

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

Corso di Campi Elettromagnetici
a.a. 2020-2021

Maxwell equations

Time domain & Phasors

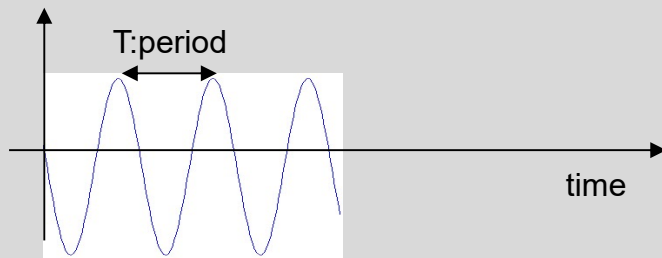


Phasors

Time domain

$$f(t)$$

Signals usually adopted in ICT applications

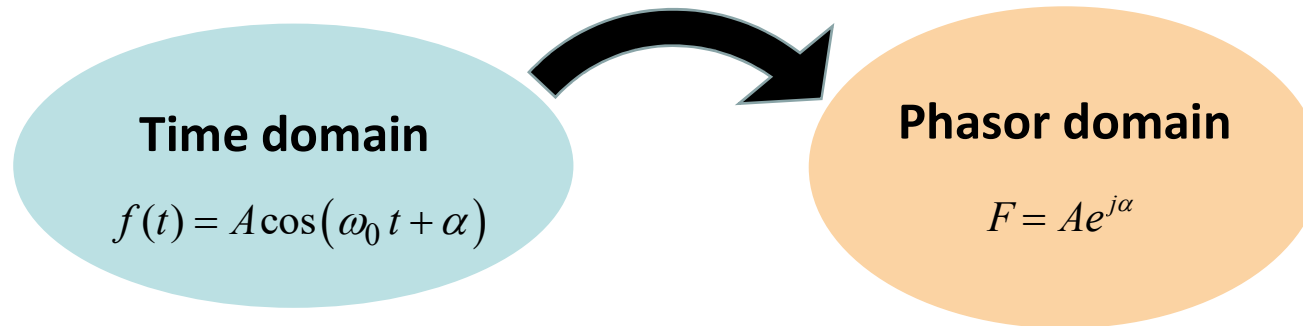


$$f(t) = A \cos(2\pi f_0 t + \alpha)$$

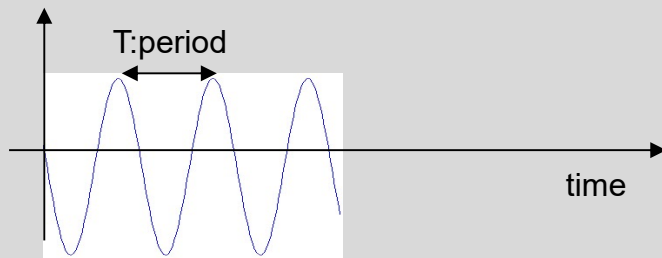
$$f_0 : \text{frequency} = \frac{1}{T}$$

$$\omega_0 : \text{angular frequency} = 2\pi f_0$$

Phasors



Signals usually adopted in ICT applications

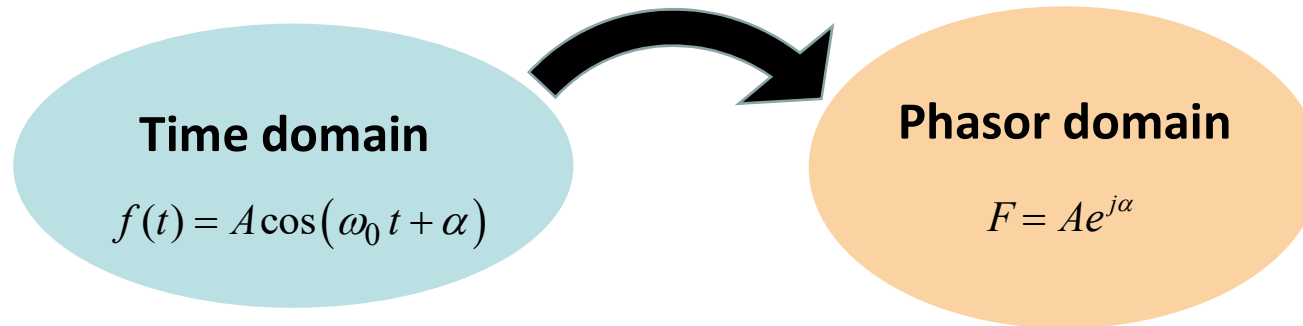


$$f(t) = A \cos(2\pi f_0 t + \alpha)$$

$$f_0 : \text{frequency} = \frac{1}{T}$$

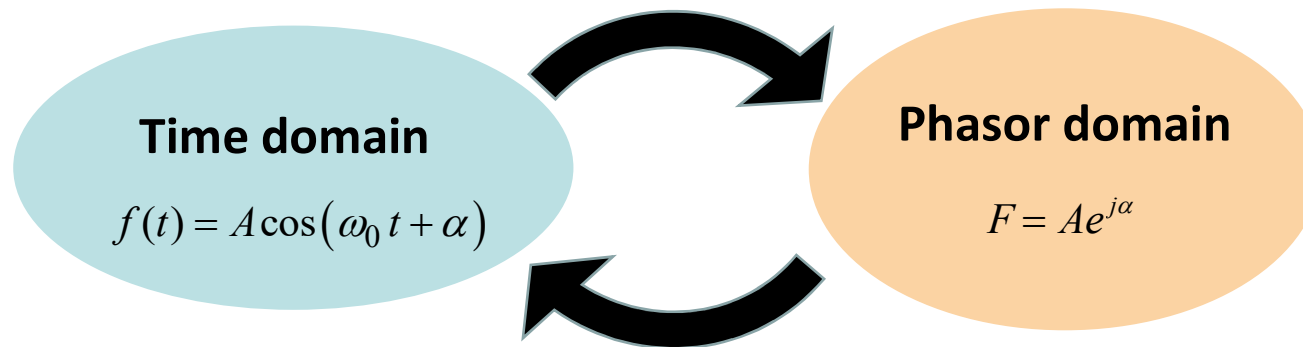
$$\omega_0 : \text{angular frequency} = 2\pi f_0$$

Phasors



- 1) How to jump back from the Phasor domain to the Time domain
- 2) Time domain derivative and Phasors

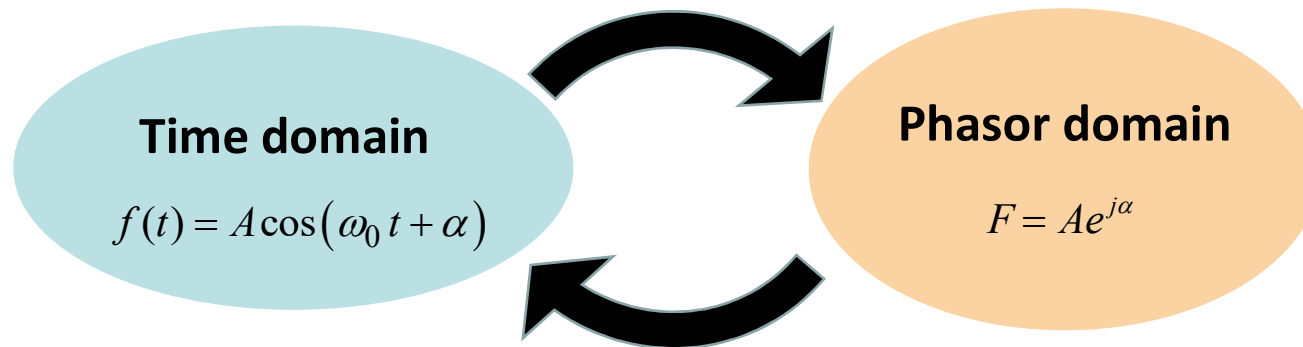
Phasors



1) How to jump back from the Phasor domain to the Time domain

$$f(t) = \operatorname{Re}\{F e^{j\omega_0 t}\} = \operatorname{Re}\{A e^{j\alpha} e^{j\omega_0 t}\} = A \cos(\omega_0 t + \alpha)$$

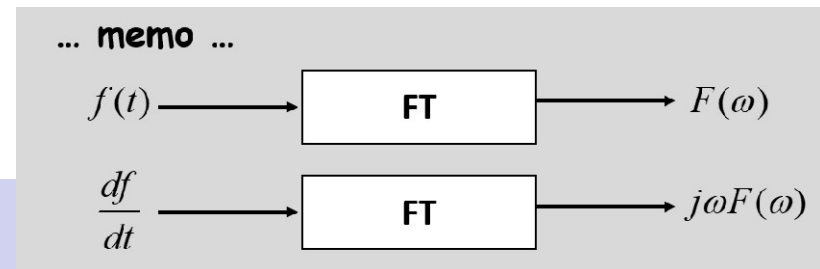
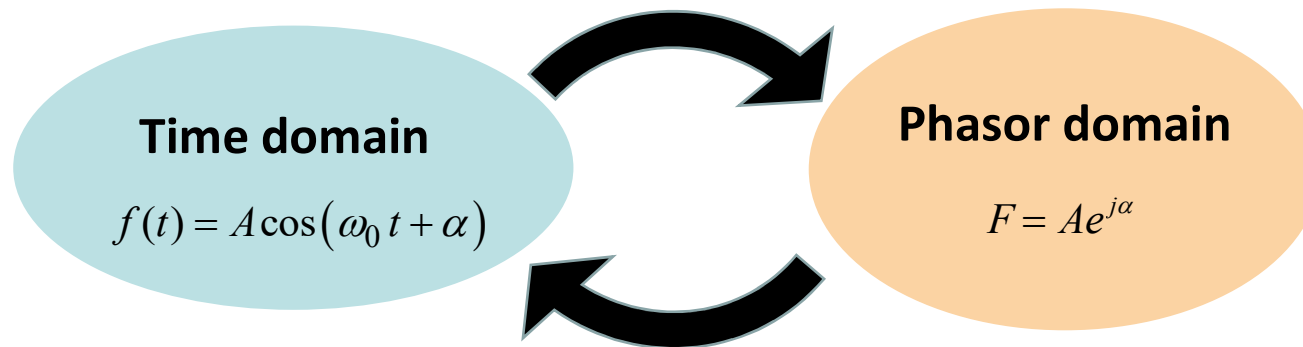
Phasors



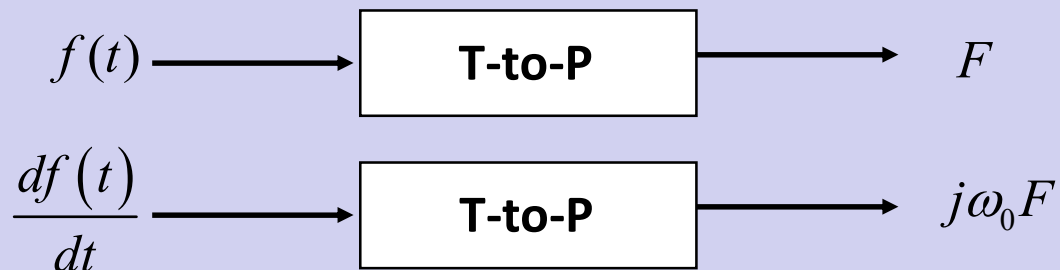
$$f(t) = A \cos(\omega_0 t + \alpha) \longrightarrow \text{T-to-P} \longrightarrow F = A e^{j\alpha}$$

$$F = A e^{j\alpha} \longrightarrow \text{P-to-T} \longrightarrow f(t) = \text{Re}\{F e^{j\omega_0 t}\} = A \cos(\omega_0 t + \alpha)$$

Phasors



2) Time domain derivative and Phasors



ω_0 now is fixed!

Phasors

- Phasors and functions of n variables
- Phasors and vector functions
- Phasors and vector functions of n variables

Phasors

- Phasors and functions of n variables
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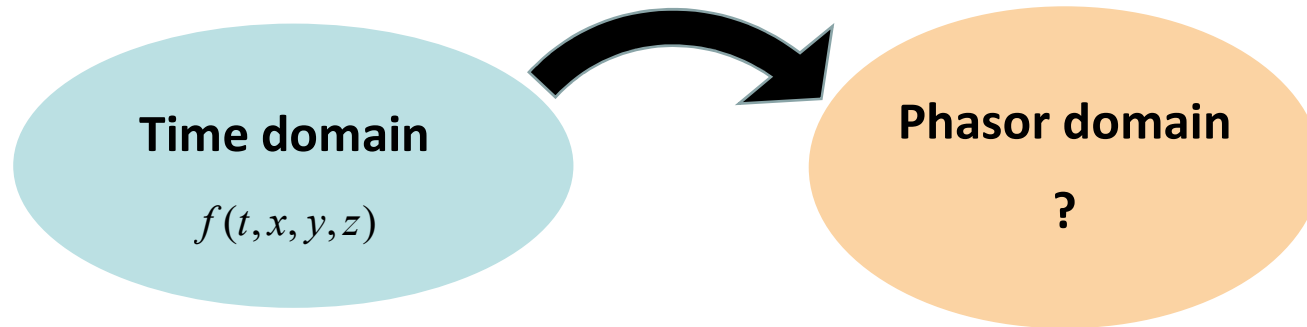
- 1) How to jump back from the Phasor domain to the Time domain
- 2) Time domain derivative and Phasors

Phasors

- **Phasors and functions of n variables**
- Phasors and vector functions
- Phasors and vector functions of n variables

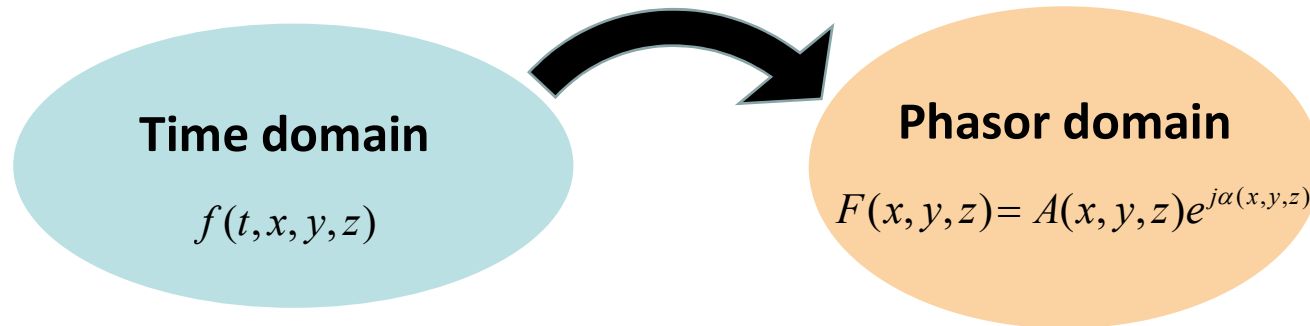
- 1) How to jump back from the Phasor domain to the Time domain**
- 2) Time domain derivative and Phasors**

Phasors and functions of n variables



$$f(t, x, y, z) = A(x, y, z) \cos(\omega_0 t + \alpha(x, y, z))$$

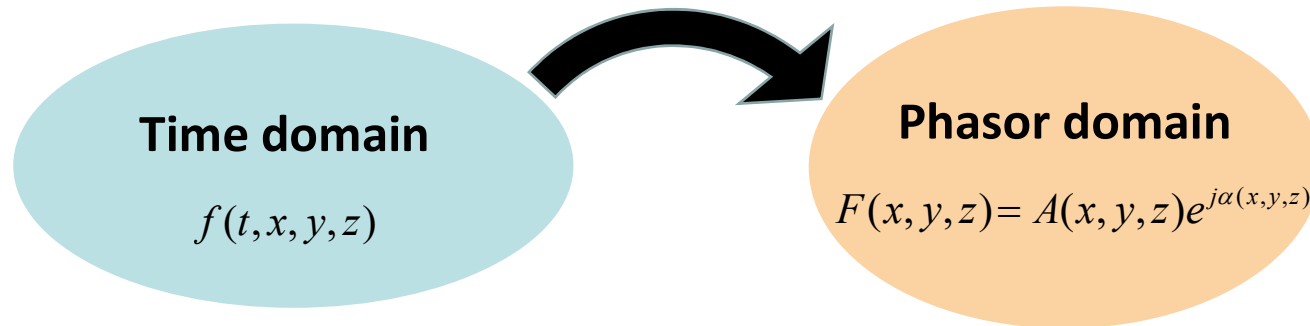
Phasors and functions of n variables



$$f(t, x, y, z) = A(x, y, z) \cos(\omega_0 t + \alpha(x, y, z))$$

$$F(x, y, z) = A(x, y, z)e^{j\alpha(x, y, z)}$$

Phasors and functions of n variables

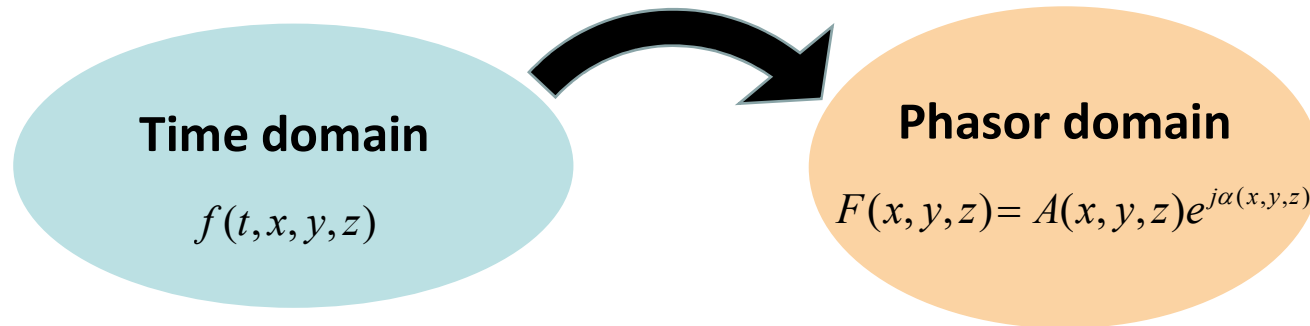


$$f(t, x, y, z) = A(x, y, z) \cos(\omega_0 t + \alpha(x, y, z))$$

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1) How to jump back from the Phasor domain to the Time domain

Phasors and functions of n variables



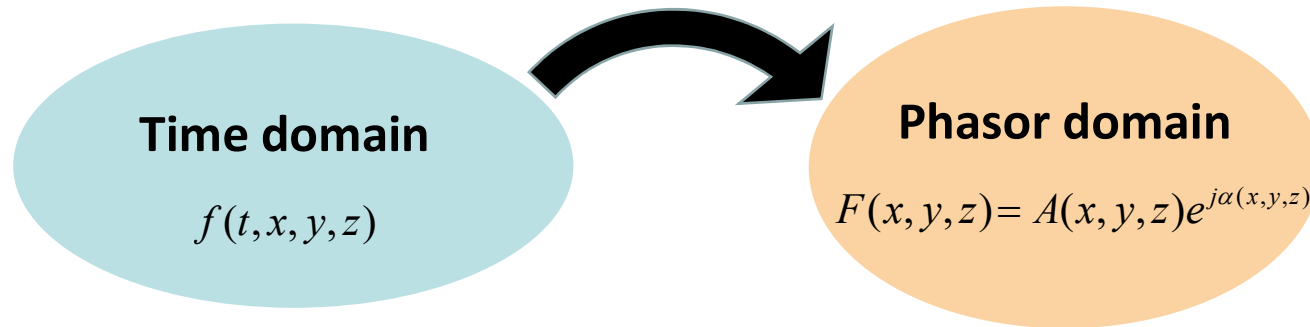
$$f(t, x, y, z) = A(x, y, z) \cos(\omega_0 t + \alpha(x, y, z))$$

$$F(x, y, z) = A(x, y, z)e^{j\alpha(x, y, z)}$$

1) How to jump back from the Phasor domain to the Time domain

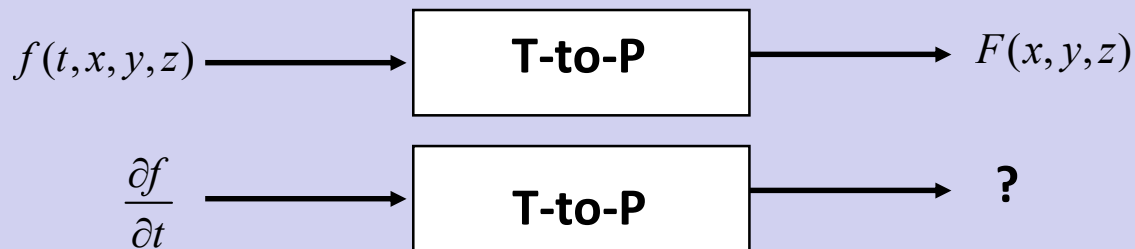
$$f(t, x, y, z) = \operatorname{Re}\{F(x, y, z)e^{j\omega_0 t}\}$$

Phasors and functions of n variables

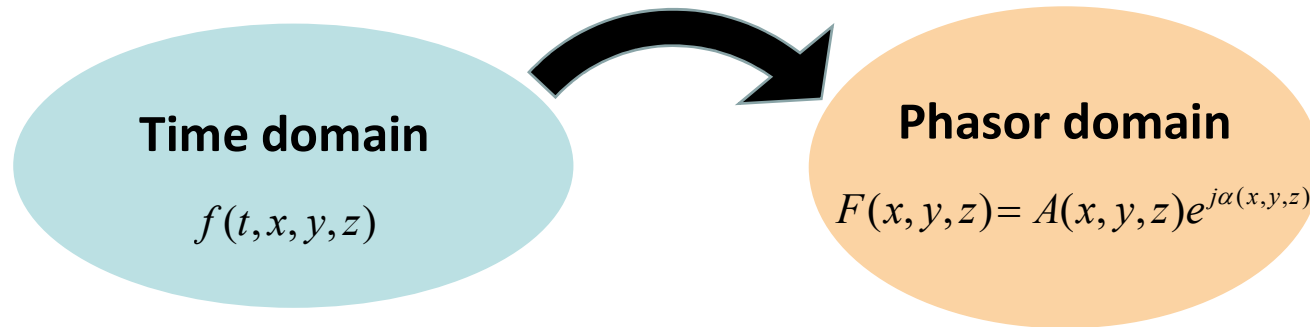


$$f(t, x, y, z) = A(x, y, z)\cos(\omega_0 t + \alpha(x, y, z))$$

2) Time domain derivative and Phasors

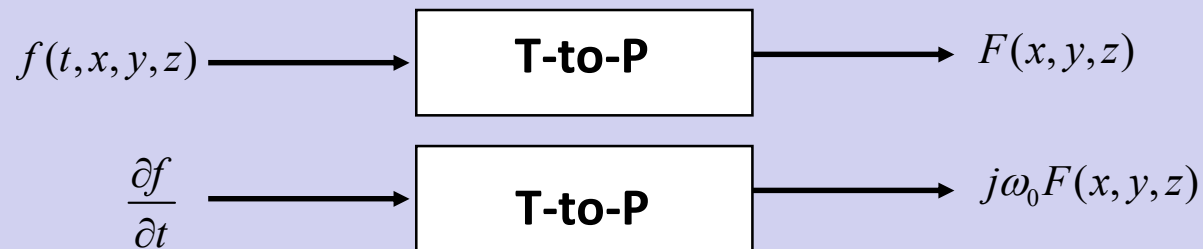


Phasors and functions of n variables

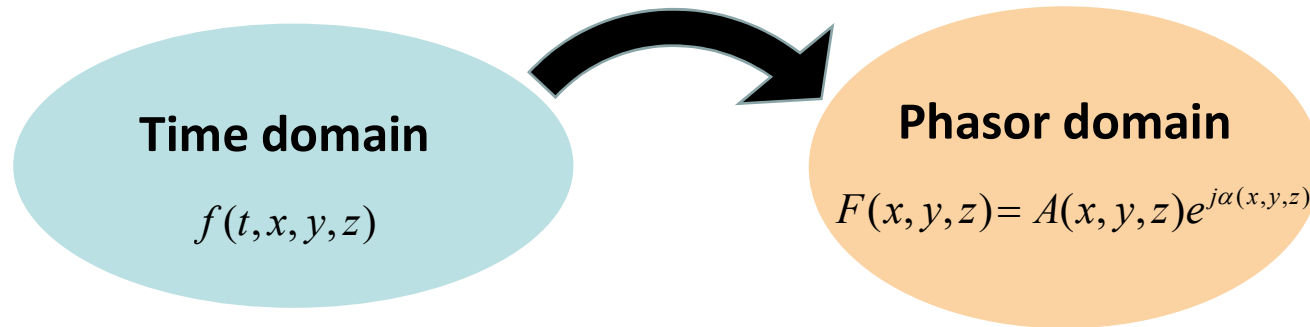


$$f(t, x, y, z) = A(x, y, z)\cos(\omega_0 t + \alpha(x, y, z))$$

2) Time domain derivative and Phasors

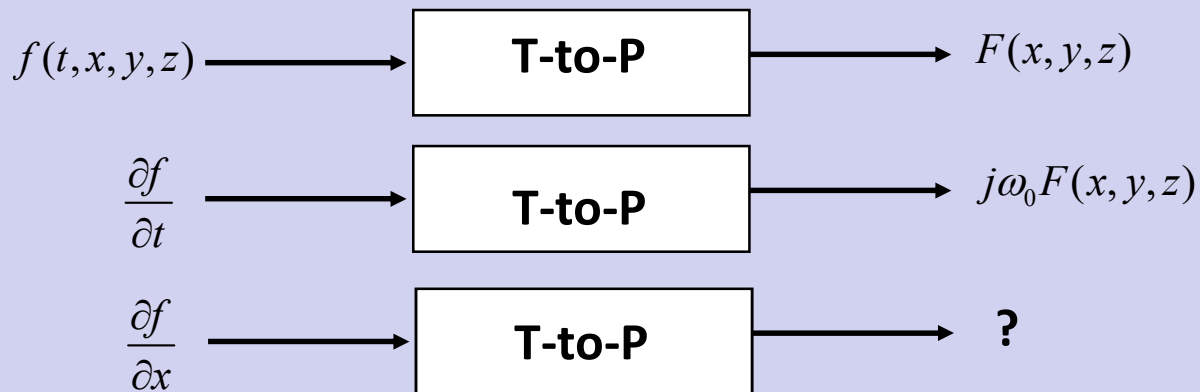


Phasors and functions of n variables

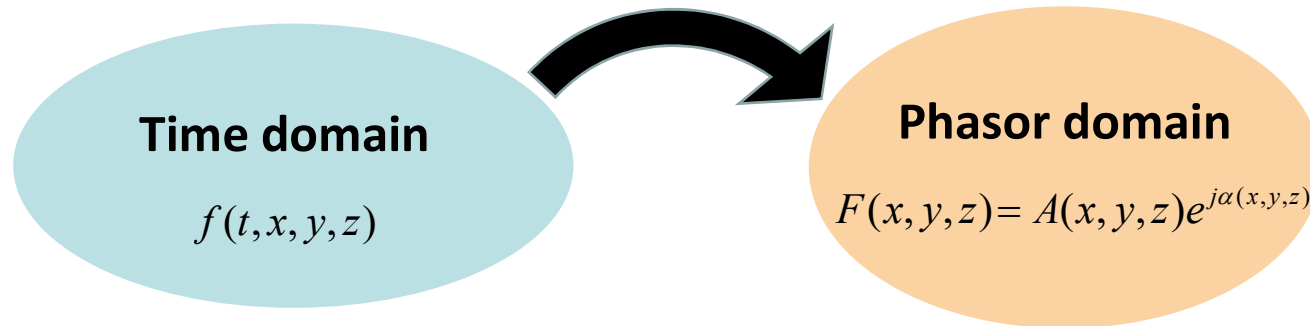


$$f(t, x, y, z) = A(x, y, z)\cos(\omega_0 t + \alpha(x, y, z))$$

2) Time domain derivative and Phasors

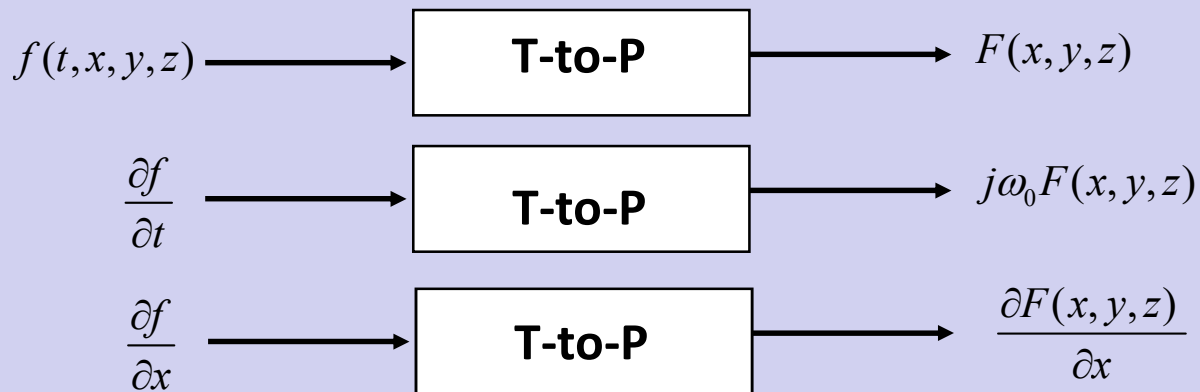


Phasors and functions of n variables



$$f(t, x, y, z) = A(x, y, z) \cos(\omega_0 t + \alpha(x, y, z))$$

2) Time domain derivative and Phasors

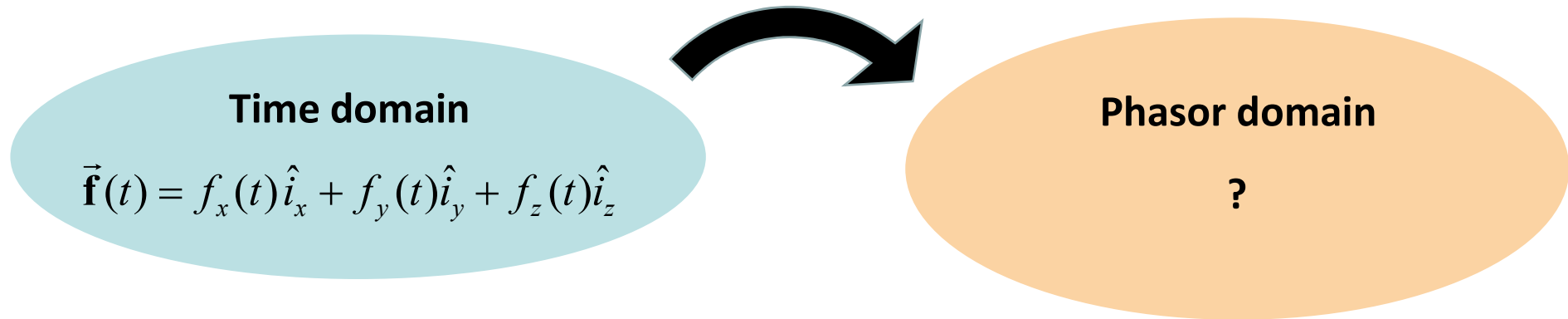


Phasors

- Phasors and functions of n variables
- **Phasors and vector functions**
- Phasors and vector functions of n variables

- 1) How to jump back from the Phasor domain to the Time domain**
- 2) Time domain derivative and Phasors**

Phasors and vector functions

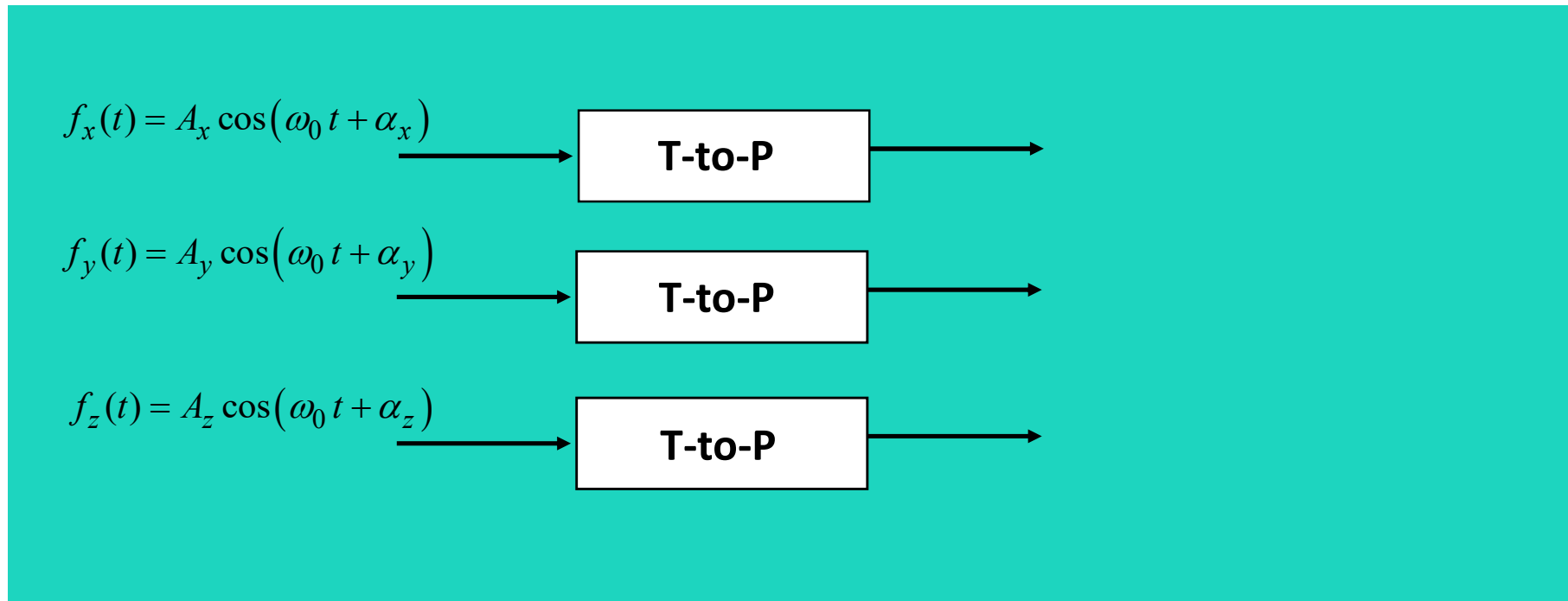
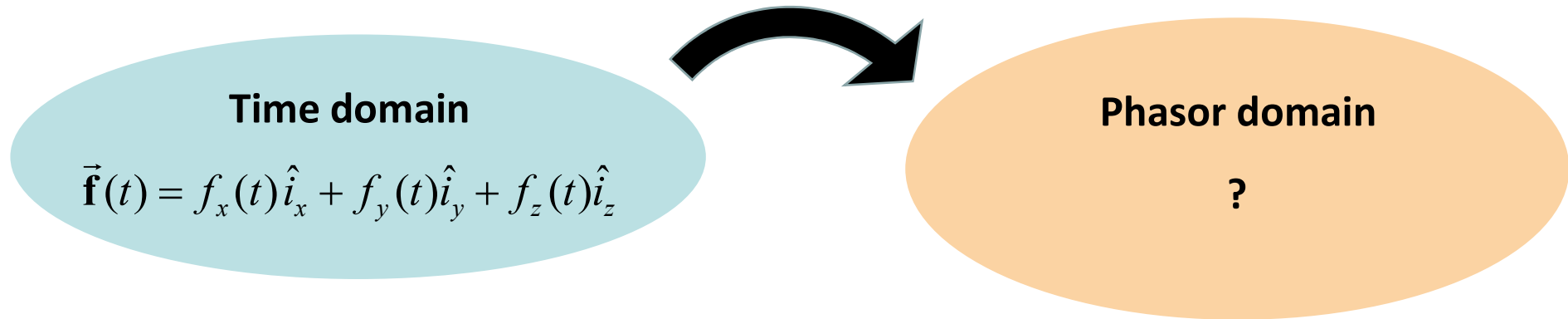


$$f_x(t) = A_x \cos(\omega_0 t + \alpha_x)$$

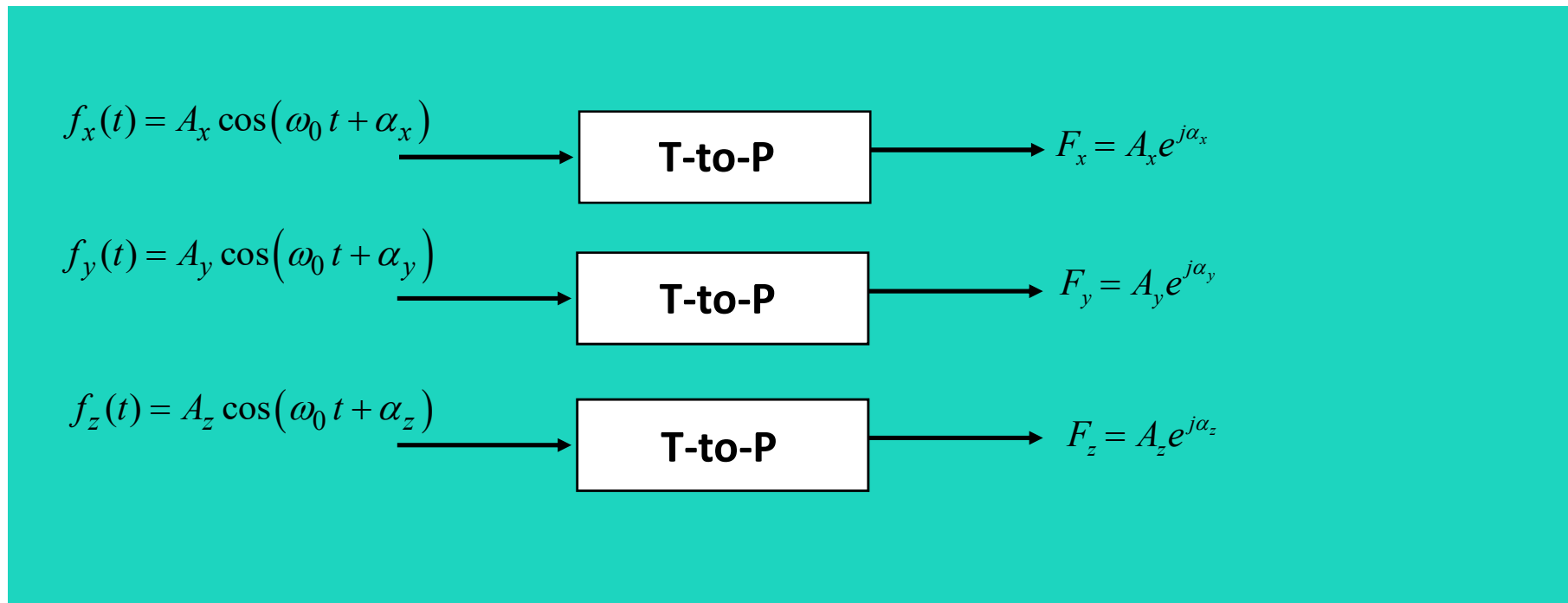
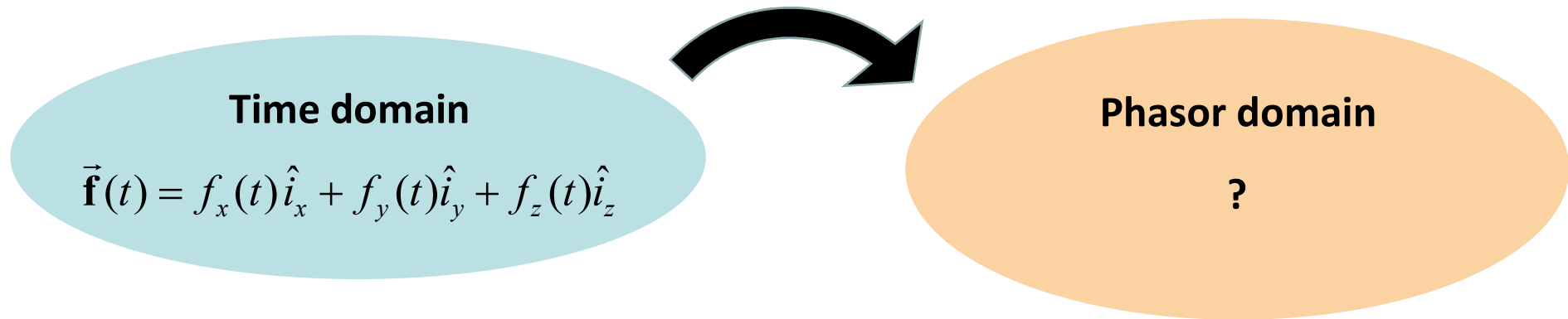
$$f_y(t) = A_y \cos(\omega_0 t + \alpha_y)$$

$$f_z(t) = A_z \cos(\omega_0 t + \alpha_z)$$

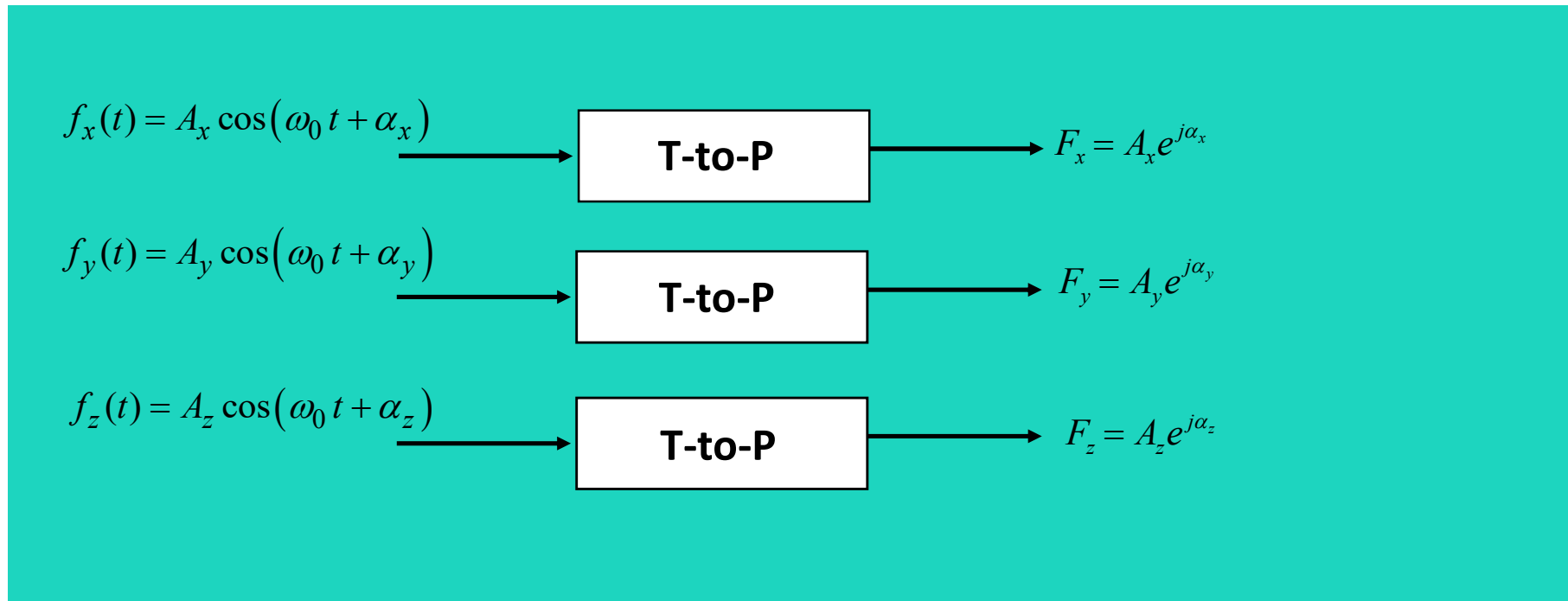
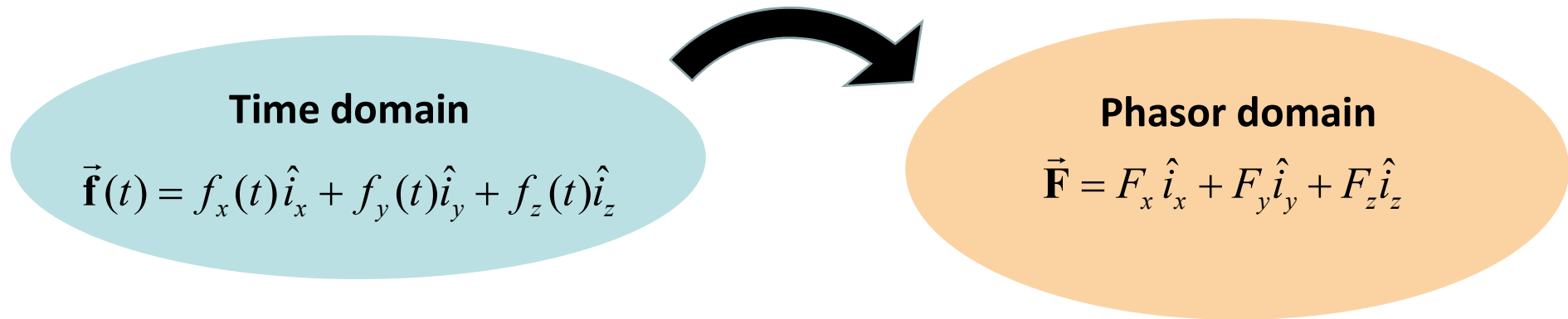
Phasors and vector functions



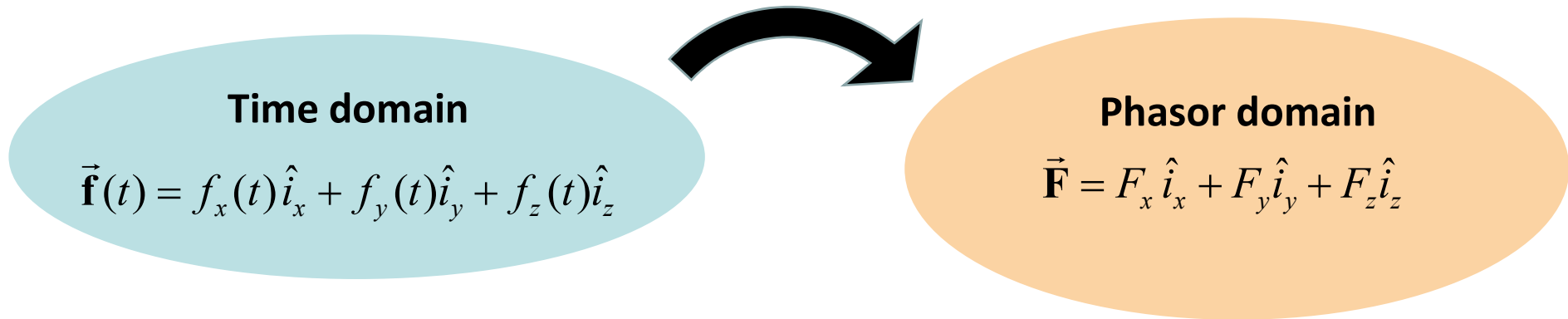
Phasors and vector functions



Phasors and vector functions



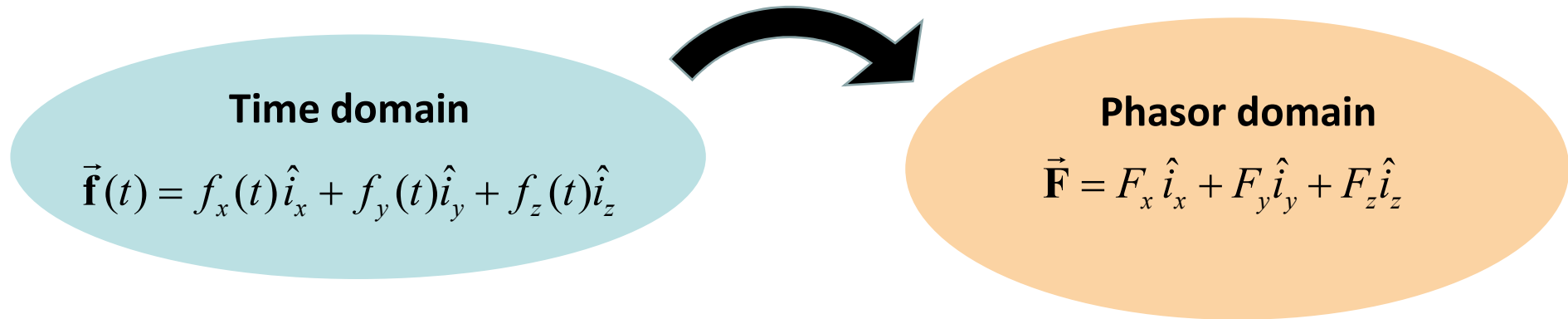
Phasors and vector functions



$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z = A_x \cos(\omega_0 t + \alpha_x)\hat{i}_x + A_y \cos(\omega_0 t + \alpha_y)\hat{i}_y + A_z \cos(\omega_0 t + \alpha_z)\hat{i}_z$$

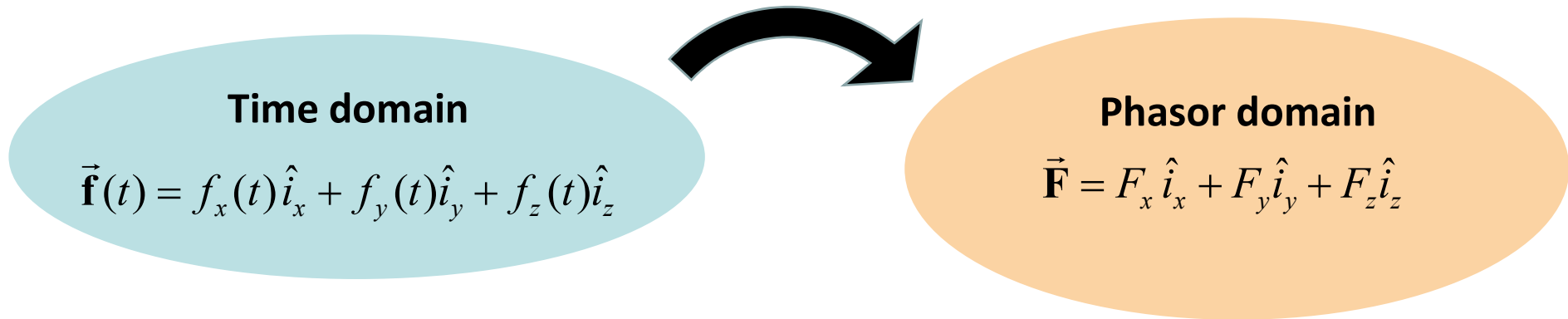
$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

Phasors and vector functions

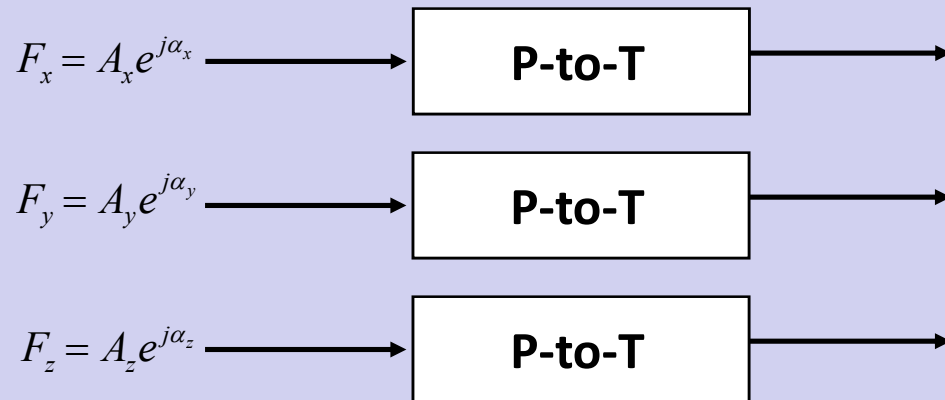


1) How to jump back from the Phasor domain to the Time domain

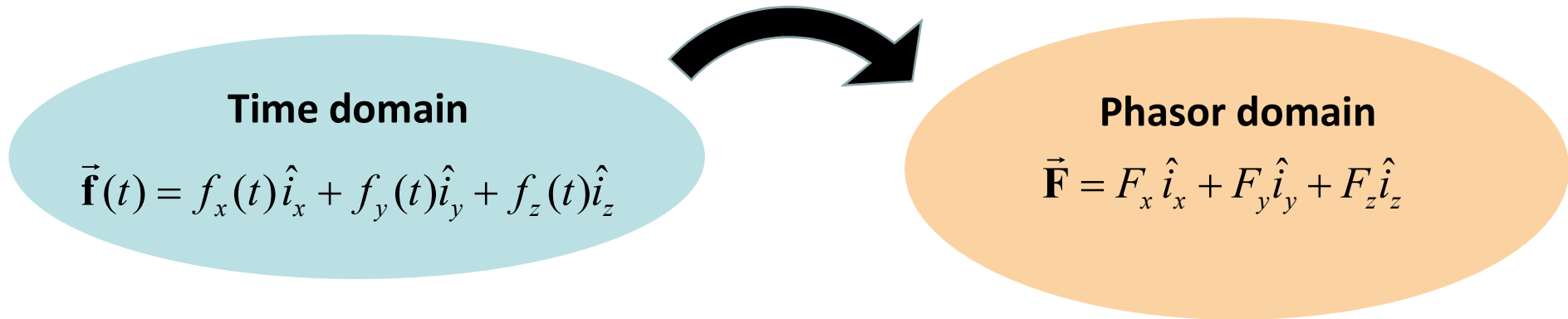
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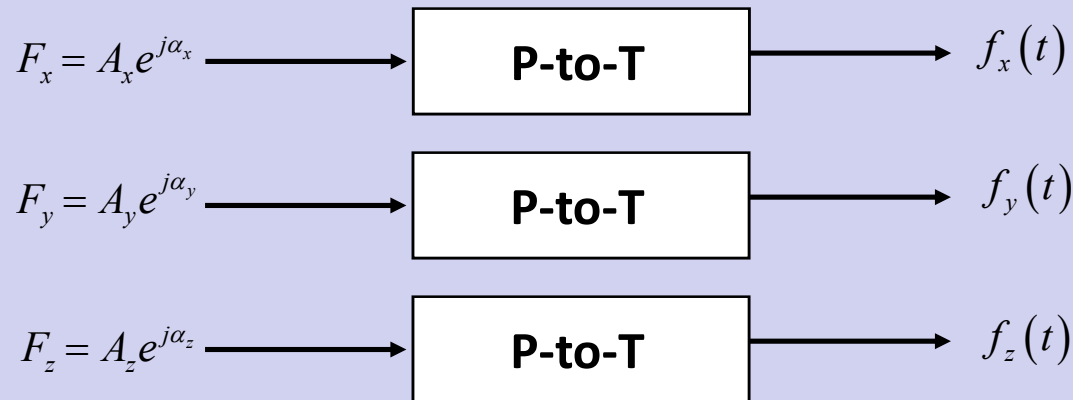
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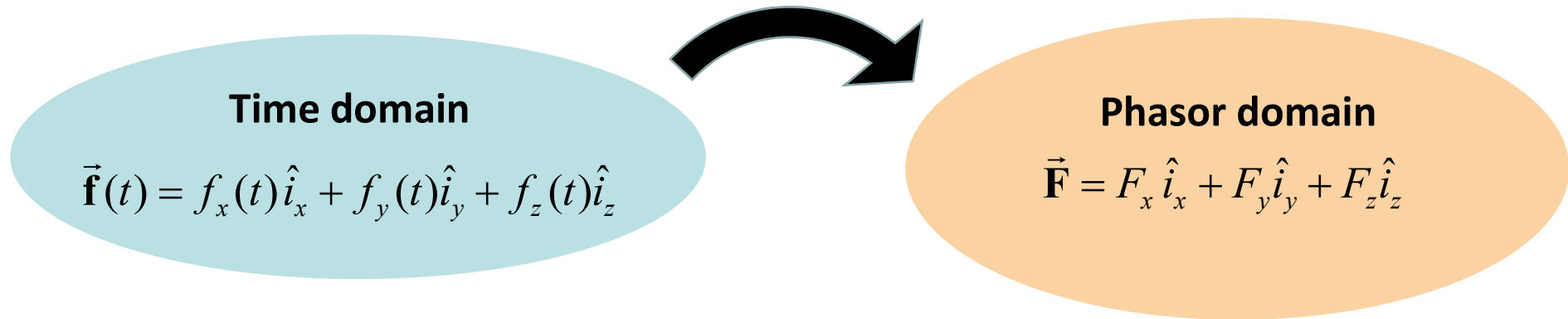
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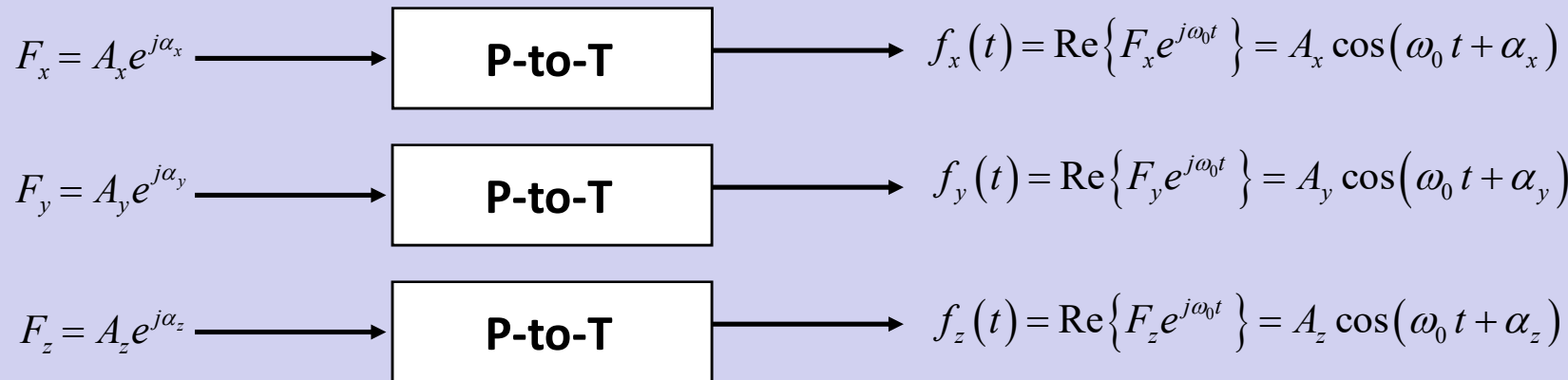
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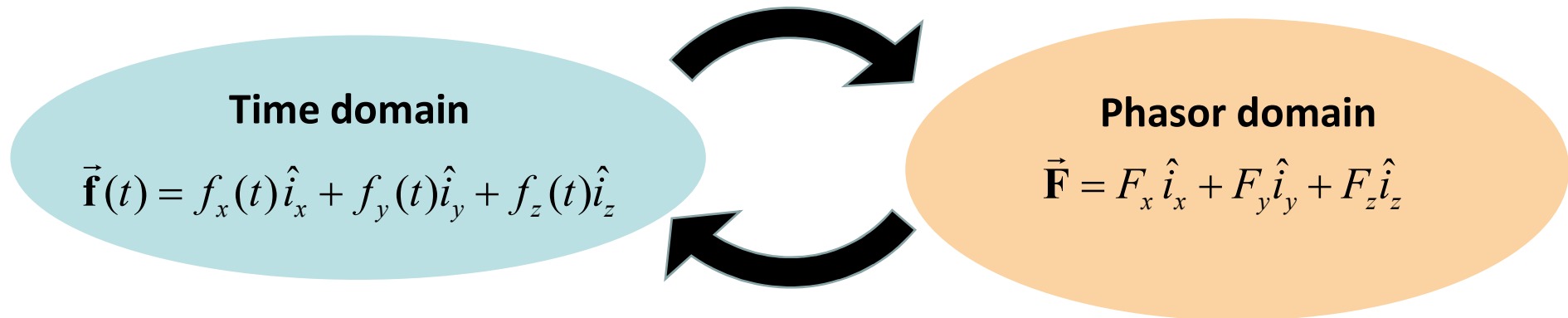
Phasors and vector functions



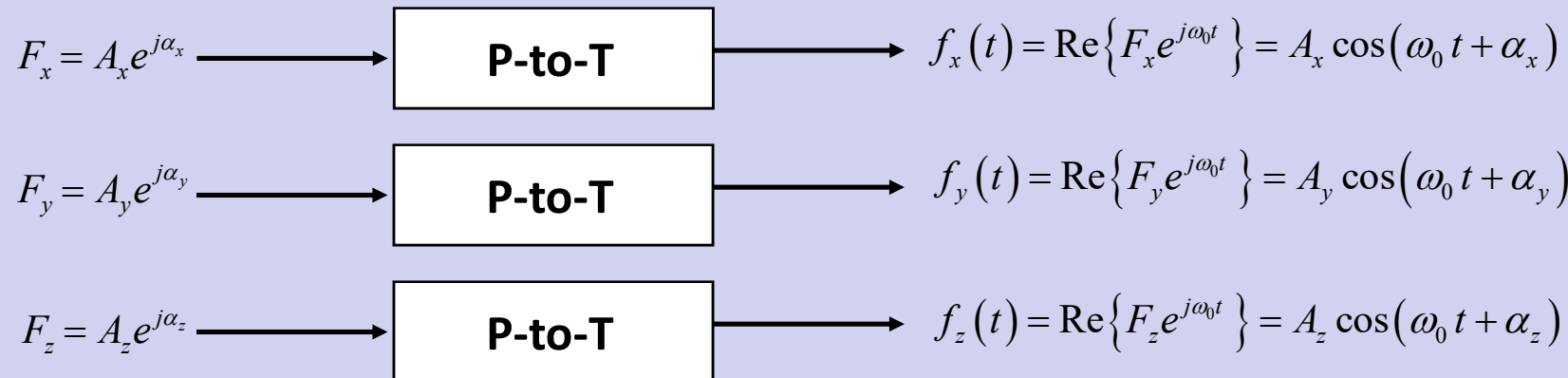
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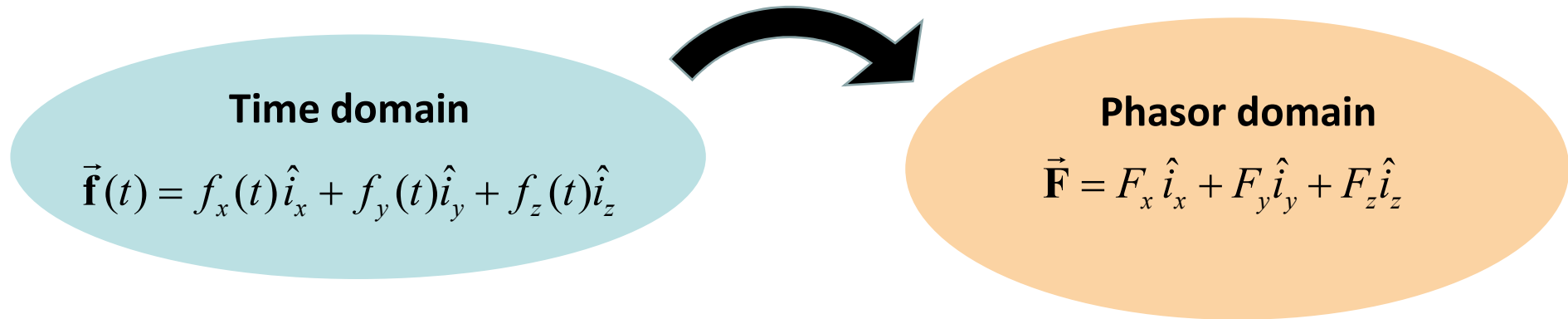
Phasors and vector functions



1) How to jump back from the Phasor domain to the Time domain

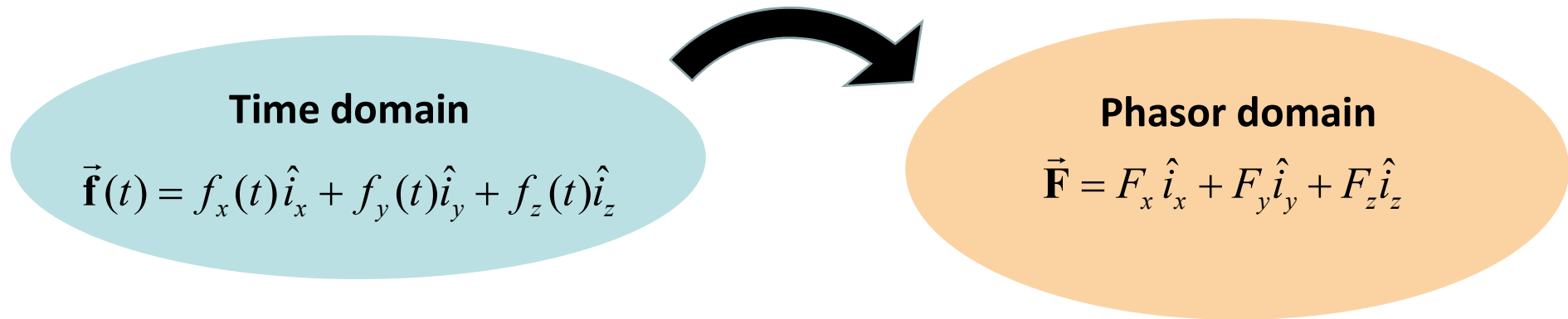


Phasors and vector functions

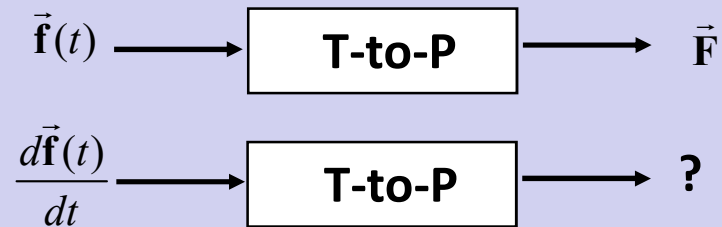


2) Time domain derivative and Phasors

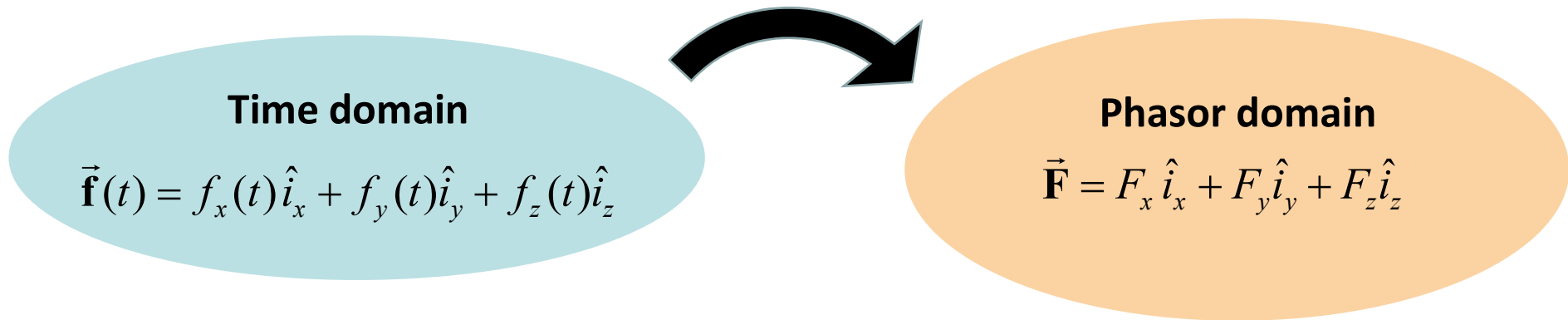
Phasors and vector functions



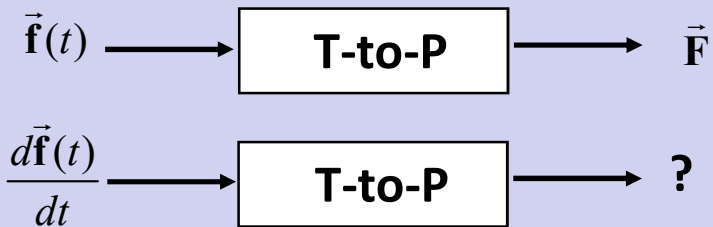
2) Time domain derivative and Phasors



Phasors and vector functions

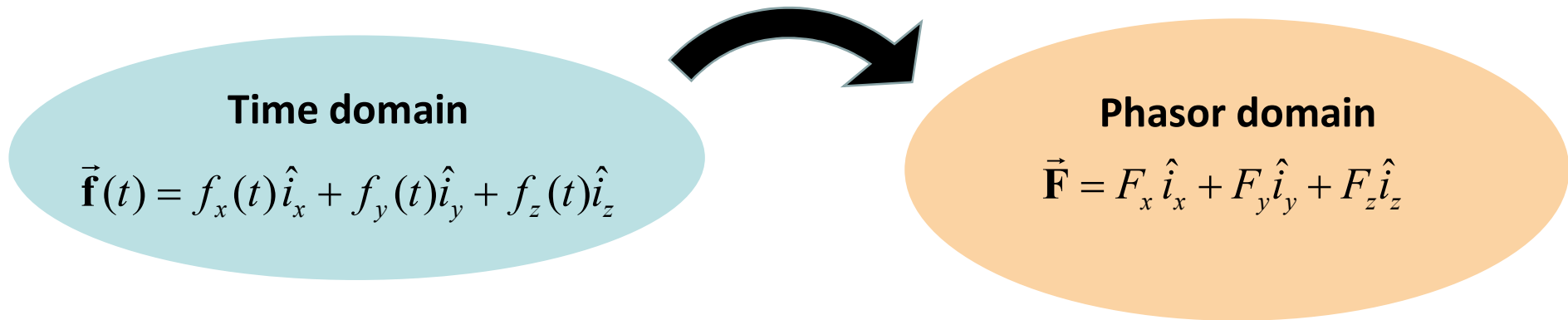


2) Time domain derivative and Phasors



$$\frac{d\vec{\mathbf{f}}(t)}{dt} = \frac{df_x(t)}{dt}\hat{i}_x + \frac{df_y(t)}{dt}\hat{i}_y + \frac{df_z(t)}{dt}\hat{i}_z$$

Phasors and vector functions



2) Time domain derivative and Phasors

$$\vec{\mathbf{f}}(t) \longrightarrow \boxed{\text{T-to-P}} \longrightarrow \vec{\mathbf{F}}$$

$$\frac{d\vec{\mathbf{f}}(t)}{dt} \longrightarrow \boxed{\text{T-to-P}} \longrightarrow ?$$

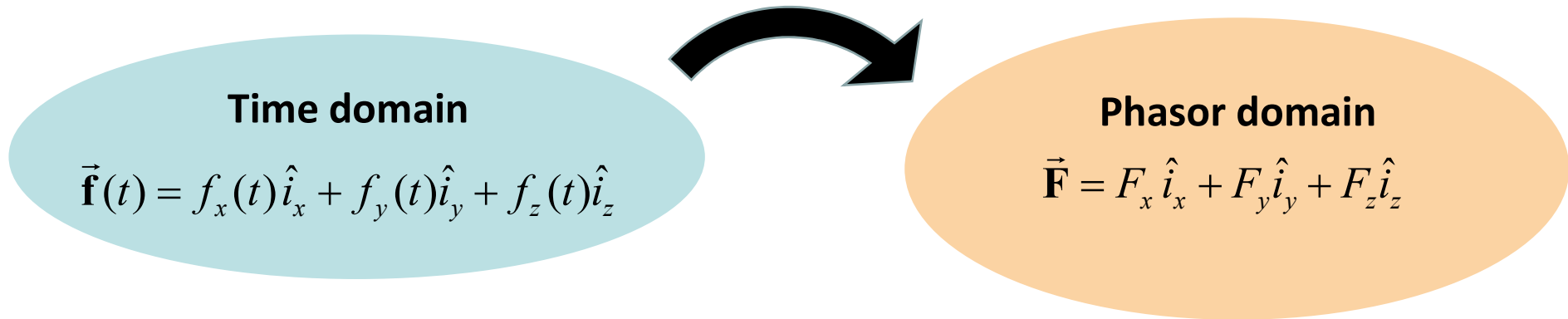
$$\frac{d\vec{\mathbf{f}}(t)}{dt} = \frac{df_x(t)}{dt}\hat{i}_x + \frac{df_y(t)}{dt}\hat{i}_y + \frac{df_z(t)}{dt}\hat{i}_z$$

$$\frac{df_x(t)}{dt} \longrightarrow \boxed{\text{T-to-P}} \longrightarrow ?$$

$$\frac{df_y(t)}{dt} \longrightarrow \boxed{\text{T-to-P}} \longrightarrow ?$$

$$\frac{df_z(t)}{dt} \longrightarrow \boxed{\text{T-to-P}} \longrightarrow ?$$

Phasors and vector functions



2) Time domain derivative and Phasors

$$\vec{\mathbf{f}}(t) \longrightarrow \boxed{\text{T-to-P}} \longrightarrow \vec{\mathbf{F}}$$

$$\frac{d\vec{\mathbf{f}}(t)}{dt} \longrightarrow \boxed{\text{T-to-P}} \longrightarrow ?$$

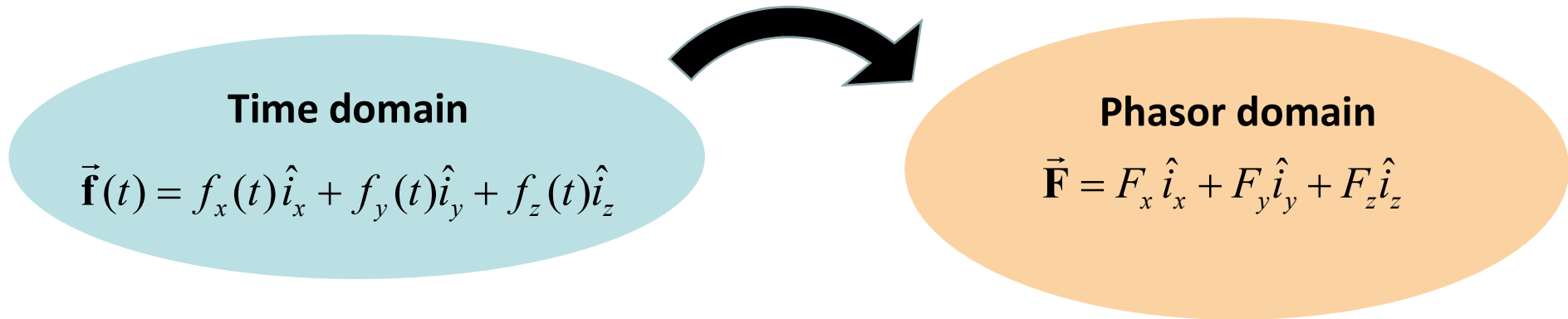
$$\frac{d\vec{\mathbf{f}}(t)}{dt} = \frac{df_x(t)}{dt}\hat{i}_x + \frac{df_y(t)}{dt}\hat{i}_y + \frac{df_z(t)}{dt}\hat{i}_z$$

$$\frac{df_x(t)}{dt} \longrightarrow \boxed{\text{T-to-P}} \longrightarrow j\omega_0 F_x$$

$$\frac{df_y(t)}{dt} \longrightarrow \boxed{\text{T-to-P}} \longrightarrow j\omega_0 F_y$$

$$\frac{df_z(t)}{dt} \longrightarrow \boxed{\text{T-to-P}} \longrightarrow j\omega_0 F_z$$

Phasors and vector functions



2) Time domain derivative and Phasors

$$\vec{\mathbf{f}}(t) \longrightarrow \boxed{\text{T-to-P}} \longrightarrow \vec{\mathbf{F}}$$

$$\frac{d\vec{\mathbf{f}}(t)}{dt} \longrightarrow \boxed{\text{T-to-P}} \longrightarrow j\omega_0 F_x \hat{i}_x + j\omega_0 F_y \hat{i}_y + j\omega_0 F_z \hat{i}_z$$

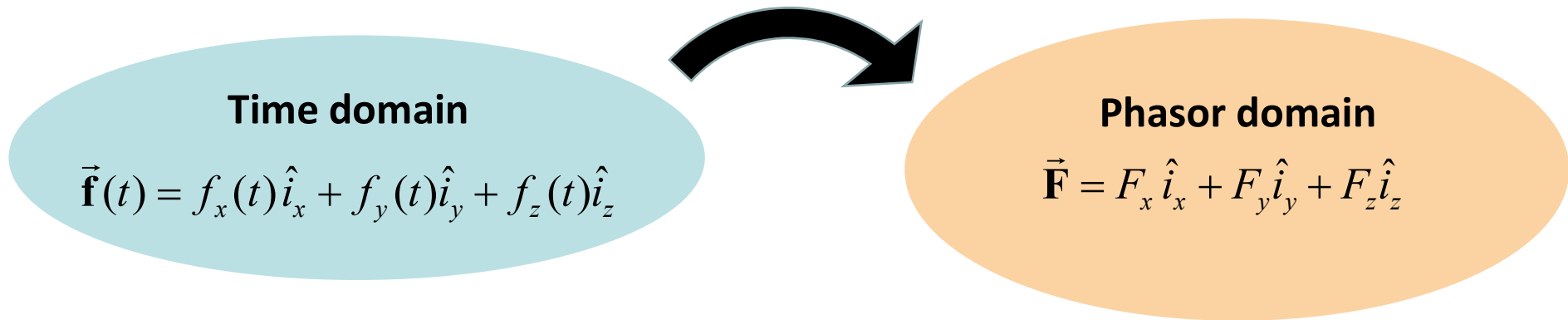
$$\frac{d\vec{\mathbf{f}}(t)}{dt} = \frac{df_x(t)}{dt}\hat{i}_x + \frac{df_y(t)}{dt}\hat{i}_y + \frac{df_z(t)}{dt}\hat{i}_z$$

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$$\frac{df_z(t)}{dt} \longrightarrow \boxed{\text{T-to-P}} \longrightarrow j\omega_0 F_z$$

Phasors and vector functions



2) Time domain derivative and Phasors

$$\vec{\mathbf{f}}(t) \longrightarrow \boxed{\text{T-to-P}} \longrightarrow \vec{\mathbf{F}}$$

$$\frac{d\vec{\mathbf{f}}(t)}{dt} \longrightarrow \boxed{\text{T-to-P}} \longrightarrow j\omega_0\vec{\mathbf{F}} = j\omega_0F_x\hat{i}_x + j\omega_0F_y\hat{i}_y + j\omega_0F_z\hat{i}_z$$

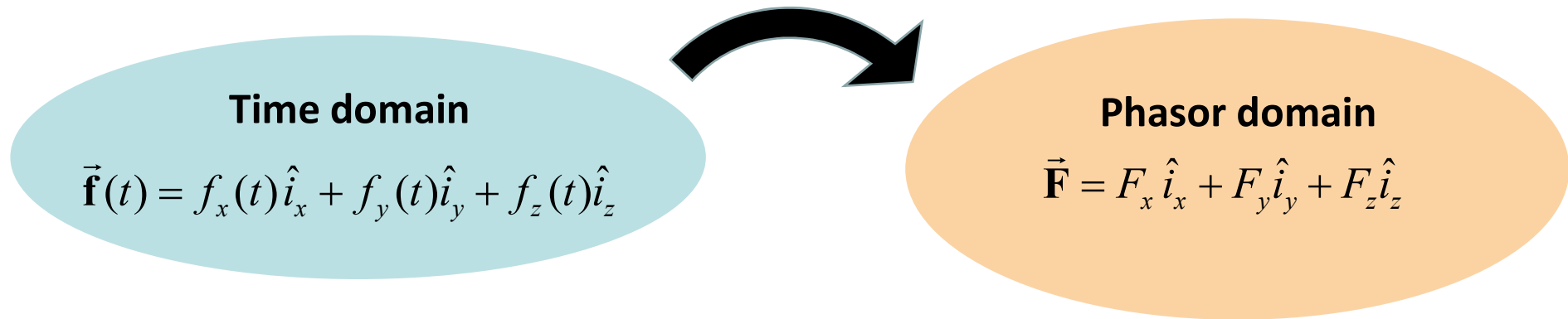
$$\frac{d\vec{\mathbf{f}}(t)}{dt} = \frac{df_x(t)}{dt}\hat{i}_x + \frac{df_y(t)}{dt}\hat{i}_y + \frac{df_z(t)}{dt}\hat{i}_z$$

$$\frac{df_x(t)}{dt} \longrightarrow \boxed{\text{T-to-P}} \longrightarrow j\omega_0F_x$$

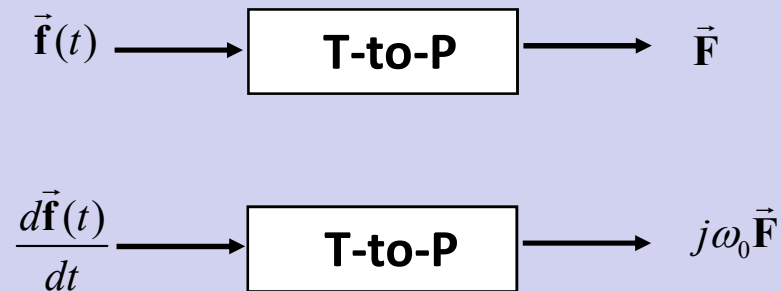
$$\frac{df_y(t)}{dt} \longrightarrow \boxed{\text{T-to-P}} \longrightarrow j\omega_0F_y$$

$$\frac{df_z(t)}{dt} \longrightarrow \boxed{\text{T-to-P}} \longrightarrow j\omega_0F_z$$

Phasors and vector functions



2) Time domain derivative and Phasors



Phasors

- Phasors and functions of n variables
- Phasors and vector functions
- **Phasors and vector functions of n variables**

- 1) How to jump back from the Phasor domain to the Time domain**
- 2) Time domain derivative and Phasors**

Phasors and vector functions of n variables

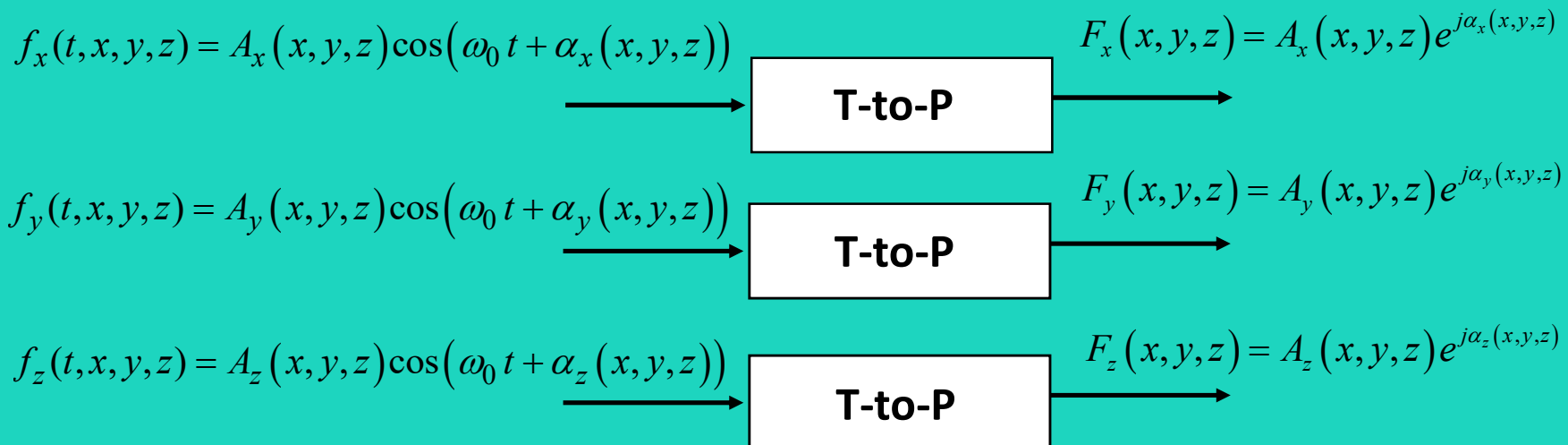
Time domain

$$\vec{\mathbf{f}}(t, x, y, z) = f_x(t, x, y, z)\hat{i}_x + f_y(t, x, y, z)\hat{i}_y + f_z(t, x, y, z)\hat{i}_z$$



Phasor domain

$$\vec{\mathbf{F}}(x, y, z) = F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z$$



Phasors and vector functions of n variables

Time domain

$$\vec{\mathbf{f}}(t, x, y, z) = f_x(t, x, y, z)\hat{i}_x + f_y(t, x, y, z)\hat{i}_y + f_z(t, x, y, z)\hat{i}_z$$



Phasor domain

$$\vec{\mathbf{F}}(x, y, z) = F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z$$

$$\vec{\mathbf{f}}(t, x, y, z) \longrightarrow \boxed{\text{T-to-P}} \longrightarrow \vec{\mathbf{F}}(x, y, z)$$

$$\begin{aligned}\vec{\mathbf{f}}(t, x, y, z) &= f_x(t, x, y, z)\hat{i}_x + f_y(t, x, y, z)\hat{i}_y + f_z(t, x, y, z)\hat{i}_z = \\ &= A_x(x, y, z)\cos(\omega_0 t + \alpha_x(x, y, z))\hat{i}_x + \\ &A_y(x, y, z)\cos(\omega_0 t + \alpha_y(x, y, z))\hat{i}_y + \\ &A_z(x, y, z)\cos(\omega_0 t + \alpha_z(x, y, z))\hat{i}_z\end{aligned}$$

$$\begin{aligned}\vec{\mathbf{F}}(x, y, z) &= F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z = \\ &= A_x(x, y, z)e^{j\alpha_x(x, y, z)}\hat{i}_x + A_y(x, y, z)e^{j\alpha_y(x, y, z)}\hat{i}_y + A_z(x, y, z)e^{j\alpha_z(x, y, z)}\hat{i}_z\end{aligned}$$

Phasors and vector functions of n variables

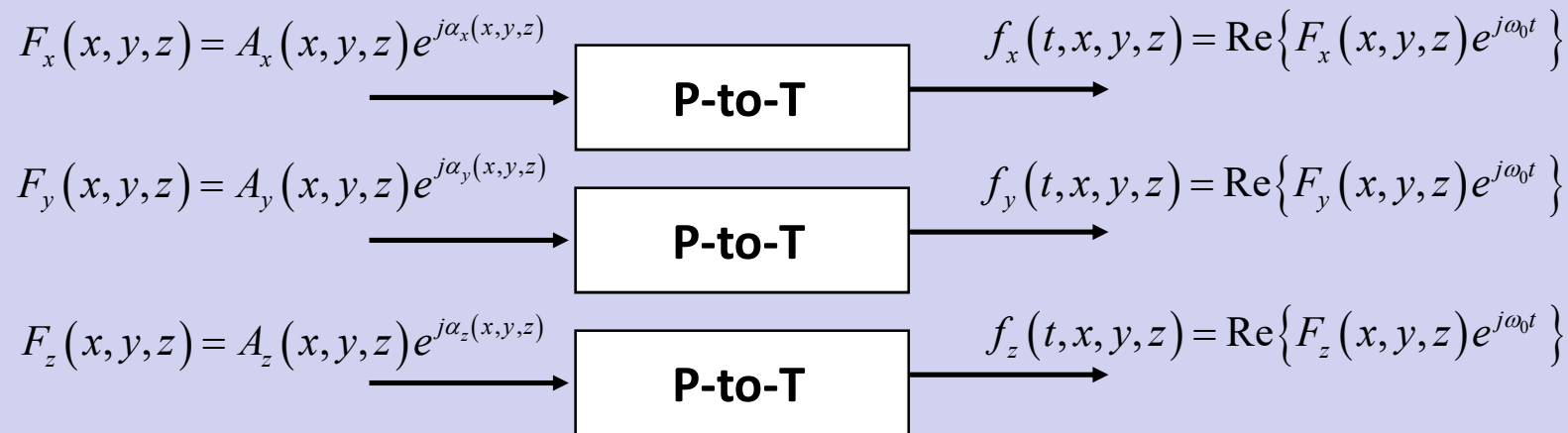
Time domain

$$\vec{\mathbf{f}}(t, x, y, z) = f_x(t, x, y, z)\hat{i}_x + f_y(t, x, y, z)\hat{i}_y + f_z(t, x, y, z)\hat{i}_z$$

Phasor domain

$$\vec{\mathbf{F}}(x, y, z) = F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z$$

1) How to jump back from the Phasor domain to the Time domain



Phasors and vector functions of n variables

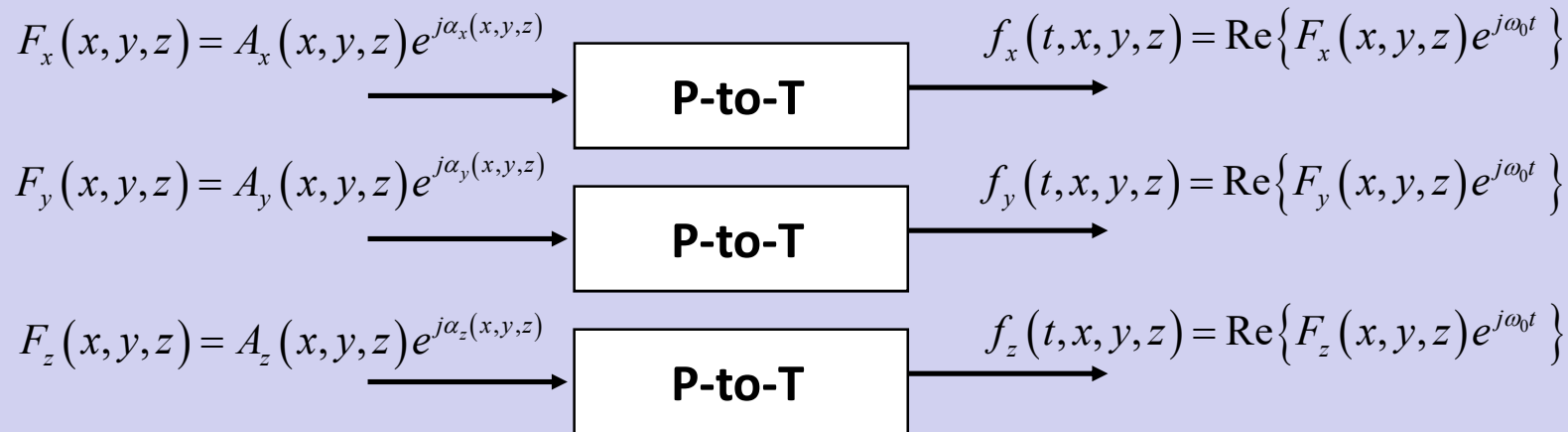
Time domain

$$\vec{\mathbf{f}}(t, x, y, z) = f_x(t, x, y, z)\hat{i}_x + f_y(t, x, y, z)\hat{i}_y + f_z(t, x, y, z)\hat{i}_z$$

Phasor domain

$$\vec{\mathbf{F}}(x, y, z) = F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z$$

1) How to jump back from the Phasor domain to the Time domain



Phasors and vector functions of n variables

Time domain

$$\vec{\mathbf{f}}(t, x, y, z) = f_x(t, x, y, z)\hat{i}_x + f_y(t, x, y, z)\hat{i}_y + f_z(t, x, y, z)\hat{i}_z$$



Phasor domain

$$\vec{\mathbf{F}}(x, y, z) = F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z$$

2) Time domain derivative and Phasors

$$\vec{\mathbf{f}}(t, x, y, z) \xrightarrow{\text{T-to-P}} \vec{\mathbf{F}}(x, y, z)$$

$$\frac{\partial \vec{\mathbf{f}}}{\partial t} \xrightarrow{\text{T-to-P}} j\omega_0 \vec{\mathbf{F}}(x, y, z) = j\omega_0 F_x(x, y, z)\hat{i}_x + j\omega_0 F_y(x, y, z)\hat{i}_y + j\omega_0 F_z(x, y, z)\hat{i}_z$$

$$\frac{\partial \vec{\mathbf{f}}(t, \vec{\mathbf{r}})}{\partial t} = \frac{\partial f_x}{\partial t}\hat{i}_x + \frac{\partial f_y}{\partial t}\hat{i}_y + \frac{\partial f_z}{\partial t}\hat{i}_z$$

$$\frac{\partial f_x(t)}{\partial t} \xrightarrow{\text{T-to-P}} j\omega_0 F_x(x, y, z)$$

$$\frac{\partial f_y(t)}{\partial t} \xrightarrow{\text{T-to-P}} j\omega_0 F_y(x, y, z)$$

$$\frac{\partial f_z(t)}{\partial t} \xrightarrow{\text{T-to-P}} j\omega_0 F_z(x, y, z)$$

Phasors and vector functions of n variables

Time domain

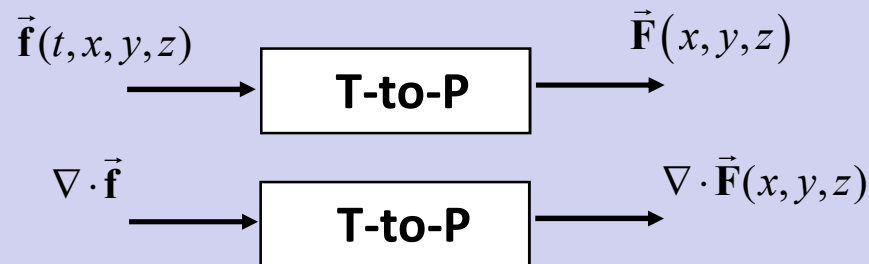
$$\vec{\mathbf{f}}(t, x, y, z) = f_x(t, x, y, z)\hat{i}_x + f_y(t, x, y, z)\hat{i}_y + f_z(t, x, y, z)\hat{i}_z$$



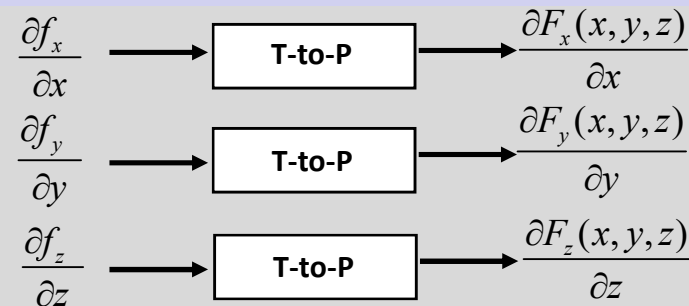
Phasor domain

$$\vec{\mathbf{F}}(x, y, z) = F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z$$

2) Time domain derivative and Phasors



$$\nabla \cdot \vec{\mathbf{f}}(t, \vec{\mathbf{r}}) = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z}$$



Phasors and vector functions of n variables

Time domain

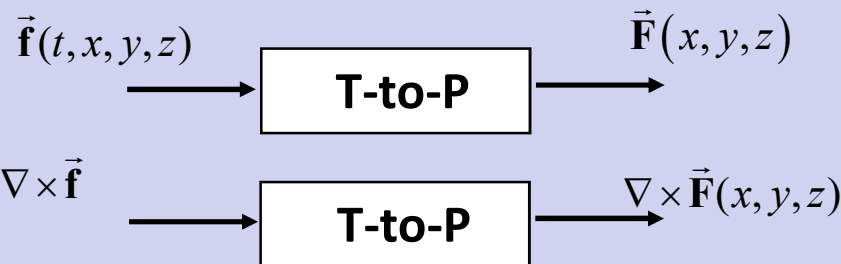
$$\vec{\mathbf{f}}(t, x, y, z) = f_x(t, x, y, z)\hat{i}_x + f_y(t, x, y, z)\hat{i}_y + f_z(t, x, y, z)\hat{i}_z$$



Phasor domain

$$\vec{\mathbf{F}}(x, y, z) = F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z$$

2) Time domain derivative and Phasors



$$\nabla \times \vec{\mathbf{f}}(t, \vec{\mathbf{r}}) = \left(\frac{\partial f_z}{\partial y} - \frac{\partial f_y}{\partial z} \right) \hat{i}_x + \left(\frac{\partial f_x}{\partial z} - \frac{\partial f_z}{\partial x} \right) \hat{i}_y + \left(\frac{\partial f_y}{\partial x} - \frac{\partial f_x}{\partial y} \right) \hat{i}_z$$

Phasors and vector functions of n variables

Time domain

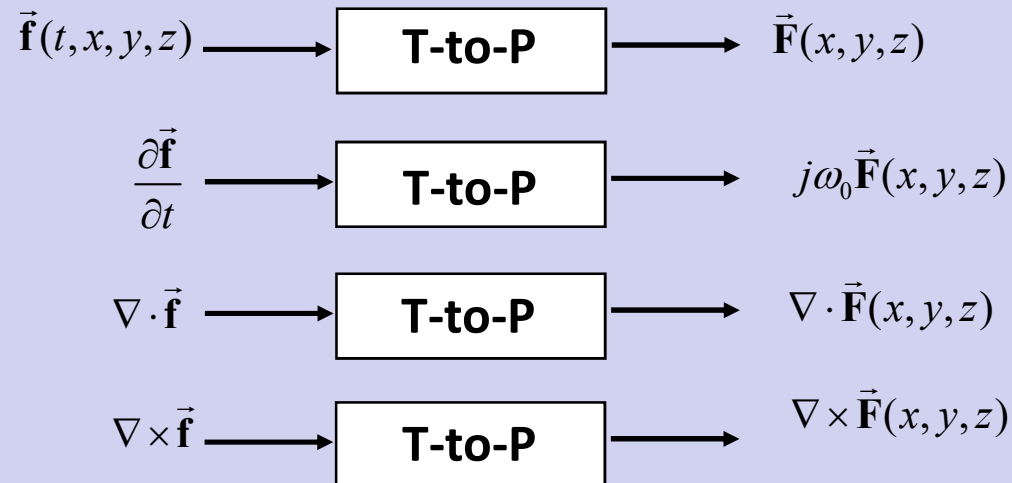
$$\vec{\mathbf{f}}(t, x, y, z) = f_x(t, x, y, z)\hat{i}_x + f_y(t, x, y, z)\hat{i}_y + f_z(t, x, y, z)\hat{i}_z$$



Phasor domain

$$\vec{\mathbf{F}}(x, y, z) = F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z$$

2) Time domain derivative and Phasors



Phasors and vector functions of n variables

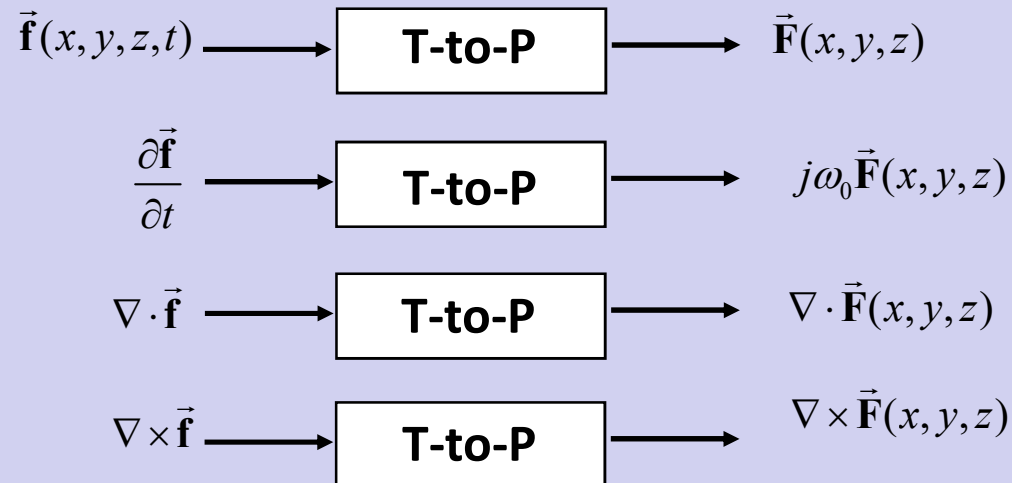
Time domain

$$\vec{\mathbf{f}}(x, y, z, t) = f_x(x, y, z, t)\hat{i}_x + f_y(x, y, z, t)\hat{i}_y + f_z(x, y, z, t)\hat{i}_z$$

Phasor domain

$$\vec{\mathbf{F}}(x, y, z) = F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z$$

2) Time domain derivative and Phasors





Maxwell equations

Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$



Maxwell equations

Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

Phasor domain





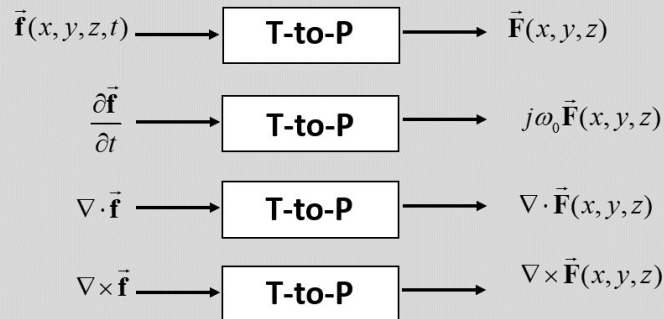
Maxwell equations

Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

Phasor domain





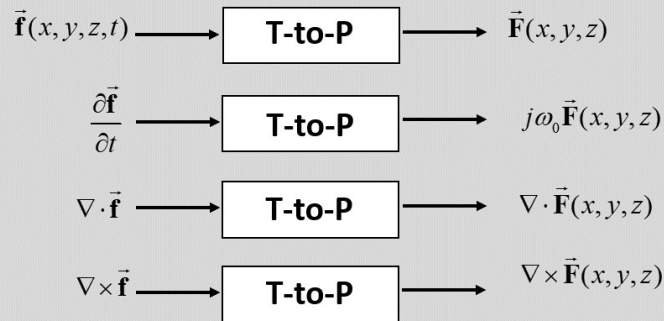
Maxwell equations

Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

Phasor domain





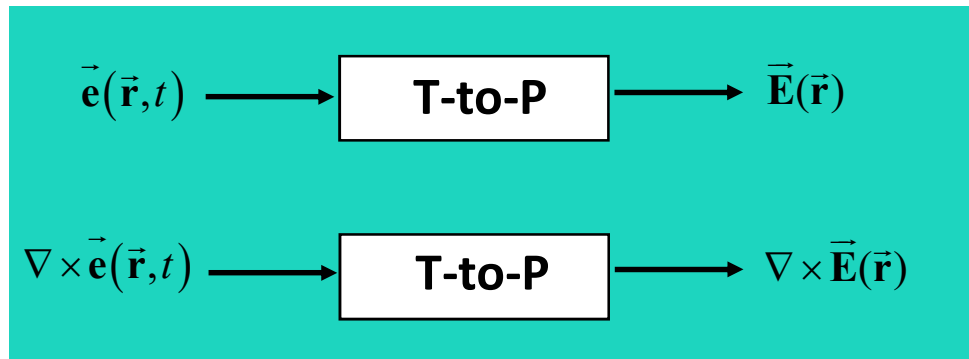
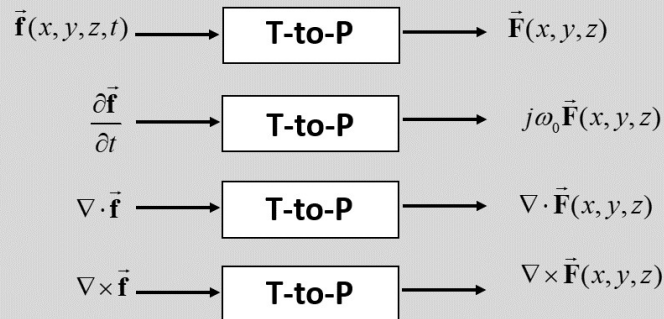
Maxwell equations

Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

Phasor domain





Maxwell equations

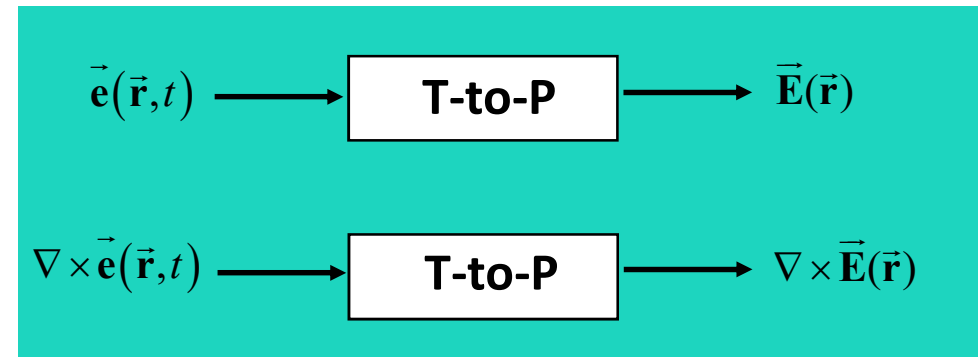
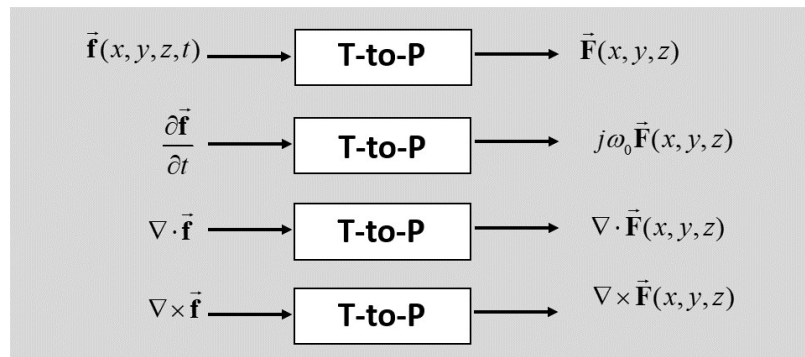
Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) \\ \vdots \end{array} \right.$$





Maxwell equations

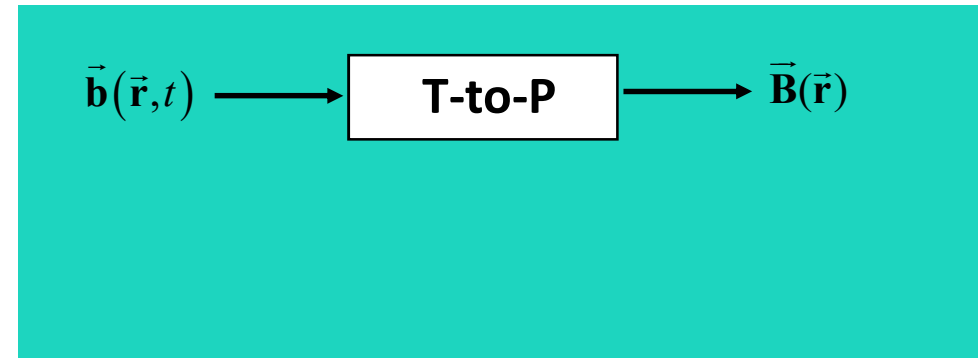
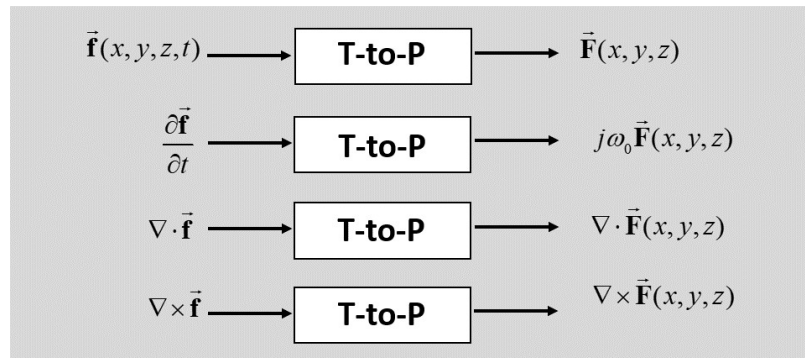
Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) \\ \vdots \end{array} \right.$$





Maxwell equations

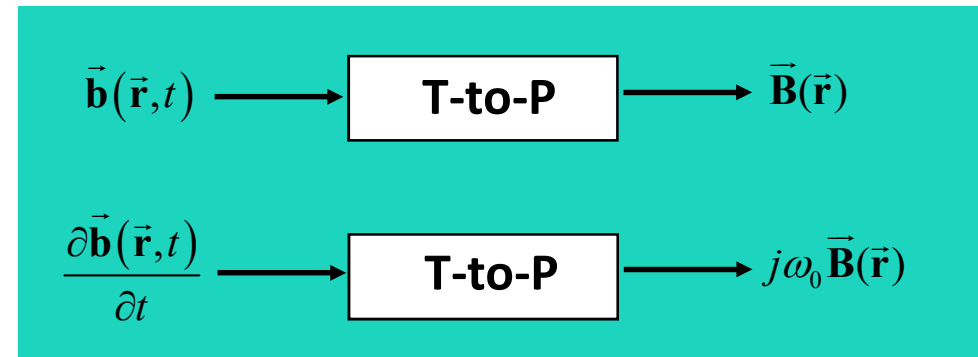
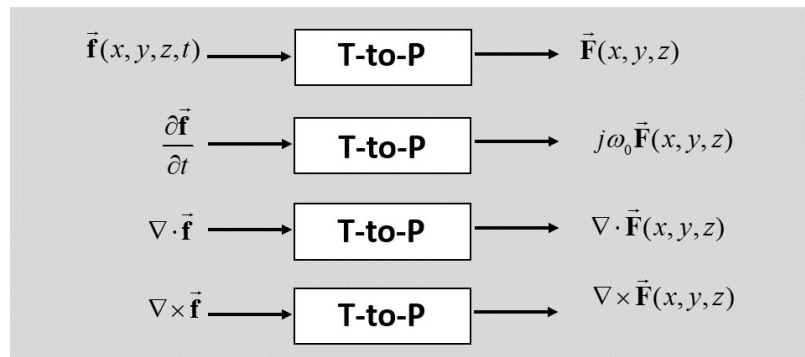
Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) \\ \vdots \end{array} \right.$$





Maxwell equations

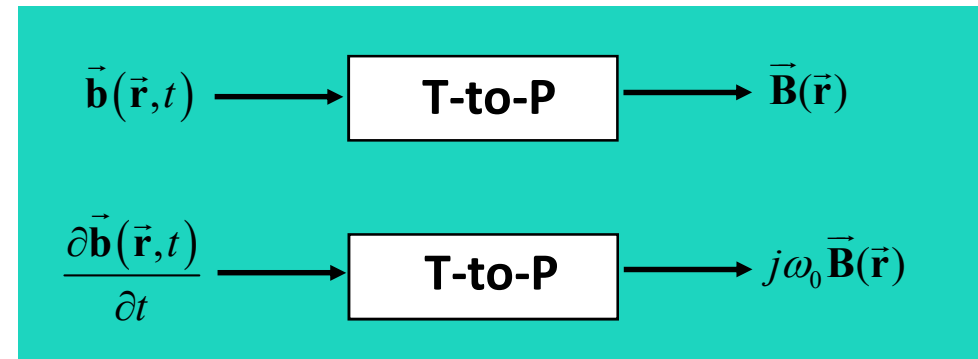
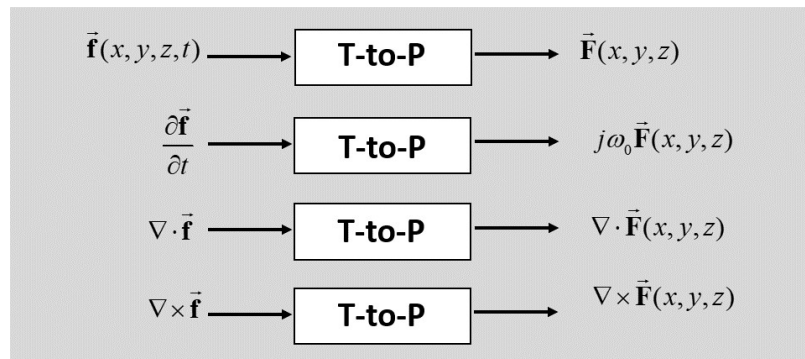
Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \end{array} \right.$$





Maxwell equations

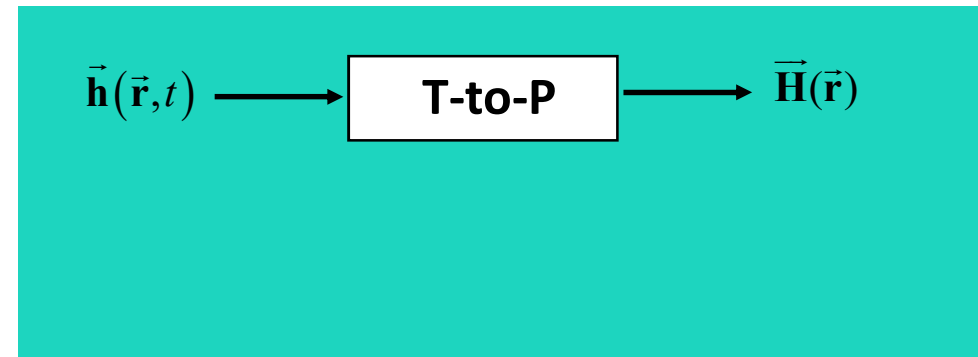
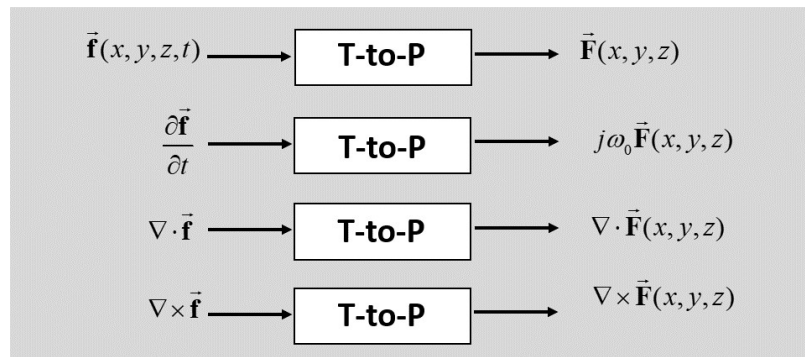
Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \end{array} \right.$$





Maxwell equations

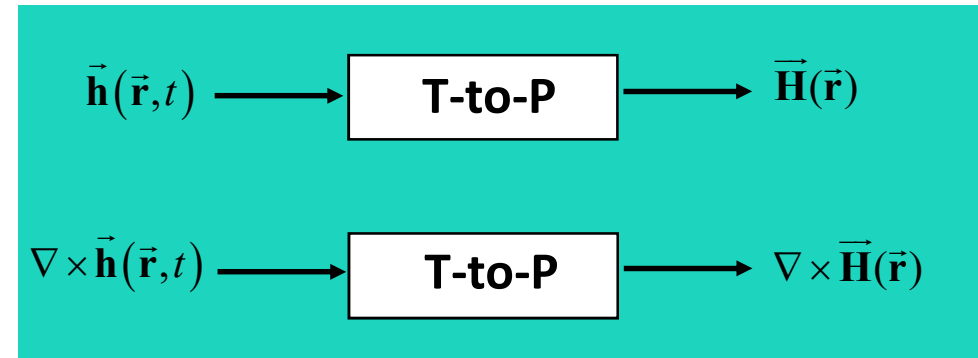
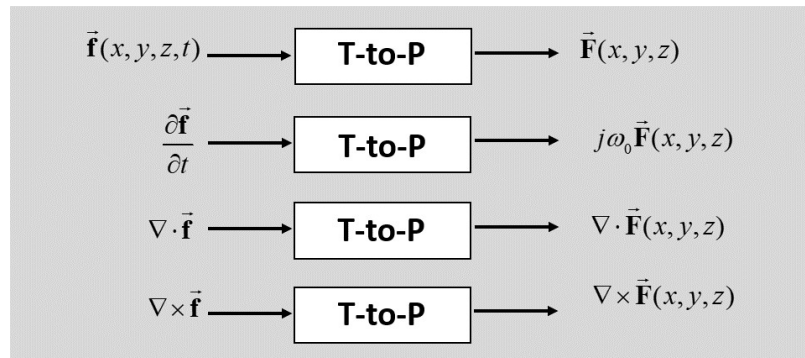
Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \end{array} \right.$$





Maxwell equations

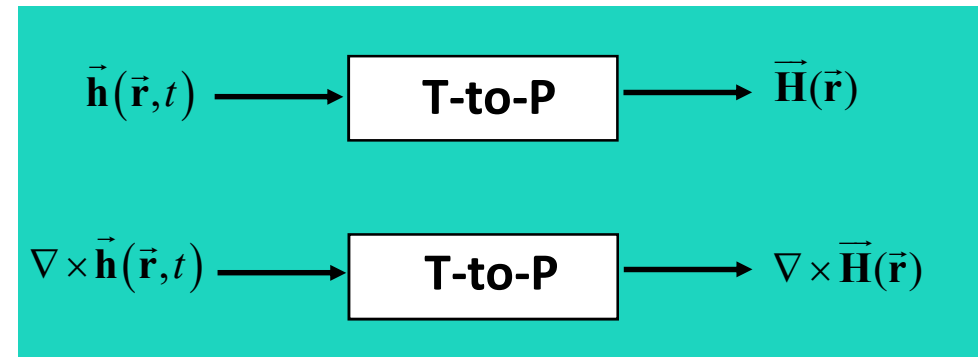
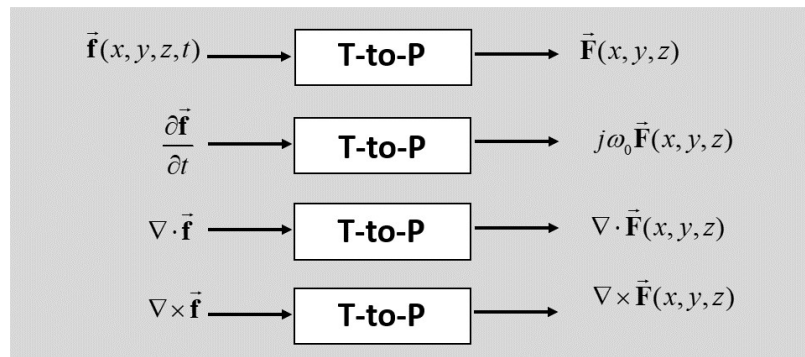
Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) \end{array} \right.$$





Maxwell equations

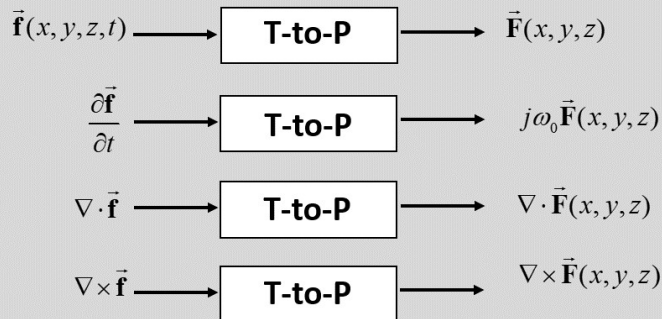
Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) \end{array} \right.$$





Maxwell equations

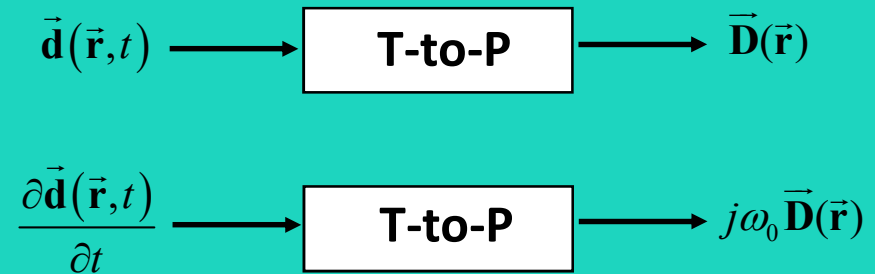
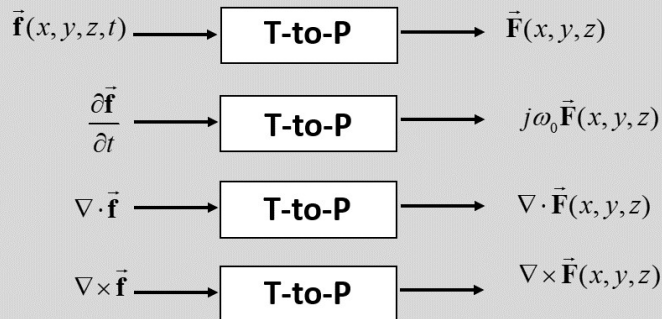
Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) \end{array} \right.$$





Maxwell equations

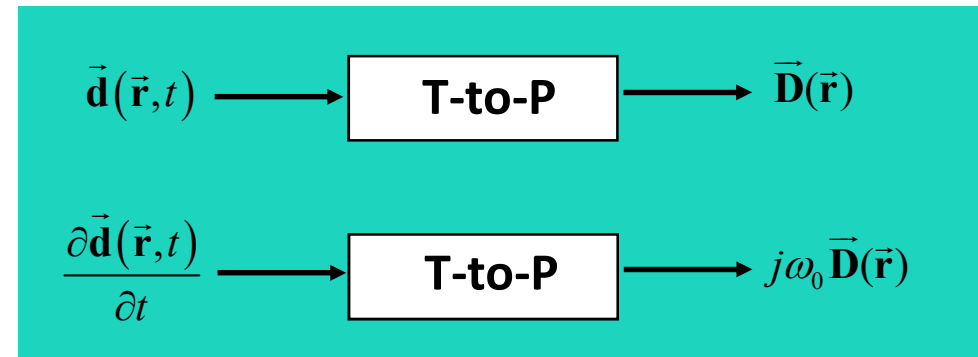
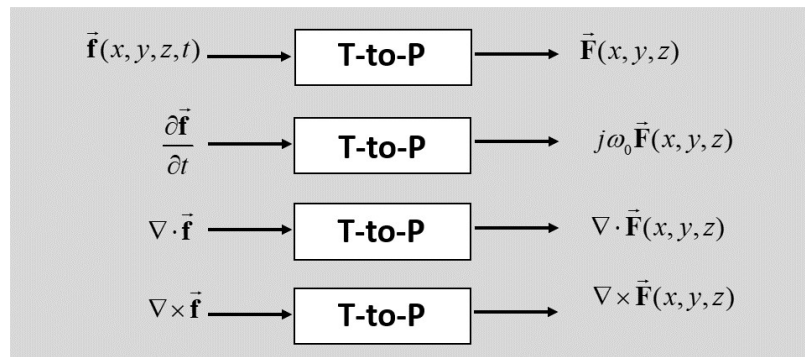
Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) \end{array} \right.$$





Maxwell equations

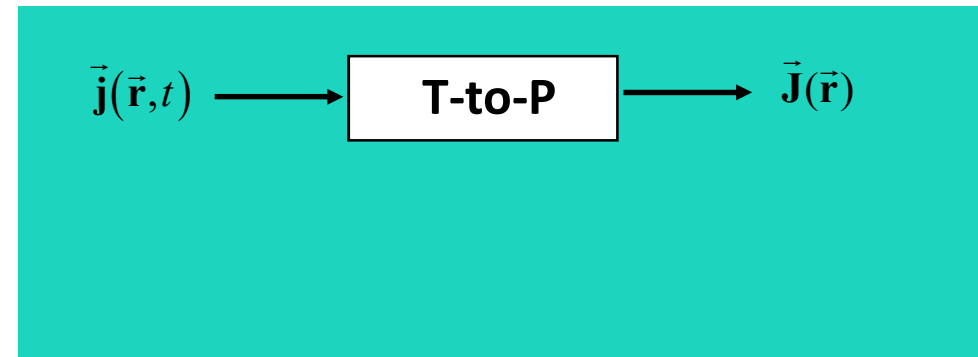
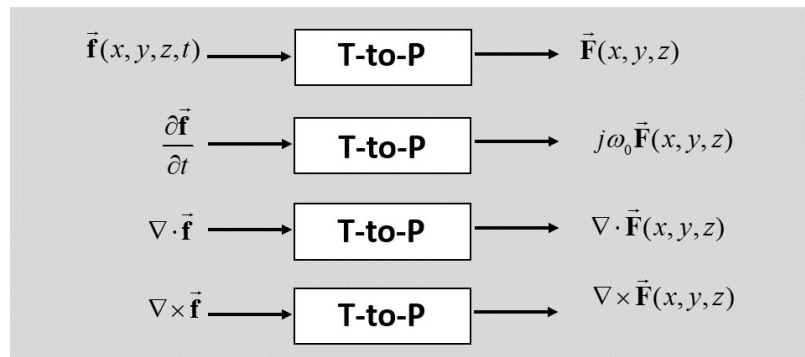
Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}) = -j\omega_0 \vec{B}(\vec{r}) \\ \nabla \times \vec{H}(\vec{r}) = j\omega_0 \vec{D}(\vec{r}) \end{array} \right.$$





Maxwell equations

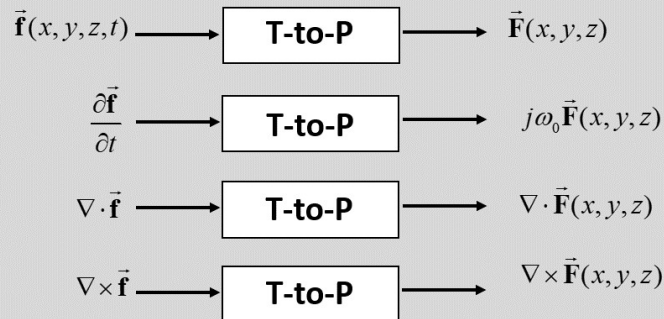
Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) \end{array} \right.$$





Maxwell equations

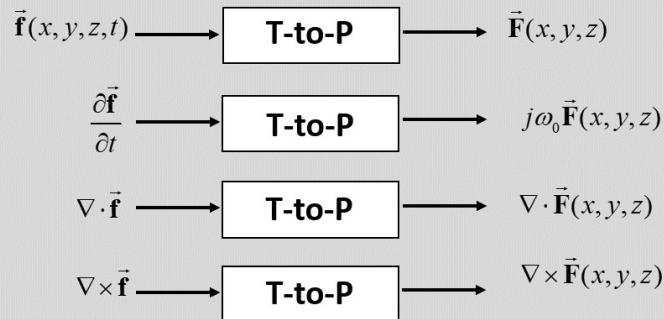
Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) \end{array} \right.$$





Maxwell equations

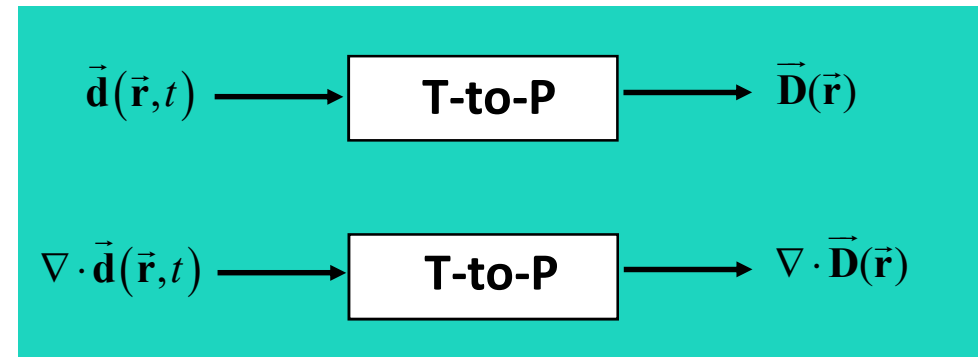
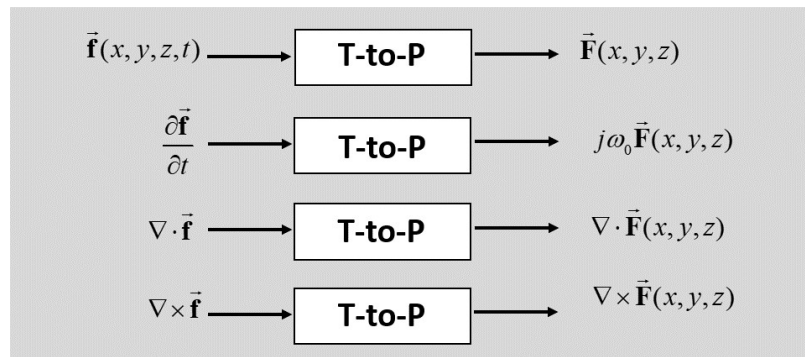
Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) \end{array} \right.$$





Maxwell equations

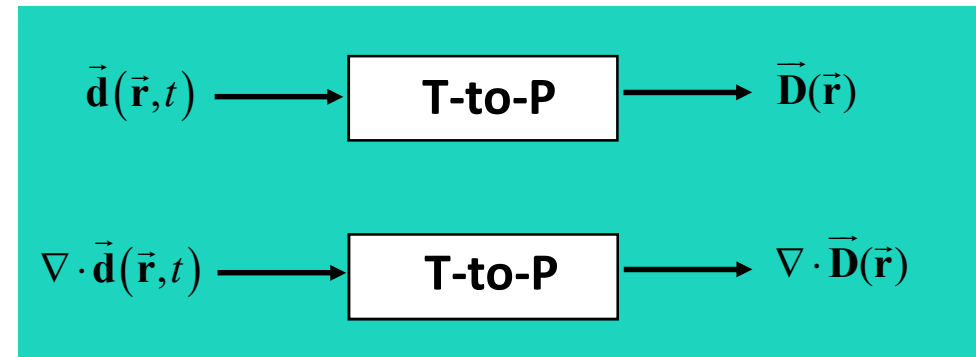
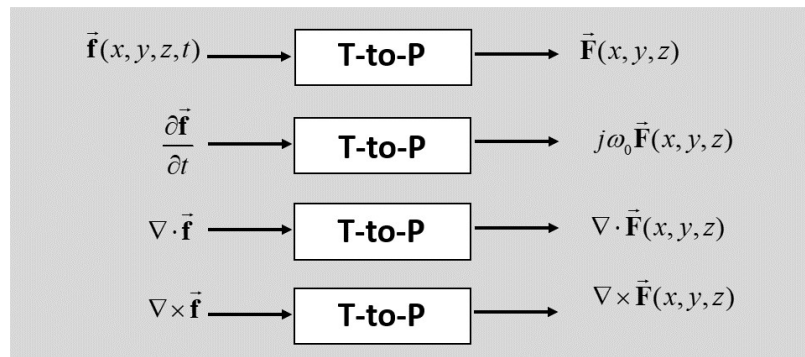
Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}) \end{array} \right.$$





Maxwell equations

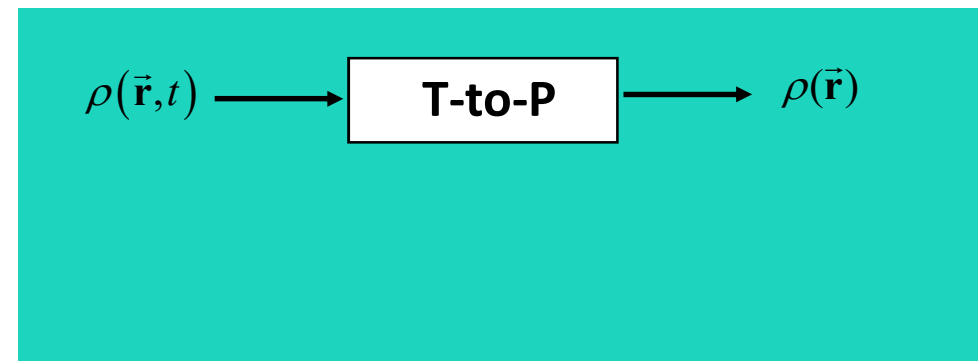
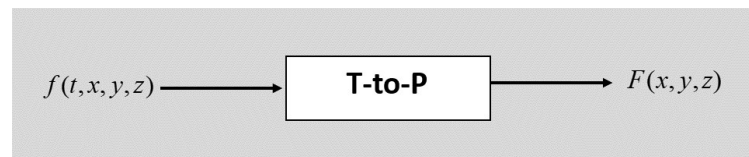
Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}) \end{array} \right.$$





Maxwell equations

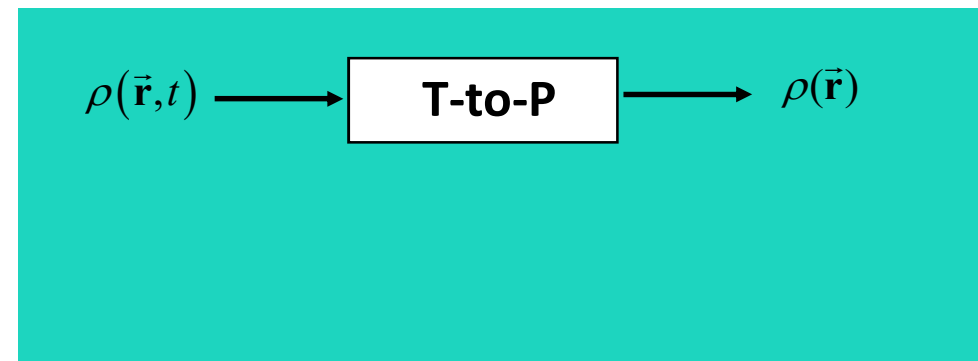
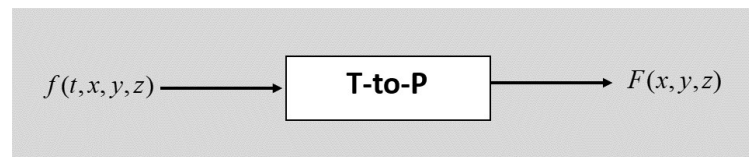
Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) \end{array} \right.$$





Maxwell equations

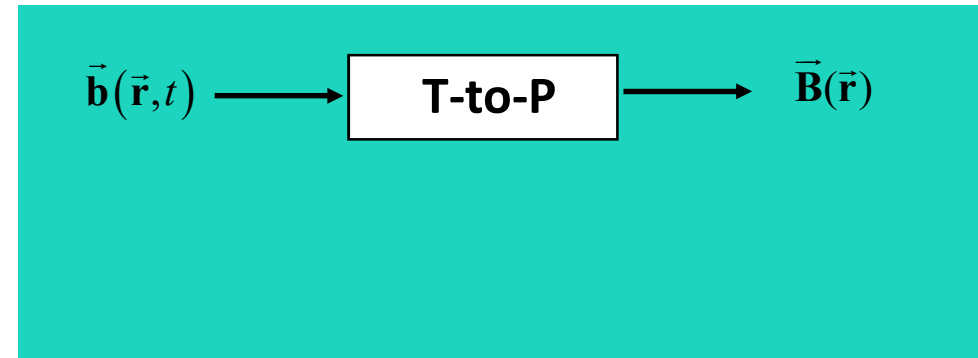
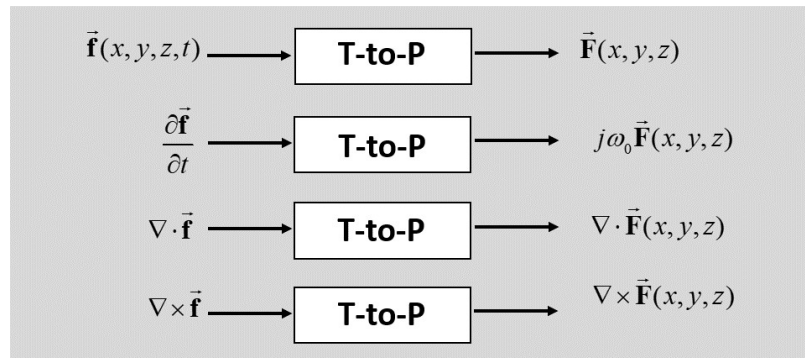
Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}) = -j\omega_0 \vec{B}(\vec{r}) \\ \nabla \times \vec{H}(\vec{r}) = j\omega_0 \vec{D}(\vec{r}) + \vec{J}(\vec{r}) \\ \nabla \cdot \vec{D}(\vec{r}) = \rho(\vec{r}) \end{array} \right.$$





Maxwell equations

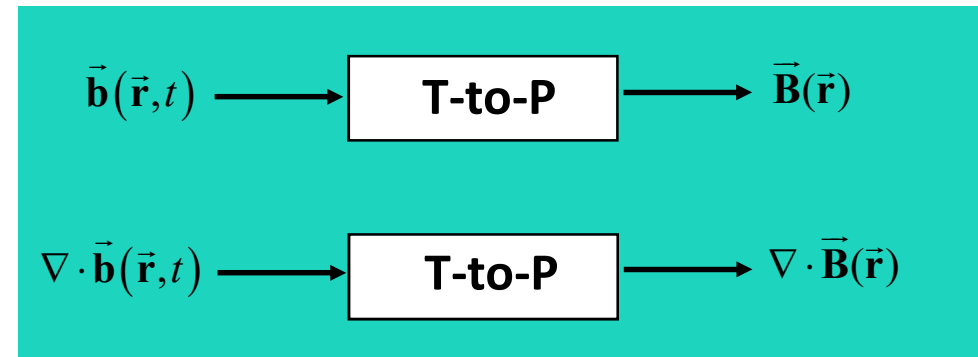
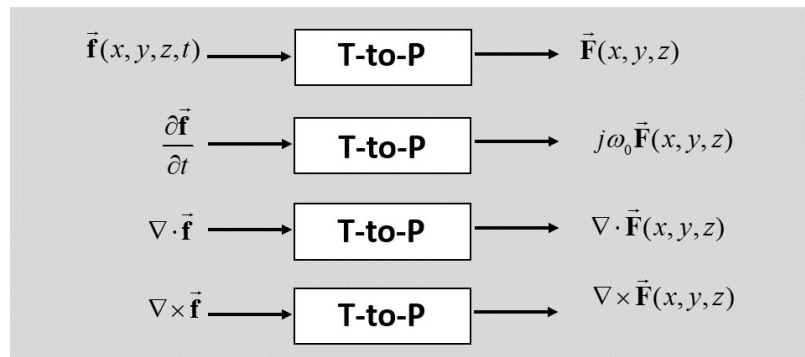
Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) \end{array} \right.$$





Maxwell equations

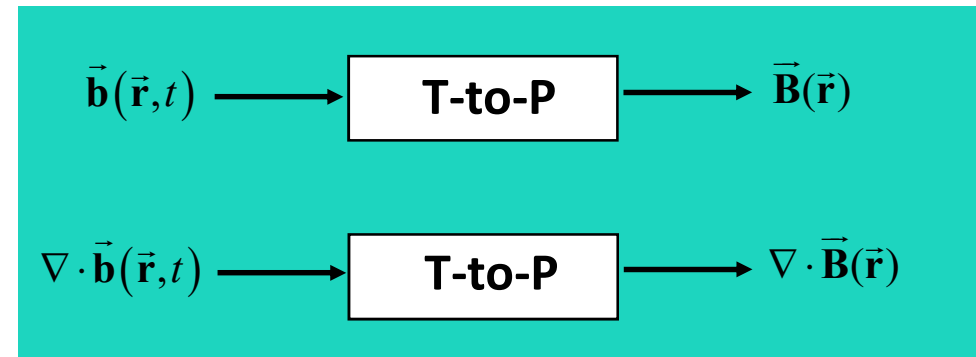
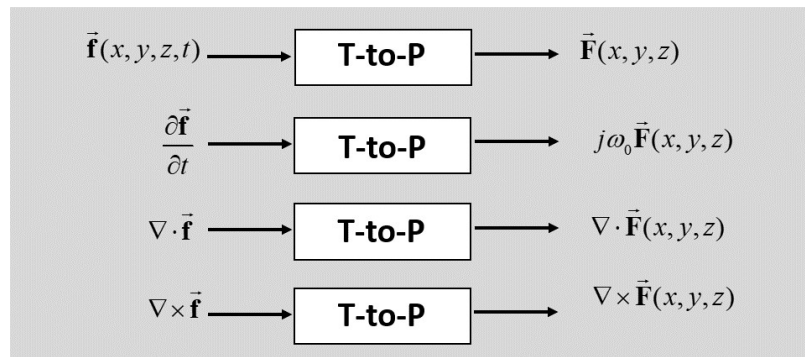
Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) = 0 \end{array} \right.$$





Maxwell equations

Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) = 0 \end{array} \right.$$



Maxwell equations

Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) = 0 \end{array} \right.$$



Maxwell equations

Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}) = -j\omega_0 \vec{B}(\vec{r}) \\ \nabla \times \vec{H}(\vec{r}) = j\omega_0 \vec{D}(\vec{r}) + \vec{J}(\vec{r}) \\ \nabla \cdot \vec{D}(\vec{r}) = \rho(\vec{r}) \\ \nabla \cdot \vec{B}(\vec{r}) = 0 \end{array} \right.$$

$\vec{e}(\vec{r}, t)$	Volt/m
$\vec{d}(\vec{r}, t)$	Coulomb/m ²
$\vec{h}(\vec{r}, t)$	Ampere/m
$\vec{b}(\vec{r}, t)$	Weber/m ²
$\vec{j}(\vec{r}, t)$	Ampere/m ²
$\rho(\vec{r}, t)$	Coulomb/m ³



Maxwell equations

Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}) = -j\omega_0 \vec{B}(\vec{r}) \\ \nabla \times \vec{H}(\vec{r}) = j\omega_0 \vec{D}(\vec{r}) + \vec{J}(\vec{r}) \\ \nabla \cdot \vec{D}(\vec{r}) = \rho(\vec{r}) \\ \nabla \cdot \vec{B}(\vec{r}) = 0 \end{array} \right.$$

$\vec{e}(\vec{r}, t)$	Volt/m
$\vec{d}(\vec{r}, t)$	Coulomb/m ²
$\vec{h}(\vec{r}, t)$	Ampere/m
$\vec{b}(\vec{r}, t)$	Weber/m ²
$\vec{j}(\vec{r}, t)$	Ampere/m ²
$\rho(\vec{r}, t)$	Coulomb/m ³

$\vec{E}(\vec{r})$
$\vec{D}(\vec{r})$
$\vec{H}(\vec{r})$
$\vec{B}(\vec{r})$
$\vec{J}(\vec{r})$
$\rho(\vec{r})$



Maxwell equations

Time domain & Phasor domain

Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

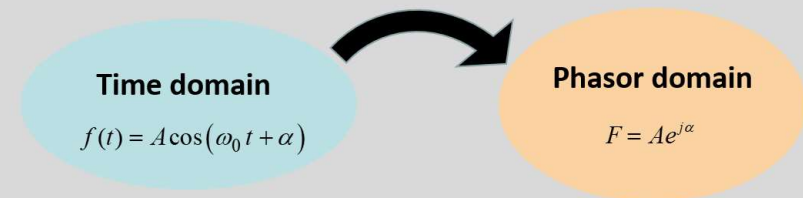
$\vec{e}(\vec{r}, t)$	Volt/m
$\vec{d}(\vec{r}, t)$	Coulomb/m ²
$\vec{h}(\vec{r}, t)$	Ampere/m
$\vec{b}(\vec{r}, t)$	Weber/m ²
$\vec{j}(\vec{r}, t)$	Ampere/m ²
$\rho(\vec{r}, t)$	Coulomb/m ³

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}) = -j\omega_0 \vec{B}(\vec{r}) \\ \nabla \times \vec{H}(\vec{r}) = j\omega_0 \vec{D}(\vec{r}) + \vec{J}(\vec{r}) \\ \nabla \cdot \vec{D}(\vec{r}) = \rho(\vec{r}) \\ \nabla \cdot \vec{B}(\vec{r}) = 0 \end{array} \right.$$

$\vec{E}(\vec{r})$

.. memo





Maxwell equations

Time domain & Phasor domain

Time domain

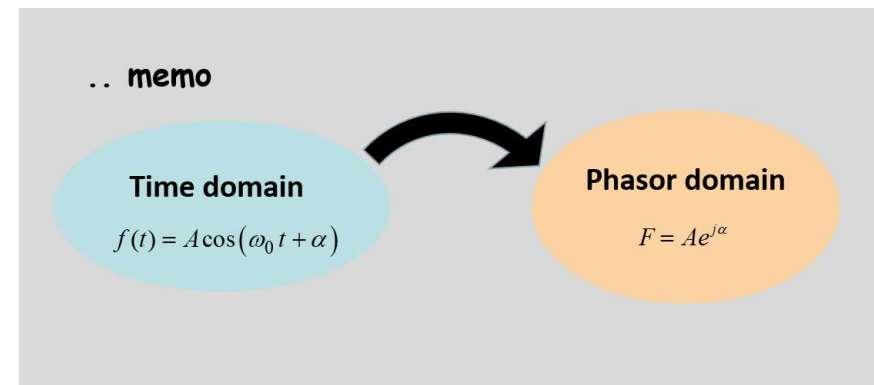
$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}) = -j\omega_0 \vec{B}(\vec{r}) \\ \nabla \times \vec{H}(\vec{r}) = j\omega_0 \vec{D}(\vec{r}) + \vec{J}(\vec{r}) \\ \nabla \cdot \vec{D}(\vec{r}) = \rho(\vec{r}) \\ \nabla \cdot \vec{B}(\vec{r}) = 0 \end{array} \right.$$

$\vec{e}(\vec{r}, t)$	Volt/m
$\vec{d}(\vec{r}, t)$	Coulomb/m ²
$\vec{h}(\vec{r}, t)$	Ampere/m
$\vec{b}(\vec{r}, t)$	Weber/m ²
$\vec{j}(\vec{r}, t)$	Ampere/m ²
$\rho(\vec{r}, t)$	Coulomb/m ³

$\vec{E}(\vec{r})$ Volt/m





Maxwell equations

Time domain & Phasor domain

Time domain

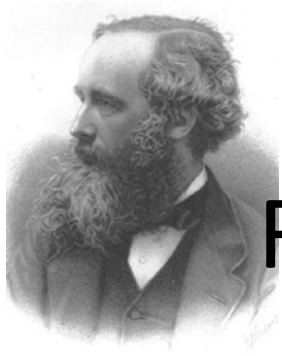
$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}) = -j\omega_0 \vec{B}(\vec{r}) \\ \nabla \times \vec{H}(\vec{r}) = j\omega_0 \vec{D}(\vec{r}) + \vec{J}(\vec{r}) \\ \nabla \cdot \vec{D}(\vec{r}) = \rho(\vec{r}) \\ \nabla \cdot \vec{B}(\vec{r}) = 0 \end{array} \right.$$

$\vec{e}(\vec{r}, t)$	Volt/m
$\vec{d}(\vec{r}, t)$	Coulomb/m ²
$\vec{h}(\vec{r}, t)$	Ampere/m
$\vec{b}(\vec{r}, t)$	Weber/m ²
$\vec{j}(\vec{r}, t)$	Ampere/m ²
$\rho(\vec{r}, t)$	Coulomb/m ³

$\vec{E}(\vec{r})$	Volt/m
$\vec{D}(\vec{r})$	Coulomb/m ²
$\vec{H}(\vec{r})$	Ampere/m
$\vec{B}(\vec{r})$	Weber/m ²
$\vec{J}(\vec{r})$	Ampere/m ²
$\rho(\vec{r})$	Coulomb/m ³



Maxwell equations

Frequency domain & Phasor domain

Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}, \omega) = -j\omega \vec{\mathbf{B}}(\vec{\mathbf{r}}, \omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}, \omega) = j\omega \vec{\mathbf{D}}(\vec{\mathbf{r}}, \omega) + \vec{\mathbf{J}}(\vec{\mathbf{r}}, \omega) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}, \omega) = \rho(\vec{\mathbf{r}}, \omega) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}, \omega) = 0 \end{array} \right.$$

Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) = 0 \end{array} \right.$$

The Maxwell equations in the Fourier domain and Phasor domain are **formally** equivalent.

However, they exhibit noticeable differences:

- i) The dimensions of the involved quantities (f.i., $\vec{\mathbf{E}}$) are different in the two domains.
- ii) In the Frequency domain ω is an independent variable, whereas in the Phasor domain ω_0 is fixed.

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics



Maxwell equations

Time domain & Phasor domain

Time domain

$$\begin{cases} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{cases}$$

$$\frac{\partial \rho(\vec{r}, t)}{\partial t} + \nabla \cdot \vec{j}(\vec{r}, t) = 0$$

Frequency domain

$$\begin{cases} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) = j\omega \vec{D}(\vec{r}, \omega) + \vec{J}(\vec{r}, \omega) \\ \nabla \cdot \vec{D}(\vec{r}, \omega) = \rho(\vec{r}, \omega) \\ \nabla \cdot \vec{B}(\vec{r}, \omega) = 0 \end{cases}$$

$$j\omega \rho(\vec{r}, \omega) + \nabla \cdot \vec{J}(\vec{r}, \omega) = 0$$

Phasor domain

$$\begin{cases} \nabla \times \vec{E}(\vec{r}) = -j\omega_0 \vec{B}(\vec{r}) \\ \nabla \times \vec{H}(\vec{r}) = j\omega_0 \vec{D}(\vec{r}) + \vec{J}(\vec{r}) \\ \nabla \cdot \vec{D}(\vec{r}) = \rho(\vec{r}) \\ \nabla \cdot \vec{B}(\vec{r}) = 0 \end{cases}$$

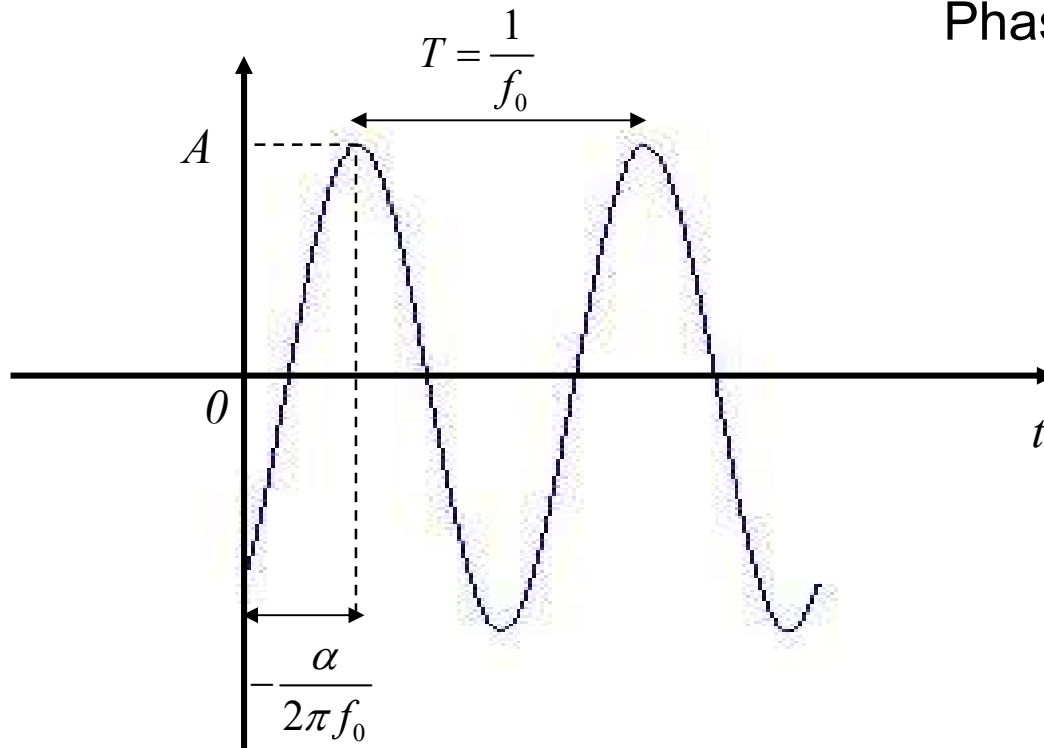
$$j\omega_0 \rho(\vec{r}) + \nabla \cdot \vec{J}(\vec{r}) = 0$$

Memo: Phasors

Phasors

$$v(t) = A \cos(2\pi f_0 t + \alpha) \longrightarrow V = Ae^{j\alpha}$$

Phasor (complex number)



Phasors

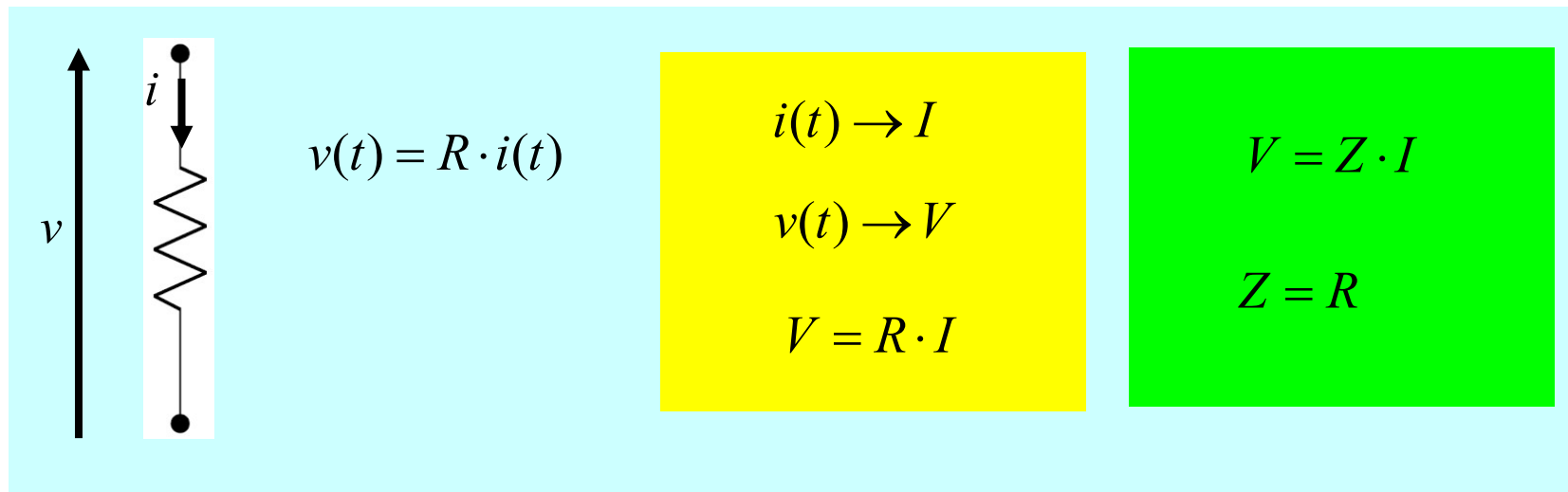
$$v(t) = A \cos(\omega_0 t + \alpha) \longrightarrow V = Ae^{j\alpha}$$

$$k \cdot v(t) \longrightarrow k \cdot V = k \cdot Ae^{j\alpha}$$

Phasors

$$v(t) = A \cos(\omega_0 t + \alpha) \longrightarrow V = Ae^{j\alpha}$$

$$k \cdot v(t) \longrightarrow k \cdot V = k \cdot Ae^{j\alpha}$$



Phasors

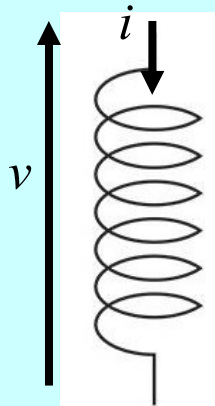
$$v(t) = A \cos(\omega_0 t + \alpha) \longrightarrow V = Ae^{j\alpha}$$

$$\frac{dv(t)}{dt} \longrightarrow j\omega_0 V = j\omega_0 Ae^{j\alpha}$$

Phasors

$$v(t) = A \cos(\omega_0 t + \alpha) \longrightarrow V = Ae^{j\alpha}$$

$$\frac{dv(t)}{dt} \longrightarrow j\omega_0 V = j\omega_0 Ae^{j\alpha}$$



$$v(t) = L \frac{di(t)}{dt}$$

$$i(t) \rightarrow I$$

$$v(t) \rightarrow V$$

$$V = j\omega_0 LI$$

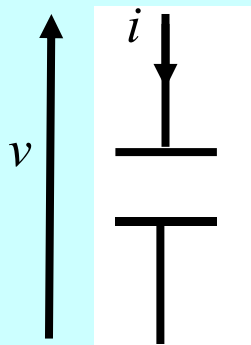
$$V = Z \cdot I$$

$$Z = j\omega_0 L$$

Phasors

$$v(t) = A \cos(\omega_0 t + \alpha) \longrightarrow V = Ae^{j\alpha}$$

$$\frac{dv(t)}{dt} \longrightarrow j\omega_0 V = j\omega_0 Ae^{j\alpha}$$



$$i(t) = C \frac{dv(t)}{dt}$$

$$i(t) \rightarrow I$$

$$v(t) \rightarrow V$$

$$I = j\omega_0 CV$$

$$V = Z \cdot I$$

$$Z = -j \frac{1}{\omega_0 C}$$

Phasors

$$v(t) = A \cos(\omega_0 t + \alpha) \longrightarrow V = Ae^{j\alpha}$$

$$i(t) = B \cos(\omega_0 t + \beta) \longrightarrow I = Be^{j\beta}$$

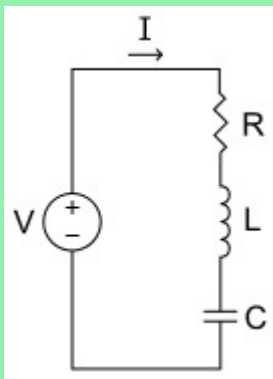
$$\langle v(t) \cdot i(t) \rangle = \frac{1}{T} \int_0^T v(t) \cdot i(t) dt = \frac{1}{2} \operatorname{Re}\{V \cdot I^*\}$$

Phasors

$$v(t) = A \cos(\omega_0 t + \alpha) \longrightarrow V = Ae^{j\alpha}$$

$$i(t) = B \cos(\omega_0 t + \beta) \longrightarrow I = Be^{j\beta}$$

$$\langle v(t) \cdot i(t) \rangle = \frac{1}{T} \int_0^T v(t) \cdot i(t) dt = \frac{1}{2} \operatorname{Re}\{V \cdot I^*\}$$



$$P = \frac{1}{2} V \cdot I^* = P_1 + jP_2$$

$$P = \frac{1}{2} V \cdot I^* = \frac{1}{2} (Z_R + Z_L + Z_C) I \cdot I^* = \frac{1}{2} \left(R + j\omega_0 L - \frac{j}{\omega_0 C} \right) |I|^2$$

$$P_1 = \frac{1}{2} R |I|^2 ; \quad P_2 = \frac{1}{2} \left(\omega_0 L - \frac{1}{\omega_0 C} \right) |I|^2$$

Phasors and vector functions of n variables

$$\vec{\mathbf{f}}_1(x, y, z, t) \longrightarrow \vec{\mathbf{F}}_1(x, y, z)$$

$$\vec{\mathbf{f}}_2(x, y, z, t) \longrightarrow \vec{\mathbf{F}}_2(x, y, z)$$

Phasors and vector functions of n variables

$$\vec{\mathbf{f}}_1(\vec{\mathbf{r}}, t) \longrightarrow \vec{\mathbf{F}}_1(\vec{\mathbf{r}})$$

$$\vec{\mathbf{f}}_2(\vec{\mathbf{r}}, t) \longrightarrow \vec{\mathbf{F}}_2(\vec{\mathbf{r}})$$

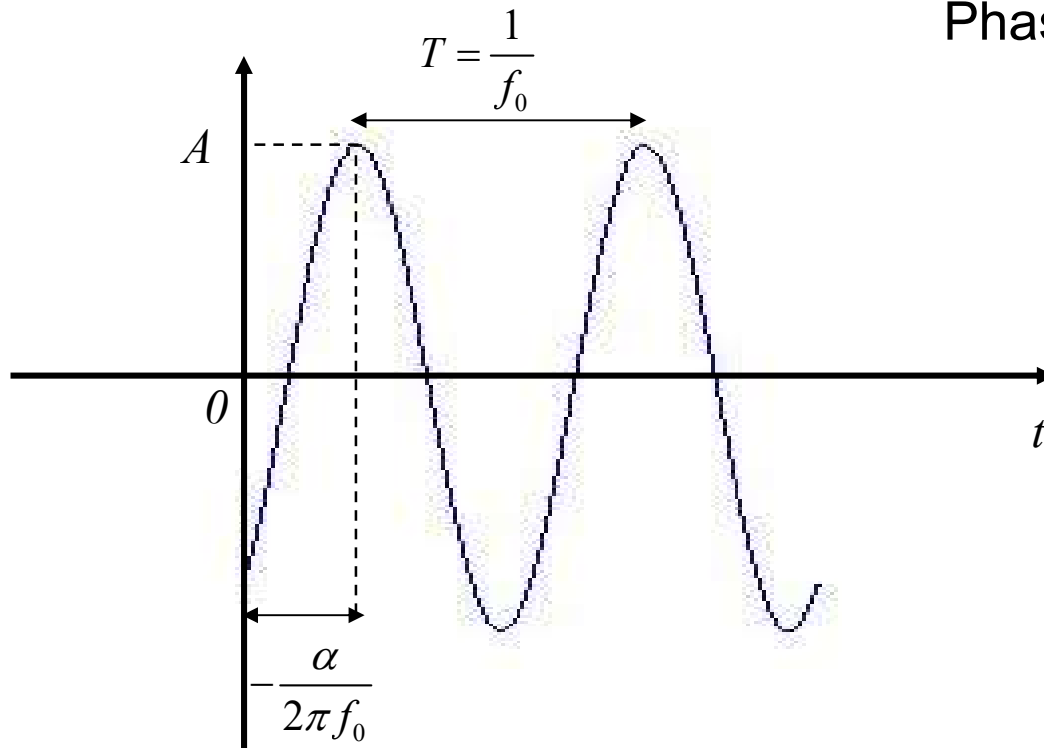
$$\langle \vec{\mathbf{f}}_1(\vec{\mathbf{r}}, t) \cdot \vec{\mathbf{f}}_2(\vec{\mathbf{r}}, t) \rangle = \frac{1}{T} \int_0^T \vec{\mathbf{f}}_1(\vec{\mathbf{r}}, t) \cdot \vec{\mathbf{f}}_2(\vec{\mathbf{r}}, t) dt = \frac{1}{2} \operatorname{Re} \{ \vec{\mathbf{F}}_1(\vec{\mathbf{r}}) \cdot \vec{\mathbf{F}}_2^*(\vec{\mathbf{r}}) \}$$

$$\langle \vec{\mathbf{f}}_1(\vec{\mathbf{r}}, t) \times \vec{\mathbf{f}}_2(\vec{\mathbf{r}}, t) \rangle = \frac{1}{T} \int_0^T \vec{\mathbf{f}}_1(\vec{\mathbf{r}}, t) \times \vec{\mathbf{f}}_2(\vec{\mathbf{r}}, t) dt = \frac{1}{2} \operatorname{Re} \{ \vec{\mathbf{F}}_1(\vec{\mathbf{r}}) \times \vec{\mathbf{F}}_2^*(\vec{\mathbf{r}}) \}$$

Phasors

$$v(t) = A \cos(2\pi f_0 t + \alpha) \longrightarrow V = Ae^{j\alpha}$$

Phasor (complex number)



Phasors

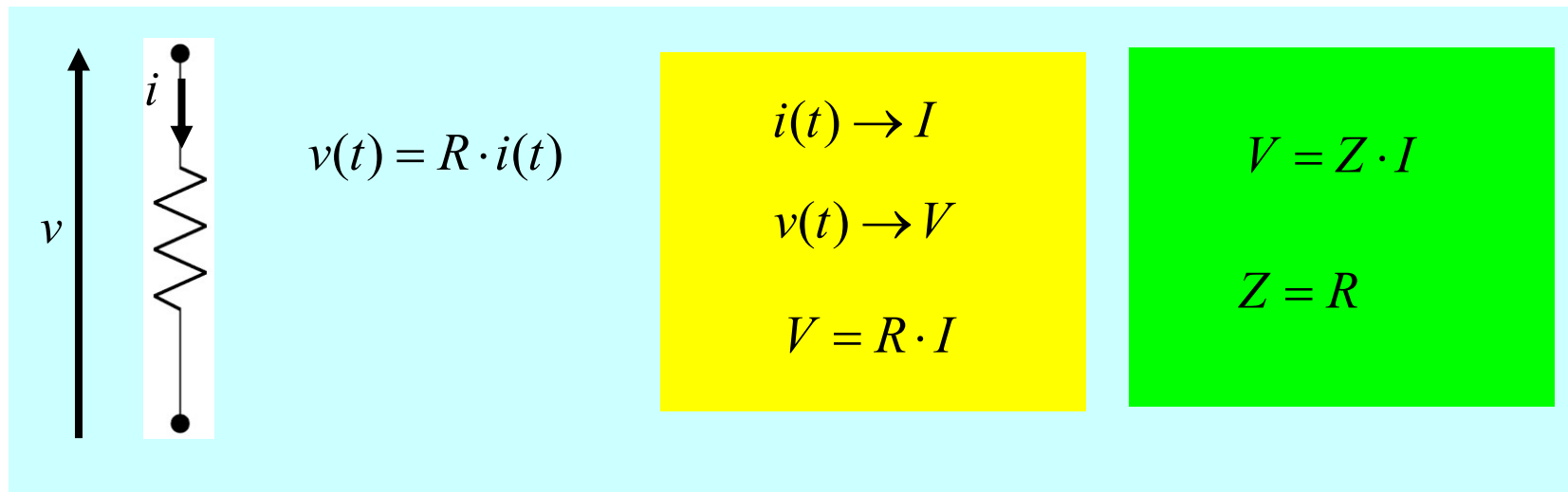
$$v(t) = A \cos(\omega_0 t + \alpha) \longrightarrow V = Ae^{j\alpha}$$

$$k \cdot v(t) \longrightarrow k \cdot V = k \cdot Ae^{j\alpha}$$

Phasors

$$v(t) = A \cos(\omega_0 t + \alpha) \longrightarrow V = A e^{j\alpha}$$

$$k \cdot v(t) \longrightarrow k \cdot V = k \cdot A e^{j\alpha}$$



Phasors

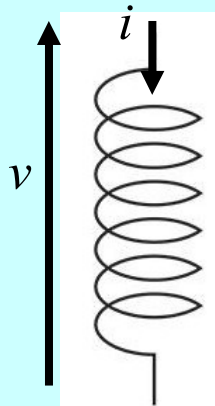
$$v(t) = A \cos(\omega_0 t + \alpha) \longrightarrow V = Ae^{j\alpha}$$

$$\frac{dv(t)}{dt} \longrightarrow j\omega_0 V = j\omega_0 Ae^{j\alpha}$$

Phasors

$$v(t) = A \cos(\omega_0 t + \alpha) \longrightarrow V = Ae^{j\alpha}$$

$$\frac{dv(t)}{dt} \longrightarrow j\omega_0 V = j\omega_0 Ae^{j\alpha}$$



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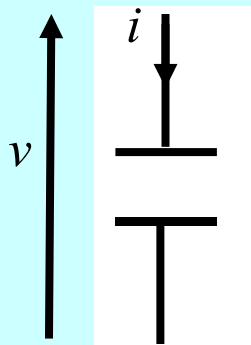
$$V = Z \cdot I$$

$$Z = j\omega_0 L$$

Phasors

$$v(t) = A \cos(\omega_0 t + \alpha) \longrightarrow V = Ae^{j\alpha}$$

$$\frac{dv(t)}{dt} \longrightarrow j\omega_0 V = j\omega_0 Ae^{j\alpha}$$



$$i(t) = C \frac{dv(t)}{dt}$$

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$$v(t) \rightarrow V$$

$$I = j\omega_0 CV$$

$$V = Z \cdot I$$

$$Z = -j \frac{1}{\omega_0 C}$$

Phasors

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$$i(t) = B \cos(\omega_0 t + \beta) \longrightarrow I = Be^{j\beta}$$

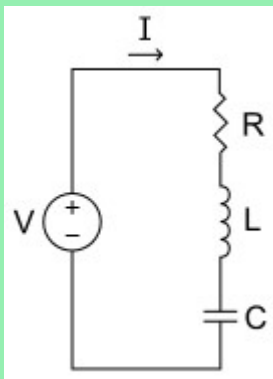
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Phasors

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$$P_1 = \frac{1}{2} R |I|^2 ; \quad P_2 = \frac{1}{2} \left(\omega_0 L - \frac{1}{\omega_0 C} \right) |I|^2$$

Phasors and vector functions of n variables

$$\vec{\mathbf{f}}_1(x, y, z, t) \longrightarrow \vec{\mathbf{F}}_1(x, y, z)$$

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Phasors and vector functions of n variables

$$\vec{\mathbf{f}}_1(\vec{\mathbf{r}}, t) \longrightarrow \vec{\mathbf{F}}_1(\vec{\mathbf{r}})$$

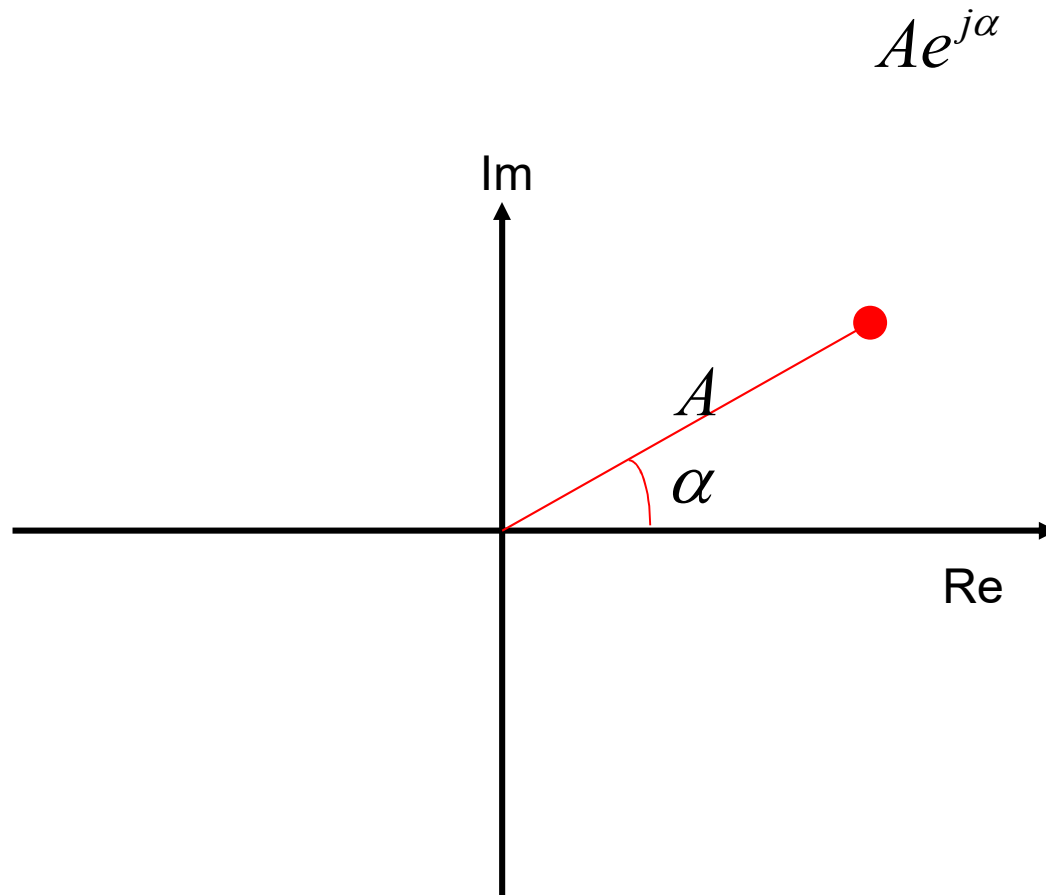
$$\vec{\mathbf{f}}_2(\vec{\mathbf{r}}, t) \longrightarrow \vec{\mathbf{F}}_2(\vec{\mathbf{r}})$$

$$\langle \vec{\mathbf{f}}_1(\vec{\mathbf{r}}, t) \cdot \vec{\mathbf{f}}_2(\vec{\mathbf{r}}, t) \rangle = \frac{1}{T} \int_0^T \vec{\mathbf{f}}_1(\vec{\mathbf{r}}, t) \cdot \vec{\mathbf{f}}_2(\vec{\mathbf{r}}, t) dt = \frac{1}{2} \operatorname{Re} \{ \vec{\mathbf{F}}_1(\vec{\mathbf{r}}) \cdot \vec{\mathbf{F}}_2^*(\vec{\mathbf{r}}) \}$$

$$\langle \vec{\mathbf{f}}_1(\vec{\mathbf{r}}, t) \times \vec{\mathbf{f}}_2(\vec{\mathbf{r}}, t) \rangle = \frac{1}{T} \int_0^T \vec{\mathbf{f}}_1(\vec{\mathbf{r}}, t) \times \vec{\mathbf{f}}_2(\vec{\mathbf{r}}, t) dt = \frac{1}{2} \operatorname{Re} \{ \vec{\mathbf{F}}_1(\vec{\mathbf{r}}) \times \vec{\mathbf{F}}_2^*(\vec{\mathbf{r}}) \}$$

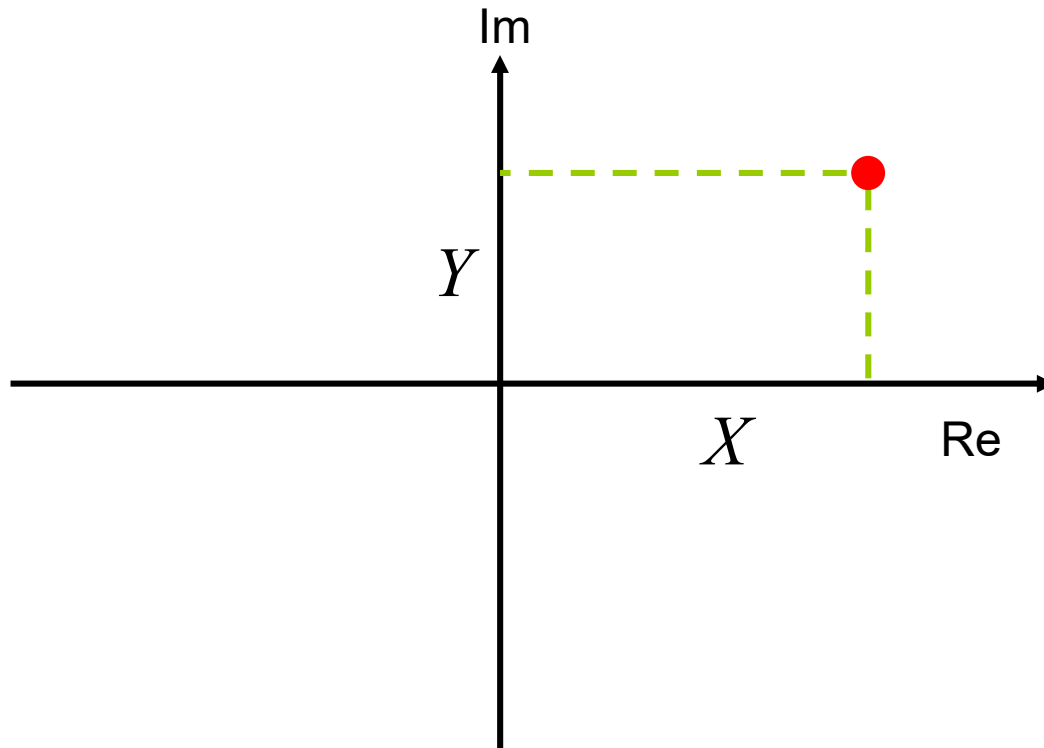
Memo: complex numbers

Complex numbers



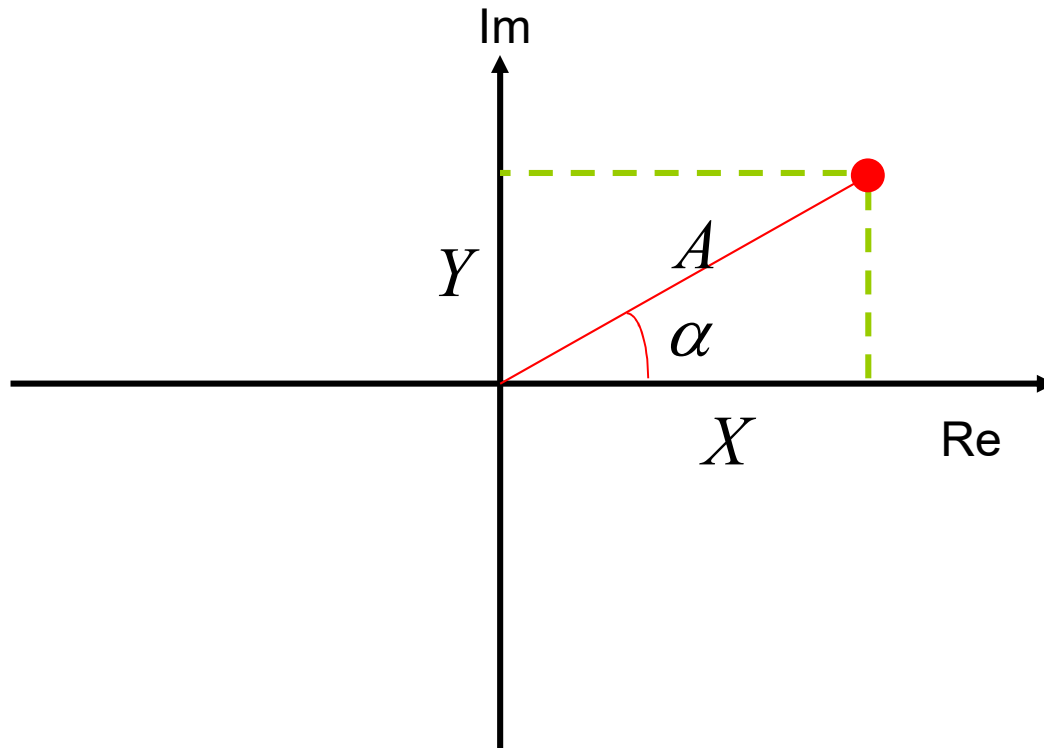
Complex numbers

$$Ae^{j\alpha} = X + jY$$



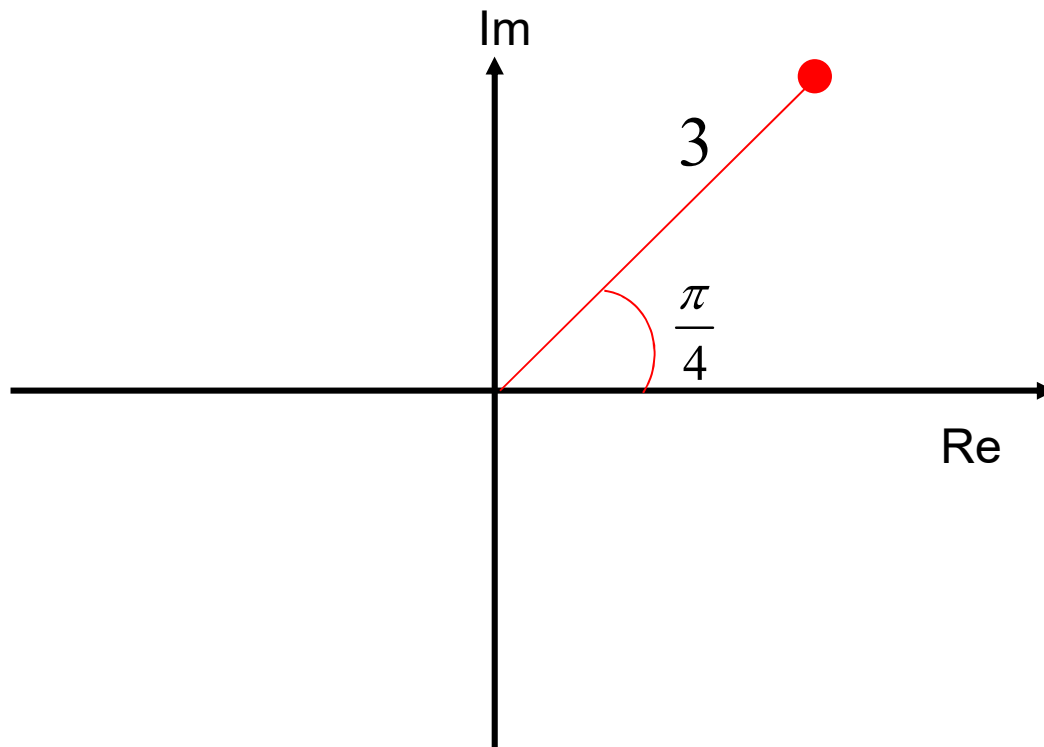
Complex numbers

$$Ae^{j\alpha} = X + jY$$

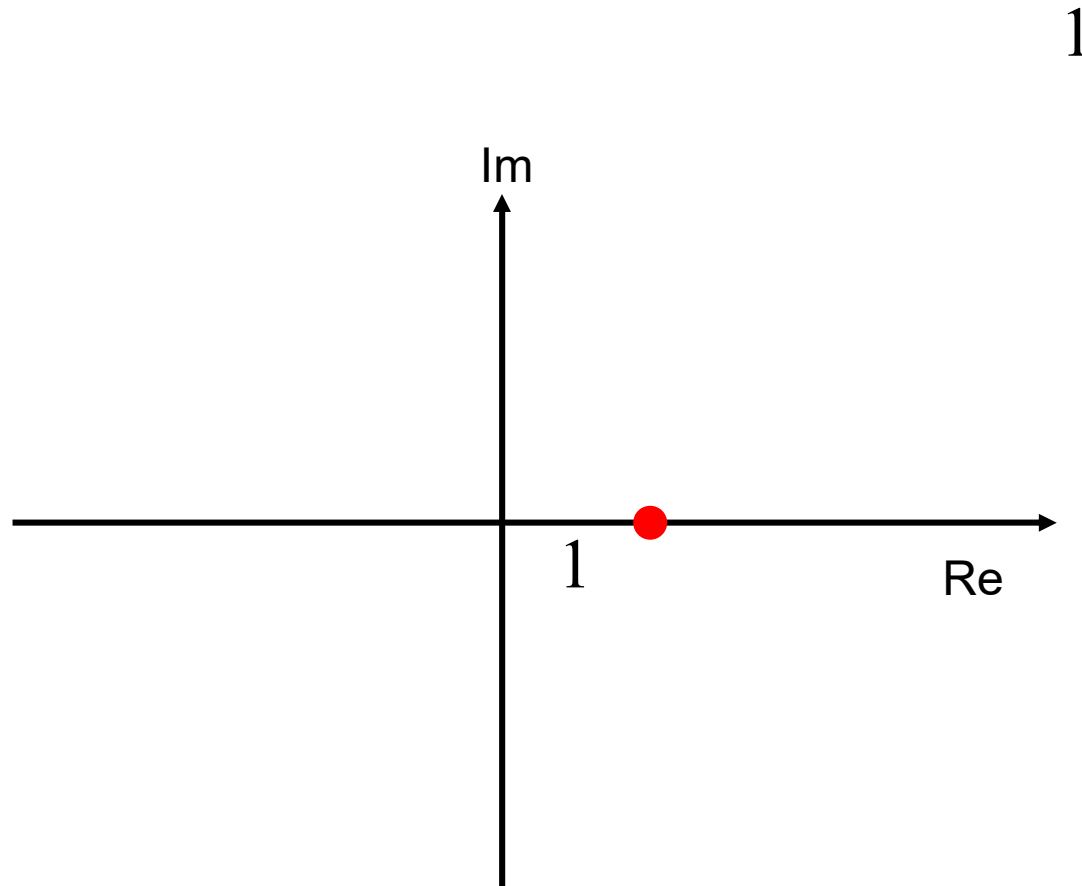


Complex numbers: some examples

$$3e^{j\frac{\pi}{4}}$$

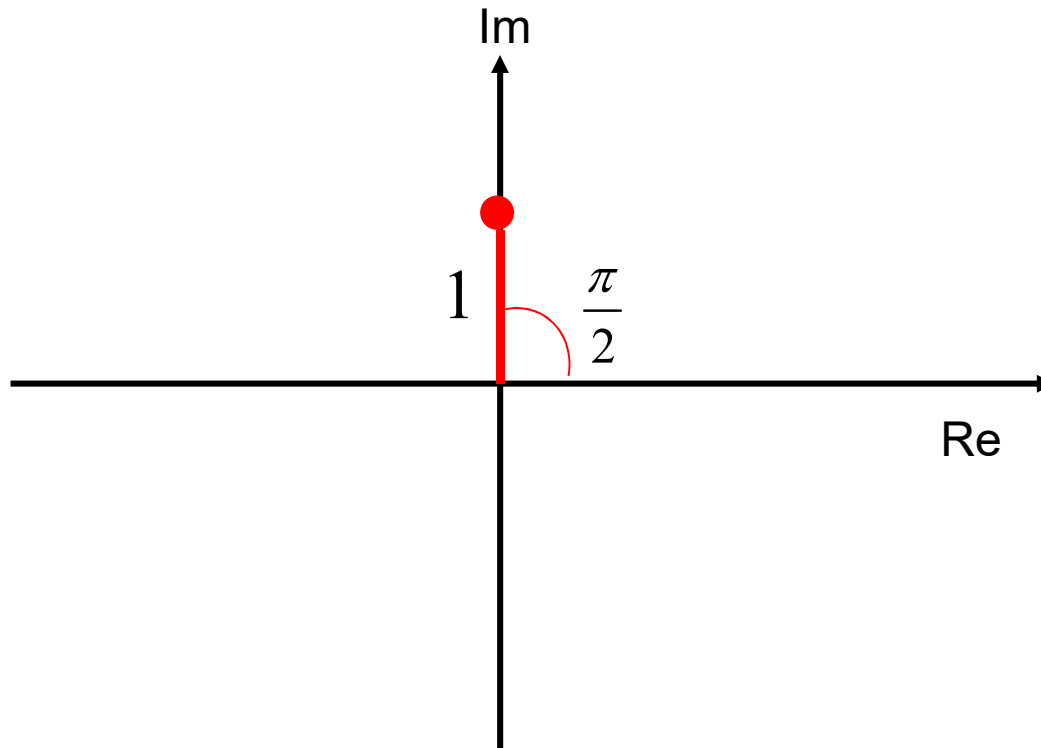


Complex numbers: some examples



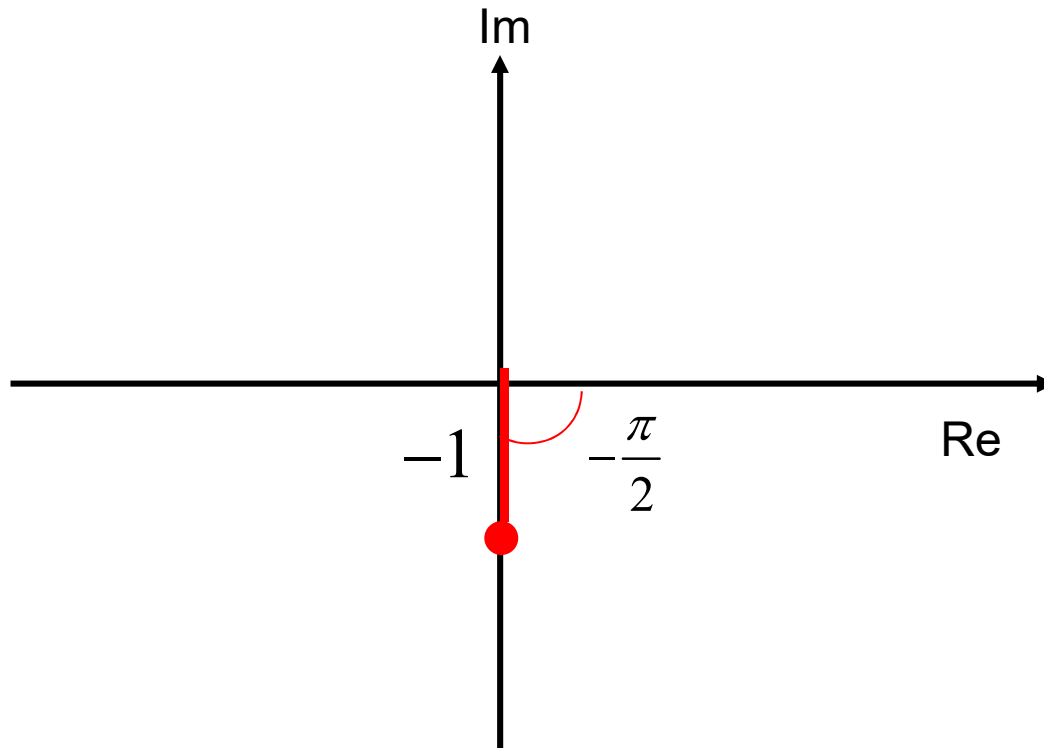
Complex numbers: some examples

$$j = e^{j\frac{\pi}{2}} = e^{j\left(\frac{\pi}{2} + 2\pi\right)}$$



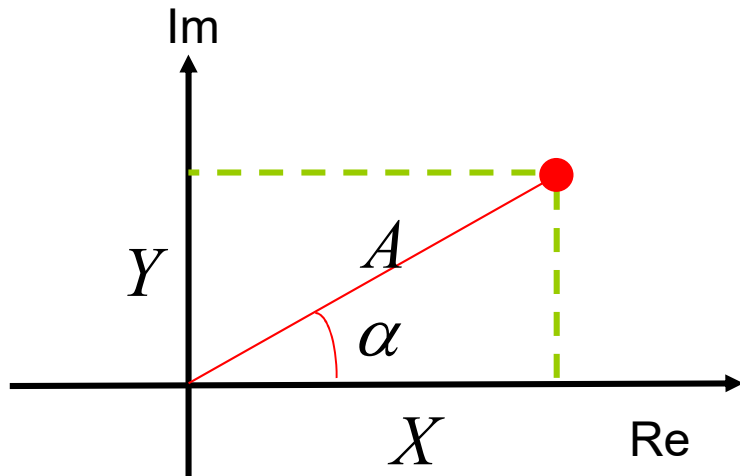
Complex numbers: some examples

$$-j = e^{-j\frac{\pi}{2}}$$



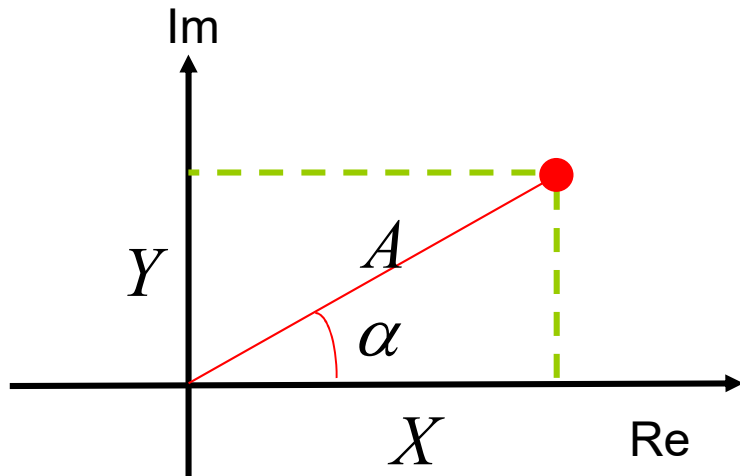
Complex numbers: conversion formulas

$$Ae^{j\alpha} = X + jY$$



Complex numbers: conversion formulas

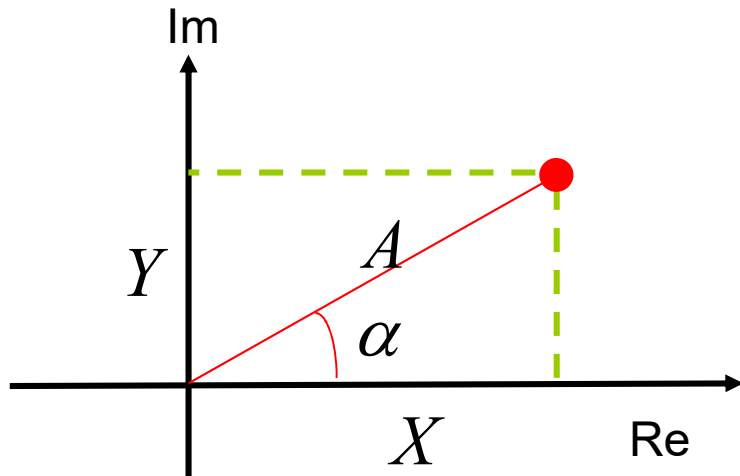
$$Ae^{j\alpha} = X + jY$$



$$Ae^{j\alpha} \longrightarrow X + jY$$

Complex numbers: conversion formulas

$$Ae^{j\alpha} = X + jY$$



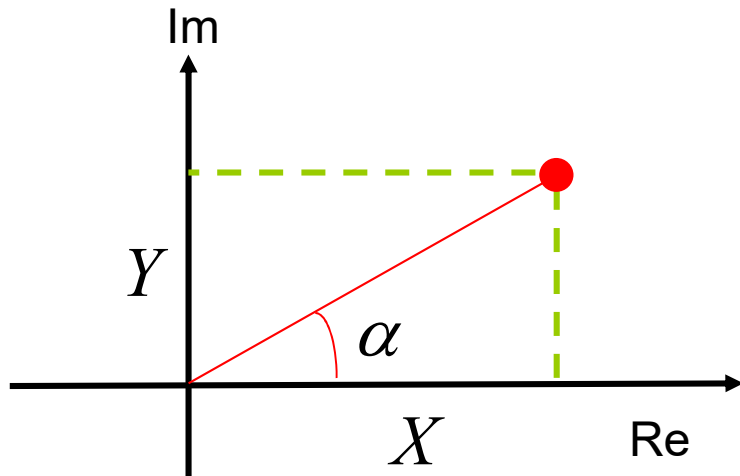
$$Ae^{j\alpha} \longrightarrow X + jY$$

$$X = A \cos \alpha$$

$$Y = A \sin \alpha$$

Complex numbers: conversion formulas

$$e^{j\alpha} = \cos \alpha + j \sin \alpha$$



$$Ae^{j\alpha} \rightarrow X + jY$$

$$X = A \cos \alpha$$

$$Y = A \sin \alpha$$

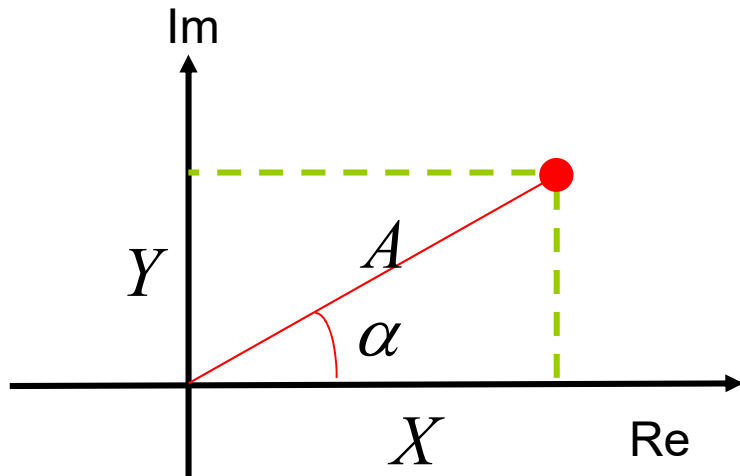
Eulero's formulas

$$\cos \alpha = \frac{e^{j\alpha} + e^{-j\alpha}}{2}$$

$$\sin \alpha = \frac{e^{j\alpha} - e^{-j\alpha}}{2j}$$

Complex numbers: conversion formulas

$$Ae^{j\alpha} = X + jY$$



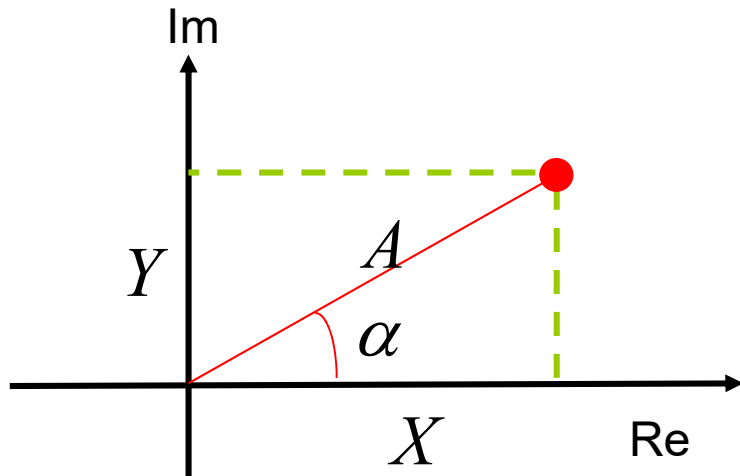
$$Ae^{j\alpha} \longrightarrow X + jY$$

$$X = A \cos \alpha$$

$$Y = A \sin \alpha$$

Complex numbers: conversion formulas

$$Ae^{j\alpha} = X + jY$$



$$X + jY \longrightarrow Ae^{j\alpha}$$

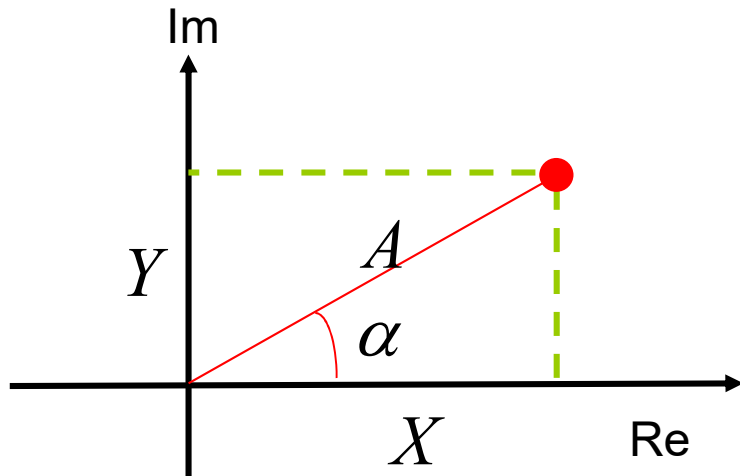
$$Ae^{j\alpha} \longrightarrow X + jY$$

$$X = A \cos \alpha$$

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Complex numbers: conversion formulas

$$Ae^{j\alpha} = X + jY$$



$$Ae^{j\alpha} \rightarrow X + jY$$

$$X = A \cos \alpha$$

$$Y = A \sin \alpha$$

$$X + jY \rightarrow Ae^{j\alpha}$$

$$A = \sqrt{X^2 + Y^2}$$

$$\begin{cases} \alpha = \arccos\left(\frac{X}{A}\right) & \text{se } Y \geq 0 \\ \alpha = -\arccos\left(\frac{X}{A}\right) & \text{se } Y < 0 \end{cases}$$

Complex numbers: conversion formulas

Some examples

$$Ae^{j\alpha} \longrightarrow X + jY$$

$$X = A \cos \alpha$$

$$Y = A \sin \alpha$$

$$2e^{j\frac{3}{4}\pi} = -\sqrt{2} + j\sqrt{2}$$

Complex numbers: conversion formulas

Some examples

$$X + jY \rightarrow Ae^{j\alpha}$$

$$A = \sqrt{X^2 + Y^2}$$

$$\begin{cases} \alpha = \arccos\left(\frac{X}{A}\right) & \text{se } Y \geq 0 \\ \alpha = -\arccos\left(\frac{X}{A}\right) & \text{se } Y < 0 \end{cases}$$

$$\sqrt{3} + j = 2e^{j\frac{\pi}{6}}$$

$$\sqrt{3} - j = 2e^{-j\frac{\pi}{6}}$$

Complex numbers: summation

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 + Z_2 = (X_1 + X_2) + j(Y_1 + Y_2)$$

Complex numbers: product

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 \cdot Z_2 = (X_1 \cdot X_2 - Y_1 \cdot Y_2) + j(X_1 \cdot Y_2 + Y_1 \cdot X_2)$$

$$Z_1 \cdot Z_2 = A_1 \cdot A_2 e^{j(\alpha_1 + \alpha_2)}$$

Complex numbers: product

Some examples

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 \cdot Z_2 = A_1 \cdot A_2 e^{j(\alpha_1 + \alpha_2)}$$

$$z_1 = 3e^{j\frac{\pi}{6}}$$

$$z_2 = 4e^{j\frac{\pi}{4}}$$

$$z_1 \cdot z_2 = 12e^{j\left(\frac{\pi}{6} + \frac{\pi}{4}\right)} = 12e^{j\left(\frac{5}{12}\pi\right)}$$

Complex numbers: product

Some examples

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 \cdot Z_2 = A_1 \cdot A_2 e^{j(\alpha_1 + \alpha_2)}$$

$$z_1 = \sqrt{3} + j = 2e^{j\frac{\pi}{6}}$$

$$z_2 = -\sqrt{2} + j\sqrt{2} = 2e^{j\frac{3\pi}{4}}$$

$$z_1 \cdot z_2 = 4e^{j\left(\frac{\pi}{6} + \frac{3\pi}{4}\right)} = 4e^{j\left(\frac{11}{12}\pi\right)}$$

Complex numbers: product

Some examples

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

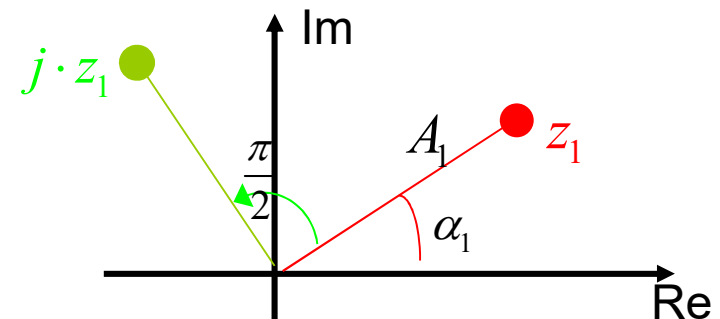
$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 \cdot Z_2 = A_1 \cdot A_2 e^{j(\alpha_1 + \alpha_2)}$$

$$z_1 = A_1 e^{j\alpha_1}$$

$$j \cdot z_1 = A_1 e^{j\left(\alpha_1 + \frac{\pi}{2}\right)}$$

$$z_2 = j = e^{j\frac{\pi}{2}}$$



Complex numbers: product

Some examples

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

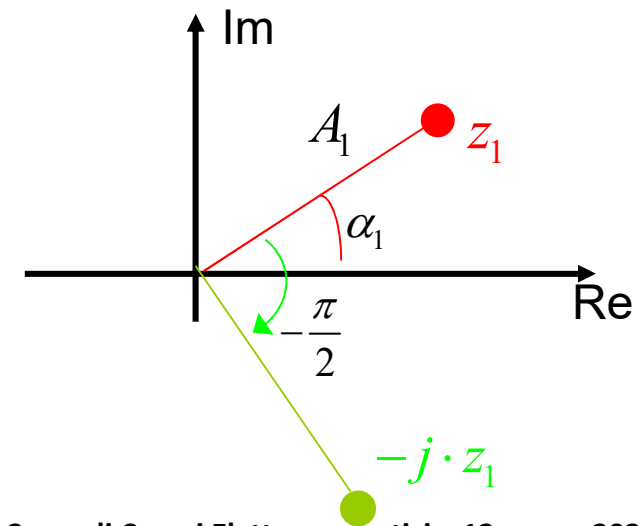
$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 \cdot Z_2 = A_1 \cdot A_2 e^{j(\alpha_1 + \alpha_2)}$$

$$z_1 = A_1 e^{j\alpha_1}$$

$$-j \cdot z_1 = A_1 e^{j\left(\alpha_1 - \frac{\pi}{2}\right)}$$

$$z_2 = -j = e^{-j\frac{\pi}{2}}$$



Complex numbers: product

Some examples

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

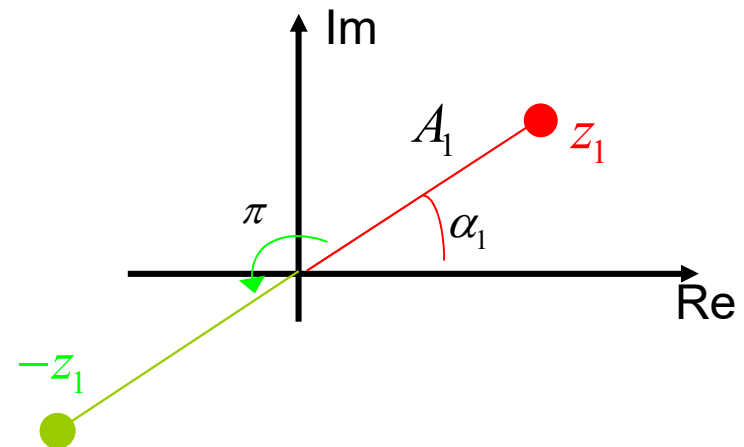
$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 \cdot Z_2 = A_1 \cdot A_2 e^{j(\alpha_1 + \alpha_2)}$$

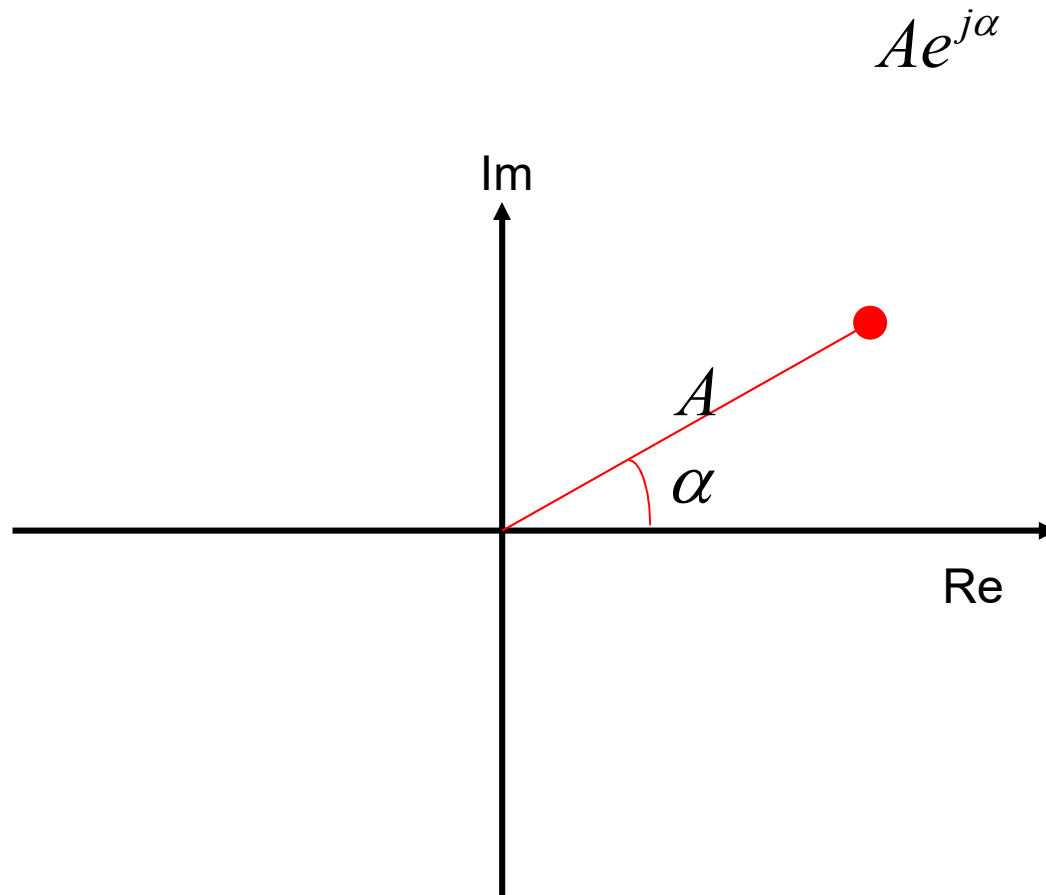
$$z_1 = A_1 e^{j\alpha_1}$$

$$-z_1 = A_1 e^{j(\alpha_1 + \pi)}$$

$$z_2 = -1 = e^{j\pi}$$

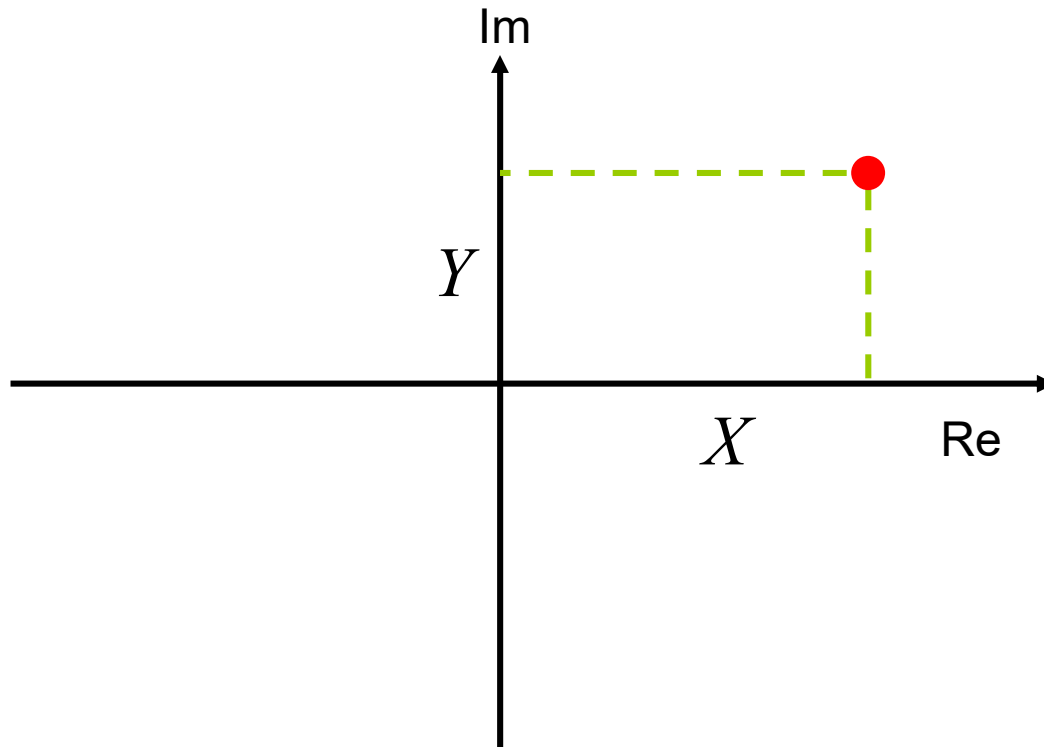


Complex numbers



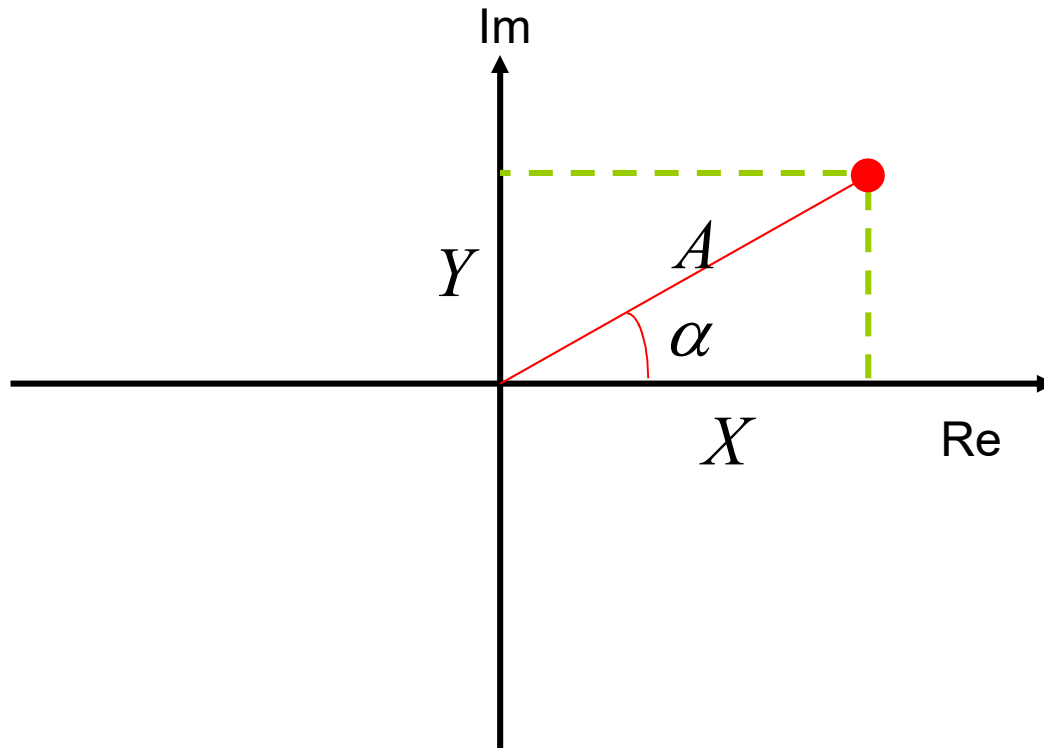
Complex numbers

$$Ae^{j\alpha} = X + jY$$



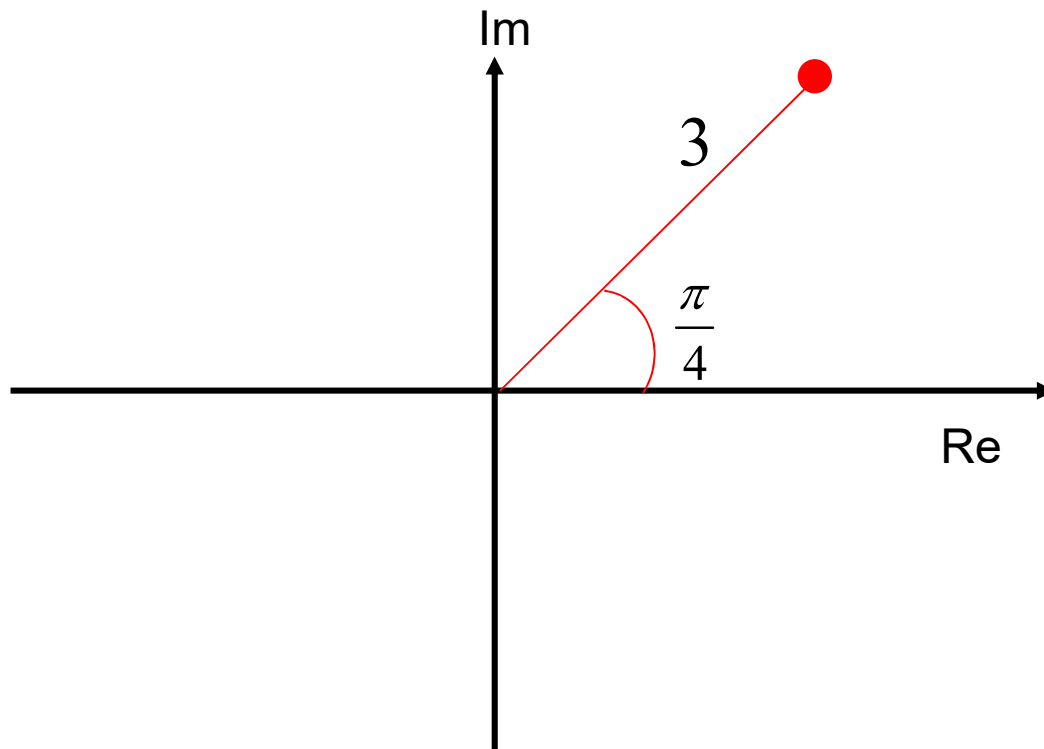
Complex numbers

$$Ae^{j\alpha} = X + jY$$

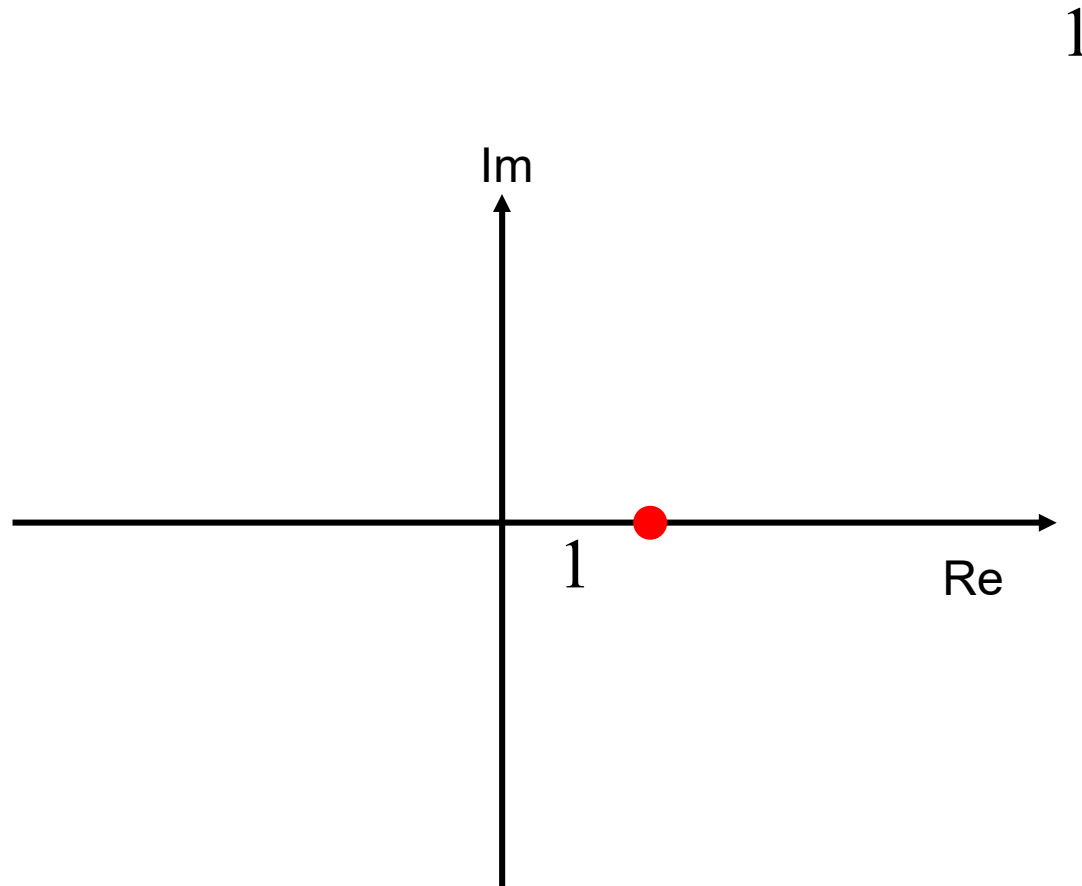


Complex numbers: some examples

$$3e^{j\frac{\pi}{4}}$$

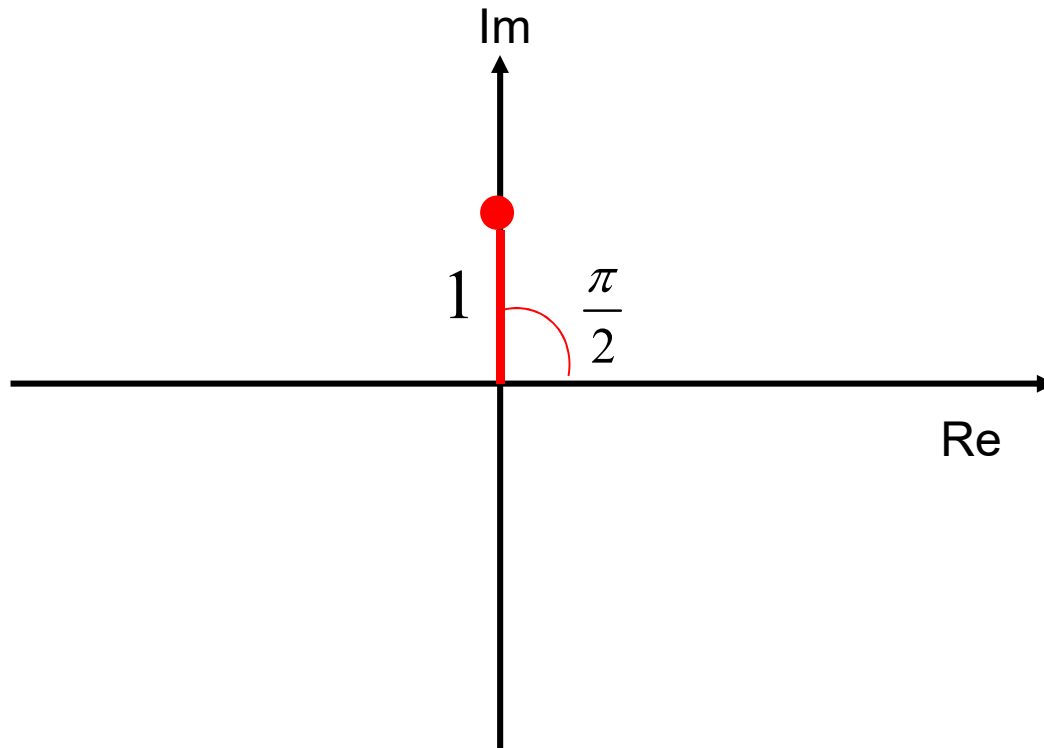


Complex numbers: some examples



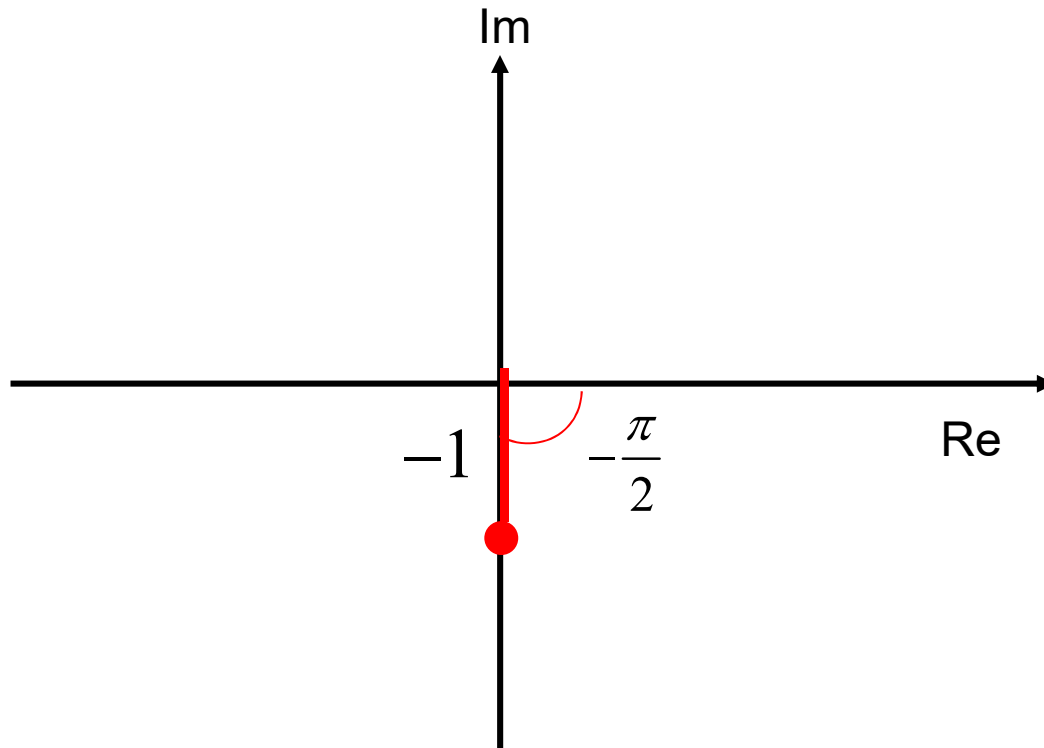
Complex numbers: some examples

$$j = e^{j\frac{\pi}{2}} = e^{j\left(\frac{\pi}{2} + 2\pi\right)}$$



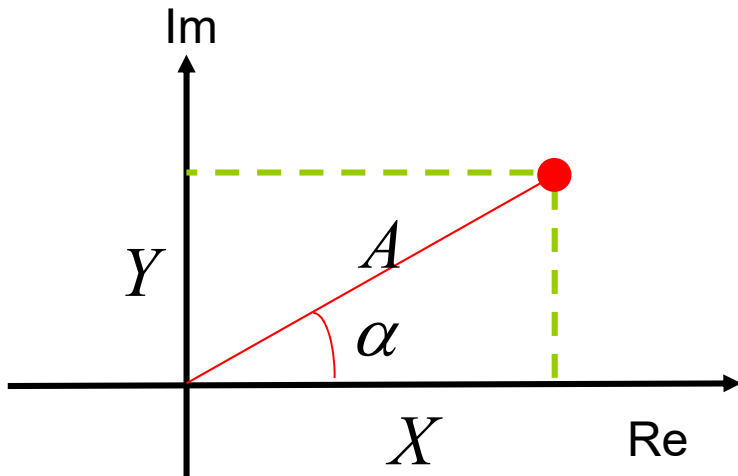
Complex numbers: some examples

$$-j = e^{-j\frac{\pi}{2}}$$



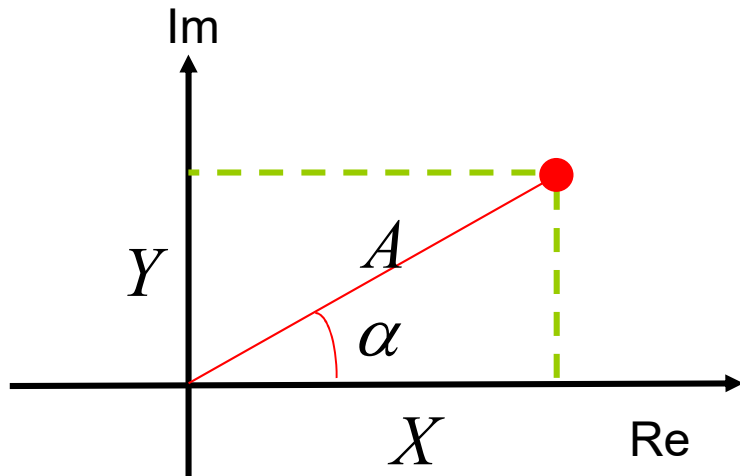
Complex numbers: conversion formulas

$$Ae^{j\alpha} = X + jY$$



Complex numbers: conversion formulas

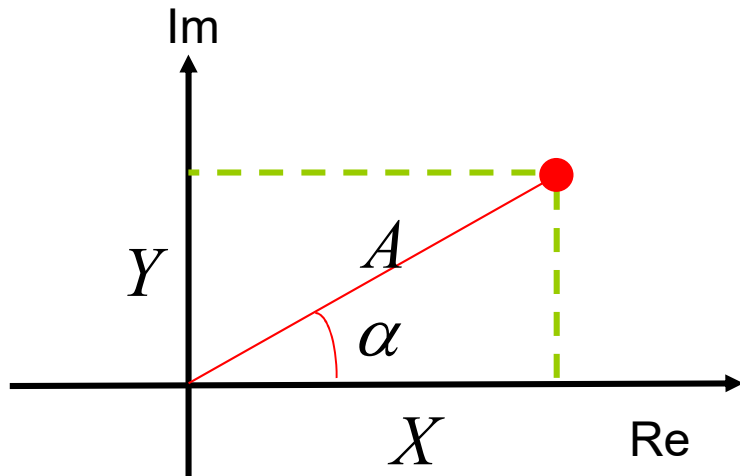
$$Ae^{j\alpha} = X + jY$$



$$Ae^{j\alpha} \longrightarrow X + jY$$

Complex numbers: conversion formulas

$$Ae^{j\alpha} = X + jY$$



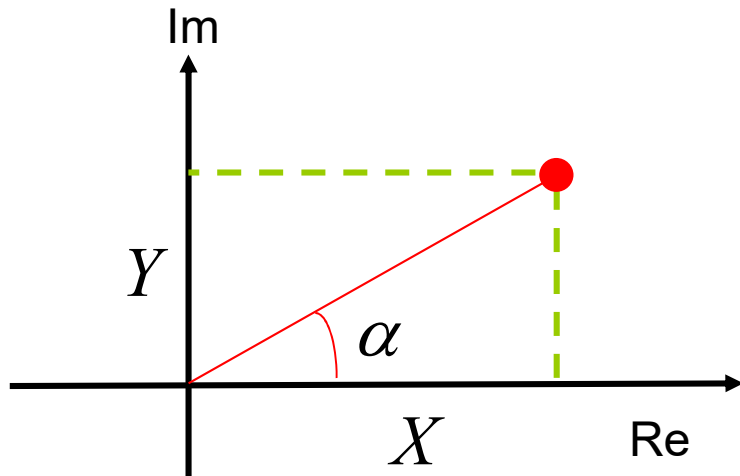
$$Ae^{j\alpha} \longrightarrow X + jY$$

$$X = A \cos \alpha$$

$$Y = A \sin \alpha$$

Complex numbers: conversion formulas

$$e^{j\alpha} = \cos \alpha + j \sin \alpha$$



$$Ae^{j\alpha} \rightarrow X + jY$$

$$X = A \cos \alpha$$

$$Y = A \sin \alpha$$

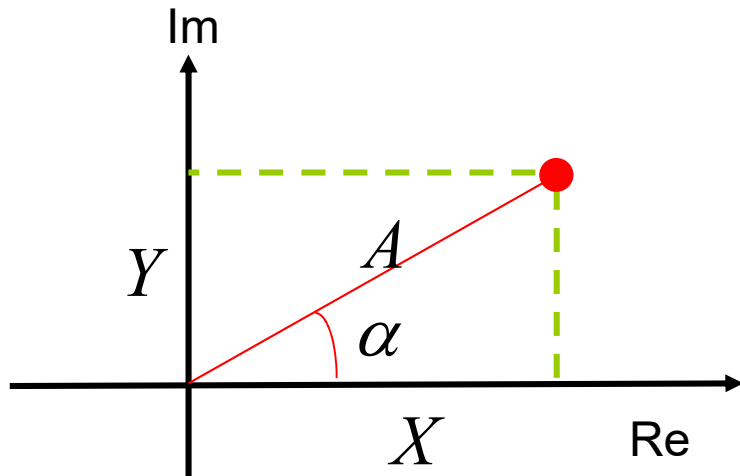
Eulero's formulas

$$\cos \alpha = \frac{e^{j\alpha} + e^{-j\alpha}}{2}$$

$$\sin \alpha = \frac{e^{j\alpha} - e^{-j\alpha}}{2j}$$

Complex numbers: conversion formulas

$$Ae^{j\alpha} = X + jY$$



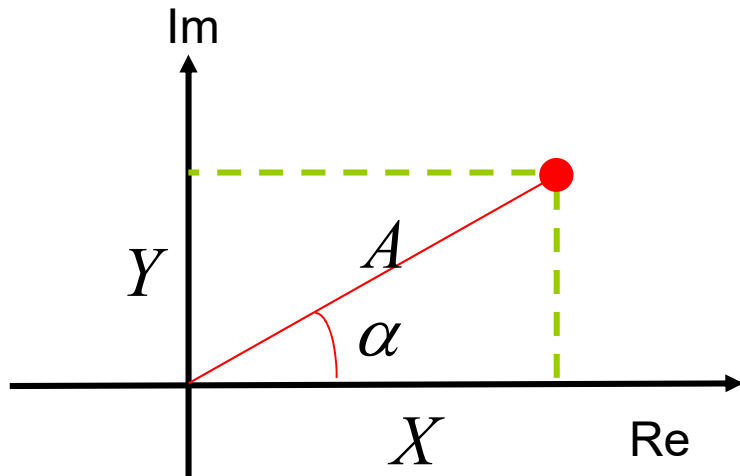
$$Ae^{j\alpha} \longrightarrow X + jY$$

$$X = A \cos \alpha$$

$$Y = A \sin \alpha$$

Complex numbers: conversion formulas

$$Ae^{j\alpha} = X + jY$$



$$X + jY \longrightarrow Ae^{j\alpha}$$

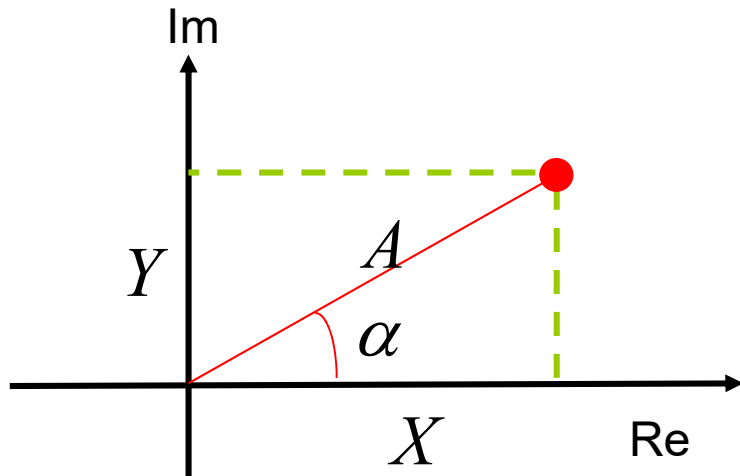
$$Ae^{j\alpha} \longrightarrow X + jY$$

$$X = A \cos \alpha$$

$$Y = A \sin \alpha$$

Complex numbers: conversion formulas

$$Ae^{j\alpha} = X + jY$$



$$Ae^{j\alpha} \rightarrow X + jY$$

$$X = A \cos \alpha$$

$$Y = A \sin \alpha$$

$$X + jY \rightarrow Ae^{j\alpha}$$

$$A = \sqrt{X^2 + Y^2}$$

$$\begin{cases} \alpha = \arccos\left(\frac{X}{A}\right) & \text{se } Y \geq 0 \\ \alpha = -\arccos\left(\frac{X}{A}\right) & \text{se } Y < 0 \end{cases}$$

Complex numbers: conversion formulas

Some examples

$$Ae^{j\alpha} \longrightarrow X + jY$$

$$X = A \cos \alpha$$

$$Y = A \sin \alpha$$

$$2e^{j\frac{3}{4}\pi} = -\sqrt{2} + j\sqrt{2}$$

Complex numbers: conversion formulas

Some examples

$$X + jY \rightarrow Ae^{j\alpha}$$

$$A = \sqrt{X^2 + Y^2}$$

$$\begin{cases} \alpha = \arccos\left(\frac{X}{A}\right) & \text{se } Y \geq 0 \\ \alpha = -\arccos\left(\frac{X}{A}\right) & \text{se } Y < 0 \end{cases}$$

$$\sqrt{3} + j = 2e^{j\frac{\pi}{6}}$$

$$\sqrt{3} - j = 2e^{-j\frac{\pi}{6}}$$

Complex numbers: summation

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 + Z_2 = (X_1 + X_2) + j(Y_1 + Y_2)$$

Complex numbers: product

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 \cdot Z_2 = (X_1 \cdot X_2 - Y_1 \cdot Y_2) + j(X_1 \cdot Y_2 + Y_1 \cdot X_2)$$

$$Z_1 \cdot Z_2 = A_1 \cdot A_2 e^{j(\alpha_1 + \alpha_2)}$$

Complex numbers: product

Some examples

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 \cdot Z_2 = A_1 \cdot A_2 e^{j(\alpha_1 + \alpha_2)}$$

$$z_1 = 3e^{j\frac{\pi}{6}}$$

$$z_2 = 4e^{j\frac{\pi}{4}}$$

$$z_1 \cdot z_2 = 12e^{j\left(\frac{\pi}{6} + \frac{\pi}{4}\right)} = 12e^{j\left(\frac{5}{12}\pi\right)}$$

Complex numbers: product

Some examples

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 \cdot Z_2 = A_1 \cdot A_2 e^{j(\alpha_1 + \alpha_2)}$$

$$z_1 = \sqrt{3} + j = 2e^{j\frac{\pi}{6}}$$

$$z_2 = -\sqrt{2} + j\sqrt{2} = 2e^{j\frac{3\pi}{4}}$$

$$z_1 \cdot z_2 = 4e^{j\left(\frac{\pi}{6} + \frac{3\pi}{4}\right)} = 4e^{j\left(\frac{11}{12}\pi\right)}$$

Complex numbers: product

Some examples

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

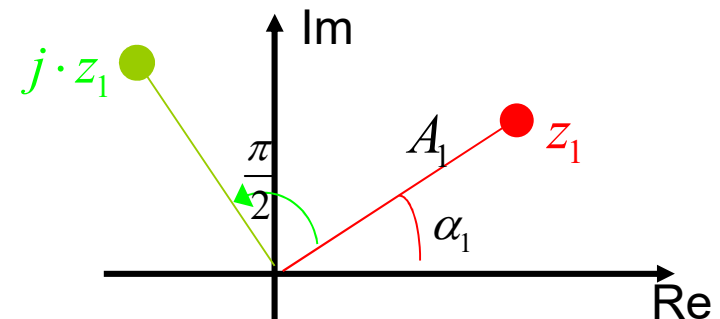
$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 \cdot Z_2 = A_1 \cdot A_2 e^{j(\alpha_1 + \alpha_2)}$$

$$z_1 = A_1 e^{j\alpha_1}$$

$$j \cdot z_1 = A_1 e^{j\left(\alpha_1 + \frac{\pi}{2}\right)}$$

$$z_2 = j = e^{j\frac{\pi}{2}}$$



Complex numbers: product

Some examples

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

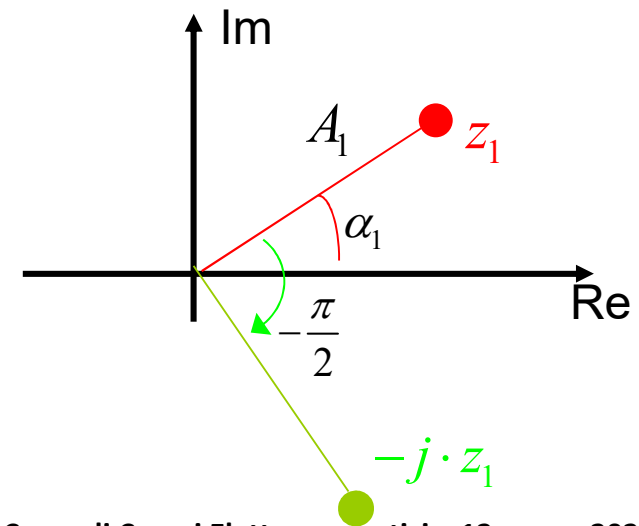
$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 \cdot Z_2 = A_1 \cdot A_2 e^{j(\alpha_1 + \alpha_2)}$$

$$z_1 = A_1 e^{j\alpha_1}$$

$$-j \cdot z_1 = A_1 e^{j\left(\alpha_1 - \frac{\pi}{2}\right)}$$

$$z_2 = -j = e^{-j\frac{\pi}{2}}$$



Complex numbers: product

Some examples

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 \cdot Z_2 = A_1 \cdot A_2 e^{j(\alpha_1 + \alpha_2)}$$

$$z_1 = A_1 e^{j\alpha_1}$$

$$-z_1 = A_1 e^{j(\alpha_1 + \pi)}$$

$$z_2 = -1 = e^{j\pi}$$

