Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2020-2021 - Laurea "Triennale" – Secondo semestre - Secondo anno

Università degli Studi di Napoli "Parthenope"

Stefano Perna

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Antenna Parameters

Parameters of the Tx Antenna

Parameters of the Rx Antenna

Parameters of the Tx Antenna

- Effective length
  - Radiation pattern
  - Radiation pattern lobes
  - Beamwidth
- Directivity
- Gain
- Radiation Resistance
- Equivalent circuit of the tx antenna
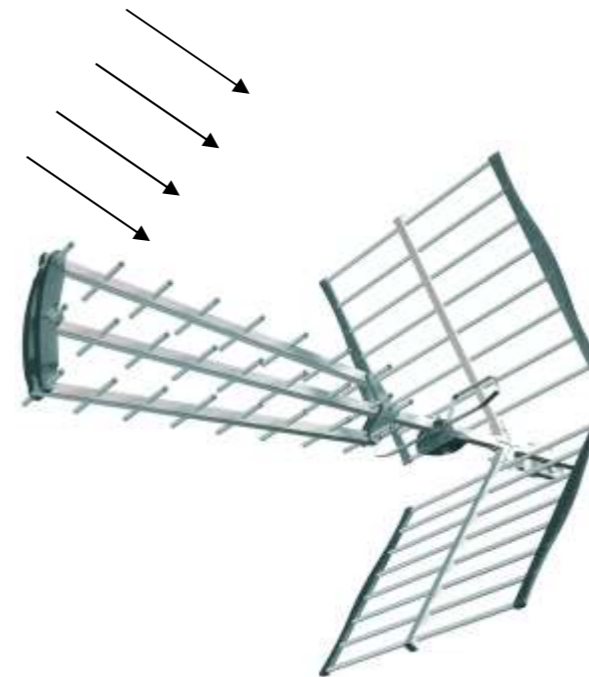- Input Impedance and Input Resistance

Antenna Parameters

Parameters of the Tx Antenna
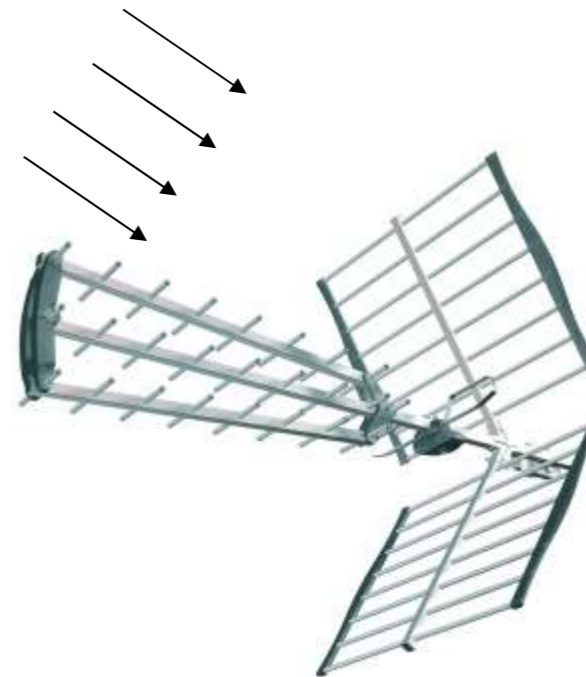
Parameters of the Rx Antenna

Parameters of the Rx Antenna

- Rx effective length
- Equivalent circuit of the rx antenna
- Effective Area

Parameters of the Rx Antenna

- Rx effective length
- Equivalent circuit of the rx antenna
- Effective Area

Rx effective length

$$|V_0| = |\mathbf{E_i} \cdot \mathbf{l}|$$

Where

$\mathbf{E_i}$ is the incident, locally plane, field

V_0 is the voltage induced at the antenna terminals, which are assumed open-circuited

$\mathbf{l}(\vartheta, \varphi) = l_\vartheta(\vartheta, \varphi)\hat{i}_\vartheta + l_\varphi(\vartheta, \varphi)\hat{i}_\varphi$ can referred to as **<u>receiving effective length</u>** of the antenna (and not only transmitting effective length)
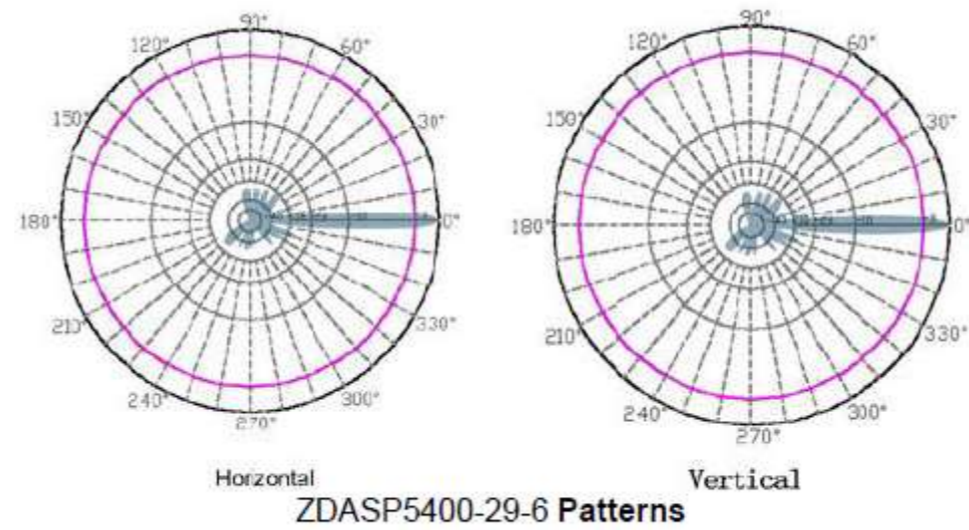
Note that this means that the behavior of an antenna when transmitting and when receiving are related.
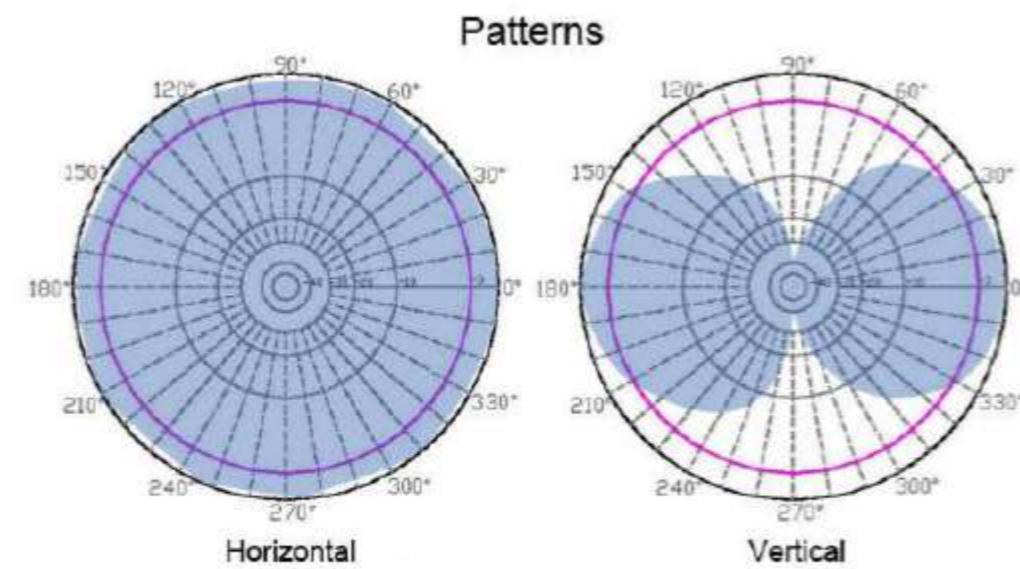
Rx effective length

three examples from the real life
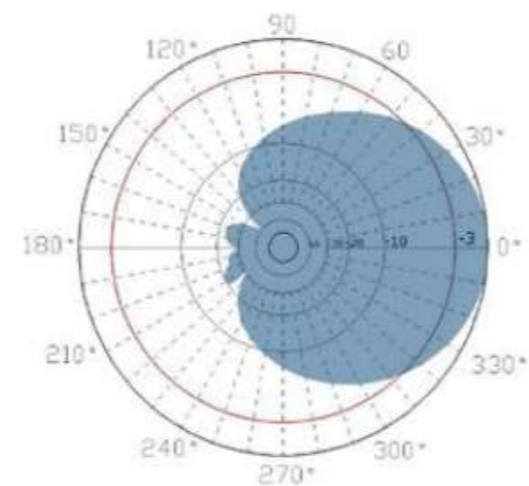
Rx effective length

Horizontal   Vertical
ZDASP5400-29-6 **Patterns**

Rx effective length

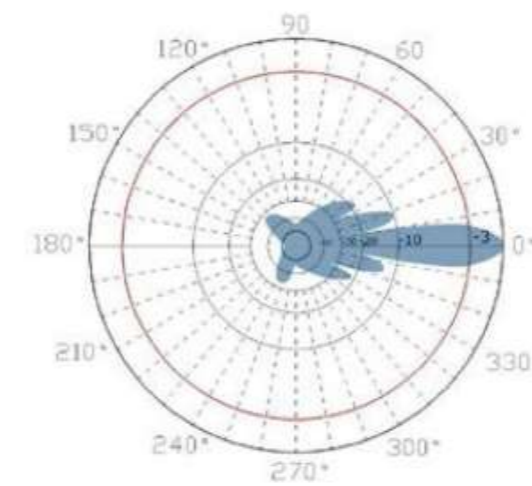
Patterns

Horizontal

Vertical

ZDAEW1900-3 **Patterns**

Rx effective length

Horizontal   Vertical

ZDADJ800-13-90 Patterns
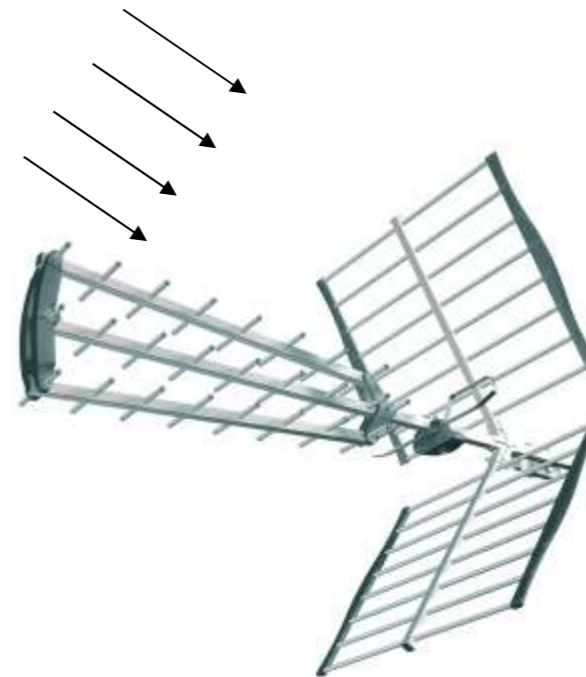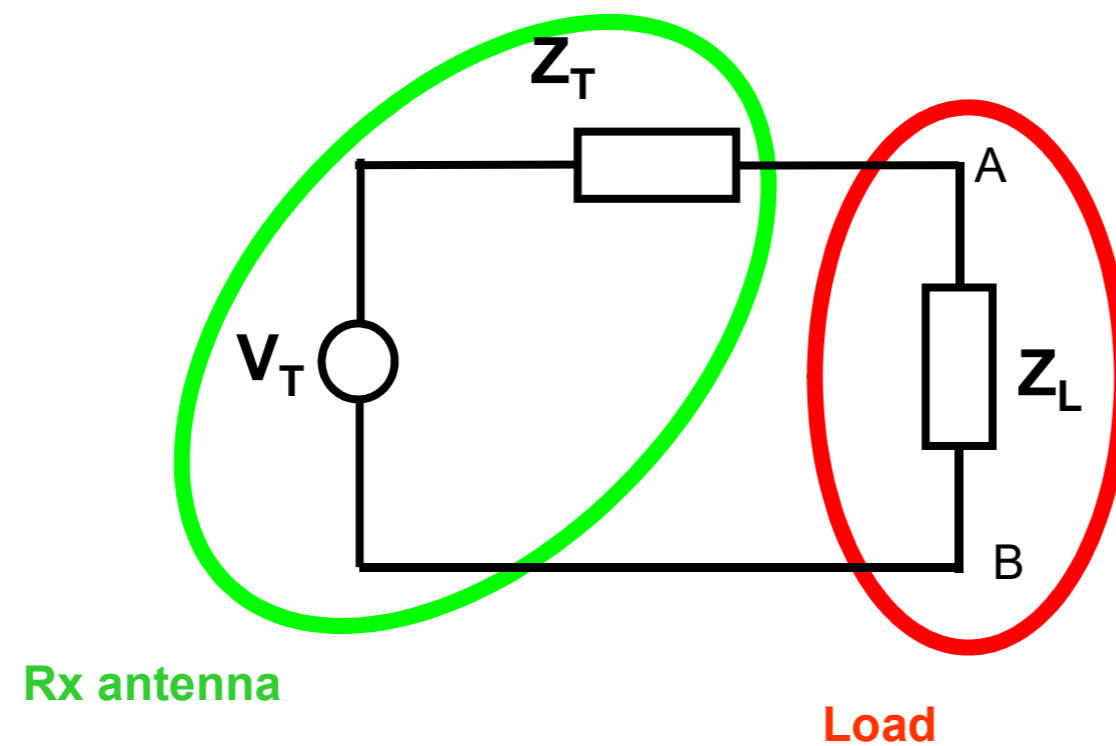
Parameters of the Rx Antenna

- Rx effective length
- Equivalent circuit of the rx antenna
- Effective Area

Equivalent circuit of the Rx antenna

$$Z_L = R_L + jX_L$$

Rx antenna

Load

The incident electric field sets up currents on the antenna. Such currents may be represented by a Thevenin-equivalent generator, which delivers power to any connected receiving load impedance.

Equivalent circuit of the Rx antenna

Rx antenna

$$|V_T| = |V_0| = |\boldsymbol{E_i} \cdot \boldsymbol{l}|$$
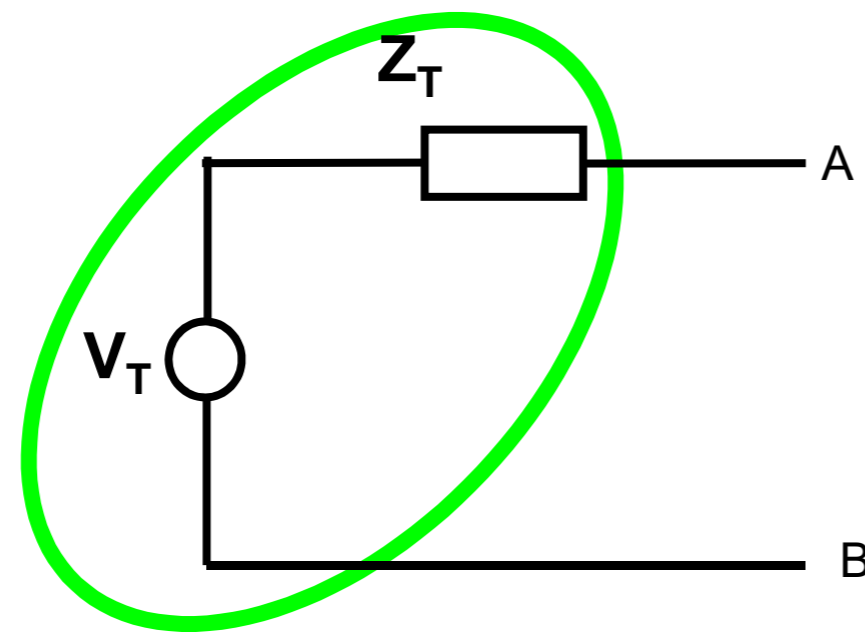
$$Z_L = R_L + jX_L$$

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Equivalent circuit of the Rx antenna

$$\left|V_T\right| = \left|V_0\right| = \left|\boldsymbol{E_i} \cdot \boldsymbol{l}\right|$$

$$Z_T = Z_{in} = R_{in} + jX_{in}$$

$$Z_L = R_L + jX_L$$

Z$_T$

A

B

Rx antenna

The incident electric field sets up currents on the antenna. Such currents may be represented by a Thevenin-equivalent generator, which delivers power to any connected receiving load impedance.
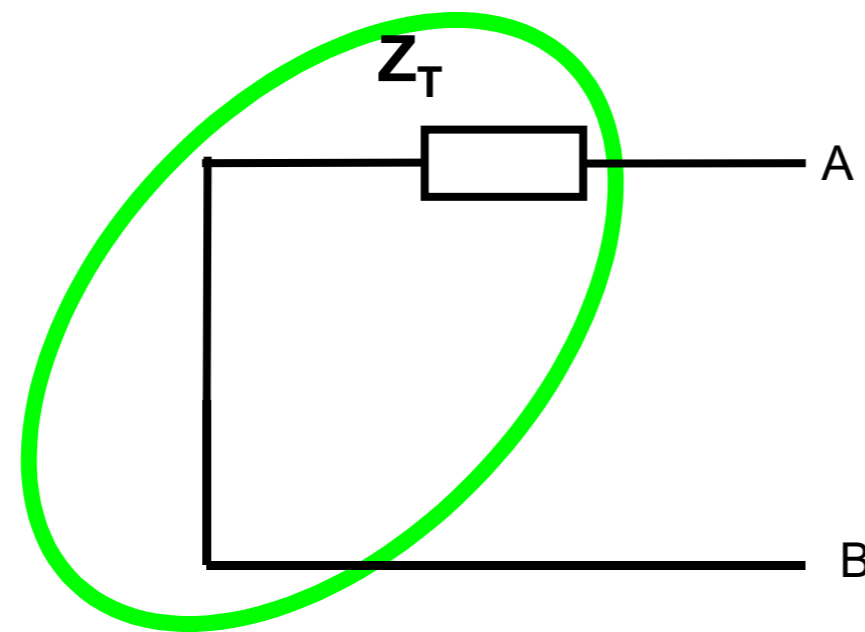
Equivalent circuit of the Rx antenna

Rx antenna

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Equivalent circuit of the Rx antenna

Rx antenna

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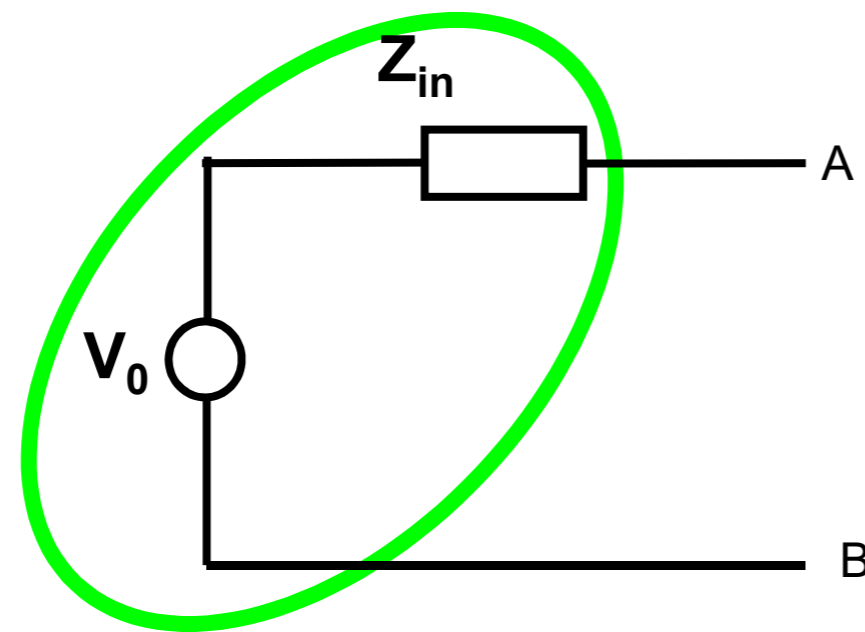
Equivalent circuit of the Rx antenna

$$\left| V_0 \right| = \left| \boldsymbol{E_i} \cdot \boldsymbol{l} \right|$$

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Rx antenna

Load

The incident electric field sets up currents on the antenna. Such currents may be represented by a Thevenin-equivalent generator, which delivers power to any connected receiving load impedance.

….MEMO..

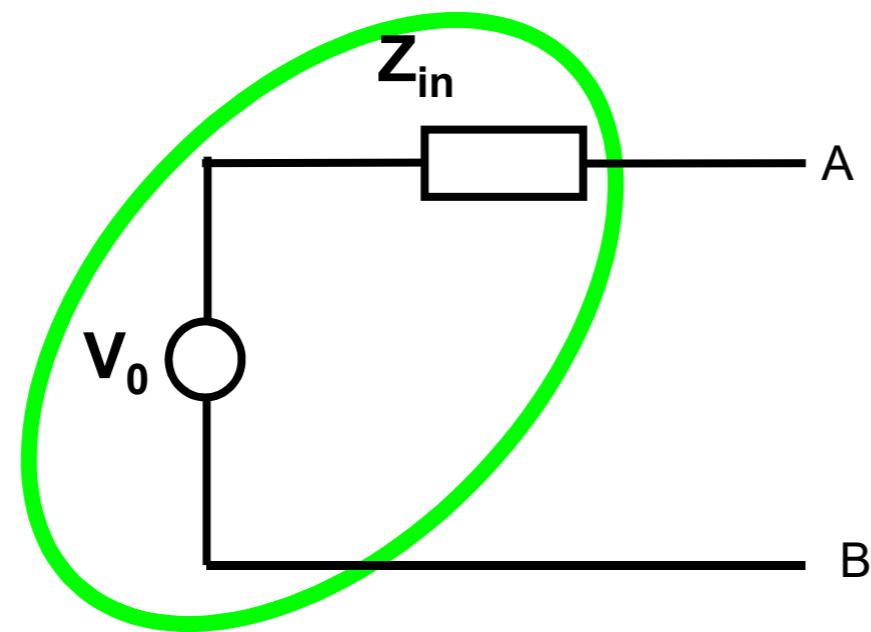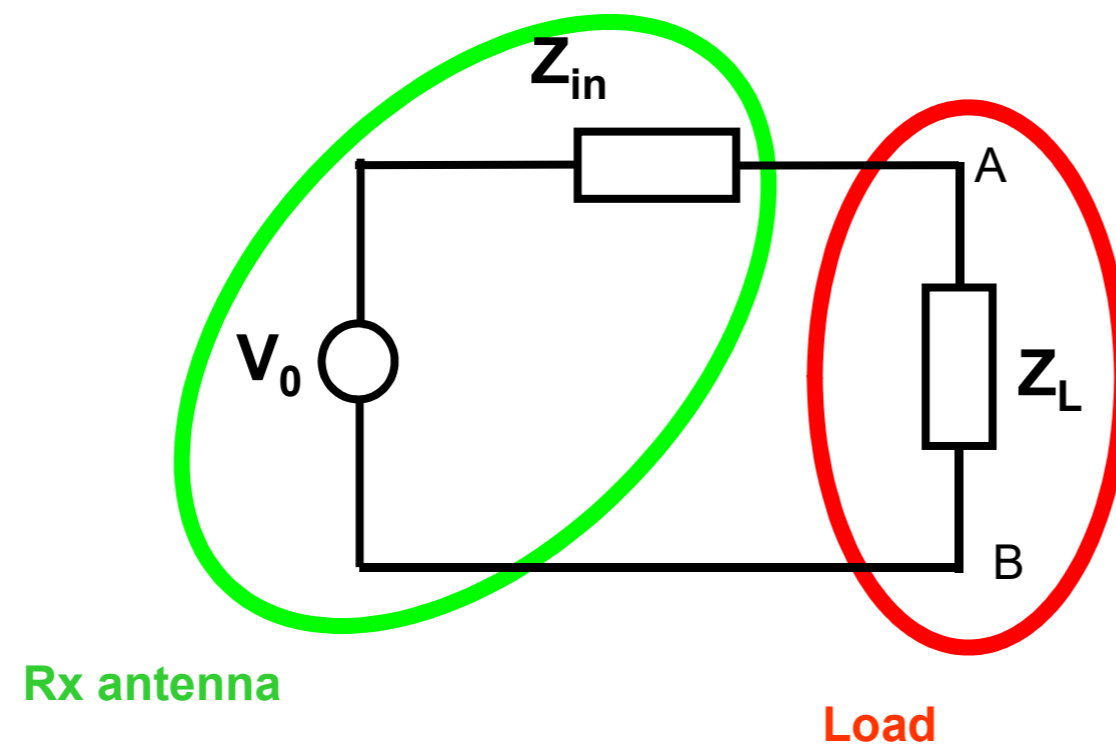## Equivalent circuit of the Tx antenna

Z_g

v_g

A

I

Z_{in}

B

Tx circuit
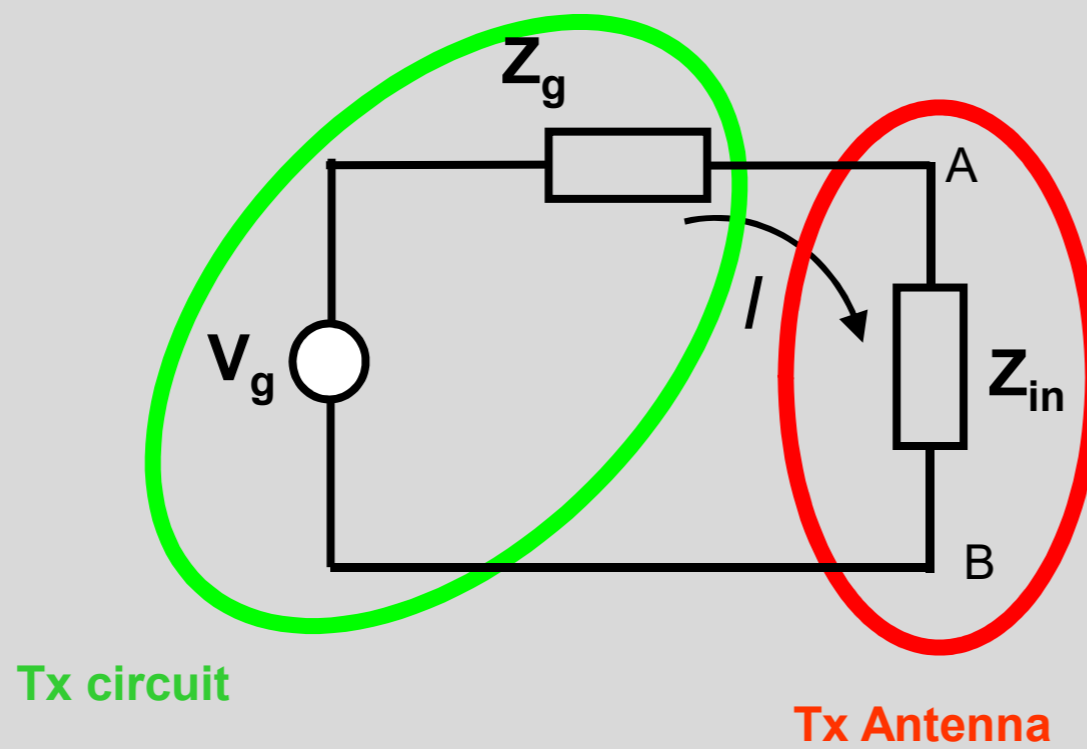
Tx Antenna

$$Z_{in} = R_{in} + jX_{in}$$

$$P_{in} = \frac{1}{2}R_{in}\left|I\right|^2$$

Equivalent circuit of the Rx antenna

Rx antenna

Load
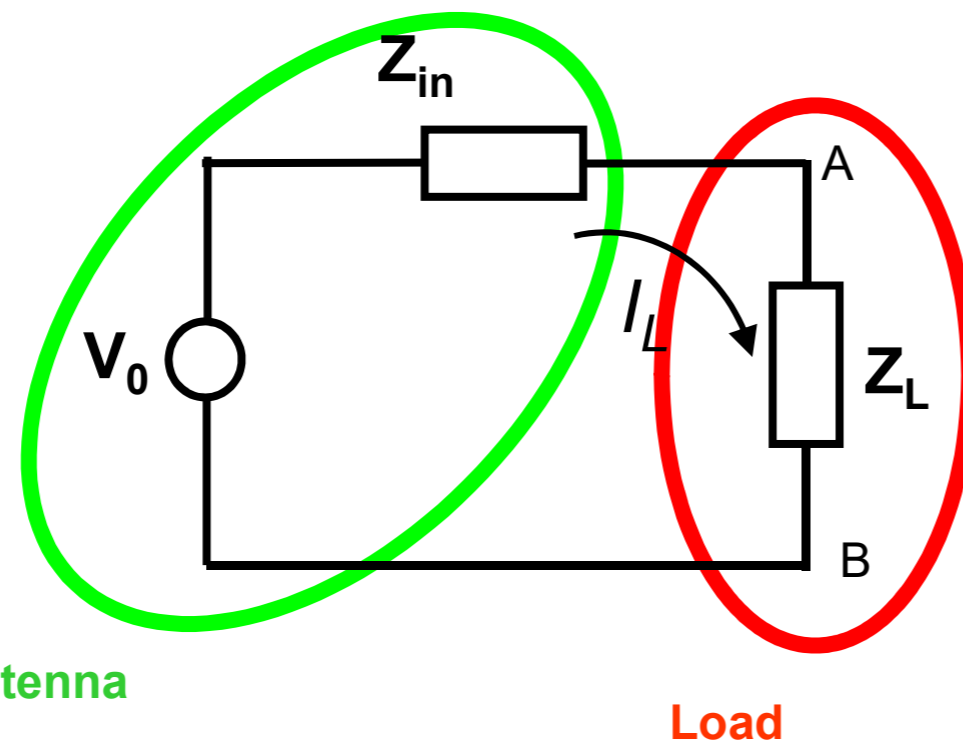
$$\left| V_0 \right| = \left| \boldsymbol{E_i} \cdot \mathbf{l} \right|$$

$$Z_{in} = R_{in} + jX_{in}$$

$$Z_L = R_L + jX_L$$

$$V_0 = (Z_{in} + Z_L) I_L \quad \Rightarrow \left| I_L \right|^2 = \frac{\left| V_0 \right|^2}{\left| Z_{in} + Z_L \right|^2}$$

$$P_L = \frac{1}{2} R_L \left| I_L \right|^2 = \frac{1}{2} \frac{R_L}{\left| Z_{in} + Z_L \right|^2} \left| V_0 \right|^2 = \frac{1}{2} \frac{R_L}{\left| Z_{in} + Z_L \right|^2} \left| \boldsymbol{E_i} \cdot \mathbf{l} \right|^2$$

Equivalent circuit of the Rx antenna

$$|V_0| = |\boldsymbol{E_i} \cdot \mathbf{l}|$$

$$Z_{in} = R_{in} + jX_{in}$$

$$Z_L = R_L + jX_L$$

Rx antenna

Load

$$P_L = \frac{1}{2} \frac{R_L}{|Z_{in} + Z_L|^2} |\boldsymbol{E_i} \cdot \mathbf{l}|^2$$

Equivalent circuit of the Rx antenna

<div style="background:yellow">

1) Polarization matching

$$\left| \boldsymbol{E_i} \cdot \boldsymbol{l} \right| = \left| \boldsymbol{E_i} \right| \left| \boldsymbol{l} \right|$$

</div>

Z$_{in}$

A

V$_0$

Z$_L$

B

Rx antenna

Load

$$P_L = \frac{1}{2} \frac{R_L}{\left| Z_{in} + Z_L \right|^2} \left| \boldsymbol{E_i} \cdot \boldsymbol{l} \right|^2$$

Equivalent circuit of the Rx antenna

1) Polarization matching

$$\left| \boldsymbol{E_i} \cdot \mathbf{l} \right| = \left| \boldsymbol{E_i} \right| \left| \mathbf{l} \right|$$

2) Power matching

$$Z_L = Z_{in}{}^* \Rightarrow \begin{cases} Z_{in} = R_{in} + jX_{in} \\ Z_L = R_{in} - jX_{in} \end{cases}$$

$$\Rightarrow \frac{R_L}{\left| Z_{in} + Z_L \right|^2} = \frac{R_{in}}{\left(2R_{in} \right)^2} = \frac{1}{4R_{in}}$$

Rx antenna

Load

$$P_L = \frac{1}{2} \frac{R_L}{\left| Z_{in} + Z_L \right|^2} \left| \boldsymbol{E_i} \cdot \mathbf{l} \right|^2$$

Equivalent circuit of the Rx antenna

1) Polarization matching

$$\left| \boldsymbol{E_i} \cdot \mathbf{l} \right| = \left| \boldsymbol{E_i} \right| \left| \mathbf{l} \right|$$

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Rx antenna

Load

$$P_L = \frac{1}{2} \frac{R_L}{\left| Z_{in} + Z_L \right|^2} \left| \boldsymbol{E_i} \cdot \mathbf{l} \right|^2$$

$$\Rightarrow P_{Lmax} = \frac{1}{2} \frac{1}{4R_{in}} \left| \boldsymbol{E_i} \right|^2 \left| \mathbf{l} \right|^2 = \frac{\zeta}{\zeta} \frac{1}{2} \frac{1}{4R_{in}} \left| \boldsymbol{E_i} \right|^2 \left| \mathbf{l} \right|^2$$

Equivalent circuit of the Rx antenna

1) Polarization matching

$$\left| \mathbf{E_i} \cdot \mathbf{l} \right| = \left| \mathbf{E_i} \right| \left| \mathbf{l} \right|$$

2) Power matching

$$Z_L = Z_{in}{}^* \Rightarrow \begin{cases} Z_{in} = R_{in} + jX_{in} \\ Z_L = R_{in} - jX_{in} \end{cases}$$

$$\Rightarrow \frac{R_L}{\left| Z_{in} + Z_L \right|^2} = \frac{R_{in}}{\left(2R_{in} \right)^2} = \frac{1}{4R_{in}}$$

$$P_{Lmax} = \frac{1}{2} \frac{\left| \mathbf{E_i} \right|^2}{\zeta} \left[\frac{\zeta \left| \mathbf{l}(\vartheta, \varphi) \right|^2}{4R_{in}} \right]$$

Effective Area

$$A_{eff}\left(\vartheta, \varphi \right)$$

$$P_L = \frac{1}{2} \frac{R_L}{\left| Z_{in} + Z_L \right|^2} \left| \mathbf{E_i} \cdot \mathbf{l} \right|^2$$

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Effective area

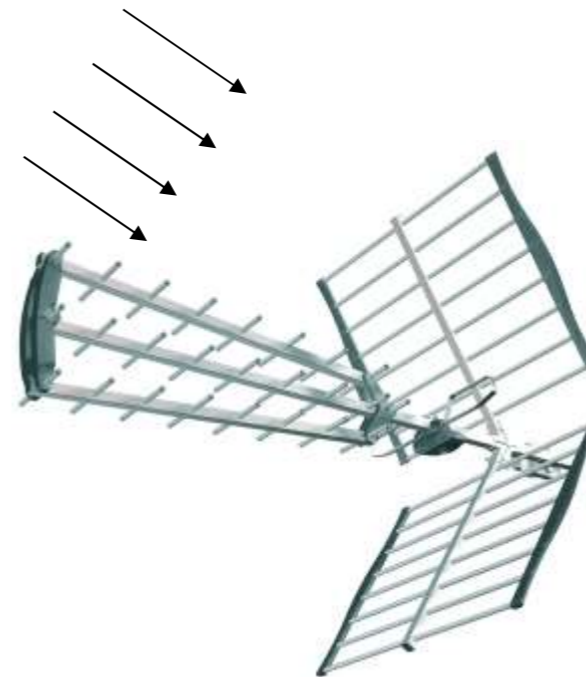In general, the aperture of an antenna is not directly related to its physical size.

However some types of antennas, for example parabolic dishes and horns, have a physical aperture (opening) which collects the radio waves. In these aperture antennas, the physical aperture A_{phys}
Provides a good approximation of the effective area A.

In any case, the effective aperture A is always less than the area of the antenna's physical aperture A_{phys}.

Parameters of the Rx Antenna

- Rx effective length
- Equivalent circuit of the rx antenna
- Effective Area

Gain and Effective Area

Gain

$$G(\vartheta,\varphi) = \frac{\dfrac{1}{2}\dfrac{|\boldsymbol{E}|^2}{\zeta}}{\dfrac{1}{4\pi r^2}P_{in}}$$

$$P_{in} = \frac{1}{2}R_{in}|I|^2$$

$$\boldsymbol{E}(r,\vartheta,\varphi) = \frac{j\zeta I}{2\lambda}\frac{e^{-j\beta r}}{r}\mathbf{l}(\vartheta,\varphi)$$

Effective Area

$$A_{eff}(\vartheta,\varphi) = \frac{\zeta|\mathbf{l}(\vartheta,\varphi)|^2}{4R_{in}}$$

$$G(\vartheta,\phi) = \frac{4\pi r^2}{2\zeta}\frac{|\boldsymbol{E}|^2}{P_{in}} = \frac{4\pi r^2}{2\zeta}\left(\frac{\zeta^2|I|^2}{4\lambda^2}\frac{|\mathbf{l}(\vartheta,\varphi)|^2}{r^2}\right)\frac{2}{R_{in}|I|^2}$$

Gain and Effective Area

Gain

$$G(\vartheta, \varphi) = \dfrac{\dfrac{1}{2}\dfrac{|\boldsymbol{E}|^2}{\zeta}}{\dfrac{1}{4\pi r^2}P_{in}}$$

$$P_{in} = \frac{1}{2}R_{in}|I|^2$$

$$\boldsymbol{E}(r, \vartheta, \varphi) = \frac{j\zeta I}{2\lambda}\frac{e^{-j\beta r}}{r}\mathbf{l}(\vartheta, \varphi)$$

Effective Area

$$A_{eff}(\vartheta, \varphi) = \frac{\zeta|\mathbf{l}(\vartheta, \varphi)|^2}{4R_{in}}$$

$$G(\vartheta, \phi) = \frac{4\pi r^2}{2\zeta}\frac{|\boldsymbol{E}|^2}{P_{in}} = \frac{4\pi r^2}{2\zeta}\left(\frac{\zeta^2|I|^2}{4\lambda^2}\frac{|\mathbf{l}(\vartheta, \varphi)|^2}{r^2}\right)\frac{1}{R_{in}|I|^2} = \frac{4\pi}{\lambda^2}\left[\frac{\zeta|\mathbf{l}(\vartheta, \varphi)|^2}{4R_{in}}\right]$$

Gain and Effective Area

Gain

$$G(\vartheta,\varphi) = \frac{\frac{1}{2}\frac{|\boldsymbol{E}|^2}{\zeta}}{\frac{1}{4\pi r^2}P_{in}}$$

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Effective Area

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$$G(\vartheta,\varphi) = \frac{4\pi}{\lambda^2}A_{eff}(\vartheta,\varphi)$$

Effective Area

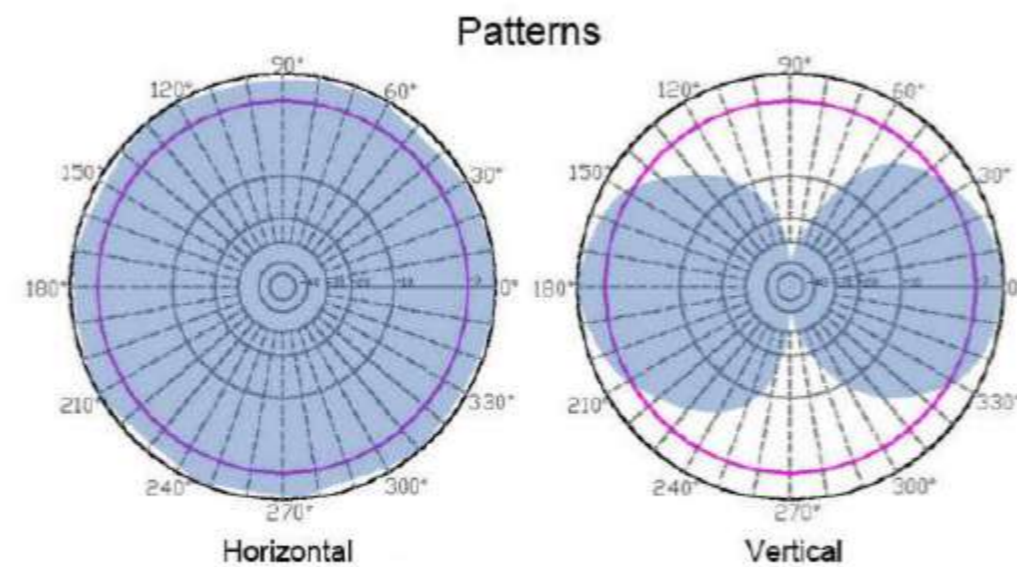## three examples from the real life

Effective Area

three examples from the real life

(maximum) Gain = 3 dB

$$G(\vartheta, \varphi) = \frac{4\pi}{\lambda^2} A_{eff}(\vartheta, \varphi)$$

Patterns

Horizontal

Vertical

ZDAEW1900-3 **Patterns**

Effective Area

three examples from the real life

(maximum) Gain = 29 dB

Horizontal   Vertical
ZDASP5400-29-6 **Patterns**

$$G(\vartheta, \varphi) = \frac{4\pi}{\lambda^2} A_{eff}(\vartheta, \varphi)$$
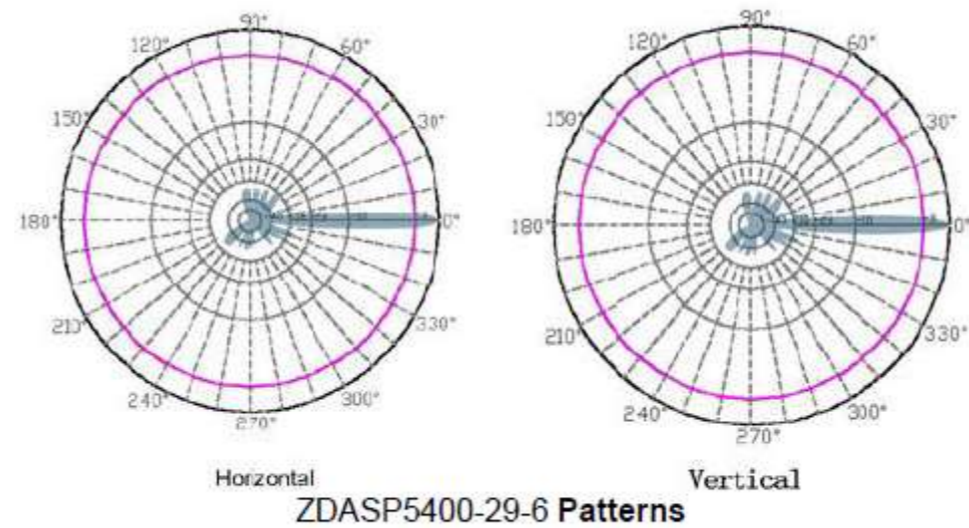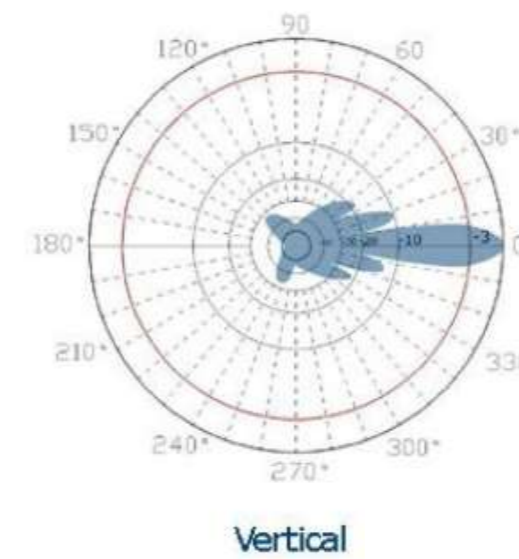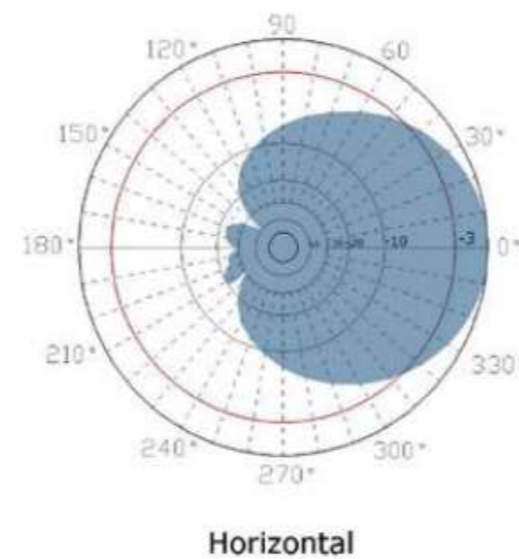
Effective Area

three examples from the real life

(maximum) Gain = 13 dB

$$G(\vartheta,\varphi) = \frac{4\pi}{\lambda^2} A_{eff}(\vartheta,\varphi)$$

Horizontal   Vertical

ZDADJ800-13-90 Patterns

Equivalent circuit of the Rx antenna

$$P_L = \frac{1}{2} \frac{R_L}{\left|Z_{in} + Z_L\right|^2} \left|\boldsymbol{E_i} \cdot \mathbf{l}\right|^2 \qquad P_{Lmax} = \frac{1}{2} \frac{\left|\boldsymbol{E_i}\right|^2 \left|\mathbf{l}\right|^2}{4R_{in}}$$

Maximum power transfer to the load is achieved when the two following conditions are simultaneously verified:

1) Polarization matching

$$\Rightarrow \left|\boldsymbol{E_i} \cdot \mathbf{l}\right| = \left|\boldsymbol{E_i}\right|\left|\mathbf{l}\right|$$

2) Power matching

$$\Rightarrow \frac{R_L}{\left|Z_{in} + Z_L\right|^2} = \frac{1}{4R_{in}}$$

$$P_{Lmax} = \frac{1}{2} \frac{\left|\boldsymbol{E_i}\right|^2}{\zeta} A_{eff}\left(\vartheta, \varphi\right)$$

Equivalent circuit of the Rx antenna

$$P_L = \frac{1}{2} \frac{R_L}{|Z_{in} + Z_L|^2} |\mathbf{E}_i \cdot \mathbf{l}|^2$$

$$P_{Lmax} = \frac{1}{2} \frac{|\mathbf{E}_i|^2 |\mathbf{l}|^2}{4R_{in}}$$
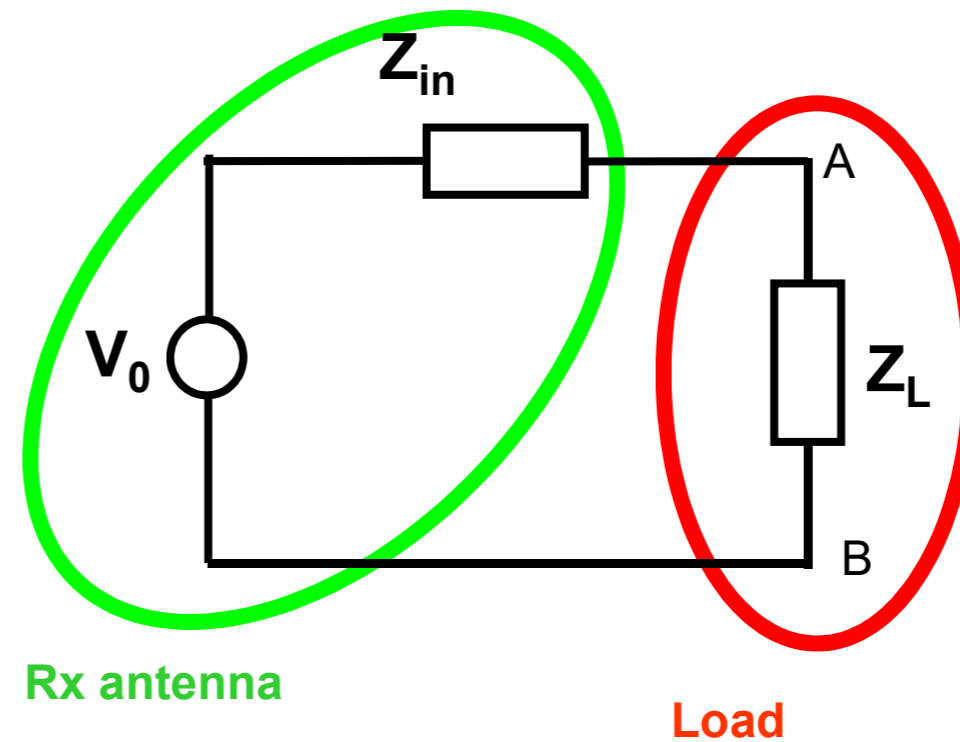
$$P_L = \frac{1}{2} \frac{R_L}{|Z_{in} + Z_L|^2} |\mathbf{E}_i \cdot \mathbf{l}|^2 = \underbrace{\frac{1}{2} \frac{|\mathbf{E}_i|^2 |\mathbf{l}|^2}{4R_{in}}}_{P_{Lmax}} \underbrace{\frac{|\mathbf{E}_i \cdot \mathbf{l}|^2}{|\mathbf{E}_i|^2 |\mathbf{l}|^2}}_{\eta_A} \underbrace{\frac{4R_{in}R_L}{|Z_{in} + Z_L|^2}}_{\eta_B}$$

$$P_L = \eta_A \, \eta_B \, P_{Lmax}$$
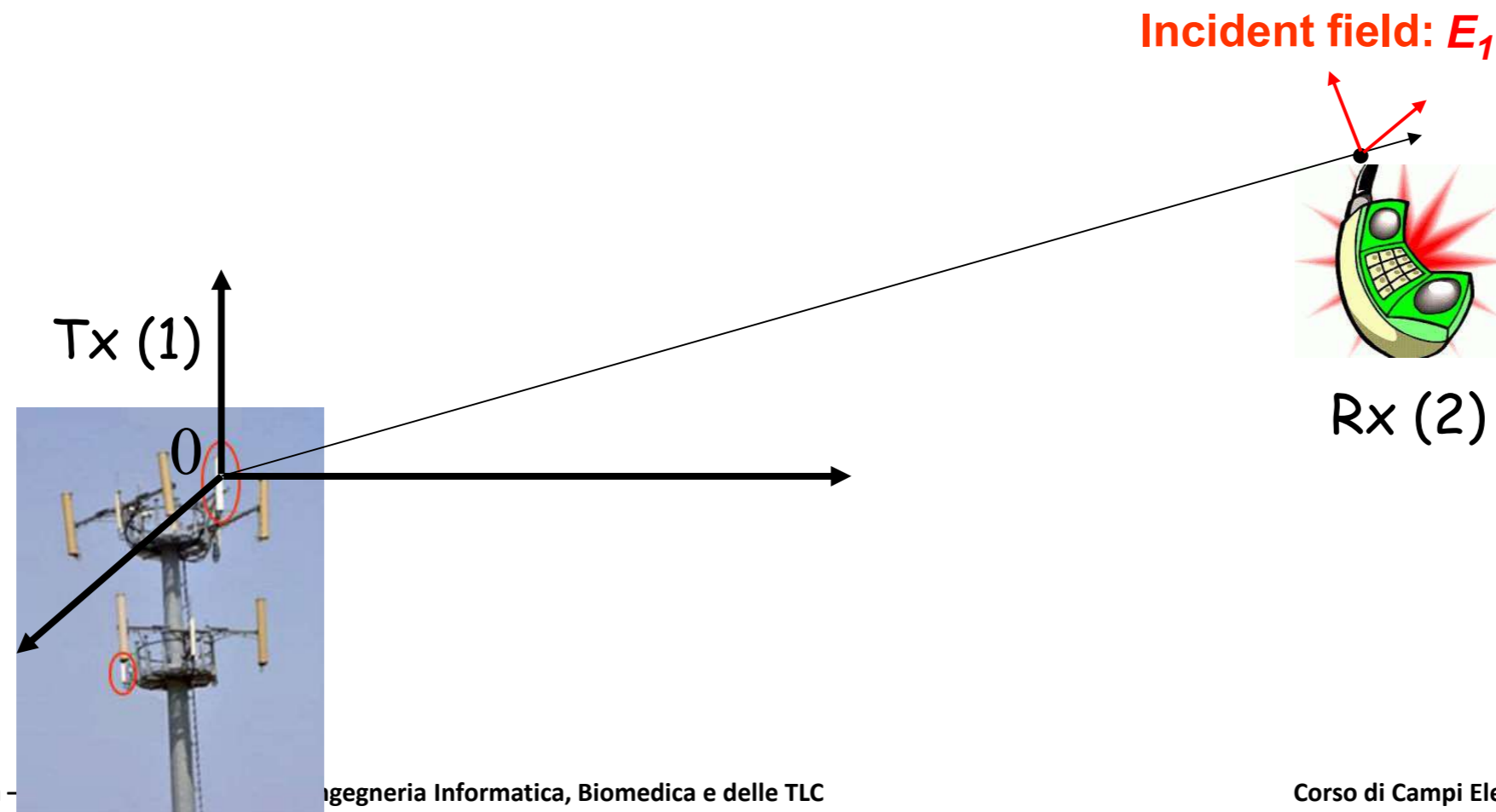
Equivalent circuit of the Rx antenna

$$P_L = \eta_A\, \eta_B\, P_{Lmax}$$

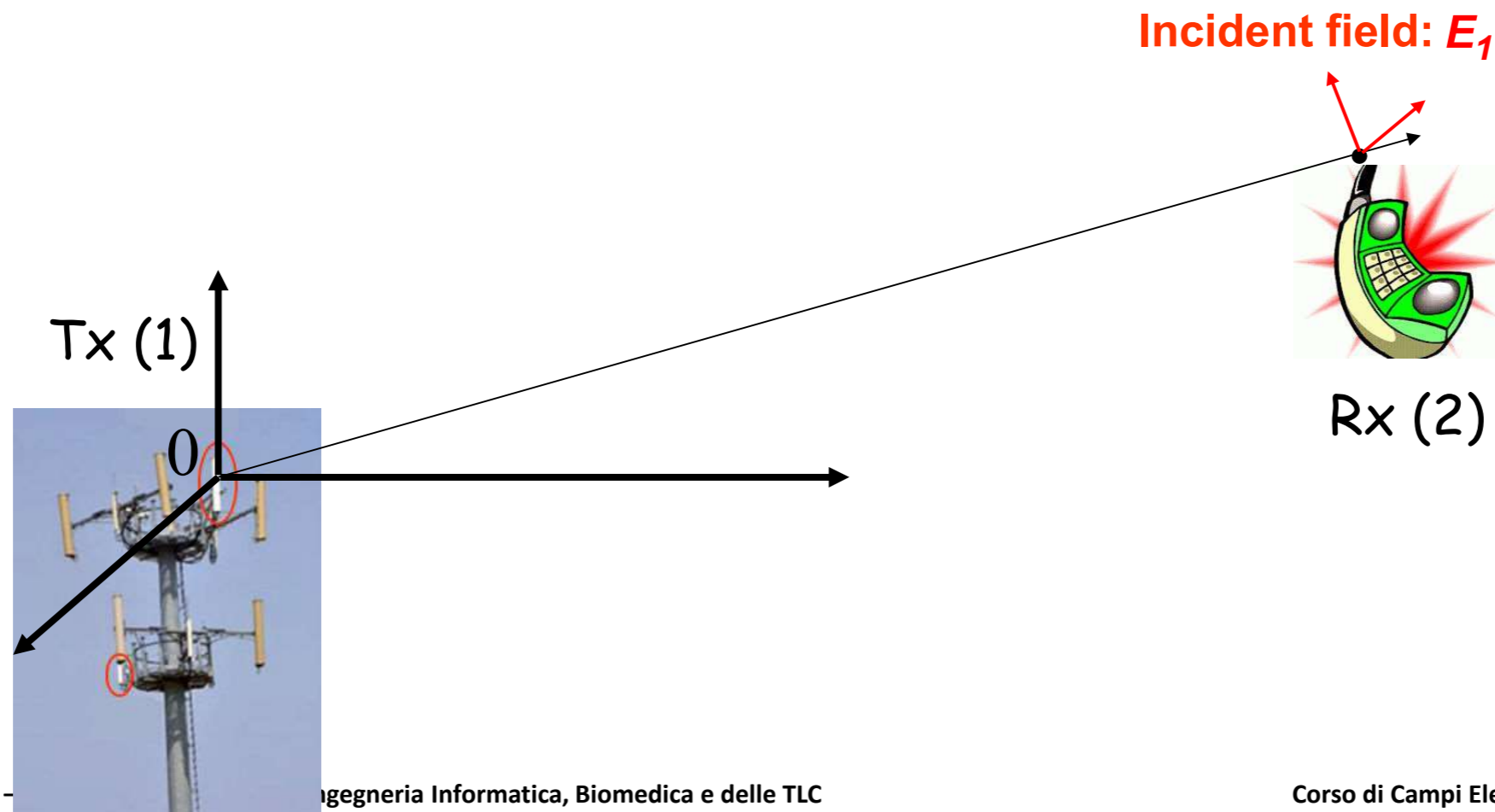$$P_{Lmax} = \frac{1}{2}\frac{|\boldsymbol{E_i}|^2}{\zeta}\, A_{eff}\left(\vartheta,\varphi\right)$$

Z$_{in}$

V$_0$

Z$_L$

A

B

Rx antenna

Load

$$P_L = \frac{1}{2}\frac{|\boldsymbol{E_i}|^2}{\zeta}\, A_{eff}\left(\vartheta,\varphi\right)\eta_A\, \eta_B$$

Radio link equation
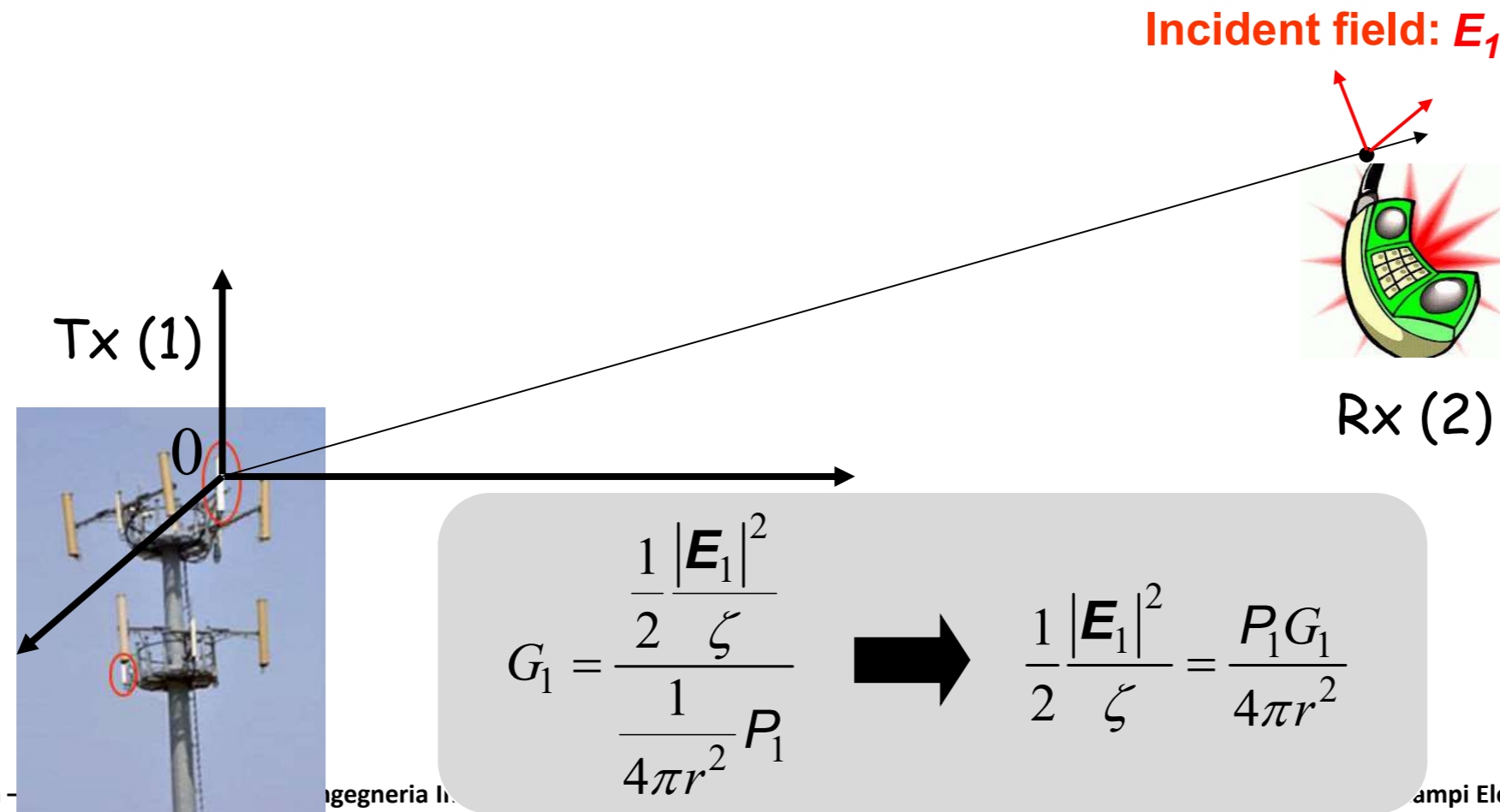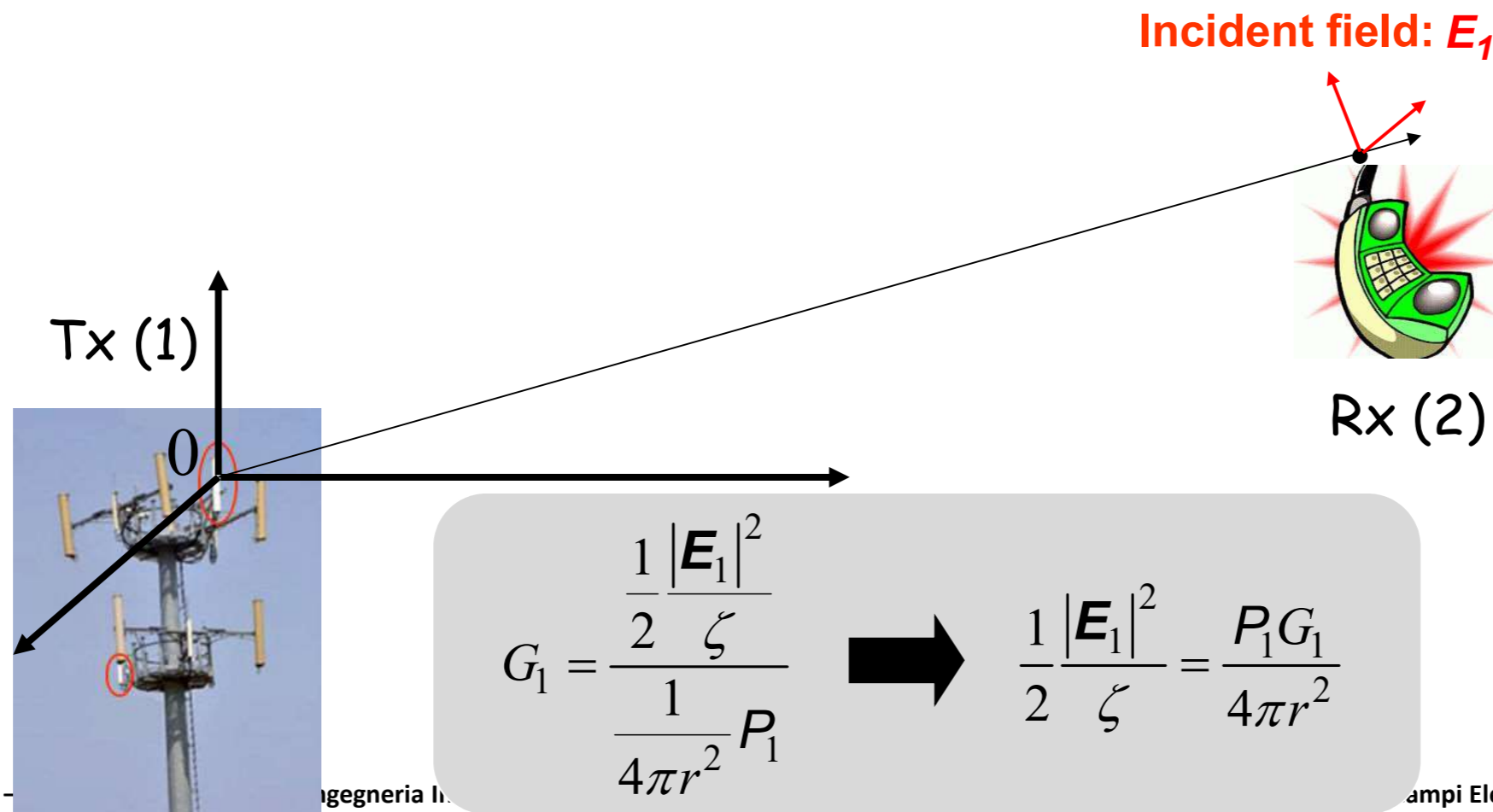
Incident field: E_1

Tx (1)

0

Rx (2)

Radio link equation

$$P_2 = \frac{1}{2} \frac{|\boldsymbol{E}_1|^2}{\zeta} A_{eff\,2} \eta_A \, \eta_B$$

Incident field: E_1

Tx (1)

0

Rx (2)

Radio link equation

$$P_2 = \frac{1}{2} \frac{|\boldsymbol{E}_1|^2}{\zeta} A_{eff\,2} \eta_A \, \eta_B$$

Incident field: E_1

Tx (1)

0

Rx (2)

$$G_1 = \frac{\dfrac{1}{2} \dfrac{|\boldsymbol{E}_1|^2}{\zeta}}{\dfrac{1}{4\pi r^2} P_1}$$

Radio link equation

$$P_2 = \frac{1}{2} \frac{|\boldsymbol{E}_1|^2}{\zeta} A_{eff\,2} \eta_A \, \eta_B$$

Incident field: $\boldsymbol{E_1}$

Tx (1)

0

Rx (2)

$$G_1 = \frac{\frac{1}{2}\dfrac{|\boldsymbol{E}_1|^2}{\zeta}}{\dfrac{1}{4\pi r^2}P_1} \qquad \Longrightarrow \qquad \frac{1}{2}\frac{|\boldsymbol{E}_1|^2}{\zeta} = \frac{P_1 G_1}{4\pi r^2}$$

Radio link equation

$$P_2 = \frac{1}{2}\frac{|\boldsymbol{E}_1|^2}{\zeta}A_{eff\,2}\eta_A\,\eta_B = P_1 G_1 A_{eff\,2}\left(\frac{1}{4\pi r^2}\right)\eta_A\,\eta_B$$

Incident field: $\boldsymbol{E_1}$

Tx (1)

0

Rx (2)

$$G_1 = \frac{\dfrac{1}{2}\dfrac{|\boldsymbol{E}_1|^2}{\zeta}}{\dfrac{1}{4\pi r^2}P_1} \quad \blacktriangleright \quad \frac{1}{2}\frac{|\boldsymbol{E}_1|^2}{\zeta} = \frac{P_1 G_1}{4\pi r^2}$$
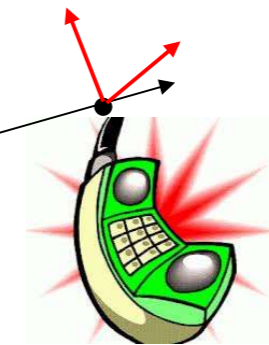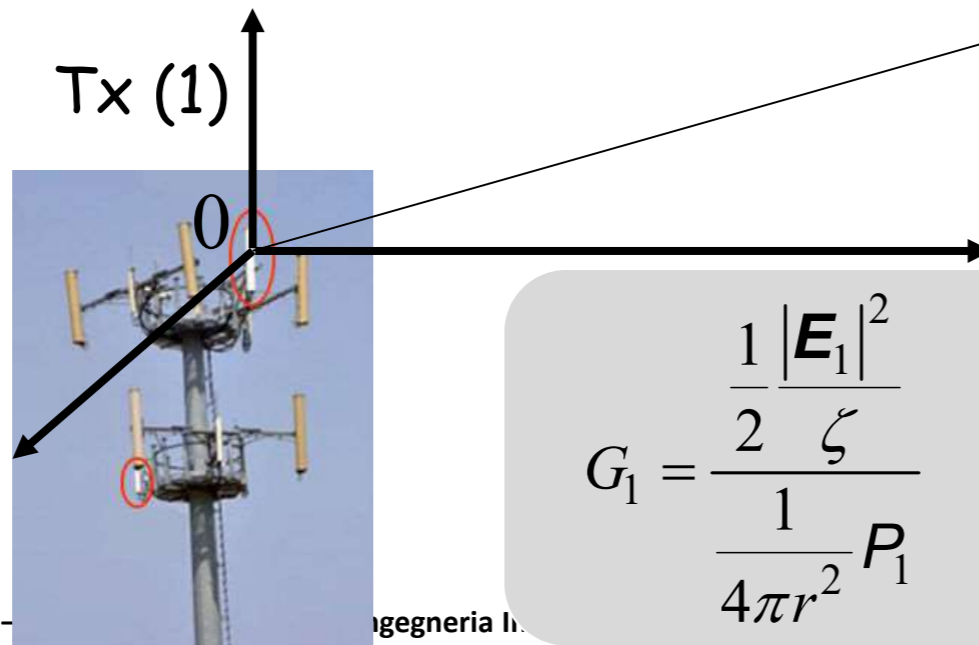
Radio link equation

$$P_2 = \frac{1}{2}\frac{|\boldsymbol{E}_1|^2}{\zeta} A_{eff\,2}\eta_A\,\eta_B = P_1 G_1 A_{eff\,2}\left(\frac{1}{4\pi r^2}\right)\eta_A\,\eta_B$$

Incident field: $\boldsymbol{E}_1$

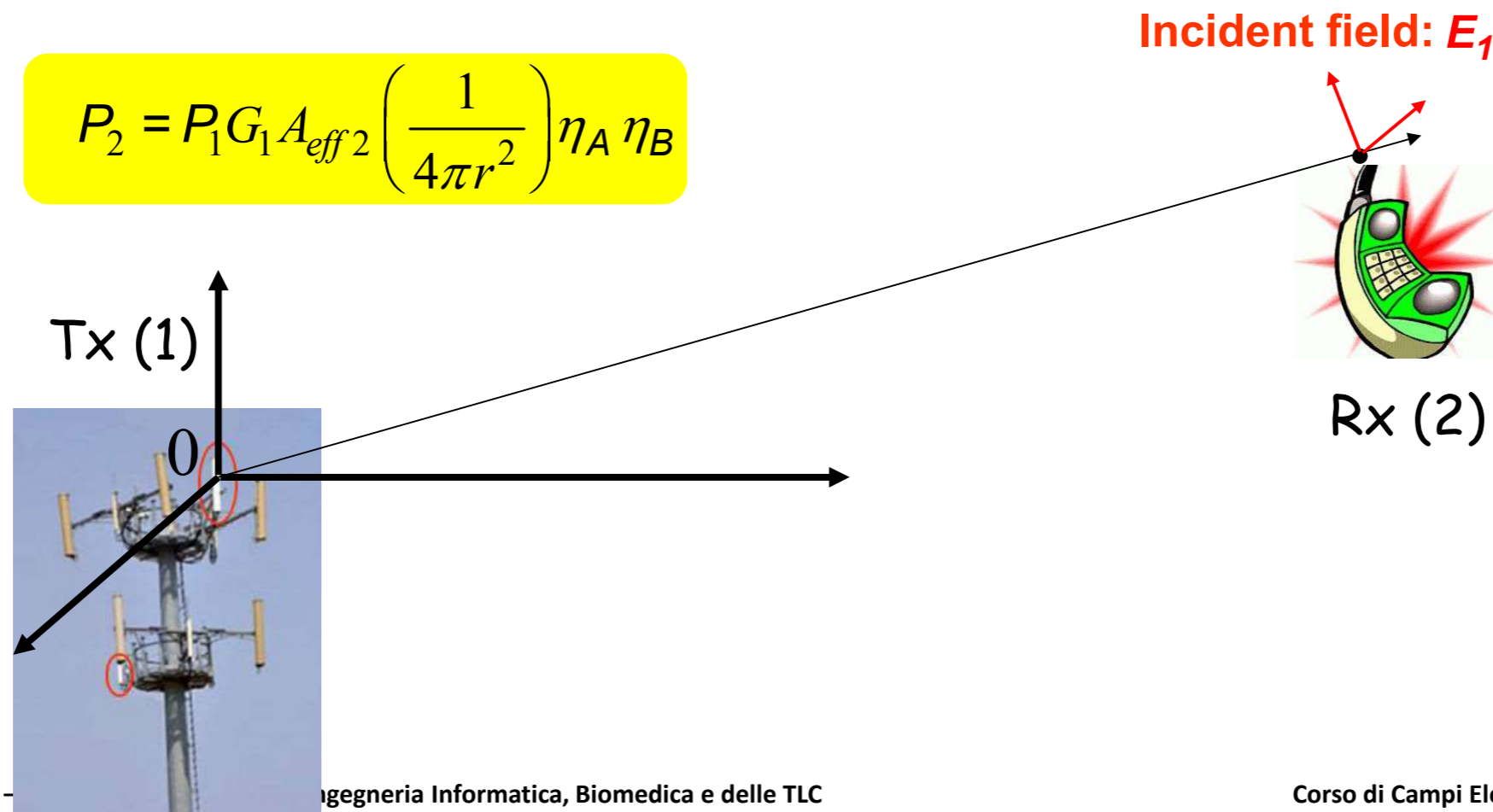$$P_2 = P_1 G_1 A_{eff\,2}\left(\frac{1}{4\pi r^2}\right)\eta_A\,\eta_B$$

Rx (2)

Tx (1)

0

$$G_1 = \frac{\dfrac{1}{2}\dfrac{|\boldsymbol{E}_1|^2}{\zeta}}{\dfrac{1}{4\pi r^2}P_1} \quad \Longrightarrow \quad \frac{1}{2}\frac{|\boldsymbol{E}_1|^2}{\zeta} = \frac{P_1 G_1}{4\pi r^2}$$

Radio link equation
(Friis equation)

$$P_2 = P_1 G_1 A_{eff\,2} \left(\frac{1}{4\pi r^2} \right) \eta_A \, \eta_B$$

Incident field: E_1

Tx (1)

0

Rx (2)

Radio link equation

(maximum) Gain = 29 dB

$$G(\vartheta, \varphi) = \frac{4\pi}{\lambda^2} A_{eff}(\vartheta, \varphi)$$

Horizontal   Vertical
ZDASP5400-29-6 **Patterns**