Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2020-2021 - Laurea “Triennale” - Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

Stefano Perna

Maxwell equations

James Clerk Maxwell 1831-1879

Differential form

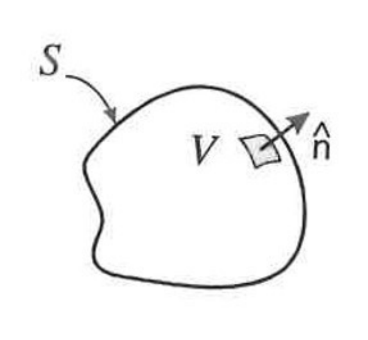
$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \end{cases}$$

Maxwell equations: integral form

… mathematical tools that we have exploited…

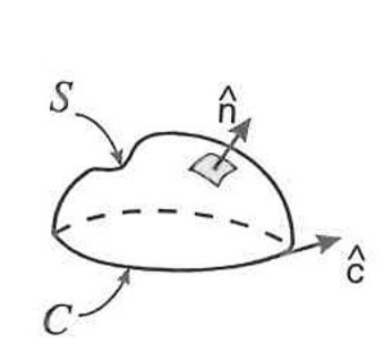
I) Gauss theorem

$$\iiint_V dV\, \nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \oiint_S dS\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}$$



II) Stokes theorem

$$\iint_S dS \left(\nabla \times \vec{\mathbf{A}}(\vec{\mathbf{r}}) \right) \cdot \hat{\mathbf{n}} = \oint_C dc\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{c}}$$



Maxwell equations

Differential form

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \end{cases}$$

Integral form

$$\begin{cases} \oint_C dc\, \vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot \hat{\mathbf{c}} = -\dfrac{d}{dt}\iint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t)\cdot \hat{\mathbf{n}} \\ \oint_C dc\, \vec{\mathbf{h}}(\vec{\mathbf{r}},t)\cdot \hat{\mathbf{c}} = \dfrac{d}{dt}\iint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t)\cdot \hat{\mathbf{n}} + i(t) \\ \oiint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t)\cdot \hat{\mathbf{n}} = q(t) \\ \oiint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t)\cdot \hat{\mathbf{n}} = 0 \end{cases}$$

… mathematical tools that we will exploit today…

$$\nabla \cdot \left(\nabla \times \vec{\mathbf{A}}(\vec{\mathbf{r}}) \right) = 0$$

Maxwell equations

Differential form

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \end{cases}$$

$\Rightarrow$

$$\frac{\partial \rho(\vec{\mathbf{r}},t)}{\partial t} + \nabla \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}},t) = 0$$

Maxwell equations

Differential form

$$\nabla \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}},t) + \frac{\partial \rho(\vec{\mathbf{r}},t)}{\partial t} = 0$$

Current density equation

Integral form

Current density equation

Maxwell equations

Differential form

$$\nabla \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}},t) + \frac{\partial \rho(\vec{\mathbf{r}},t)}{\partial t} = 0$$

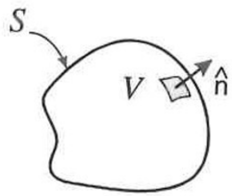
Current density equation

Integral form

Current density equation

Gauss theorem

$$\iiint_V dV\, \nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \oiint_S dS\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}$$



Maxwell equations

Differential form

$$\nabla \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}},t)+\frac{\partial \rho(\vec{\mathbf{r}},t)}{\partial t}=0$$

Current density equation

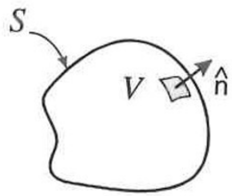
Integral form

$$\oiint_S dS\,\vec{\mathbf{j}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}+\frac{dq(t)}{dt}=0$$

Current density equation

Gauss theorem

$$\iiint_V dV\,\nabla\cdot\vec{\mathbf{A}}(\vec{\mathbf{r}})=\oiint_S dS\,\vec{\mathbf{A}}(\vec{\mathbf{r}})\cdot\hat{\mathbf{n}}$$



Maxwell equations

Integral form

$$\oiint_S dS\,\vec{\mathbf{j}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}} + \frac{dq(t)}{dt} = 0$$

Current density equation

….considerations

Stationary fields $\left(\frac{d}{dt}=0\right)$ ➡ $\oiint_S dS\,\vec{\mathbf{j}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}} = 0$ **Kirchhoff's first law**

Maxwell equations

Differential form

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \end{cases}$$

$$\nabla \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}},t) + \frac{\partial \rho(\vec{\mathbf{r}},t)}{\partial t} = 0$$

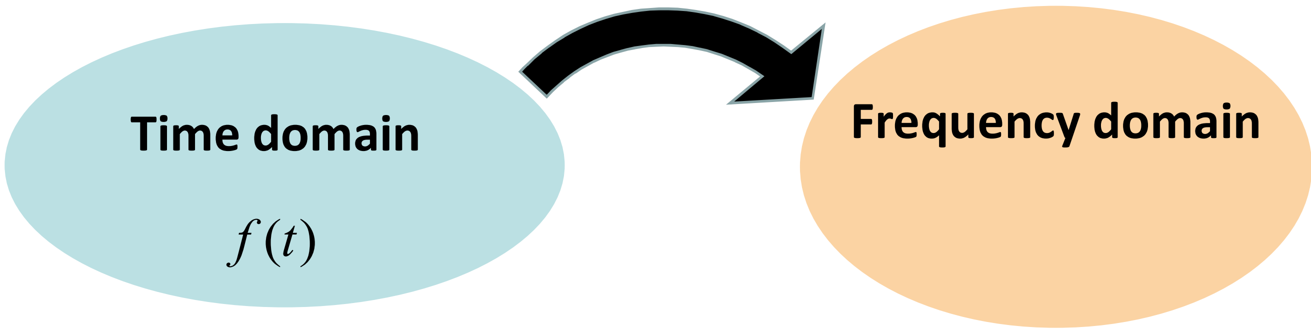
Integral form

$$\begin{cases} \oint_C dc\, \vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{c}} = -\dfrac{d}{dt}\iint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}} \\ \oint_C dc\, \vec{\mathbf{h}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{c}} = \dfrac{d}{dt}\iint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}} + i(t) \\ \oiint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}} = q(t) \\ \oiint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}} = 0 \end{cases}$$

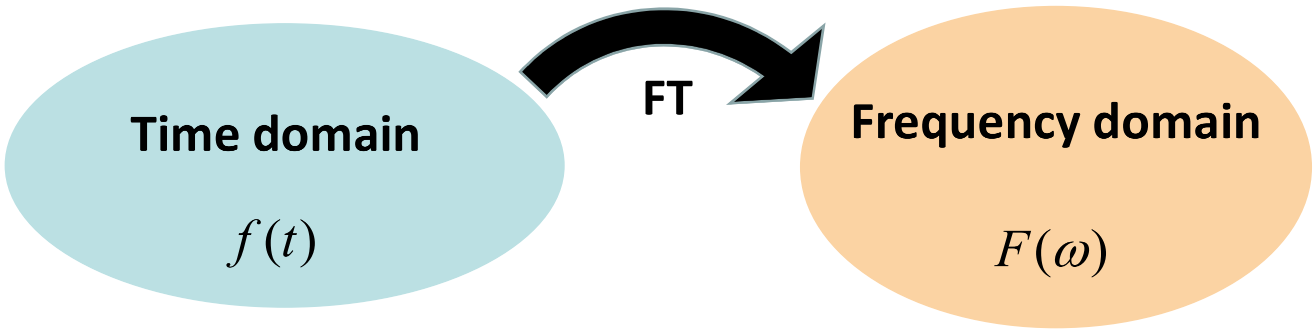
$$\oiint_S dS\, \vec{\mathbf{j}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}} + \frac{dq(t)}{dt} = 0$$

Maxwell equations: Time domain, Frequency domain, Phasors

Frequency domain



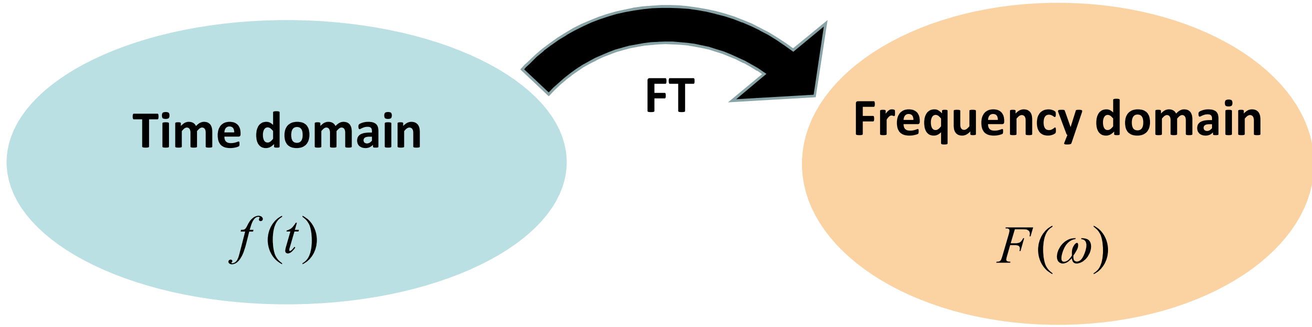
Frequency domain



$$F(\omega)=\int_{-\infty}^{+\infty} f(t)\, e^{-j\omega t}\, dt$$

Fourier Transform (FT)

Frequency domain



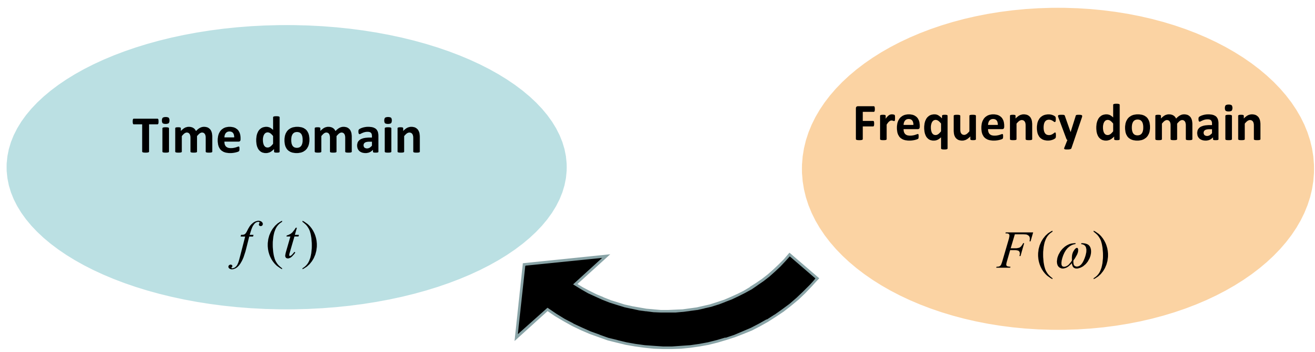
$$F(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt$$

Fourier Transform (FT)

1) How to jump back from the Frequency domain to the Time domain

2) Time domain derivative and Fourier Transform

Frequency domain

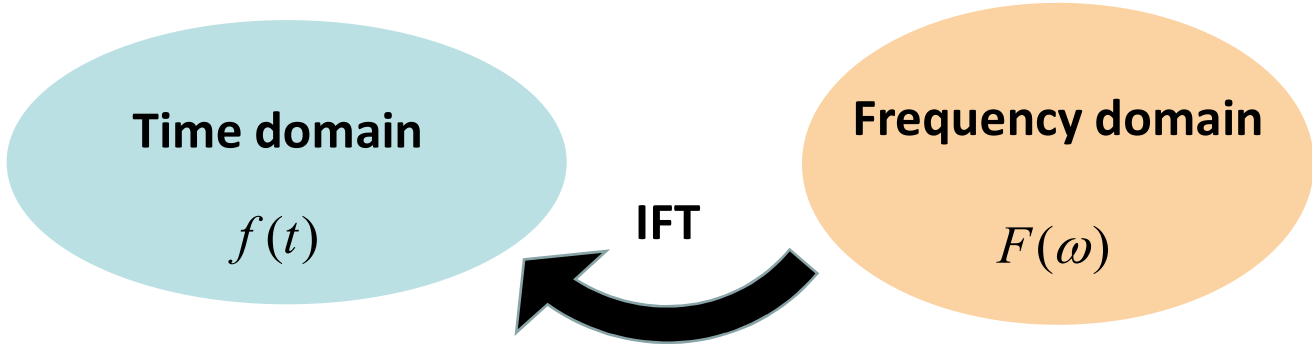


$$F(\omega)=\int_{-\infty}^{+\infty} f(t)\,e^{-j\omega t}\,dt$$

Fourier Transform (FT)

1) How to jump back from the Spectral domain to the Time domain

Frequency domain



$$F(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt$$

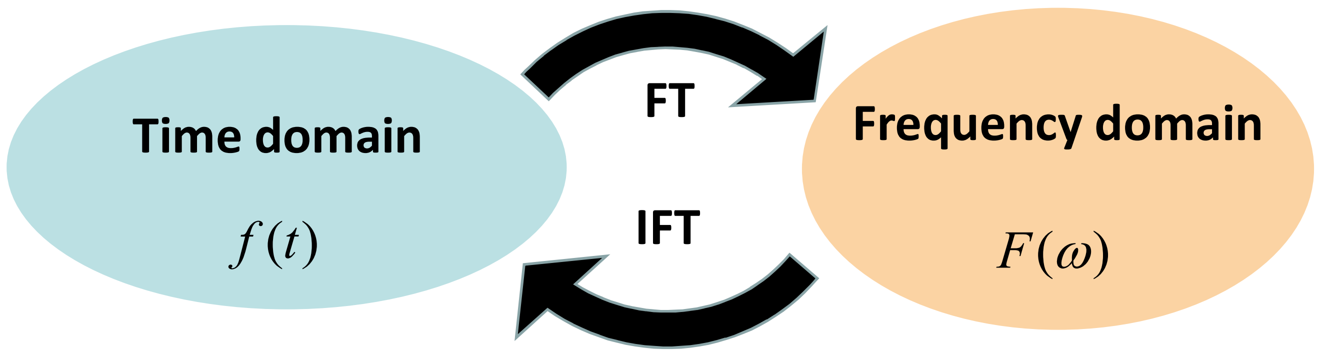
Fourier Transform (FT)

1) How to jump back from the Spectral domain to the Time domain

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(\omega) e^{j\omega t} d\omega$$

Inverse Fourier Transform (IFT)

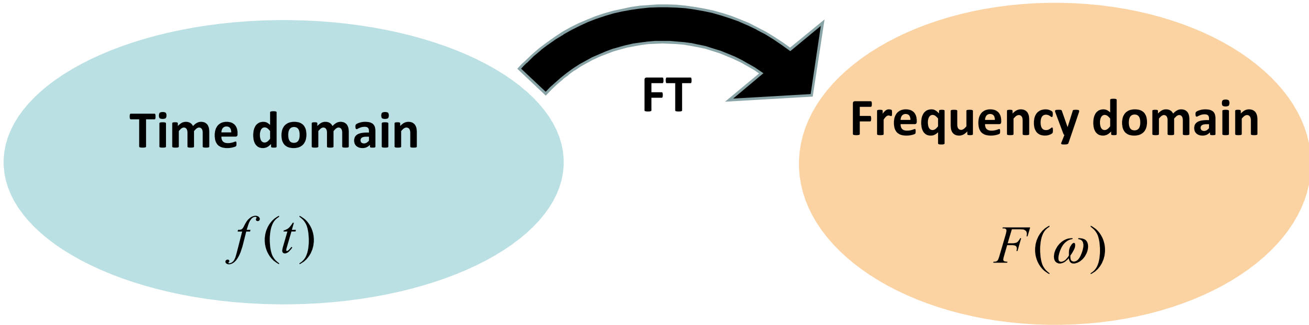
Frequency domain



$f(t)$ → **FT** → $$F(\omega)=\int_{-\infty}^{+\infty} f(t)\,e^{-j\omega t}\,dt$$

$F(\omega)$ → **IFT** → $$f(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} F(\omega)\,e^{j\omega t}\,d\omega$$

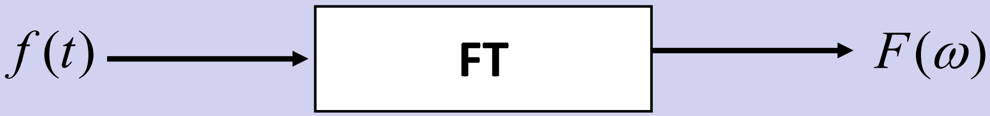
Frequency domain



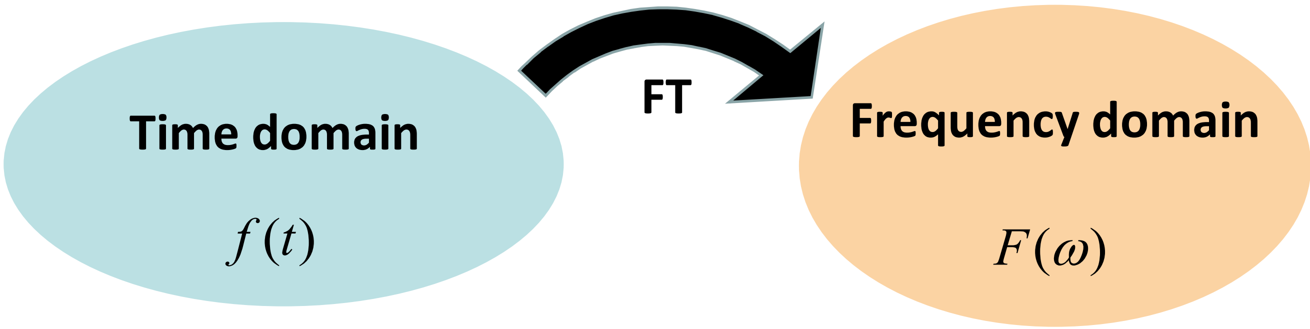
$$F(\omega)=\int_{-\infty}^{+\infty} f(t)e^{-j\omega t}dt$$

Fourier Transform (FT)

2) Time-domain derivative and Fourier Transform



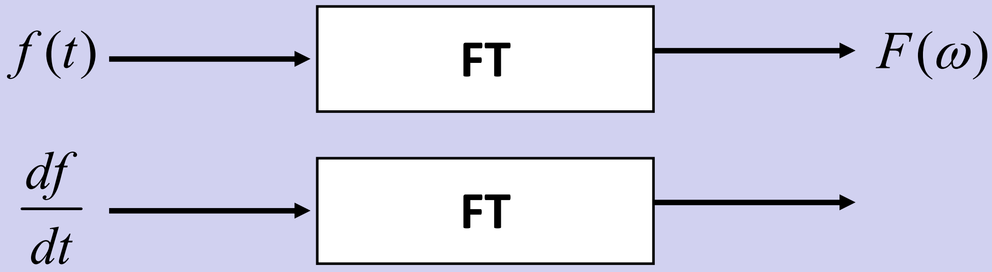
Frequency domain



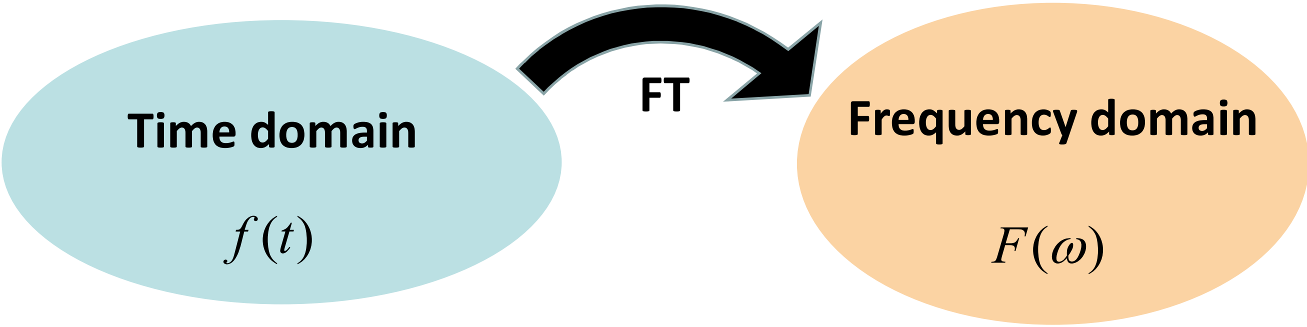
$$F(\omega)=\int_{-\infty}^{+\infty} f(t)e^{-j\omega t}dt$$

Fourier Transform (FT)

2) Time-domain derivative and Fourier Transform



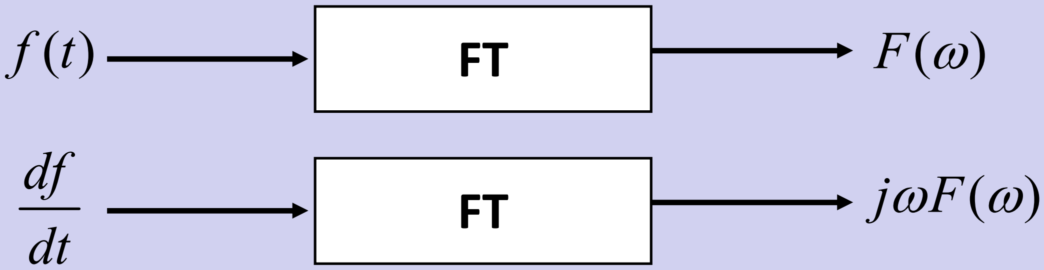
Frequency domain



$$F(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt$$

Fourier Transform (FT)

2) Time-domain derivative and Fourier Transform



Frequency domain

- Fourier Transform and functions of *n* variables
- Fourier Transform and vector functions
- Fourier Transform and vector functions of *n* variables

Frequency domain

- Fourier Transform and functions of *n* variables

- Fourier Transform and vector functions

- Fourier Transform and vector functions of *n* variables

1) How to jump back from the Frequency domain to the Time domain

2) Time domain derivative and Fourier Transform

Frequency domain

- Fourier Transform and functions of *n* variables

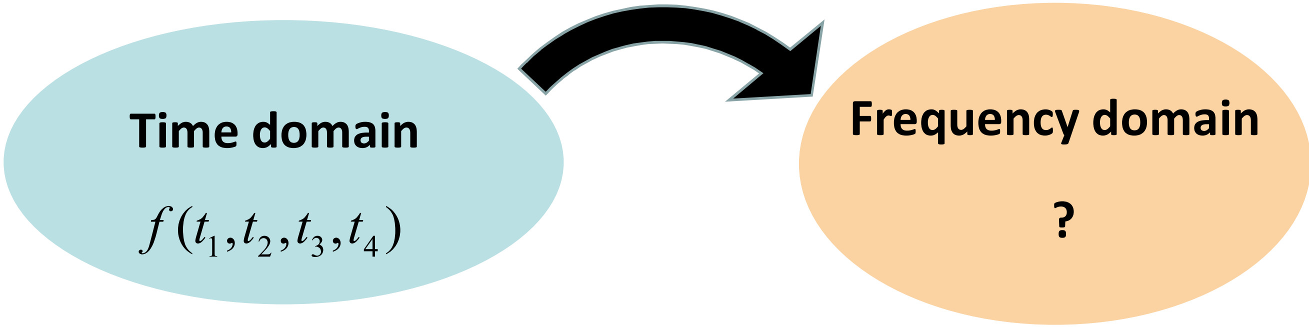
- Fourier Transform and vector functions

- Fourier Transform and vector functions of *n* variables

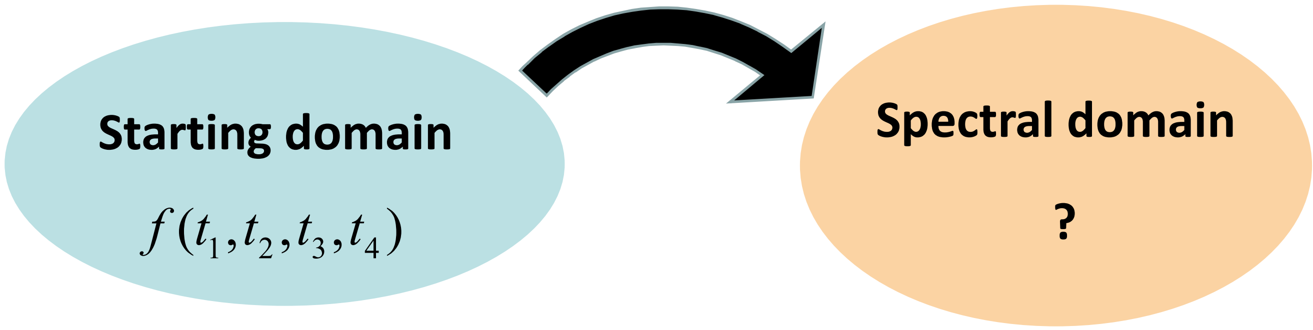
1) How to jump back from the Frequency domain to the Time domain

2) Time domain derivative and Fourier Transform

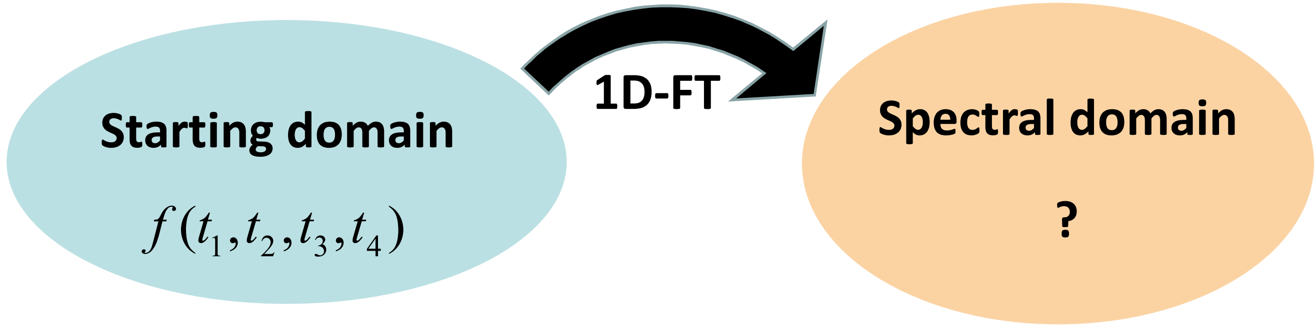
Fourier Transform and functions of *n* variables



Fourier Transform and functions of *n* variables



Fourier Transform and functions of *n* variables



One Dimensional Fourier Transform (1D-FT)

Fourier Transform and functions of *n* variables

Starting domain

$f(t_1, t_2, t_3, t_4)$

1D-FT

Spectral domain

$F(\omega_1, t_2, t_3, t_4)$

One Dimensional Fourier Transform (1D-FT)

$$F(\omega_1, t_2, t_3, t_4) = \int_{-\infty}^{+\infty} f(t_1, t_2, t_3, t_4)\, e^{-j\omega_1 t_1}\, dt_1$$

Fourier Transform and functions of *n* variables

Starting domain

$f(t_1, t_2, t_3, t_4)$

Spectral domain

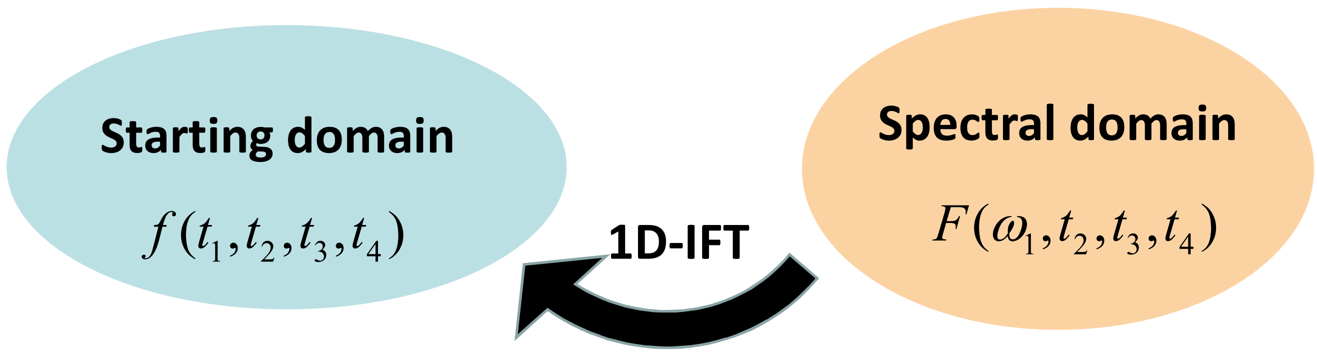
$F(\omega_1, t_2, t_3, t_4)$

One Dimensional Fourier Transform (1D-FT)

$$F(\omega_1, t_2, t_3, t_4) = \int_{-\infty}^{+\infty} f(t_1, t_2, t_3, t_4)\, e^{-j\omega_1 t_1}\, dt_1$$

1) How to jump back from the Spectral domain to the Time domain

Fourier Transform and functions of *n* variables



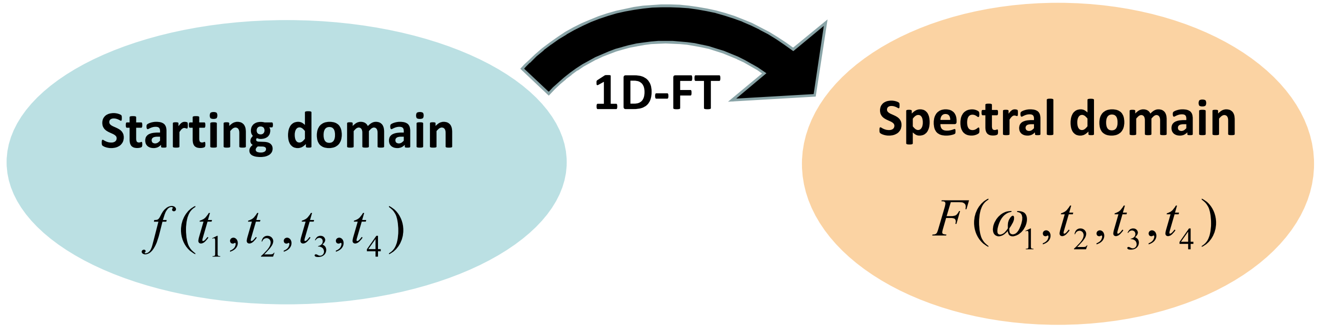
One Dimensional Fourier Transform (1D-FT)

$$F(\omega_1, t_2, t_3, t_4) = \int_{-\infty}^{+\infty} f(t_1, t_2, t_3, t_4)\, e^{-j\omega_1 t_1}\, dt_1$$

1) How to jump back from the Spectral domain to the Time domain

$$f(t_1, t_2, t_3, t_4) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(\omega_1, t_2, t_3, t_4)\, e^{j\omega_1 t_1}\, d\omega_1 \quad \textbf{1D-IFT}$$

Fourier Transform and functions of *n* variables



$f(t_1,t_2,t_3,t_4)$ → **1D-FT** →

$$F(\omega_1,t_2,t_3,t_4)=\int_{-\infty}^{+\infty} f(t_1,t_2,t_3,t_4)\,e^{-j\omega_1 t_1}\,dt_1$$

$F(\omega_1,t_2,t_3,t_4)$ → **1D-IFT** →

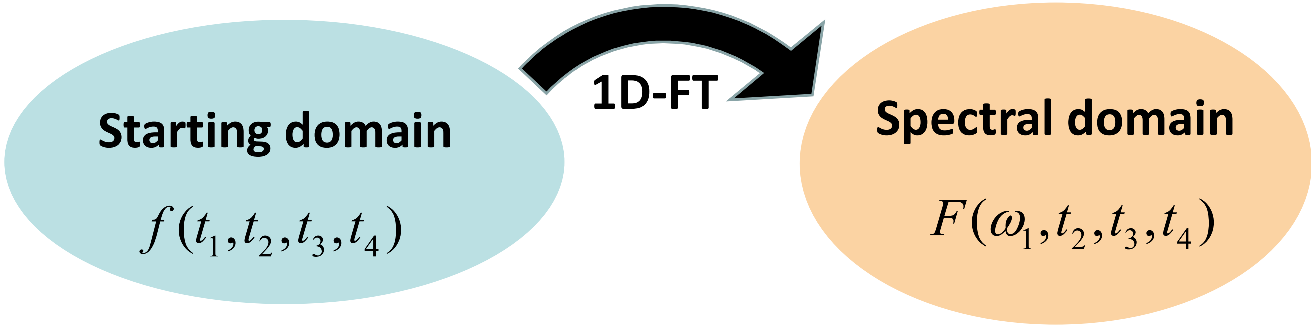
$$f(t_1,t_2,t_3,t_4)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} F(\omega_1,t_2,t_3,t_4)\,e^{j\omega_1 t_1}\,d\omega_1$$

Fourier Transform and functions of *n* variables

$$\frac{\partial f(t_1,t_2,t_3,t_4)}{\partial t_1}=\frac{1}{2\pi}\int_{-\infty}^{+\infty} j\omega_1 F(\omega_1,t_2,t_3,t_4) e^{j\omega_1 t_1} d\omega_1$$

$$\frac{\partial f(t_1,t_2,t_3,t_4)}{\partial t_2}=\frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{\partial F(\omega_1,t_2,t_3,t_4)}{\partial t_2} e^{j\omega_1 t_1} d\omega_1$$

Fourier Transform and functions of *n* variables



2) Time domain derivative and Fourier Transform

Fourier Transform and functions of *n* variables

Starting domain

$f(t_1, t_2, t_3, t_4)$

1D-FT

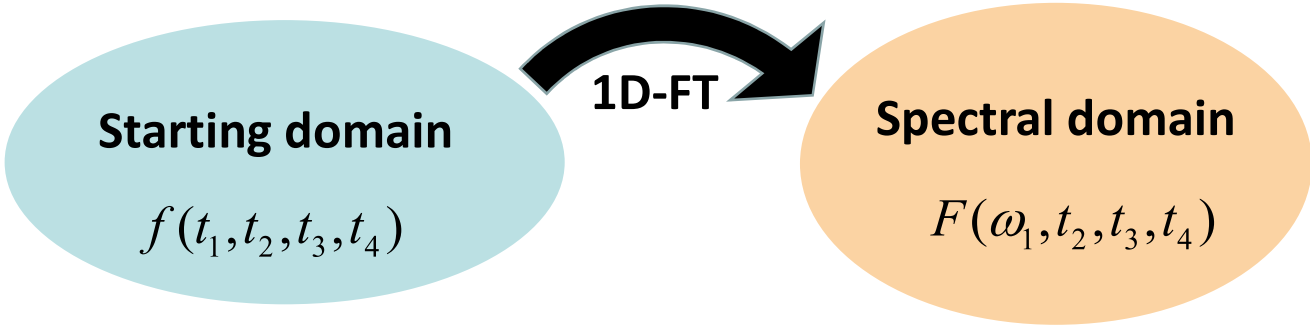
Spectral domain

$F(\omega_1, t_2, t_3, t_4)$

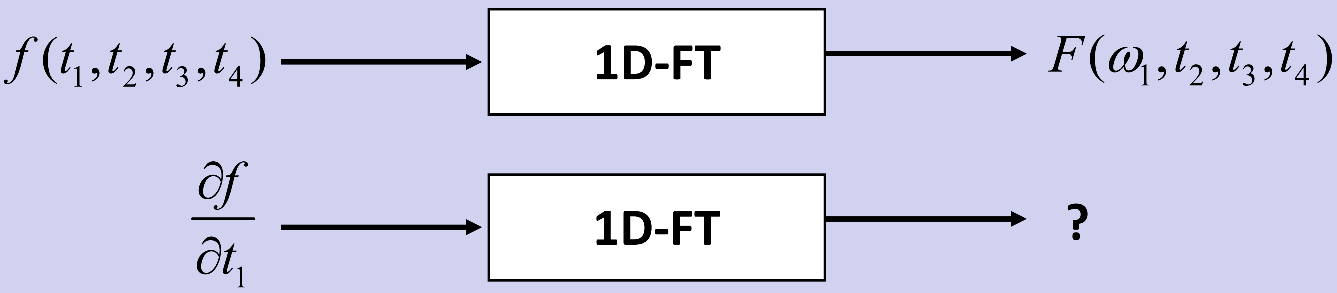
2) Time domain derivative and Fourier Transform

$f(t_1, t_2, t_3, t_4) \longrightarrow$ **1D-FT** $\longrightarrow F(\omega_1, t_2, t_3, t_4)$

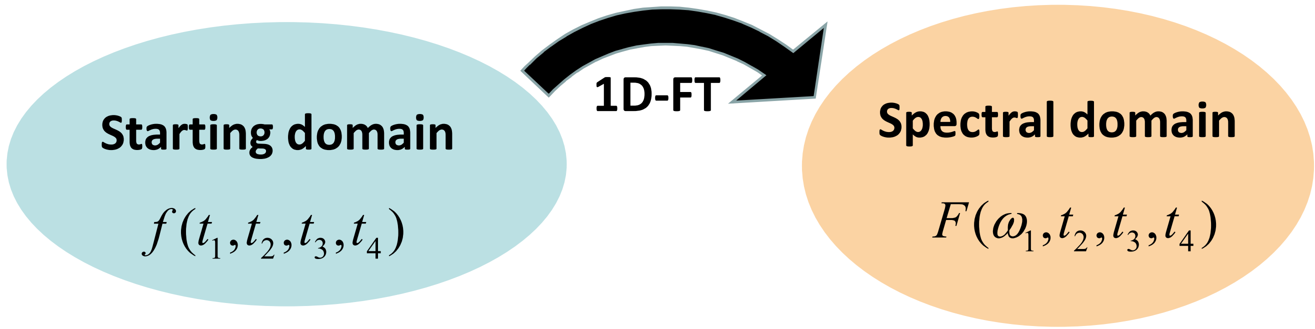
Fourier Transform and functions of *n* variables



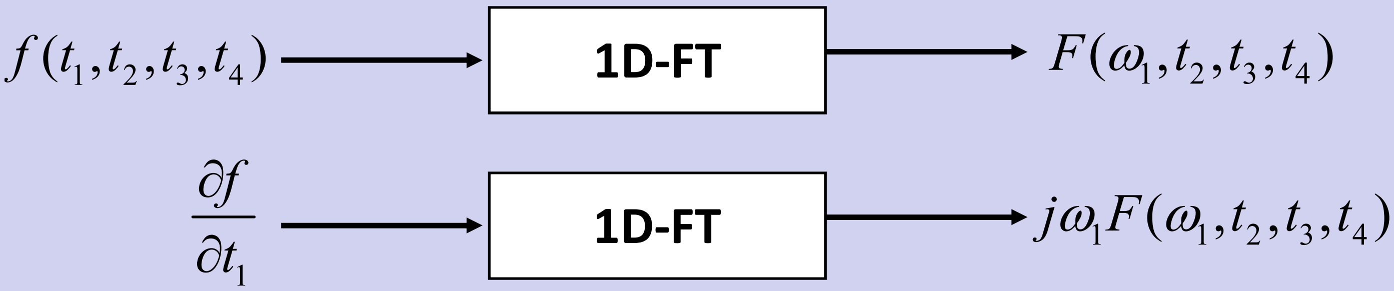
2) Time domain derivative and Fourier Transform



Fourier Transform and functions of *n* variables



2) Time domain derivative and Fourier Transform



Fourier Transform and functions of *n* variables

Starting domain $f(t_1, t_2, t_3, t_4)$ — **1D-FT** → **Spectral domain** $F(\omega_1, t_2, t_3, t_4)$

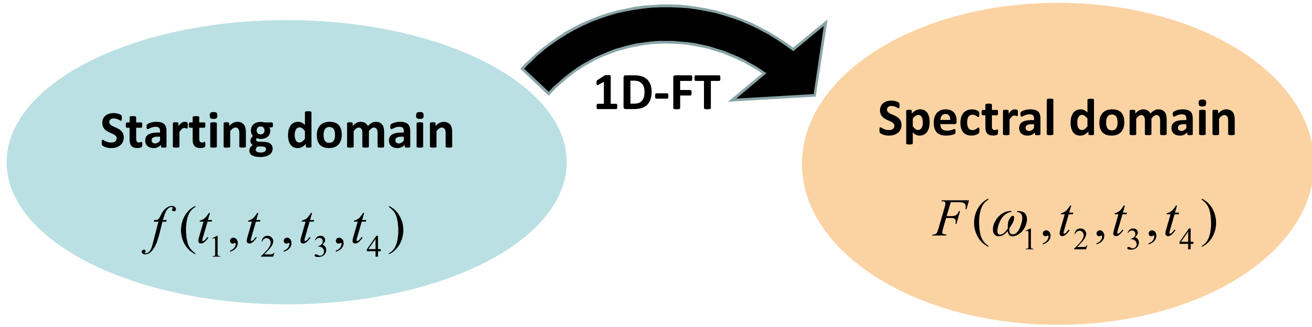
2) Time domain derivative and Fourier Transform

$f(t_1, t_2, t_3, t_4)$ → **1D-FT** → $F(\omega_1, t_2, t_3, t_4)$

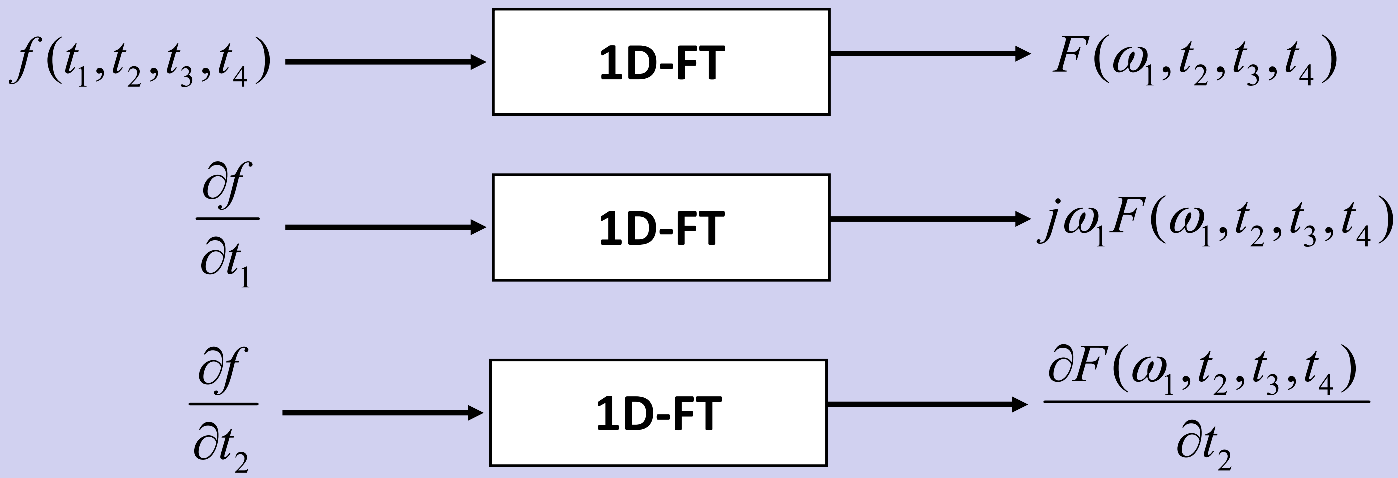
$\dfrac{\partial f}{\partial t_1}$ → **1D-FT** → $j\omega_1 F(\omega_1, t_2, t_3, t_4)$

$\dfrac{\partial f}{\partial t_2}$ → **1D-FT** → **?**

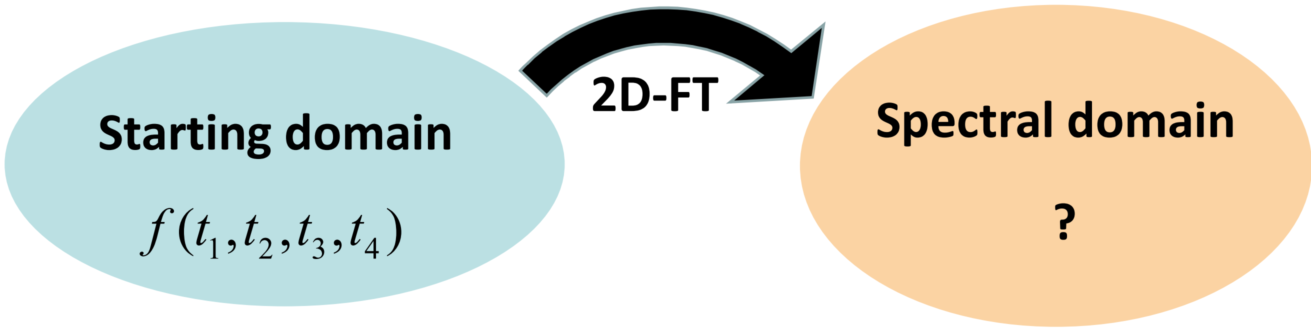
Fourier Transform and functions of *n* variables



2) Time domain derivative and Fourier Transform

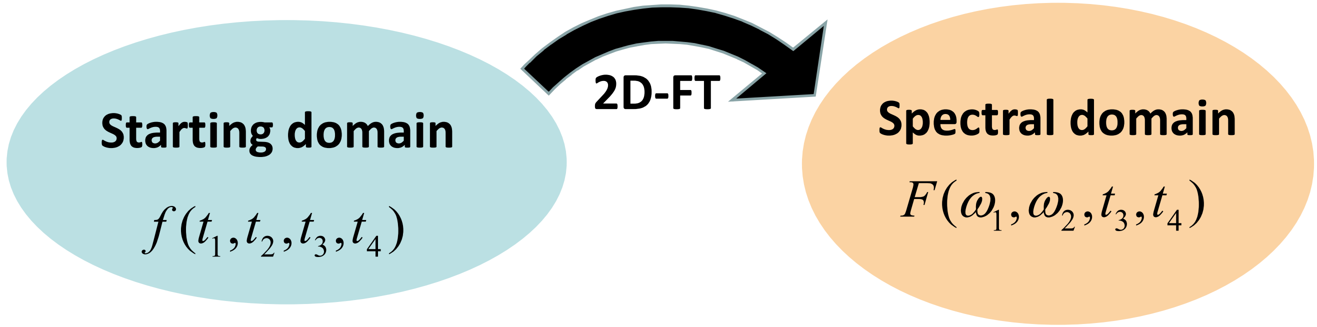


Fourier Transform and functions of *n* variables



Two Dimensional Fourier Transform (2D-FT)

Fourier Transform and functions of *n* variables



Two Dimensional Fourier Transform (2D-FT)

$$F(\omega_1, \omega_2, t_3, t_4) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} dt_1 dt_2 f(t_1, t_2, t_3, t_4) e^{-j\omega_1 t_1} e^{-j\omega_2 t_2}$$

Fourier Transform and functions of *n* variables

Starting domain

$f(t_1, t_2, t_3, t_4)$

Spectral domain

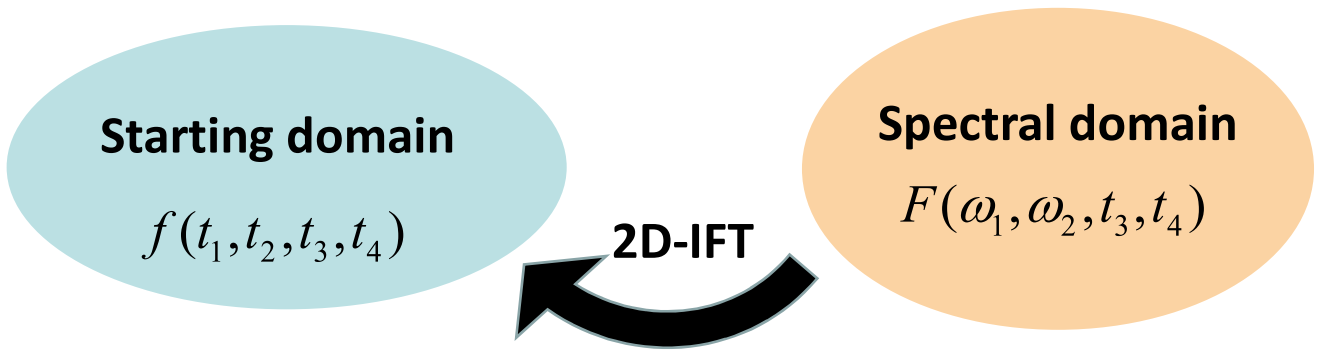
$F(\omega_1, \omega_2, t_3, t_4)$

Two Dimensional Fourier Transform (2D-FT)

$$F(\omega_1, \omega_2, t_3, t_4) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dt_1 dt_2 f(t_1, t_2, t_3, t_4)\, e^{-j\omega_1 t_1} e^{-j\omega_2 t_2}$$

1) How to jump back from the Spectral domain to the Time domain

Fourier Transform and functions of *n* variables



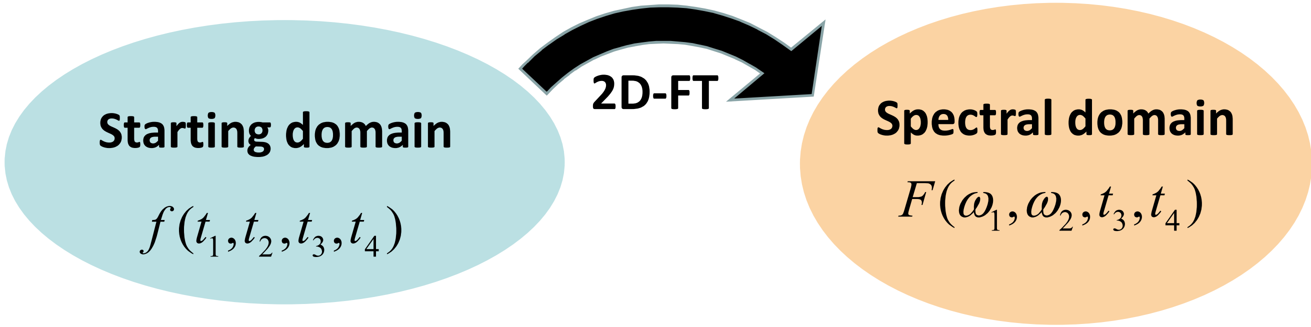
Two Dimensional Fourier Transform (2D-FT)

$$F(\omega_1,\omega_2,t_3,t_4)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dt_1 dt_2 f(t_1,t_2,t_3,t_4)\,e^{-j\omega_1 t_1}e^{-j\omega_2 t_2}$$

1) How to jump back from the Spectral domain to the Time domain

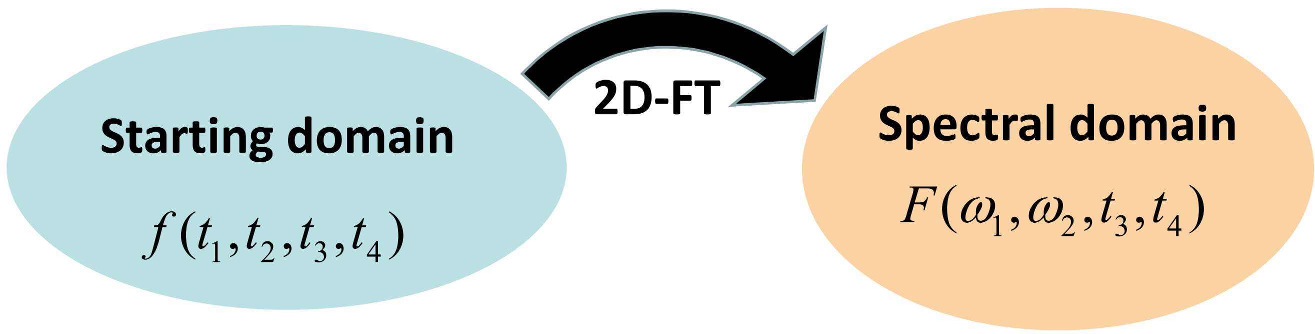
$$f(t_1,t_2,t_3,t_4)=\left(\frac{1}{2\pi}\right)^2\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} F(\omega_1,\omega_2,t_3,t_4)e^{j\omega_1 t_1}e^{j\omega_2 t_2}d\omega_1 d\omega_2 \quad \textbf{2D-IFT}$$

Fourier Transform and functions of *n* variables

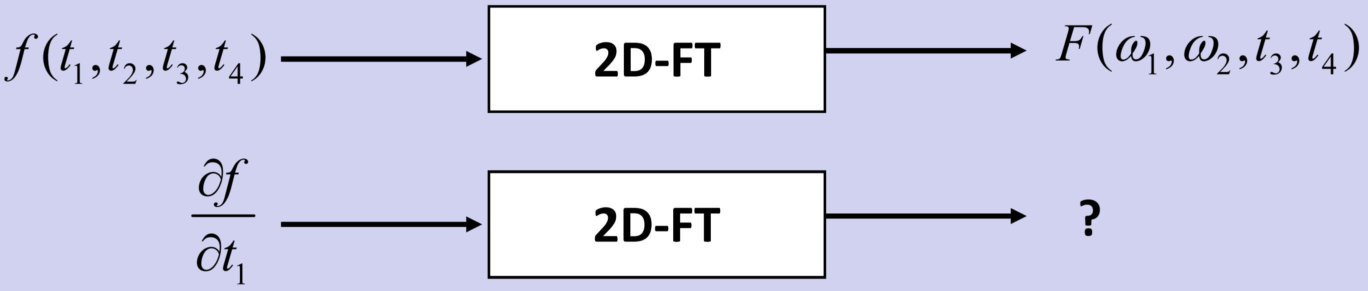


2) Time domain derivative and Fourier Transform

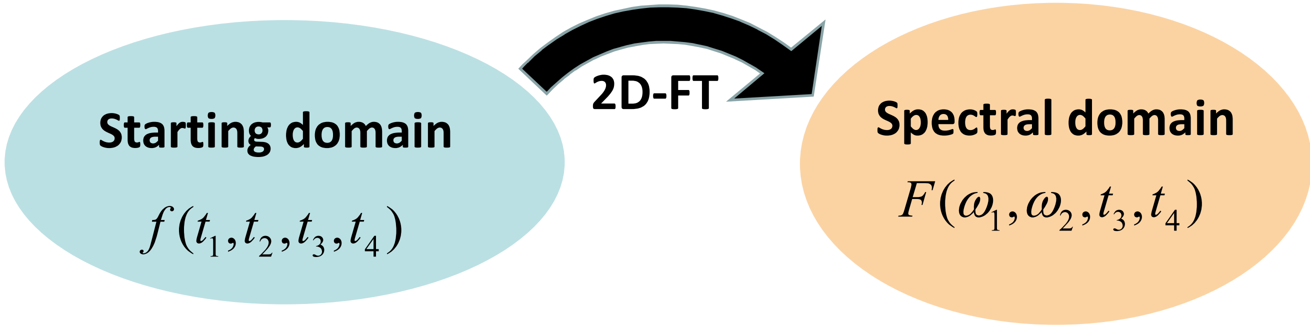
Fourier Transform and functions of *n* variables



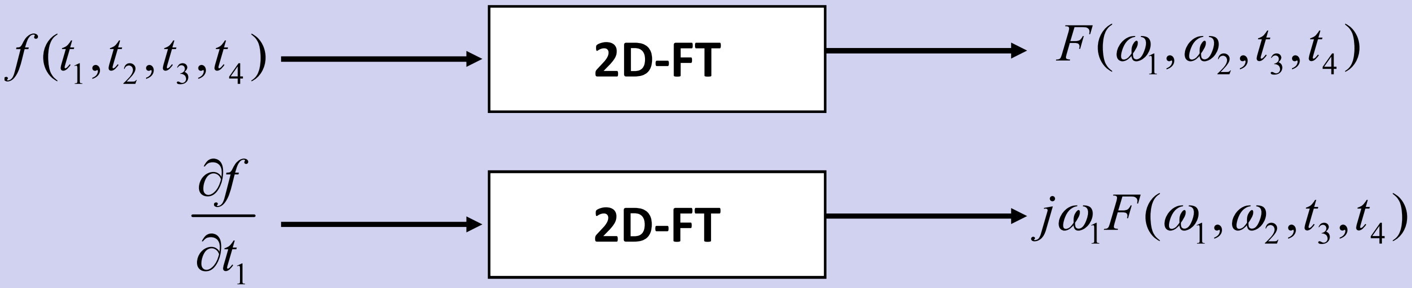
2) Time domain derivative and Fourier Transform



Fourier Transform and functions of *n* variables



2) Time domain derivative and Fourier Transform



Fourier Transform and functions of *n* variables

Starting domain

$f(t_1, t_2, t_3, t_4)$

2D-FT →

Spectral domain

$F(\omega_1, \omega_2, t_3, t_4)$

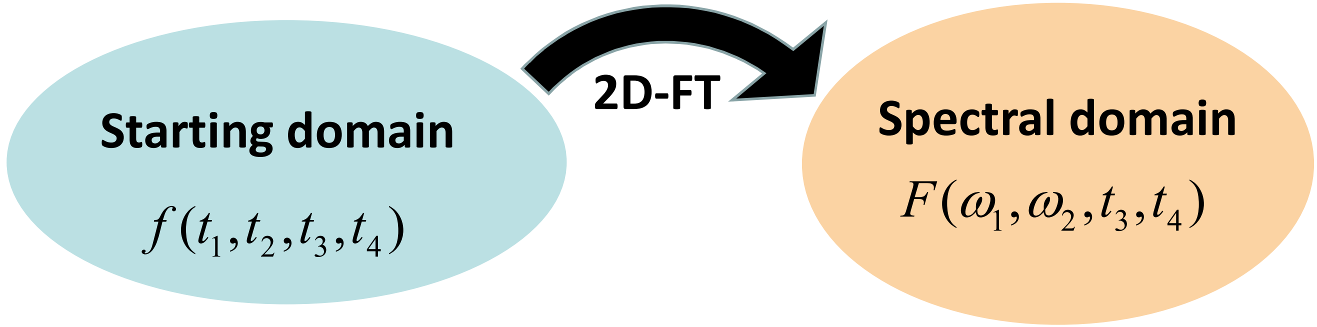
2) Time domain derivative and Fourier Transform

$f(t_1, t_2, t_3, t_4)$ → **2D-FT** → $F(\omega_1, \omega_2, t_3, t_4)$

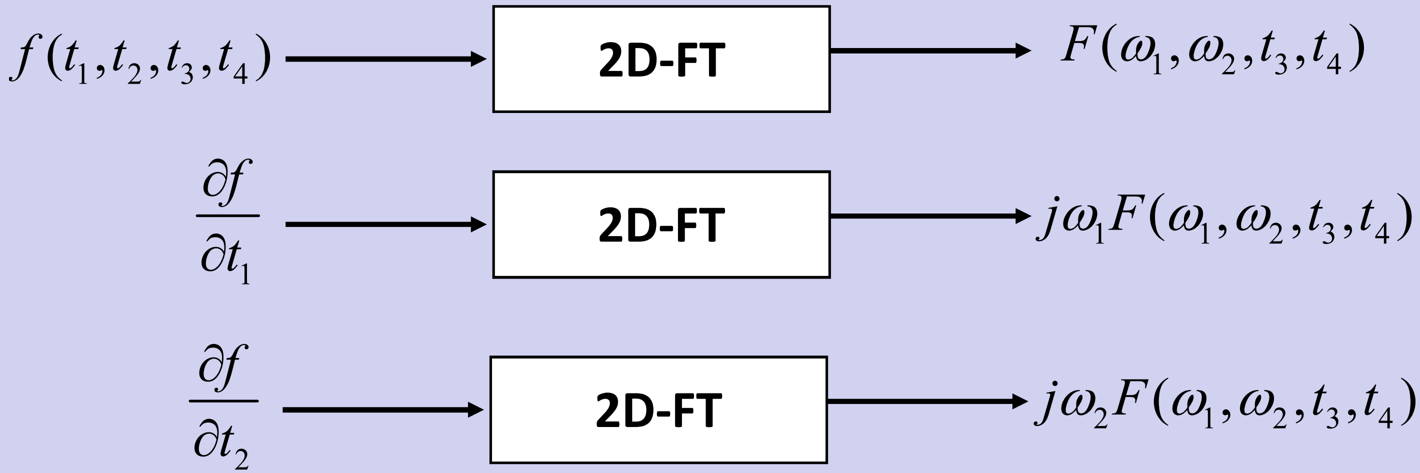
$\dfrac{\partial f}{\partial t_1}$ → **2D-FT** → $j\omega_1 F(\omega_1, \omega_2, t_3, t_4)$

$\dfrac{\partial f}{\partial t_2}$ → **2D-FT** → **?**

Fourier Transform and functions of *n* variables



2) Time domain derivative and Fourier Transform



Fourier Transform and functions of *n* variables

Starting domain

$f(t_1, t_2, t_3, t_4)$

2D-FT →

Spectral domain

$F(\omega_1, \omega_2, t_3, t_4)$

2) Time domain derivative and Fourier Transform

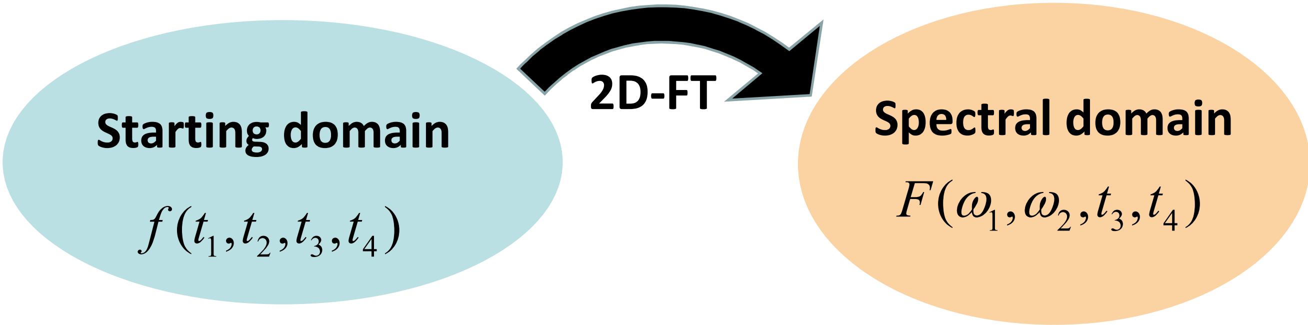
$f(t_1, t_2, t_3, t_4)$ → **2D-FT** → $F(\omega_1, \omega_2, t_3, t_4)$

$\dfrac{\partial f}{\partial t_1}$ → **2D-FT** → $j\omega_1 F(\omega_1, \omega_2, t_3, t_4)$

$\dfrac{\partial f}{\partial t_2}$ → **2D-FT** → $j\omega_2 F(\omega_1, \omega_2, t_3, t_4)$

$\dfrac{\partial f}{\partial t_3}$ → **2D-FT** → **?**

Fourier Transform and functions of *n* variables



2) Time domain derivative and Fourier Transform

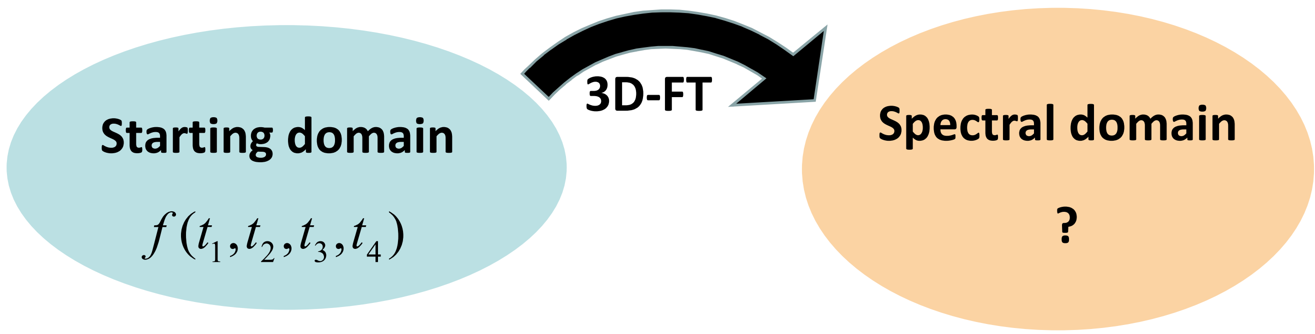
$$f(t_1,t_2,t_3,t_4) \longrightarrow \boxed{\text{2D-FT}} \longrightarrow F(\omega_1,\omega_2,t_3,t_4)$$

$$\frac{\partial f}{\partial t_1} \longrightarrow \boxed{\text{2D-FT}} \longrightarrow j\omega_1 F(\omega_1,\omega_2,t_3,t_4)$$

$$\frac{\partial f}{\partial t_2} \longrightarrow \boxed{\text{2D-FT}} \longrightarrow j\omega_2 F(\omega_1,\omega_2,t_3,t_4)$$

$$\frac{\partial f}{\partial t_3} \longrightarrow \boxed{\text{2D-FT}} \longrightarrow \frac{\partial F(\omega_1,\omega_2,t_3,t_4)}{\partial t_3}$$

Fourier Transform and functions of *n* variables



Three Dimensional Fourier Transform (3D-FT)

Fourier Transform and functions of *n* variables

Starting domain

$f(t_1, t_2, t_3, t_4)$

3D-FT →

Spectral domain

$F(\omega_1, \omega_2, \omega_3, t_4)$

Three Dimensional Fourier Transform (3D-FT)

$$F(\omega_1, \omega_2, \omega_3, t_4) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dt_1 dt_2 dt_3 \, f(t_1, t_2, t_3, t_4)\, e^{-j\omega_1 t_1} e^{-j\omega_2 t_2} e^{-j\omega_3 t_3}$$

Fourier Transform and functions of *n* variables

Starting domain

$f(t_1,t_2,t_3,t_4)$

Spectral domain

$F(\omega_1,\omega_2,\omega_3,t_4)$

Three Dimensional Fourier Transform (3D-FT)

$$F(\omega_1,\omega_2,\omega_3,t_4)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dt_1 dt_2 dt_3 f(t_1,t_2,t_3,t_4)\, e^{-j\omega_1 t_1} e^{-j\omega_2 t_2} e^{-j\omega_3 t_3}$$

1) How to jump back from the Spectral domain to the Time domain

Fourier Transform and functions of *n* variables

Starting domain

$f(t_1,t_2,t_3,t_4)$

3D-IFT

Spectral domain

$F(\omega_1,\omega_2,\omega_3,t_4)$

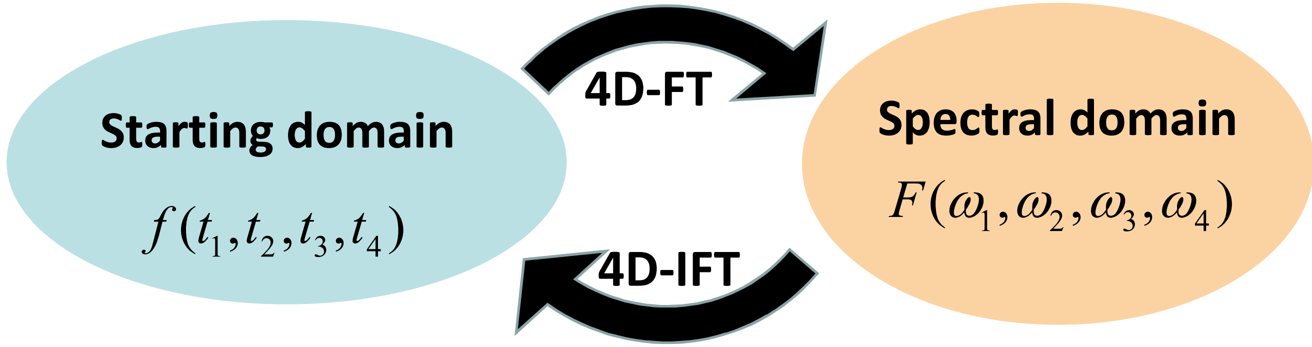
Three Dimensional Fourier Transform (3D-FT)

$$F(\omega_1,\omega_2,\omega_3,t_4)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dt_1 dt_2 dt_3 f(t_1,t_2,t_3,t_4)\, e^{-j\omega_1 t_1} e^{-j\omega_2 t_2} e^{-j\omega_3 t_3}$$

1) How to jump back from the Spectral domain to the Time domain

$$f(t_1,t_2,t_3,t_4)=\left(\frac{1}{2\pi}\right)^3\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} F(\omega_1,\omega_2,\omega_3,t_4)e^{j\omega_1 t_1} e^{j\omega_2 t_2} e^{j\omega_3 t_3}\, d\omega_1 d\omega_2 d\omega_3$$ **3D-IFT**

Fourier Transform and functions of *n* variables



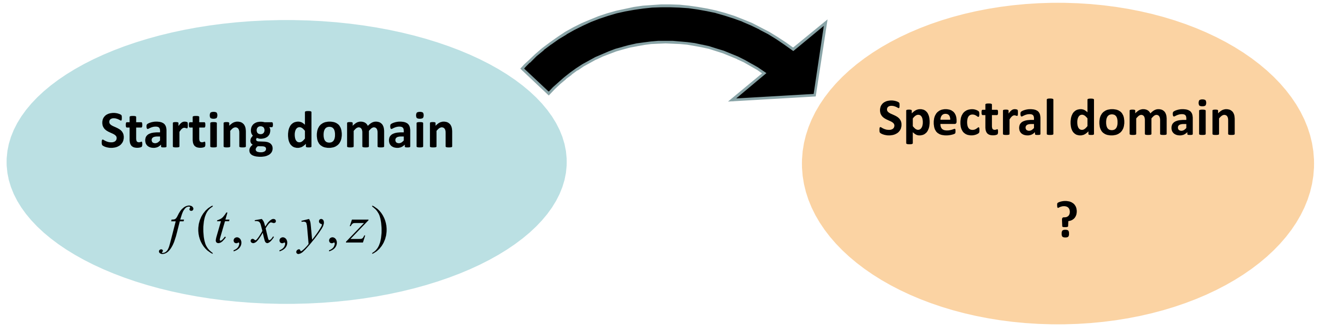
Four Dimensional Fourier Transform (4D-FT)

$$F(\omega_1,\omega_2,\omega_3,\omega_4)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dt_1 dt_2 dt_3 dt_4 f(t_1,t_2,t_3,t_4)e^{-j\omega_1 t_1}e^{-j\omega_2 t_2}e^{-j\omega_3 t_3}e^{-j\omega_4 t_4}$$

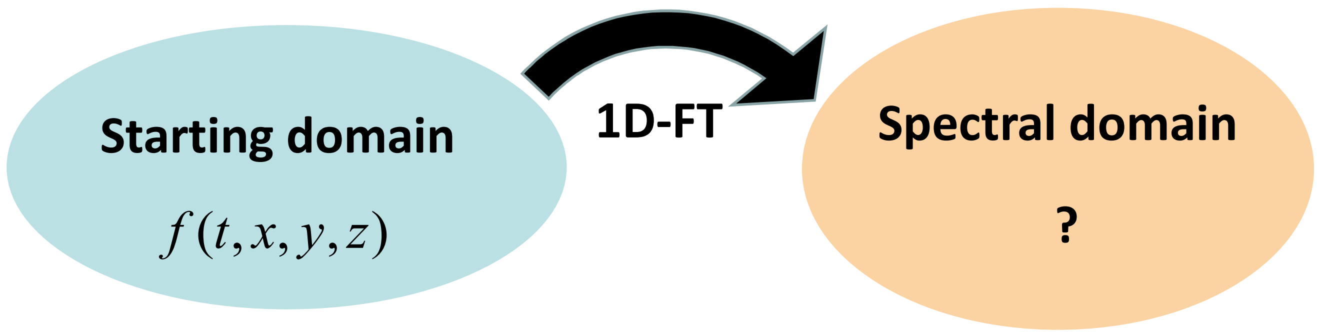
1) How to jump back from the Spectral domain to the Time domain

$$f(t_1,t_2,t_3,t_4)=\left(\frac{1}{2\pi}\right)^4\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} F(\omega_1,\omega_2,\omega_3,\omega_4)e^{j\omega_1 t_1}e^{j\omega_2 t_2}e^{j\omega_3 t_3}e^{j\omega_4 t_4}d\omega_1 d\omega_2 d\omega_3 d\omega_4$$ **4D-IFT**

Fourier Transform and functions of *n* variables



Fourier Transform and functions of *n* variables



One Dimensional Fourier Transform (1D-FT)

Fourier Transform and functions of *n* variables

Starting domain

$f(t,x,y,z)$

1D-FT

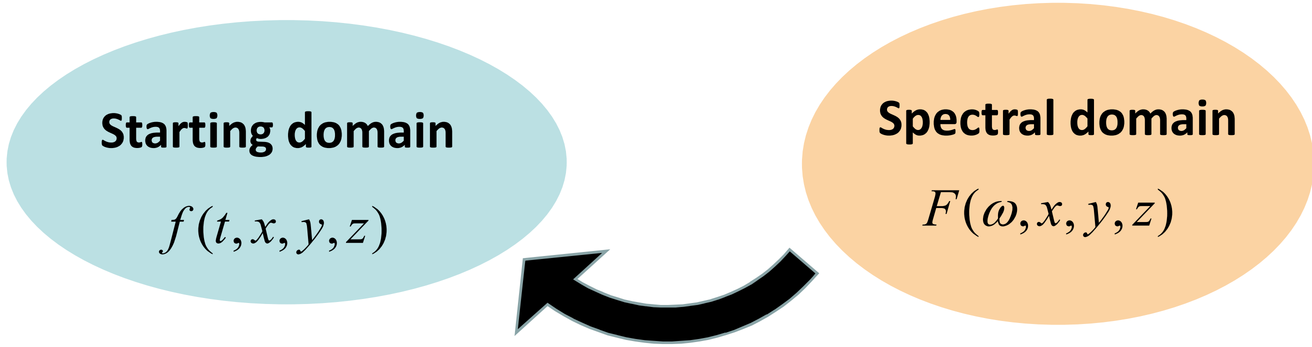
Spectral domain

$F(\omega,x,y,z)$

One Dimensional Fourier Transform (1D-FT)

$$F(\omega,x,y,z)=\int_{-\infty}^{+\infty} f(t,x,y,z)\,e^{-j\omega t}\,dt$$

Fourier Transform and functions of *n* variables



One Dimensional Fourier Transform (1D-FT)

$$F(\omega,x,y,z)=\int_{-\infty}^{+\infty} f(t,x,y,z)e^{-j\omega t}dt$$

1) How to jump back from the Spectral domain to the Time domain

Fourier Transform and functions of *n* variables

Starting domain

$f(t,x,y,z)$

1D-IFT

Spectral domain

$F(\omega,x,y,z)$

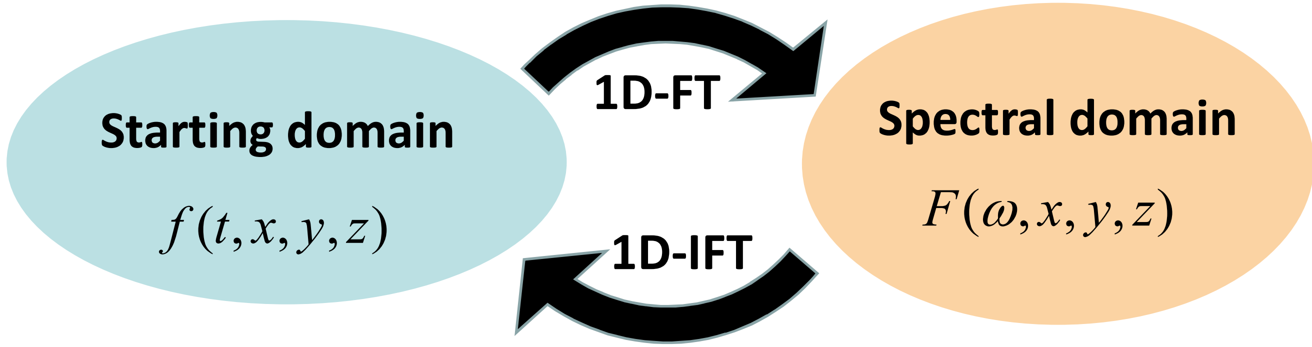
One Dimensional Fourier Transform (1D-FT)

$$F(\omega,x,y,z)=\int_{-\infty}^{+\infty} f(t,x,y,z)\,e^{-j\omega t}\,dt$$

1) How to jump back from the Spectral domain to the Time domain

$$f(t,x,y,z)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} F(\omega,x,y,z)\,e^{j\omega t}\,d\omega$$ **1D-IFT**

Fourier Transform and functions of *n* variables



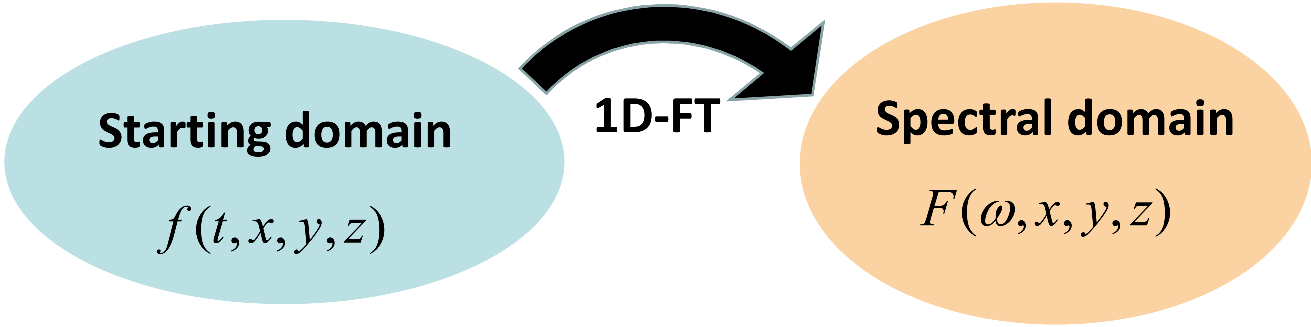
$f(t,x,y,z)$ → **1D-FT** →

$$F(\omega,x,y,z) = \int_{-\infty}^{+\infty} f(t,x,y,z)\, e^{-j\omega t}\, dt$$

$F(\omega,x,y,z)$ → **1D-IFT** →

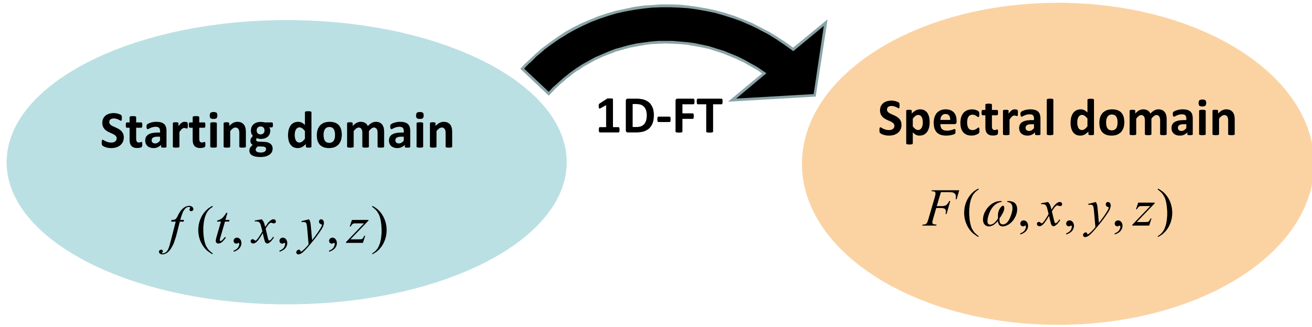
$$f(t,x,y,z) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(\omega,x,y,z)\, e^{j\omega t}\, d\omega$$

Fourier Transform and functions of *n* variables

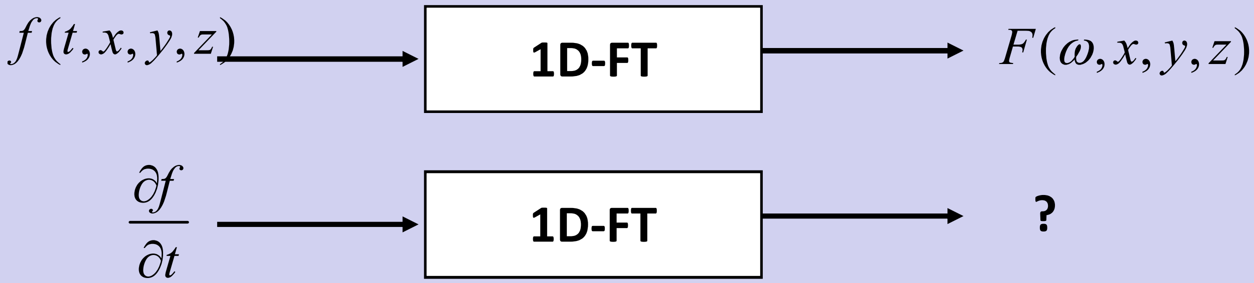


2) Time domain derivative and Fourier Transform

Fourier Transform and functions of *n* variables



2) Time domain derivative and Fourier Transform



Fourier Transform and functions of *n* variables

Starting domain $f(t, x, y, z)$ → **1D-FT** → **Spectral domain** $F(\omega, x, y, z)$

2) Time domain derivative and Fourier Transform

$f(t, x, y, z)$ → **1D-FT** → $F(\omega, x, y, z)$

$\dfrac{\partial f}{\partial t}$ → **1D-FT** → $j\omega F(\omega, x, y, z)$

Fourier Transform and functions of *n* variables

Starting domain $f(t,x,y,z)$ → **1D-FT** → **Spectral domain** $F(\omega,x,y,z)$

2) Time domain derivative and Fourier Transform

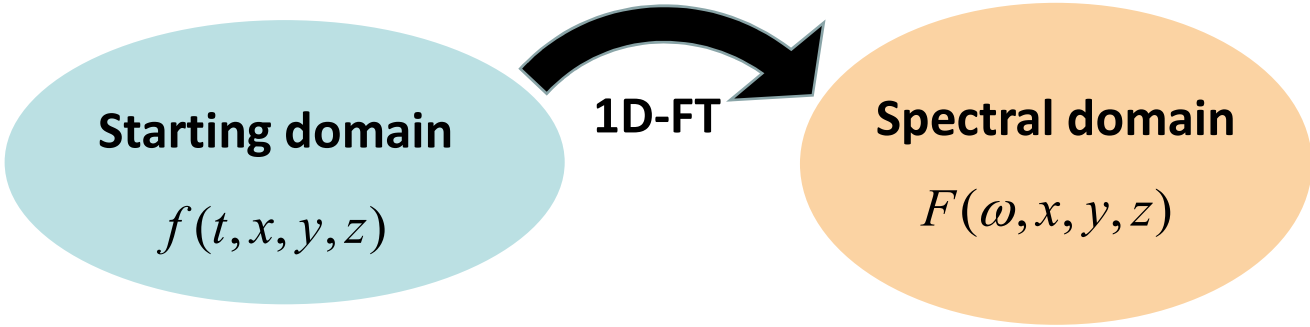
$f(t,x,y,z)$ → **1D-FT** → $F(\omega,x,y,z)$

$\frac{\partial f}{\partial t}$ → **1D-FT** → $j\omega F(\omega,x,y,z)$

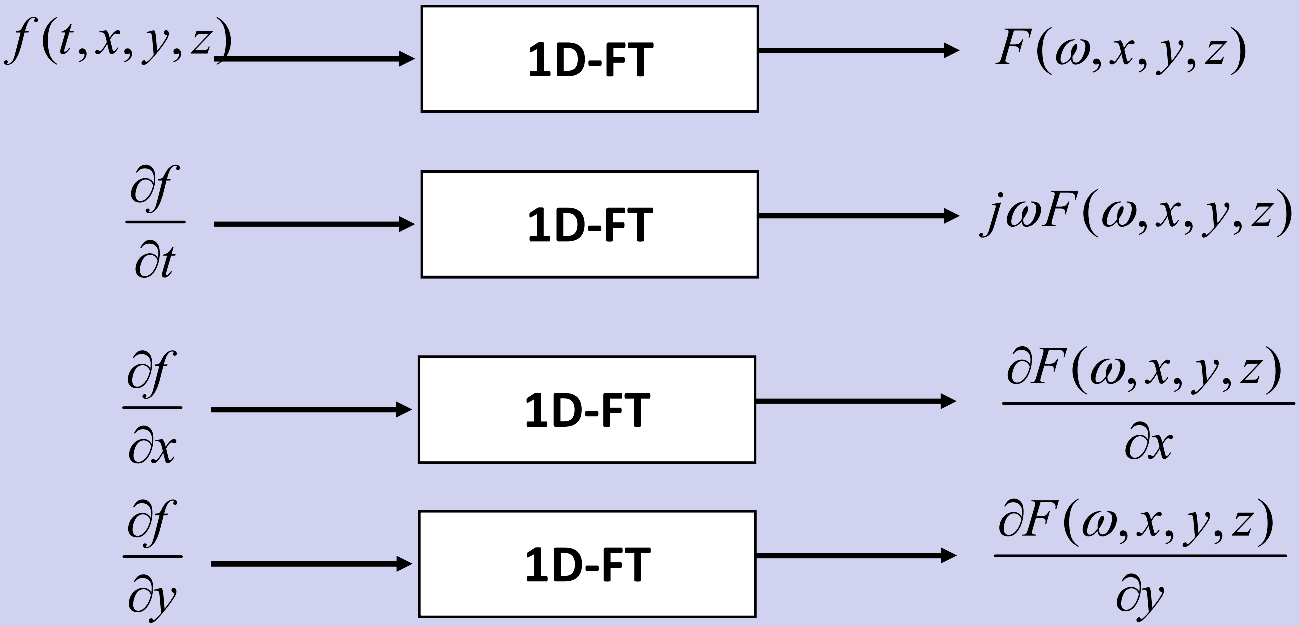
$\frac{\partial f}{\partial x}$ → **1D-FT** → **?**

$\frac{\partial f}{\partial y}$ → **1D-FT** → **?**

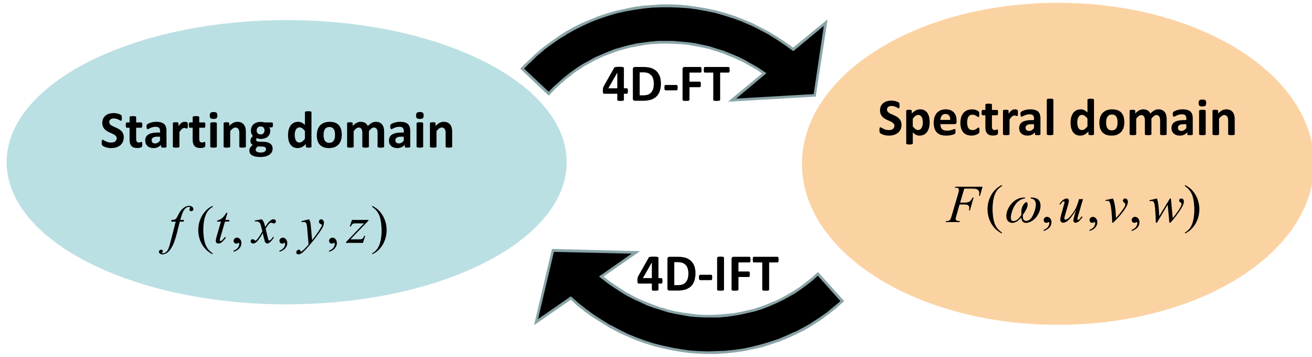
Fourier Transform and functions of *n* variables



2) Time domain derivative and Fourier Transform



Fourier Transform and functions of *n* variables



Four Dimensional Fourier Transform (4D-FT)

$$F(\omega,u,v,w)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(t,x,y,z)\,e^{-j\omega t}e^{-jux}e^{-jvy}e^{-jwz}\,dt\,dx\,dy\,dz$$

1) How to jump back from the Spectral domain to the Time domain

$$f(t,x,y,z)=\left(\frac{1}{2\pi}\right)^4\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} F(\omega,u,v,w)\,e^{j\omega t}e^{jux}e^{jvy}e^{jwz}\,d\omega\,du\,dv\,dw$$ **4D-IFT**

Frequency domain

- Fourier Transform and functions of n variables

- Fourier Transform and vector functions

- Fourier Transform and vector functions of n variables

1) How to jump back from the Frequency domain to the Time domain

2) Time domain derivative and Fourier Transform

Fourier Transform and vector functions

Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Frequency domain

?

Fourier Transform and vector functions

Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Frequency domain

?

$f_x(t)$ → **1D-FT** → $F_x(\omega)$

$f_y(t)$ → **1D-FT** → $F_y(\omega)$

$f_z(t)$ → **1D-FT** → $F_z(\omega)$

Fourier Transform and vector functions

Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Frequency domain

?

$f_x(t)$ → **1D-FT** → $F_x(\omega) = \int_{-\infty}^{+\infty} f_x(t)e^{-j\omega t}dt$

$f_y(t)$ → **1D-FT** → $F_y(\omega) = \int_{-\infty}^{+\infty} f_y(t)e^{-j\omega t}dt$

$f_z(t)$ → **1D-FT** → $F_z(\omega) = \int_{-\infty}^{+\infty} f_z(t)e^{-j\omega t}dt$

Fourier Transform and vector functions

Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Frequency domain

$$\vec{\mathbf{F}}(\omega) = F_x(\omega)\hat{i}_x + F_y(\omega)\hat{i}_y + F_z(\omega)\hat{i}_z$$

$f_x(t) \rightarrow$ **1D-FT** $\rightarrow F_x(\omega) = \int_{-\infty}^{+\infty} f_x(t) e^{-j\omega t} dt$

$f_y(t) \rightarrow$ **1D-FT** $\rightarrow F_y(\omega) = \int_{-\infty}^{+\infty} f_y(t) e^{-j\omega t} dt$

$f_z(t) \rightarrow$ **1D-FT** $\rightarrow F_z(\omega) = \int_{-\infty}^{+\infty} f_z(t) e^{-j\omega t} dt$

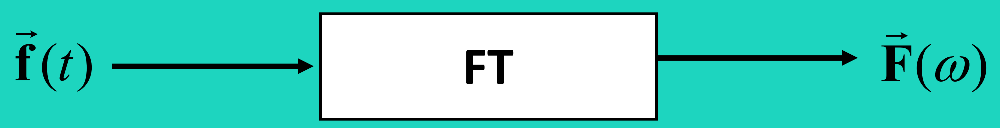
Fourier Transform and vector functions

Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Frequency domain

$$\vec{\mathbf{F}}(\omega) = F_x(\omega)\hat{i}_x + F_y(\omega)\hat{i}_y + F_z(\omega)\hat{i}_z$$



Fourier Transform and vector functions

Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Frequency domain

$$\vec{\mathbf{F}}(\omega) = F_x(\omega)\hat{i}_x + F_y(\omega)\hat{i}_y + F_z(\omega)\hat{i}_z$$

1) How to jump back from the Spectral domain to the Time domain

Fourier Transform and vector functions

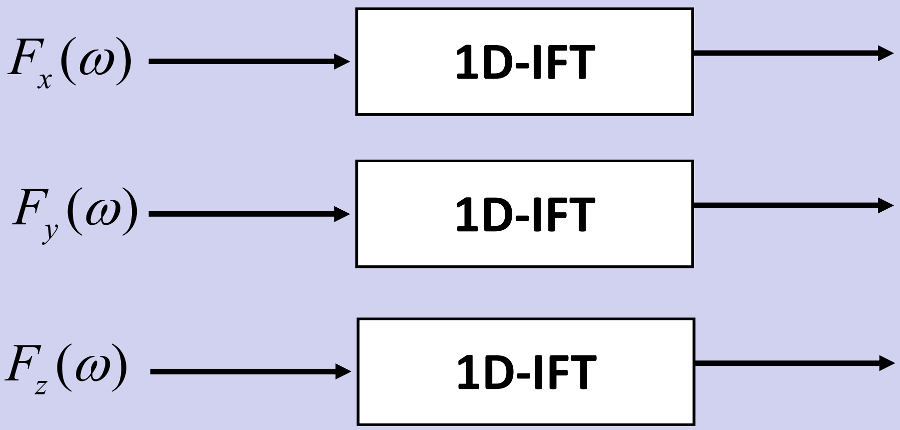
Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Frequency domain

$$\vec{\mathbf{F}}(\omega) = F_x(\omega)\hat{i}_x + F_y(\omega)\hat{i}_y + F_z(\omega)\hat{i}_z$$

1) How to jump back from the Spectral domain to the Time domain



Fourier Transform and vector functions

Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Frequency domain

$$\vec{\mathbf{F}}(\omega) = F_x(\omega)\hat{i}_x + F_y(\omega)\hat{i}_y + F_z(\omega)\hat{i}_z$$

1) How to jump back from the Spectral domain to the Time domain

$F_x(\omega)$ → **1D-IFT** → $f_x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F_x(\omega)e^{j\omega t}d\omega$

$F_y(\omega)$ → **1D-IFT** → $f_y(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F_y(\omega)e^{j\omega t}d\omega$

$F_z(\omega)$ → **1D-IFT** → $f_z(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F_z(\omega)e^{j\omega t}d\omega$

Fourier Transform and vector functions

Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Frequency domain

$$\vec{\mathbf{F}}(\omega) = F_x(\omega)\hat{i}_x + F_y(\omega)\hat{i}_y + F_z(\omega)\hat{i}_z$$

2) Time domain derivative and Fourier Transform

Fourier Transform and vector functions

Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Frequency domain

$$\vec{\mathbf{F}}(\omega) = F_x(\omega)\hat{i}_x + F_y(\omega)\hat{i}_y + F_z(\omega)\hat{i}_z$$

2) Time domain derivative and Fourier Transform

$\vec{\mathbf{f}}(t)$ → **FT** → $\vec{\mathbf{F}}(\omega)$

$\dfrac{d\vec{\mathbf{f}}(t)}{dt}$ → **FT** → **?**

Fourier Transform and vector functions

Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Frequency domain

$$\vec{\mathbf{F}}(\omega) = F_x(\omega)\hat{i}_x + F_y(\omega)\hat{i}_y + F_z(\omega)\hat{i}_z$$

2) Time domain derivative and Fourier Transform

$\vec{\mathbf{f}}(t)$ → **FT** → $\vec{\mathbf{F}}(\omega)$

$\dfrac{d\vec{\mathbf{f}}(t)}{dt}$ → **FT** → **?**

$$\frac{d\vec{\mathbf{f}}(t)}{dt} = \frac{df_x(t)}{dt}\hat{i}_x + \frac{df_y(t)}{dt}\hat{i}_y + \frac{df_z(t)}{dt}\hat{i}_z$$

Fourier Transform and vector functions

Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Frequency domain

$$\vec{\mathbf{F}}(\omega) = F_x(\omega)\hat{i}_x + F_y(\omega)\hat{i}_y + F_z(\omega)\hat{i}_z$$

2) Time domain derivative and Fourier Transform

$\vec{\mathbf{f}}(t)$ → **FT** → $\vec{\mathbf{F}}(\omega)$

$\dfrac{d\vec{\mathbf{f}}(t)}{dt}$ → **FT** → **?**

$$\frac{d\vec{\mathbf{f}}(t)}{dt} = \frac{df_x(t)}{dt}\hat{i}_x + \frac{df_y(t)}{dt}\hat{i}_y + \frac{df_z(t)}{dt}\hat{i}_z$$

$\dfrac{df_x(t)}{dt}$ → **1D-FT** → **?**

$\dfrac{df_y(t)}{dt}$ → **1D-FT** → **?**

$\dfrac{df_z(t)}{dt}$ → **1D-FT** → **?**

Fourier Transform and vector functions

Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Frequency domain

$$\vec{\mathbf{F}}(\omega) = F_x(\omega)\hat{i}_x + F_y(\omega)\hat{i}_y + F_z(\omega)\hat{i}_z$$

2) Time domain derivative and Fourier Transform

$\vec{\mathbf{f}}(t)$ → **FT** → $\vec{\mathbf{F}}(\omega)$

$\frac{d\vec{\mathbf{f}}(t)}{dt}$ → **FT** → **?**

$$\frac{d\vec{\mathbf{f}}(t)}{dt} = \frac{df_x(t)}{dt}\hat{i}_x + \frac{df_y(t)}{dt}\hat{i}_y + \frac{df_z(t)}{dt}\hat{i}_z$$

$\frac{df_x(t)}{dt}$ → **1D-FT** → $j\omega F_x(\omega)$

$\frac{df_y(t)}{dt}$ → **1D-FT** → $j\omega F_y(\omega)$

$\frac{df_z(t)}{dt}$ → **1D-FT** → $j\omega F_z(\omega)$

Fourier Transform and vector functions

Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Frequency domain

$$\vec{\mathbf{F}}(\omega) = F_x(\omega)\hat{i}_x + F_y(\omega)\hat{i}_y + F_z(\omega)\hat{i}_z$$

2) Time domain derivative and Fourier Transform

$$\vec{\mathbf{f}}(t) \longrightarrow \boxed{\textbf{FT}} \longrightarrow \vec{\mathbf{F}}(\omega)$$

$$\frac{d\vec{\mathbf{f}}(t)}{dt} \longrightarrow \boxed{\textbf{FT}} \longrightarrow \quad j\omega F_x(\omega)\hat{i}_x + j\omega F_y(\omega)\hat{i}_y + j\omega F_z(\omega)\hat{i}_z$$

$$\frac{d\vec{\mathbf{f}}(t)}{dt} = \frac{df_x(t)}{dt}\hat{i}_x + \frac{df_y(t)}{dt}\hat{i}_y + \frac{df_z(t)}{dt}\hat{i}_z$$

$$\frac{df_x(t)}{dt} \longrightarrow \boxed{\textbf{1D-FT}} \longrightarrow j\omega F_x(\omega)$$

$$\frac{df_y(t)}{dt} \longrightarrow \boxed{\textbf{1D-FT}} \longrightarrow j\omega F_y(\omega)$$

$$\frac{df_z(t)}{dt} \longrightarrow \boxed{\textbf{1D-FT}} \longrightarrow j\omega F_z(\omega)$$

Fourier Transform and vector functions

Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Frequency domain

$$\vec{\mathbf{F}}(\omega) = F_x(\omega)\hat{i}_x + F_y(\omega)\hat{i}_y + F_z(\omega)\hat{i}_z$$

2) Time domain derivative and Fourier Transform

$\vec{\mathbf{f}}(t)$ → **FT** → $\vec{\mathbf{F}}(\omega)$

$\dfrac{d\vec{\mathbf{f}}(t)}{dt}$ → **FT** → $j\omega\vec{\mathbf{F}}(\omega) = j\omega F_x(\omega)\hat{i}_x + j\omega F_y(\omega)\hat{i}_y + j\omega F_z(\omega)\hat{i}_z$

$$\frac{d\vec{\mathbf{f}}(t)}{dt} = \frac{df_x(t)}{dt}\hat{i}_x + \frac{df_y(t)}{dt}\hat{i}_y + \frac{df_z(t)}{dt}\hat{i}_z$$

$\dfrac{df_x(t)}{dt}$ → **1D-FT** → $j\omega F_x(\omega)$

$\dfrac{df_y(t)}{dt}$ → **1D-FT** → $j\omega F_y(\omega)$

$\dfrac{df_z(t)}{dt}$ → **1D-FT** → $j\omega F_z(\omega)$

Fourier Transform and vector functions

Time domain

$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

Frequency domain

$$\vec{\mathbf{F}}(\omega) = F_x(\omega)\hat{i}_x + F_y(\omega)\hat{i}_y + F_z(\omega)\hat{i}_z$$

2) Time domain derivative and Fourier Transform

$$\vec{\mathbf{f}}(t) \longrightarrow \boxed{\text{FT}} \longrightarrow \vec{\mathbf{F}}(\omega)$$

$$\frac{d\vec{\mathbf{f}}(t)}{dt} \longrightarrow \boxed{\text{FT}} \longrightarrow j\omega\vec{\mathbf{F}}(\omega)$$

Frequency domain

- Fourier Transform and functions of n variables

- Fourier Transform and vector functions

- Fourier Transform and vector functions of n variables

1) How to jump back from the Frequency domain to the Time domain

2) Time domain derivative and Fourier Transform