta \end{cases}$$

$$\left|\vec{\mathbf{F}}\right|=\sqrt{\vec{\mathbf{F}}\cdot\vec{\mathbf{F}}^{*}}$$

$$\sin\left(\alpha+\frac{\pi}{2}\right)=\cos\alpha\ ;\ \sin\left(\alpha-\frac{\pi}{2}\right)=-\cos\alpha$$

$$\left[\sin\left(\frac{\vartheta}{2}\right)\right]^{2}=\frac{1-\cos(\vartheta)}{2};\left[\cos\left(\frac{\vartheta}{2}\right)\right]^{2}=\frac{1+\cos(\vartheta)}{2}$$

$$\cos\phi=\frac{e^{j\phi}+e^{-j\phi}}{2}$$

$$\sin\phi=\frac{e^{j\phi}-e^{-j\phi}}{2j}$$

$$e^{j\phi}=\cos\phi+j\sin\phi$$

Phasors and vector functions

Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Phasor domain

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z$$

$$f_x(t) = A_x \cos\left(\omega_0 t + \alpha_x\right) \xrightarrow{\text{T-to-P}} F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y \cos\left(\omega_0 t + \alpha_y\right) \xrightarrow{\text{T-to-P}} F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z \cos\left(\omega_0 t + \alpha_z\right) \xrightarrow{\text{T-to-P}} F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = \left(A_x e^{j\alpha_x}\right)\hat{i}_x + \left(A_y e^{j\alpha_y}\right)\hat{i}_y + \left(A_z e^{j\alpha_z}\right)\hat{i}_z = \left(R_x + jI_x\right)\hat{i}_x + \left(R_y + jI_y\right)\hat{i}_y + \left(R_z + jI_z\right)\hat{i}_z$$

$$= \underbrace{\left[R_x\hat{i}_x + R_y\hat{i}_y + R_z\hat{i}_z\right]}_{F_p\hat{\mathbf{p}}} + j\underbrace{\left[I_x\hat{i}_x + I_y\hat{i}_y + I_z\hat{i}_z\right]}_{F_q\hat{\mathbf{q}}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

F_p and F_q are real!

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\,\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

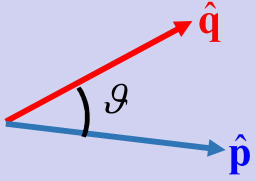
$$f_x(t) = A_x\cos\left(\omega_0\, t + \alpha_x\right) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos\left(\omega_0\, t + \alpha_y\right) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos\left(\omega_0\, t + \alpha_z\right) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

Polarization plane



Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

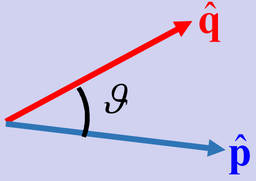
$$f_x(t) = A_x\cos\left(\omega_0 t + \alpha_x\right) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos\left(\omega_0 t + \alpha_y\right) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos\left(\omega_0 t + \alpha_z\right) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

Polarization plane



$$\begin{cases} \hat{\mathbf{p}}\cdot\hat{\mathbf{p}} = 1 \\ \hat{\mathbf{q}}\cdot\hat{\mathbf{q}} = 1 \\ \hat{\mathbf{p}}\cdot\hat{\mathbf{q}} = \hat{\mathbf{q}}\cdot\hat{\mathbf{p}} = \cos\vartheta \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{\vec{\mathbf{F}}\cdot\vec{\mathbf{F}}^*} = \sqrt{\left(F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}\right)\cdot\left(F_p\hat{\mathbf{p}} - jF_q\hat{\mathbf{q}}\right)} = \sqrt{\left(F_p^2\hat{\mathbf{p}}\cdot\hat{\mathbf{p}} - jF_pF_q\hat{\mathbf{p}}\cdot\hat{\mathbf{q}} + jF_qF_p\hat{\mathbf{q}}\cdot\hat{\mathbf{p}} + F_q^2\hat{\mathbf{q}}\cdot\hat{\mathbf{q}}\right)} = \sqrt{F_p^{\ 2} + F_q^{\ 2}}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{\vec{\mathbf{F}}\cdot\vec{\mathbf{F}}^*} = \sqrt{F_xF_x^* + F_yF_y^* + F_zF_z^*} = \sqrt{\left|F_x\right|^2 + \left|F_y\right|^2 + \left|F_z\right|^2}$$

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x \cos\left(\omega_0 t + \alpha_x\right) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

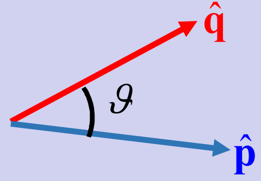
$$f_y(t) = A_y \cos\left(\omega_0 t + \alpha_y\right) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z \cos\left(\omega_0 t + \alpha_z\right) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p{}^2 + F_q{}^2}$$

Polarization plane



$$C_1 = X + jY = \left|C_1\right| e^{j\alpha}$$

$$\left|C_1\right| = \sqrt{X^2 + Y^2}$$

$$\begin{cases} \mathrm{Re}\left\{C_1\right\} = X = \left|C_1\right| \cos\alpha \\ \mathrm{Im}\left\{C_1\right\} = Y = \left|C_1\right| \sin\alpha \end{cases}$$

$$C_2 = F_p + jF_q = \left|C_2\right| e^{j\phi}$$

$$\left|C_2\right| = \sqrt{F_p{}^2 + F_q{}^2}$$

$$\begin{cases} \mathrm{Re}\left\{C_2\right\} = F_p = \left|C_2\right| \cos\phi \\ \mathrm{Im}\left\{C_2\right\} = F_q = \left|C_2\right| \sin\phi \end{cases}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right| \cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right| \sin\phi \end{cases}$$

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos\left(\omega_0 t + \alpha_x\right) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos\left(\omega_0 t + \alpha_y\right) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

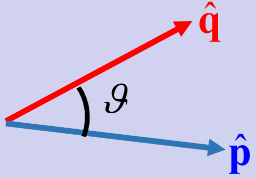
$$f_z(t) = A_z\cos\left(\omega_0 t + \alpha_z\right) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\ 2} + F_q^{\ 2}}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

Polarization plane



$$C_2 = F_p + jF_q = \left|C_2\right|e^{j\phi}$$

$$\left|C_2\right| = \sqrt{F_p^{\ 2} + F_q^{\ 2}}$$

$$\begin{cases} \mathrm{Re}\left\{C_2\right\} = F_p = \left|C_2\right|\cos\phi \\ \mathrm{Im}\left\{C_2\right\} = F_q = \left|C_2\right|\sin\phi \end{cases}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

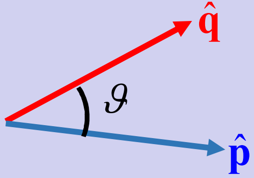
$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

Polarization plane



$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\left(\frac{\hat{\mathbf{p}} + \hat{\mathbf{q}}}{2}\right) + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\left(\frac{\hat{\mathbf{p}} - \hat{\mathbf{q}}}{2}\right)$$

$$\cos\phi = \frac{e^{j\phi} + e^{-j\phi}}{2}$$

$$\sin\phi = \frac{e^{j\phi} - e^{-j\phi}}{2j}$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}} \ = \left|\vec{\mathbf{F}}\right|\cos\phi\hat{\mathbf{p}} + j\left|\vec{\mathbf{F}}\right|\sin\phi\hat{\mathbf{q}}$$

$$= \left|\vec{\mathbf{F}}\right|\left[\frac{e^{j\phi} + e^{-j\phi}}{2}\right]\hat{\mathbf{p}} + j\left|\vec{\mathbf{F}}\right|\left[\frac{e^{j\phi} - e^{-j\phi}}{2j}\right]\hat{\mathbf{q}} \ = \left|\vec{\mathbf{F}}\right| e^{j\phi}\left(\frac{\hat{\mathbf{p}} + \hat{\mathbf{q}}}{2}\right) + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\left(\frac{\hat{\mathbf{p}} - \hat{\mathbf{q}}}{2}\right)$$

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos\left(\omega_0 t + \alpha_x\right) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos\left(\omega_0 t + \alpha_y\right) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

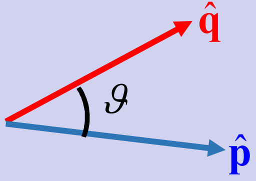
$$f_z(t) = A_z\cos\left(\omega_0 t + \alpha_z\right) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

Polarization plane



$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\left(\frac{\hat{\mathbf{p}} + \hat{\mathbf{q}}}{2}\right) + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\left(\frac{\hat{\mathbf{p}} - \hat{\mathbf{q}}}{2}\right)$$

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos\left(\omega_0 t + \alpha_x\right) \xrightarrow{\qquad} F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos\left(\omega_0 t + \alpha_y\right) \xrightarrow{\qquad} F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

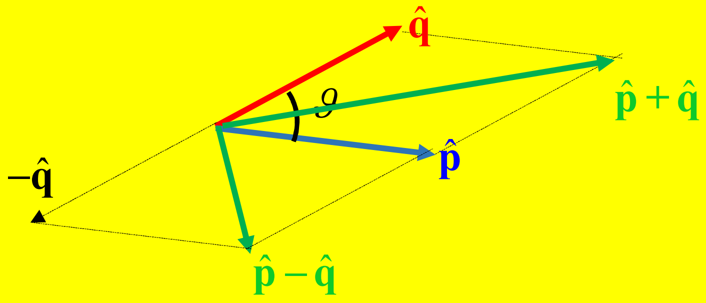
$$f_z(t) = A_z\cos\left(\omega_0 t + \alpha_z\right) \xrightarrow{\qquad} F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p{}^2 + F_q{}^2}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

Polarization plane



$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\underbrace{\left(\frac{\hat{\mathbf{p}}+\hat{\mathbf{q}}}{2}\right)}_{\vec{\mathbf{U}}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\underbrace{\left(\frac{\hat{\mathbf{p}}-\hat{\mathbf{q}}}{2}\right)}_{\vec{\mathbf{V}}}$$

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos\left(\omega_0 t + \alpha_x\right) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos\left(\omega_0 t + \alpha_y\right) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

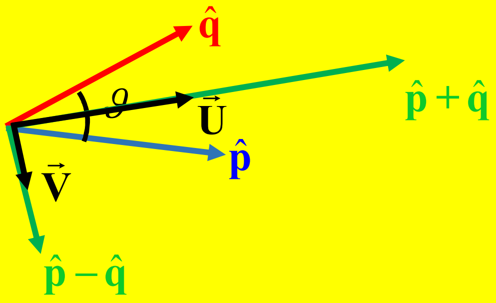
$$f_z(t) = A_z\cos\left(\omega_0 t + \alpha_z\right) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\ 2} + F_q^{\ 2}}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

Polarization plane



$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\underbrace{\left(\frac{\hat{\mathbf{p}} + \hat{\mathbf{q}}}{2}\right)}_{\vec{\mathbf{U}}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\underbrace{\left(\frac{\hat{\mathbf{p}} - \hat{\mathbf{q}}}{2}\right)}_{\vec{\mathbf{V}}}$$

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

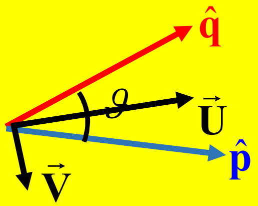
$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p{}^2 + F_q{}^2}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

Polarization plane



$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\underbrace{\left(\frac{\hat{\mathbf{p}} + \hat{\mathbf{q}}}{2}\right)}_{\vec{\mathbf{U}}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\underbrace{\left(\frac{\hat{\mathbf{p}} - \hat{\mathbf{q}}}{2}\right)}_{\vec{\mathbf{V}}}$$

$$\vec{\mathbf{U}} \cdot \vec{\mathbf{V}} = 0$$

$$\left|\vec{\mathbf{U}}\right| \neq 1$$

$$\left|\vec{\mathbf{V}}\right| \neq 1$$

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos\left(\omega_0 t + \alpha_x\right) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos\left(\omega_0 t + \alpha_y\right) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

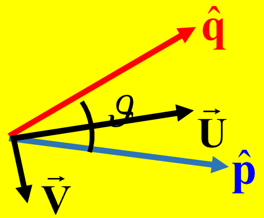
$$f_z(t) = A_z\cos\left(\omega_0 t + \alpha_z\right) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

Polarization plane



$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\underbrace{\left(\frac{\hat{\mathbf{p}} + \hat{\mathbf{q}}}{2}\right)}_{\vec{\mathbf{U}}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\underbrace{\left(\frac{\hat{\mathbf{p}} - \hat{\mathbf{q}}}{2}\right)}_{\vec{\mathbf{V}}}$$

$$\vec{\mathbf{U}}\cdot\vec{\mathbf{V}} = 0 \quad \left|\vec{\mathbf{U}}\right| \neq 1 \quad \left|\vec{\mathbf{V}}\right| \neq 1$$

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos\left(\omega_0 t + \alpha_x\right) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos\left(\omega_0 t + \alpha_y\right) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

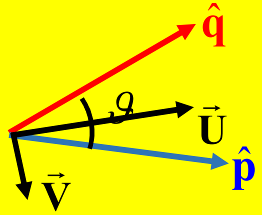
$$f_z(t) = A_z\cos\left(\omega_0 t + \alpha_z\right) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

Polarization plane



$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\underbrace{\left(\frac{\hat{\mathbf{p}} + \hat{\mathbf{q}}}{2}\right)}_{\vec{\mathbf{U}}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\underbrace{\left(\frac{\hat{\mathbf{p}} - \hat{\mathbf{q}}}{2}\right)}_{\vec{\mathbf{V}}}$$

$$\vec{\mathbf{U}}\cdot\vec{\mathbf{V}} = 0 \qquad \left|\vec{\mathbf{U}}\right| \neq 1 \qquad \left|\vec{\mathbf{V}}\right| \neq 1$$

$$\vec{\mathbf{U}} = \frac{\hat{\mathbf{p}} + \hat{\mathbf{q}}}{2}$$

$$\vec{\mathbf{V}} = \frac{\hat{\mathbf{p}} - \hat{\mathbf{q}}}{2}$$

$$\vec{\mathbf{U}}\cdot\vec{\mathbf{V}} = \left(\frac{\hat{\mathbf{p}} + \hat{\mathbf{q}}}{2}\right)\cdot\left(\frac{\hat{\mathbf{p}} - \hat{\mathbf{q}}}{2}\right) = \frac{\hat{\mathbf{p}}\cdot\hat{\mathbf{p}} - \hat{\mathbf{p}}\cdot\hat{\mathbf{q}} + \hat{\mathbf{q}}\cdot\hat{\mathbf{p}} - \hat{\mathbf{q}}\cdot\hat{\mathbf{q}}}{4} = 0$$

$\vec{\mathbf{U}}$ and $\vec{\mathbf{V}}$ are orthogonal

$$\begin{cases} \hat{\mathbf{p}}\cdot\hat{\mathbf{p}} = 1 \\ \hat{\mathbf{q}}\cdot\hat{\mathbf{q}} = 1 \end{cases}$$

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos\left(\omega_0 t + \alpha_x\right) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos\left(\omega_0 t + \alpha_y\right) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos\left(\omega_0 t + \alpha_z\right) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

$$\left[\cos\left(\frac{\vartheta}{2}\right)\right]^2 = \frac{1+\cos(\vartheta)}{2}$$

$$\left[\sin\left(\frac{\vartheta}{2}\right)\right]^2 = \frac{1-\cos(\vartheta)}{2}$$

$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\underbrace{\left(\frac{\hat{\mathbf{p}}+\hat{\mathbf{q}}}{2}\right)}_{\vec{\mathbf{U}}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\underbrace{\left(\frac{\hat{\mathbf{p}}-\hat{\mathbf{q}}}{2}\right)}_{\vec{\mathbf{V}}}$$

$$\vec{\mathbf{U}}\cdot\vec{\mathbf{V}} = 0 \qquad \left|\vec{\mathbf{U}}\right| \neq 1 \qquad \left|\vec{\mathbf{V}}\right| \neq 1$$

$$\vec{\mathbf{U}} = \frac{\hat{\mathbf{p}}+\hat{\mathbf{q}}}{2} \qquad \left|\vec{\mathbf{U}}\right|^2 = \left(\frac{\hat{\mathbf{p}}+\hat{\mathbf{q}}}{2}\right)\cdot\left(\frac{\hat{\mathbf{p}}+\hat{\mathbf{q}}}{2}\right) = \frac{\left(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}} + \hat{\mathbf{p}}\cdot\hat{\mathbf{q}} + \hat{\mathbf{q}}\cdot\hat{\mathbf{p}} + \hat{\mathbf{q}}\cdot\hat{\mathbf{q}}\right)}{4} = \frac{2+2\cos\vartheta}{4} = \frac{1+\cos\vartheta}{2} = \left[\cos\left(\frac{\vartheta}{2}\right)\right]^2$$

$$\vec{\mathbf{V}} = \frac{\hat{\mathbf{p}}-\hat{\mathbf{q}}}{2} \qquad \left|\vec{\mathbf{V}}\right|^2 = \left(\frac{\hat{\mathbf{p}}-\hat{\mathbf{q}}}{2}\right)\cdot\left(\frac{\hat{\mathbf{p}}-\hat{\mathbf{q}}}{2}\right) = \frac{\left(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}} - \hat{\mathbf{p}}\cdot\hat{\mathbf{q}} - \hat{\mathbf{q}}\cdot\hat{\mathbf{p}} + \hat{\mathbf{q}}\cdot\hat{\mathbf{q}}\right)}{4} = \frac{2-2\cos\vartheta}{4} = \frac{1-\cos\vartheta}{2} = \left[\sin\left(\frac{\vartheta}{2}\right)\right]^2$$

$$\begin{cases} \hat{\mathbf{p}}\cdot\hat{\mathbf{p}} = 1 \\ \hat{\mathbf{q}}\cdot\hat{\mathbf{q}} = 1 \\ \hat{\mathbf{p}}\cdot\hat{\mathbf{q}} = \hat{\mathbf{q}}\cdot\hat{\mathbf{p}} = \cos\vartheta \end{cases}$$

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos\left(\omega_0 t + \alpha_x\right) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos\left(\omega_0 t + \alpha_y\right) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

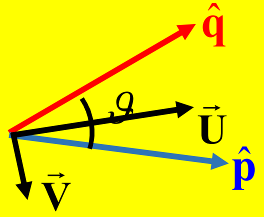
$$f_z(t) = A_z\cos\left(\omega_0 t + \alpha_z\right) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\ 2} + F_q^{\ 2}}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

Polarization plane



$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\underbrace{\left(\frac{\hat{\mathbf{p}}+\hat{\mathbf{q}}}{2}\right)}_{\vec{\mathbf{U}}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\underbrace{\left(\frac{\hat{\mathbf{p}}-\hat{\mathbf{q}}}{2}\right)}_{\vec{\mathbf{V}}}$$

$$\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} \qquad \sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}$$

$$\vec{\mathbf{U}}\cdot\vec{\mathbf{V}} = 0 \qquad \left|\vec{\mathbf{U}}\right| \neq 1 \qquad \left|\vec{\mathbf{V}}\right| \neq 1$$

$$\vec{\mathbf{U}} = \frac{\hat{\mathbf{p}}+\hat{\mathbf{q}}}{2} \qquad \left|\vec{\mathbf{U}}\right|^2 = \left[\cos\left(\frac{\vartheta}{2}\right)\right]^2$$

$$\vec{\mathbf{V}} = \frac{\hat{\mathbf{p}}-\hat{\mathbf{q}}}{2} \qquad \left|\vec{\mathbf{V}}\right|^2 = \left[\sin\left(\frac{\vartheta}{2}\right)\right]^2$$

$$\hat{\mathbf{u}} = \frac{\vec{\mathbf{U}}}{\left|\vec{\mathbf{U}}\right|} = \frac{\hat{\mathbf{p}}+\hat{\mathbf{q}}}{2}\frac{1}{\cos\left(\frac{\vartheta}{2}\right)}$$

$$\boxed{\vec{\mathbf{U}} = \frac{\hat{\mathbf{p}}+\hat{\mathbf{q}}}{2} = \cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}}}$$

$$\hat{\mathbf{v}} = \frac{\vec{\mathbf{V}}}{\left|\vec{\mathbf{V}}\right|} = \frac{\hat{\mathbf{p}}-\hat{\mathbf{q}}}{2}\frac{1}{\sin\left(\frac{\vartheta}{2}\right)}$$

$$\boxed{\vec{\mathbf{V}} = \frac{\hat{\mathbf{p}}-\hat{\mathbf{q}}}{2} = \sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}}$$

Phasors and vector functions

$$\vec{\mathbf{f}}(t)=f_x(t)\hat{i}_x+f_y(t)\hat{i}_y+f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}}=F_x\hat{i}_x+F_y\hat{i}_y+F_z\hat{i}_z=A_xe^{j\alpha_x}\hat{i}_x+A_ye^{j\alpha_y}\hat{i}_y+A_ze^{j\alpha_z}\hat{i}_z$$

$$f_x(t)=A_x\cos\left(\omega_0 t+\alpha_x\right)\longrightarrow F_x=A_xe^{j\alpha_x}=R_x+jI_x$$

$$f_y(t)=A_y\cos\left(\omega_0 t+\alpha_y\right)\longrightarrow F_y=A_ye^{j\alpha_y}=R_y+jI_y$$

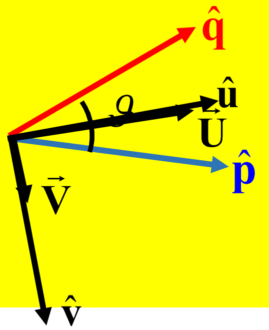
$$f_z(t)=A_z\cos\left(\omega_0 t+\alpha_z\right)\longrightarrow F_z=A_ze^{j\alpha_z}=R_z+jI_z$$

$$\vec{\mathbf{F}}=F_p\hat{\mathbf{p}}+jF_q\hat{\mathbf{q}}$$

$$\left|\vec{\mathbf{F}}\right|=\sqrt{F_p^{\ 2}+F_q^{\ 2}}$$

$$\begin{cases}F_p=\left|\vec{\mathbf{F}}\right|\cos\phi\\ F_q=\left|\vec{\mathbf{F}}\right|\sin\phi\end{cases}$$

Polarization plane



$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}}=0$$

$$\left|\hat{\mathbf{u}}\right|=1$$

$$\left|\hat{\mathbf{v}}\right|=1$$

$$\vec{\mathbf{F}}=\left|\vec{\mathbf{F}}\right|e^{j\phi}\underbrace{\left(\frac{\hat{\mathbf{p}}+\hat{\mathbf{q}}}{2}\right)}_{\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}}}+\left|\vec{\mathbf{F}}\right|e^{-j\phi}\underbrace{\left(\frac{\hat{\mathbf{p}}-\hat{\mathbf{q}}}{2}\right)}_{\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}}$$

$$\vec{\mathbf{U}}\cdot\vec{\mathbf{V}}=0\qquad\left|\vec{\mathbf{U}}\right|\neq1\qquad\left|\vec{\mathbf{V}}\right|\neq1$$

$$\vec{\mathbf{U}}=\frac{\hat{\mathbf{p}}+\hat{\mathbf{q}}}{2}\qquad\left|\vec{\mathbf{U}}\right|^2=\left[\cos\left(\frac{\vartheta}{2}\right)\right]^2$$

$$\vec{\mathbf{V}}=\frac{\hat{\mathbf{p}}-\hat{\mathbf{q}}}{2}\qquad\left|\vec{\mathbf{V}}\right|^2=\left[\sin\left(\frac{\vartheta}{2}\right)\right]^2$$

$$\hat{\mathbf{u}}=\frac{\vec{\mathbf{U}}}{\left|\vec{\mathbf{U}}\right|}=\frac{\hat{\mathbf{p}}+\hat{\mathbf{q}}}{2}\frac{1}{\cos\left(\frac{\vartheta}{2}\right)}$$

$$\boxed{\vec{\mathbf{U}}=\frac{\hat{\mathbf{p}}+\hat{\mathbf{q}}}{2}=\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}}}$$

$$\hat{\mathbf{v}}=\frac{\vec{\mathbf{V}}}{\left|\vec{\mathbf{V}}\right|}=\frac{\hat{\mathbf{p}}-\hat{\mathbf{q}}}{2}\frac{1}{\sin\left(\frac{\vartheta}{2}\right)}$$

$$\boxed{\vec{\mathbf{V}}=\frac{\hat{\mathbf{p}}-\hat{\mathbf{q}}}{2}=\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}}$$

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

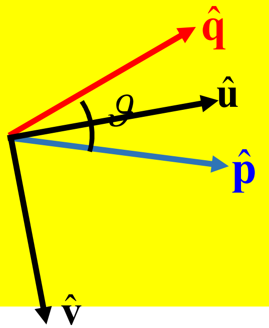
$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\boxed{\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

Polarization plane



$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

$$\left|\hat{\mathbf{u}}\right| = 1$$

$$\left|\hat{\mathbf{v}}\right| = 1$$

$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right|e^{j\phi}\underbrace{\left(\frac{\hat{\mathbf{p}}+\hat{\mathbf{q}}}{2}\right)}_{\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}}} + \left|\vec{\mathbf{F}}\right|e^{-j\phi}\underbrace{\left(\frac{\hat{\mathbf{p}}-\hat{\mathbf{q}}}{2}\right)}_{\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}}$$

$$\vec{\mathbf{U}}\cdot\vec{\mathbf{V}} = 0 \qquad \left|\vec{\mathbf{U}}\right| \neq 1 \qquad \left|\vec{\mathbf{V}}\right| \neq 1$$

$$\vec{\mathbf{U}} = \frac{\hat{\mathbf{p}}+\hat{\mathbf{q}}}{2} \qquad \left|\vec{\mathbf{U}}\right|^2 = \left[\cos\left(\frac{\vartheta}{2}\right)\right]^2$$

$$\vec{\mathbf{V}} = \frac{\hat{\mathbf{p}}-\hat{\mathbf{q}}}{2} \qquad \left|\vec{\mathbf{V}}\right|^2 = \left[\sin\left(\frac{\vartheta}{2}\right)\right]^2$$

$$\boxed{\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right|e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right|e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}}$$

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\,\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos\left(\omega_0\,t + \alpha_x\right) \xrightarrow{\qquad} F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos\left(\omega_0\,t + \alpha_y\right) \xrightarrow{\qquad} F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

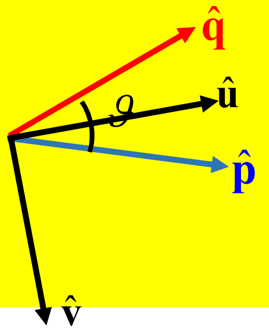
$$f_z(t) = A_z\cos\left(\omega_0\,t + \alpha_z\right) \xrightarrow{\qquad} F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\boxed{\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\ 2} + F_q^{\ 2}}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

Polarization plane



$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

$$\left|\hat{\mathbf{u}}\right| = 1$$

$$\left|\hat{\mathbf{v}}\right| = 1$$

$$\boxed{\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\vec{\mathbf{v}}}$$

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x \cos\left(\omega_0 t + \alpha_x\right) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y \cos\left(\omega_0 t + \alpha_y\right) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z \cos\left(\omega_0 t + \alpha_z\right) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

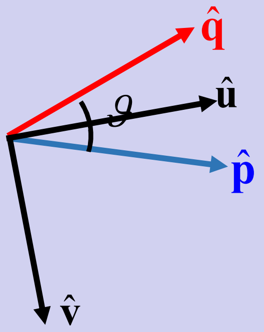
$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi} \cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi} \sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p{}^2 + F_q{}^2}$$

$$\hat{\mathbf{u}} \cdot \hat{\mathbf{v}} = 0$$

Polarization plane



Phasors and vector functions

$$\vec{\mathbf{f}}(t)=f_x(t)\hat{i}_x+f_y(t)\hat{i}_y+f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}}=F_x\hat{i}_x+F_y\hat{i}_y+F_z\hat{i}_z=A_xe^{j\alpha_x}\hat{i}_x+A_ye^{j\alpha_y}\hat{i}_y+A_ze^{j\alpha_z}\hat{i}_z$$

$$f_x(t)=A_x\cos\left(\omega_0t+\alpha_x\right)\longrightarrow F_x=A_xe^{j\alpha_x}=R_x+jI_x$$

$$f_y(t)=A_y\cos\left(\omega_0t+\alpha_y\right)\longrightarrow F_y=A_ye^{j\alpha_y}=R_y+jI_y$$

$$f_z(t)=A_z\cos\left(\omega_0t+\alpha_z\right)\longrightarrow F_z=A_ze^{j\alpha_z}=R_z+jI_z$$

$$\vec{\mathbf{F}}=F_p\hat{\mathbf{p}}+jF_q\hat{\mathbf{q}}$$

$$\vec{\mathbf{f}}(t)=F_p\cos\left(\omega_0t\right)\hat{\mathbf{p}}-F_q\sin\left(\omega_0t\right)\hat{\mathbf{q}}$$

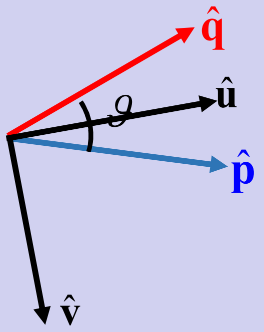
$$\vec{\mathbf{F}}=\left|\vec{\mathbf{F}}\right|e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}}+\left|\vec{\mathbf{F}}\right|e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}$$

$$\begin{cases}F_p=\left|\vec{\mathbf{F}}\right|\cos\phi\\ F_q=\left|\vec{\mathbf{F}}\right|\sin\phi\end{cases}$$

$$\left|\vec{\mathbf{F}}\right|=\sqrt{F_p^{\ 2}+F_q^{\ 2}}$$

$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}}=0$$

Polarization plane



$$\vec{\mathbf{f}}(t)=\mathrm{Re}\left\{\vec{\mathbf{F}}e^{j\omega_0t}\right\}=\mathrm{Re}\left\{\left[F_p\hat{\mathbf{p}}+jF_q\hat{\mathbf{q}}\right]e^{j\omega_0t}\right\}$$

$$=\mathrm{Re}\left\{\left[F_p\hat{\mathbf{p}}+jF_q\hat{\mathbf{q}}\right]\left[\cos\left(\omega_0t\right)+j\sin\left(\omega_0t\right)\right]\right\}$$

$$=\mathrm{Re}\left\{F_p\cos\left(\omega_0t\right)\hat{\mathbf{p}}-F_q\sin\left(\omega_0t\right)\hat{\mathbf{q}}+jF_p\sin\left(\omega_0t\right)\hat{\mathbf{p}}+jF_q\cos\left(\omega_0t\right)\hat{\mathbf{q}}\right\}$$

$$=F_p\cos\left(\omega_0t\right)\hat{\mathbf{p}}-F_q\sin\left(\omega_0t\right)\hat{\mathbf{q}}$$

$$e^{j\omega_0t}=\cos\left(\omega_0t\right)+j\sin\left(\omega_0t\right)$$

Phasors and vector functions

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos\left(\omega_0 t + \alpha_x\right) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos\left(\omega_0 t + \alpha_y\right) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos\left(\omega_0 t + \alpha_z\right) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\vec{\mathbf{f}}(t) = F_p\cos\left(\omega_0 t\right)\hat{\mathbf{p}} - F_q\sin\left(\omega_0 t\right)\hat{\mathbf{q}}$$

$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}$$

$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right|\cos\left(\frac{\vartheta}{2}\right)\cos\left(\omega_0 t + \phi\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right|\sin\left(\frac{\vartheta}{2}\right)\cos\left(\omega_0 t - \phi\right)\hat{\mathbf{v}}$$

$$e^{j\omega_0 t}e^{j\phi} = \cos\left(\omega_0 t + \phi\right) + j\sin\left(\omega_0 t + \phi\right)$$

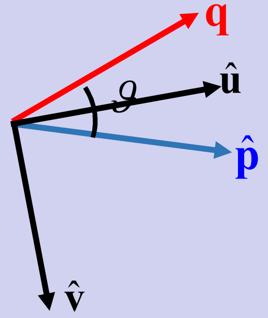
$$e^{j\omega_0 t}e^{-j\phi} = \cos\left(\omega_0 t - \phi\right) + j\sin\left(\omega_0 t - \phi\right)$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

Polarization plane



$$\vec{\mathbf{f}}(t) = \mathrm{Re}\left\{\vec{\mathbf{F}}e^{j\omega_0 t}\right\} = \mathrm{Re}\left\{\left[\left|\vec{\mathbf{F}}\right| e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}\right]e^{j\omega_0 t}\right\}$$

$$= \mathrm{Re}\left\{\left|\vec{\mathbf{F}}\right|\cos\left(\frac{\vartheta}{2}\right)e^{j\omega_0 t}e^{j\phi}\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right|\sin\left(\frac{\vartheta}{2}\right)e^{j\omega_0 t}e^{-j\phi}\hat{\mathbf{v}}\right\}$$

$$= \left|\vec{\mathbf{F}}\right|\cos\left(\frac{\vartheta}{2}\right)\cos\left(\omega_0 t + \phi\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right|\sin\left(\frac{\vartheta}{2}\right)\cos\left(\omega_0 t - \phi\right)\hat{\mathbf{v}}$$

Polarization plane

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x \cos\left(\omega_0 t + \alpha_x\right) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y \cos\left(\omega_0 t + \alpha_y\right) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z \cos\left(\omega_0 t + \alpha_z\right) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\vec{\mathbf{f}}(t) = F_p\cos\left(\omega_0 t\right)\hat{\mathbf{p}} - F_q\sin\left(\omega_0 t\right)\hat{\mathbf{q}}$$

$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}$$

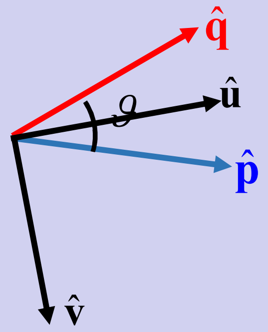
$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right|\cos\left(\frac{\vartheta}{2}\right)\cos\left(\omega_0 t + \phi\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right|\sin\left(\frac{\vartheta}{2}\right)\cos\left(\omega_0 t - \phi\right)\hat{\mathbf{v}}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

Polarization plane



The vector $\vec{\mathbf{f}}(t)$ lies in the polarization plane $(\hat{\mathbf{p}}, \hat{\mathbf{q}})$ (which is coincident with the plane $(\hat{\mathbf{u}}, \hat{\mathbf{v}})$), and it changes in general both its amplitude and its direction as the time elapses.

Two cases are of particular interest:

- **Linear polarization**
- **Circular polarization**

Polarization plane

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos(\omega_0 t + \alpha_x) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos(\omega_0 t + \alpha_y) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos(\omega_0 t + \alpha_z) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\vec{\mathbf{f}}(t) = F_p\cos(\omega_0 t)\hat{\mathbf{p}} - F_q\sin(\omega_0 t)\hat{\mathbf{q}}$$

$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}$$

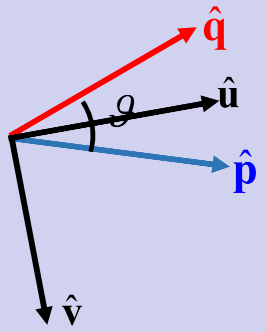
$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right|\cos\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t + \phi)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right|\sin\left(\frac{\vartheta}{2}\right)\cos(\omega_0 t - \phi)\hat{\mathbf{v}}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

Polarization plane



The vector $\vec{\mathbf{f}}(t)$ lies in the polarization plane $(\hat{\mathbf{p}},\hat{\mathbf{q}})$ (which is coincident with the plane $(\hat{\mathbf{u}},\hat{\mathbf{v}})$), and it changes in general both its amplitude and its direction as the time elapses.

Two cases are of particular interest:

- **Linear polarization**
- Circular polarization

Linear Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos\left(\omega_0 t + \alpha_x\right) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos\left(\omega_0 t + \alpha_y\right) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos\left(\omega_0 t + \alpha_z\right) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\vec{\mathbf{f}}(t) = F_p\cos\left(\omega_0 t\right)\hat{\mathbf{p}} - F_q\sin\left(\omega_0 t\right)\hat{\mathbf{q}}$$

$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}$$

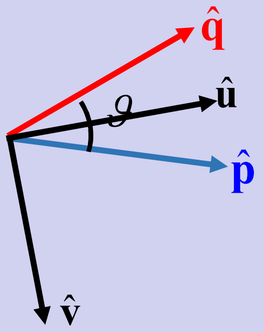
$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right|\cos\left(\frac{\vartheta}{2}\right)\cos\left(\omega_0 t + \phi\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right|\sin\left(\frac{\vartheta}{2}\right)\cos\left(\omega_0 t - \phi\right)\hat{\mathbf{v}}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\,2} + F_q^{\,2}}$$

$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

Polarization plane



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line:

Linear Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos\left(\omega_0 t + \alpha_x\right) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos\left(\omega_0 t + \alpha_y\right) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos\left(\omega_0 t + \alpha_z\right) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\vec{\mathbf{f}}(t) = F_p\cos\left(\omega_0 t\right)\hat{\mathbf{p}} - F_q\sin\left(\omega_0 t\right)\hat{\mathbf{q}}$$

$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}$$

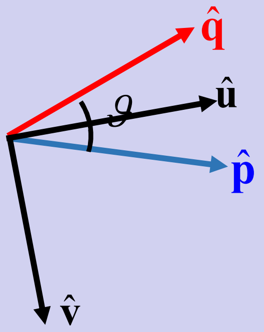
$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right|\cos\left(\frac{\vartheta}{2}\right)\cos\left(\omega_0 t + \phi\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right|\sin\left(\frac{\vartheta}{2}\right)\cos\left(\omega_0 t - \phi\right)\hat{\mathbf{v}}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\;2} + F_q^{\;2}}$$

$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

Polarization plane



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line:

- $\hat{\mathbf{p}} = \hat{\mathbf{q}}$

Linear Polarization

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

$$f_x(t) = A_x\cos\left(\omega_0 t + \alpha_x\right) \longrightarrow F_x = A_x e^{j\alpha_x} = R_x + jI_x$$

$$f_y(t) = A_y\cos\left(\omega_0 t + \alpha_y\right) \longrightarrow F_y = A_y e^{j\alpha_y} = R_y + jI_y$$

$$f_z(t) = A_z\cos\left(\omega_0 t + \alpha_z\right) \longrightarrow F_z = A_z e^{j\alpha_z} = R_z + jI_z$$

$$\vec{\mathbf{F}} = F_p\hat{\mathbf{p}} + jF_q\hat{\mathbf{q}}$$

$$\vec{\mathbf{f}}(t) = F_p\cos\left(\omega_0 t\right)\hat{\mathbf{p}} - F_q\sin\left(\omega_0 t\right)\hat{\mathbf{q}}$$

$$\vec{\mathbf{F}} = \left|\vec{\mathbf{F}}\right| e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right| e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}$$

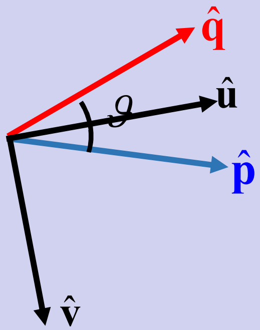
$$\boxed{\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right|\cos\left(\frac{\vartheta}{2}\right)\cos\left(\omega_0 t + \phi\right)\hat{\mathbf{u}} + \left|\vec{\mathbf{F}}\right|\sin\left(\frac{\vartheta}{2}\right)\cos\left(\omega_0 t - \phi\right)\hat{\mathbf{v}}}$$

$$\begin{cases} F_p = \left|\vec{\mathbf{F}}\right|\cos\phi \\ F_q = \left|\vec{\mathbf{F}}\right|\sin\phi \end{cases}$$

$$\left|\vec{\mathbf{F}}\right| = \sqrt{F_p^{\ 2} + F_q^{\ 2}}$$

$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}} = 0$$

Polarization plane



$$\boxed{\hat{\mathbf{p}} = \hat{\mathbf{q}}}$$

$$\Downarrow$$

$$\vartheta = 0 \Rightarrow \boxed{\cos\left(\frac{\vartheta}{2}\right) = 1} \qquad \sin\left(\frac{\vartheta}{2}\right) = 0$$

$$\Downarrow$$

$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right|\cos\left(\omega_0 t + \phi\right)\hat{\mathbf{u}}$$

$$\boxed{\frac{\hat{\mathbf{p}} + \hat{\mathbf{q}}}{2} = \cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}}}$$

$$\frac{\hat{\mathbf{p}} - \hat{\mathbf{q}}}{2} = \sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}$$

$$\hat{\mathbf{u}} = \hat{\mathbf{p}} = \hat{\mathbf{q}}$$

Linear Polarization

$$\vec{\mathbf{f}}(t)=f_x(t)\hat{i}_x+f_y(t)\hat{i}_y+f_z(t)\hat{i}_z$$

$$\vec{\mathbf{F}}=F_x\hat{i}_x+F_y\hat{i}_y+F_z\hat{i}_z=A_xe^{j\alpha_x}\hat{i}_x+A_ye^{j\alpha_y}\hat{i}_y+A_ze^{j\alpha_z}\hat{i}_z$$

$$f_x(t)=A_x\cos\left(\omega_0t+\alpha_x\right)\longrightarrow F_x=A_xe^{j\alpha_x}=R_x+jI_x$$

$$f_y(t)=A_y\cos\left(\omega_0t+\alpha_y\right)\longrightarrow F_y=A_ye^{j\alpha_y}=R_y+jI_y$$

$$f_z(t)=A_z\cos\left(\omega_0t+\alpha_z\right)\longrightarrow F_z=A_ze^{j\alpha_z}=R_z+jI_z$$

$$\vec{\mathbf{F}}=F_p\hat{\mathbf{p}}+jF_q\hat{\mathbf{q}}$$

$$\vec{\mathbf{f}}(t)=F_p\cos\left(\omega_0t\right)\hat{\mathbf{p}}-F_q\sin\left(\omega_0t\right)\hat{\mathbf{q}}$$

$$\vec{\mathbf{F}}=\left|\vec{\mathbf{F}}\right|e^{j\phi}\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}}+\left|\vec{\mathbf{F}}\right|e^{-j\phi}\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}$$

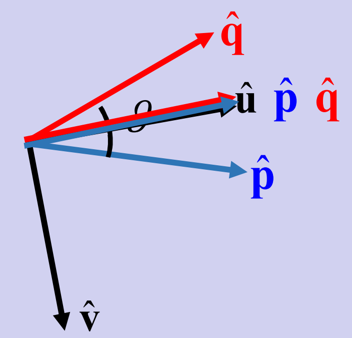
$$\vec{\mathbf{f}}(t)=\left|\vec{\mathbf{F}}\right|\cos\left(\frac{\vartheta}{2}\right)\cos\left(\omega_0t+\phi\right)\hat{\mathbf{u}}+\left|\vec{\mathbf{F}}\right|\sin\left(\frac{\vartheta}{2}\right)\cos\left(\omega_0t-\phi\right)\hat{\mathbf{v}}$$

$$\begin{cases}F_p=\left|\vec{\mathbf{F}}\right|\cos\phi\\F_q=\left|\vec{\mathbf{F}}\right|\sin\phi\end{cases}$$

$$\left|\vec{\mathbf{F}}\right|=\sqrt{F_p^{\ 2}+F_q^{\ 2}}$$

$$\hat{\mathbf{u}}\cdot\hat{\mathbf{v}}=0$$

Polarization plane



$$\boxed{\hat{\mathbf{p}}=\hat{\mathbf{q}}}$$

$$\Downarrow$$

$$\vartheta=0\Rightarrow\boxed{\cos\left(\frac{\vartheta}{2}\right)=1}\quad\sin\left(\frac{\vartheta}{2}\right)=0$$

$$\Downarrow$$

$$\vec{\mathbf{f}}(t)=\left|\vec{\mathbf{F}}\right|\cos\left(\omega_0t+\phi\right)\hat{\mathbf{u}}$$

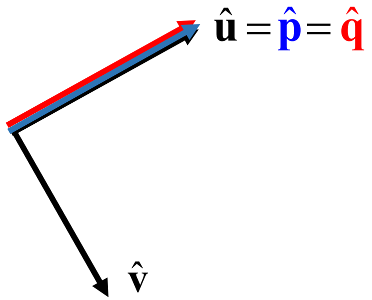
$$\boxed{\frac{\hat{\mathbf{p}}+\hat{\mathbf{q}}}{2}=\cos\left(\frac{\vartheta}{2}\right)\hat{\mathbf{u}}}$$

$$\frac{\hat{\mathbf{p}}-\hat{\mathbf{q}}}{2}=\sin\left(\frac{\vartheta}{2}\right)\hat{\mathbf{v}}$$

$$\hat{\mathbf{u}}=\hat{\mathbf{p}}=\hat{\mathbf{q}}$$

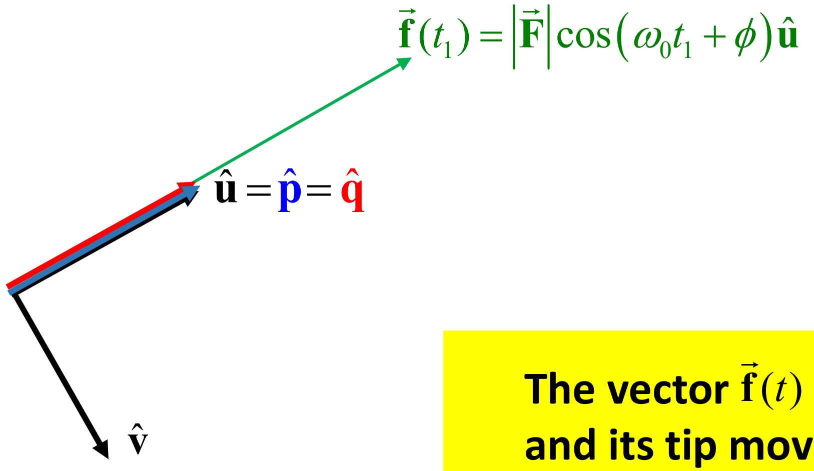
Linear Polarization

$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right| \cos\left(\omega_0 t + \phi\right) \hat{\mathbf{u}}$$



Linear Polarization

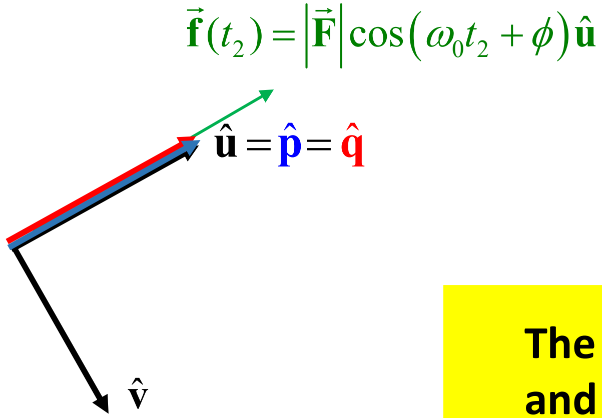
$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right| \cos\left(\omega_0 t + \phi\right) \hat{\mathbf{u}}$$



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line

Linear Polarization

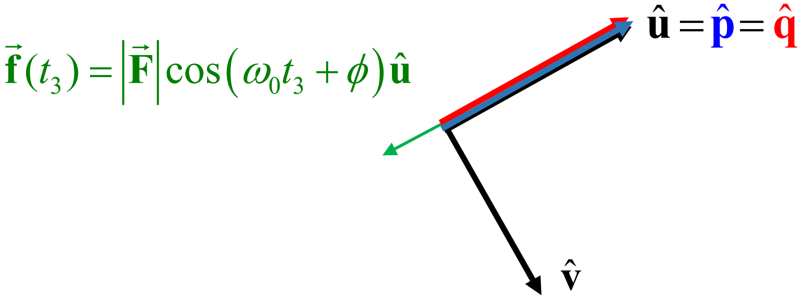
$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right| \cos\left(\omega_0 t + \phi\right) \hat{\mathbf{u}}$$



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line

Linear Polarization

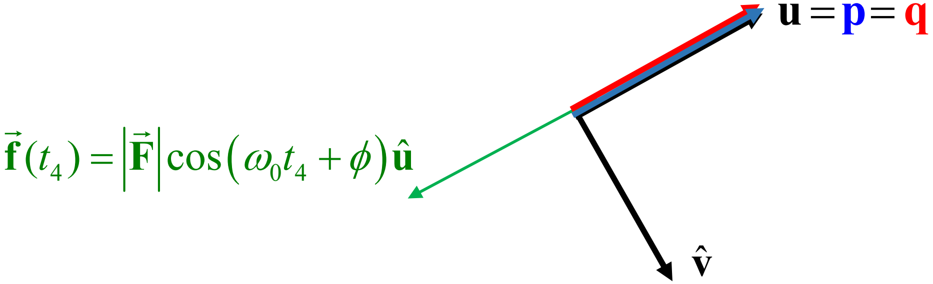
$$\vec{\mathbf{f}}(t)=\left|\vec{\mathbf{F}}\right|\cos\left(\omega_0 t+\phi\right)\hat{\mathbf{u}}$$



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line

Linear Polarization

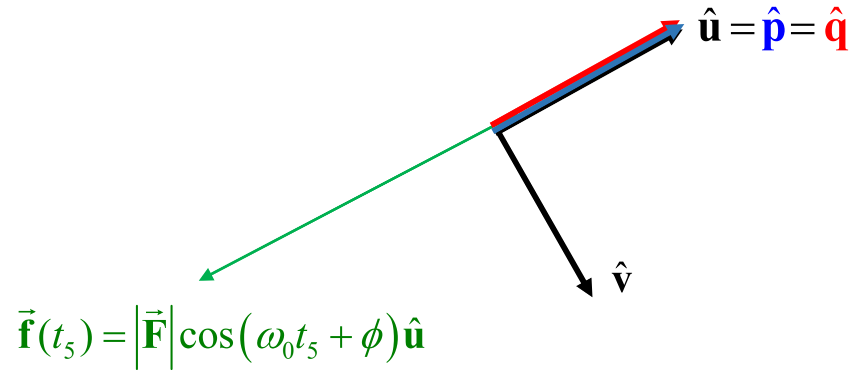
$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right| \cos\left(\omega_0 t + \phi\right) \hat{\mathbf{u}}$$



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line

Linear Polarization

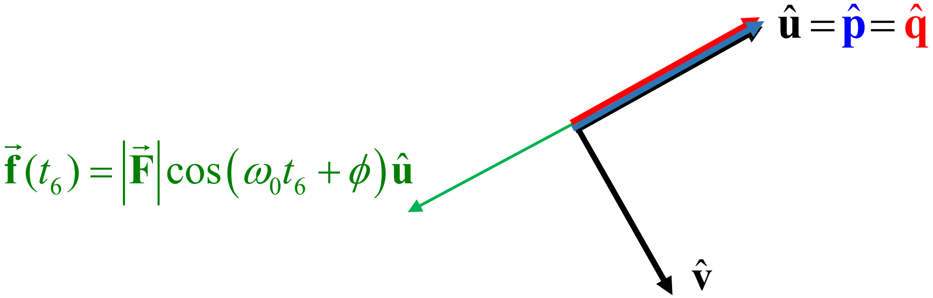
$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right| \cos\left(\omega_0 t + \phi\right) \hat{\mathbf{u}}$$



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line

Linear Polarization

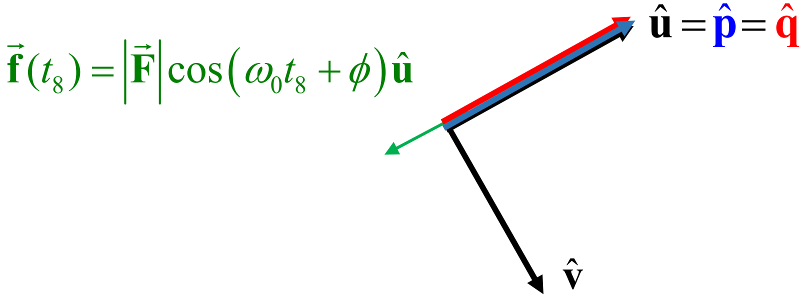
$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right| \cos\left(\omega_0 t + \phi\right)\hat{\mathbf{u}}$$



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line

Linear Polarization

$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right| \cos\left(\omega_0 t + \phi\right) \hat{\mathbf{u}}$$

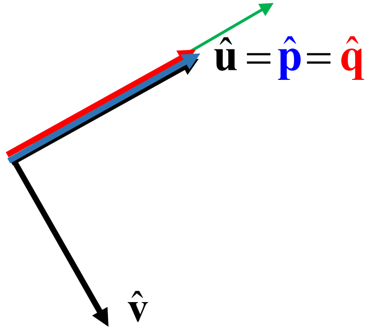


The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line

Linear Polarization

$$\vec{\mathbf{f}}(t) = \left|\vec{\mathbf{F}}\right| \cos\left(\omega_0 t + \phi\right) \hat{\mathbf{u}}$$

$$\vec{\mathbf{f}}(t_9) = \left|\vec{\mathbf{F}}\right| \cos\left(\omega_0 t_9 + \phi\right) \hat{\mathbf{u}}$$



The vector $\vec{\mathbf{f}}(t)$ does not change its direction and its tip moves along a straight line