Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2020-2021 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

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Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Introduction

- To describe the performance of an antenna, definitions of various parameters are necessary.
- Some of the parameters are interrelated and not all of them need be specified for complete description of the antenna performance.

Antenna Parameters

Parameters of the Tx Antenna

Parameters of the Rx Antenna

Parameters of the Tx Antenna

- Effective length
  - Radiation pattern
  - Radiation pattern lobes
  - Beamwidth
- Directivity
- Gain
- Radiation Resistance
- Equivalent circuit of the tx antenna
- Input Impedance and Input Resistance

Effective Length

$$r >> D$$

$$r > \frac{2D^2}{\lambda}$$

$$r >> \lambda$$

$$\mathbf{E}(\vec{\mathbf{r}}) = \mathbf{E}(r,\vartheta,\varphi) = \frac{j\zeta I}{2\lambda}\frac{e^{-j\beta r}}{r}\mathbf{l}(\vartheta,\varphi)$$

$$\zeta\mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

$$\mathbf{l}(\vartheta,\varphi) = l_\vartheta(\vartheta,\varphi)\hat{i}_\vartheta + l_\varphi(\vartheta,\varphi)\hat{i}_\varphi$$

$$\vec{\mathbf{S}} = \frac{1}{2\zeta}\left|\vec{\mathbf{E}}\right|^2\hat{i}_r = \frac{\zeta}{2}\left|\vec{\mathbf{H}}\right|^2\hat{i}_r$$

$\mathbf{l}(\vartheta,\varphi) = l_\vartheta(\vartheta,\varphi)\hat{i}_\vartheta + l_\varphi(\vartheta,\varphi)\hat{i}_\varphi$ **effective length** of the antenna

Effective Length

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$$\vec{\mathbf{S}} = \frac{1}{2\zeta}\left|\vec{\mathbf{E}}\right|^2\hat{i}_r = \frac{\zeta}{2}\left|\vec{\mathbf{H}}\right|^2\hat{i}_r$$

Elementary electrical dipole

$$\mathbf{E}(\vec{\mathbf{r}}) = \frac{j\zeta I}{2\lambda}\frac{e^{-j\beta r}}{r}\Delta z \sin\vartheta\,\hat{i}_\vartheta$$

$$\zeta\mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

$$\mathbf{l}(\vartheta,\varphi) = \Delta z \sin\vartheta\,\hat{i}_\vartheta$$

Small loop antenna

$$\mathbf{E}(\vec{\mathbf{r}}) = \frac{j\zeta I}{2\lambda}\frac{e^{-j\beta r}}{r}(-j\beta\Delta S)\sin\vartheta\,\hat{i}_\varphi$$

$$\zeta\mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

$$\mathbf{l}(\vartheta,\varphi) = -j\beta\Delta S \sin\vartheta\,\hat{i}_\varphi$$

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Radiation pattern

$$r >> D$$

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$$\mathbf{l}(\vartheta,\varphi) = l_\vartheta(\vartheta,\varphi)\hat{i}_\vartheta + l_\varphi(\vartheta,\varphi)\hat{i}_\varphi$$

$$\vec{\mathbf{S}} = \frac{1}{2\zeta}\left|\vec{\mathbf{E}}\right|^2\hat{i}_r = \frac{\zeta}{2}\left|\vec{\mathbf{H}}\right|^2\hat{i}_r$$

An antenna *radiation pattern* or *antenna pattern* is defined as “a mathematical function or a graphical representation of the radiation properties of the antenna as a function of space coordinates”.

In most cases, the radiation pattern is determined in the *far- field region* and is represented as a function of the directional coordinates.

We can describe the angular behavior of the field radiated by the antenna by representing its effective length.

Radiation pattern

a. *field* pattern (*in linear scale*) typically represents a plot of the magnitude of the electric or magnetic field as a function of the angular space.

b. *power* pattern (*in linear scale*) typically represents a plot of the square of the magnitude of the electric or magnetic field as a function of the angular space.

c. *power* pattern (*in dB*) represents the magnitude of the electric or magnetic field, in decibels, as a function of the angular space.

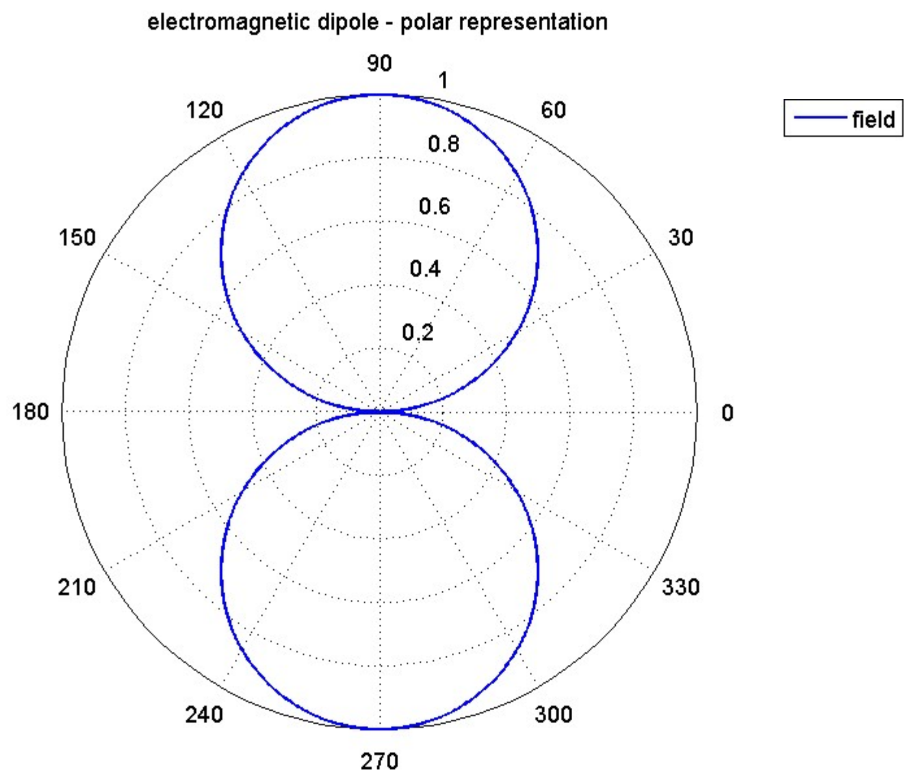
Often the *field* and *power* patterns are properly normalized, yielding *normalized field* and *power patterns*

Radiation pattern

an example: the electrical elementary dipole

$$\mathbf{l}\left(\vartheta,\varphi\right)=\Delta z\sin\vartheta\,\hat{i}_{\vartheta}$$

Vertical plane (φ=0)

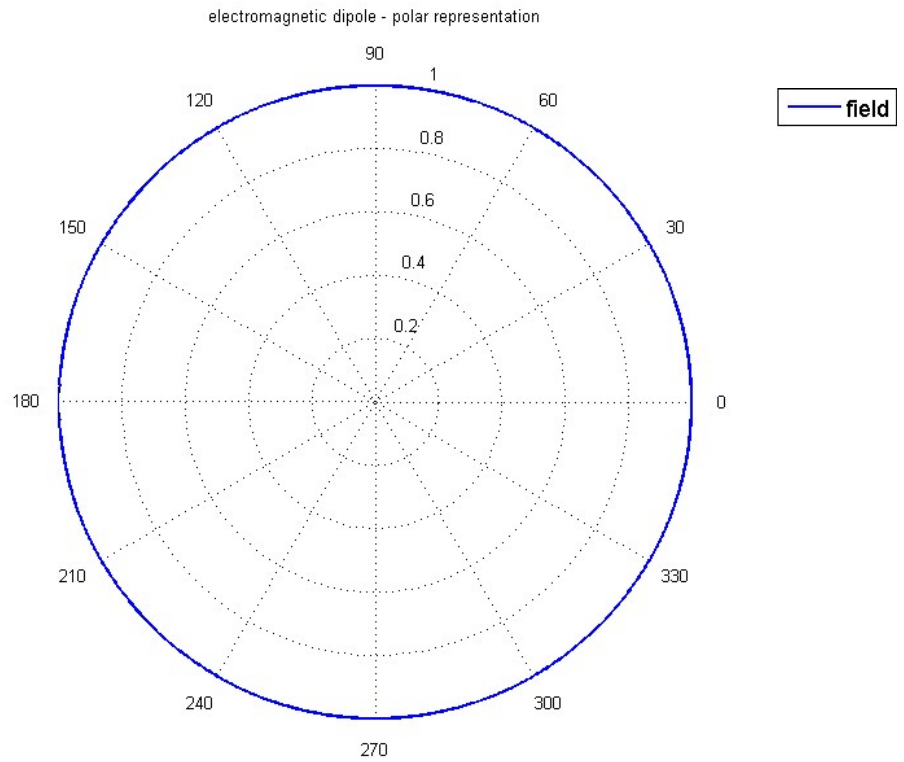


Radiation pattern

an example: the electrical elementary dipole

$$\mathbf{l}(\vartheta,\varphi) = \Delta z \sin\vartheta\, \hat{i}_\vartheta$$

Horizontal plane ($\theta=\pi/2$)



Parameters of the Tx Antenna

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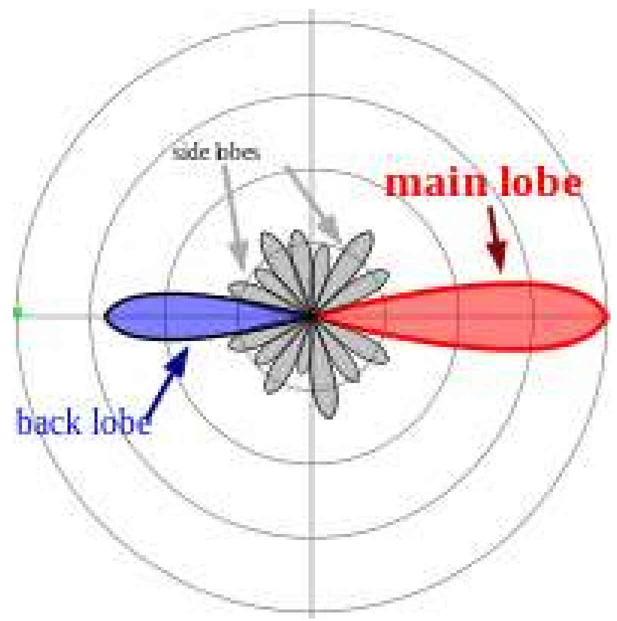
Radiation pattern lobes

- In some very specific directions there are zeros, or *nulls*, in the pattern indicating no radiation.
- The protuberances between the nulls are referred to as *lobes*, and the main, or major, lobe is in the direction of maximum radiation.

- There are also *side lobes* and *back lobes*.
  - A *back lobe* is “a radiation lobe whose axis makes an angle of approximately 180◦ with respect to the beam of an antenna.” Usually it refers to a minor lobe that occupies the hemisphere in a direction opposite to that of the major (main) lobe.
  - *Side lobes* and *back lobes* divert power away from the main beam and are desired as small as possible.



Radiation pattern

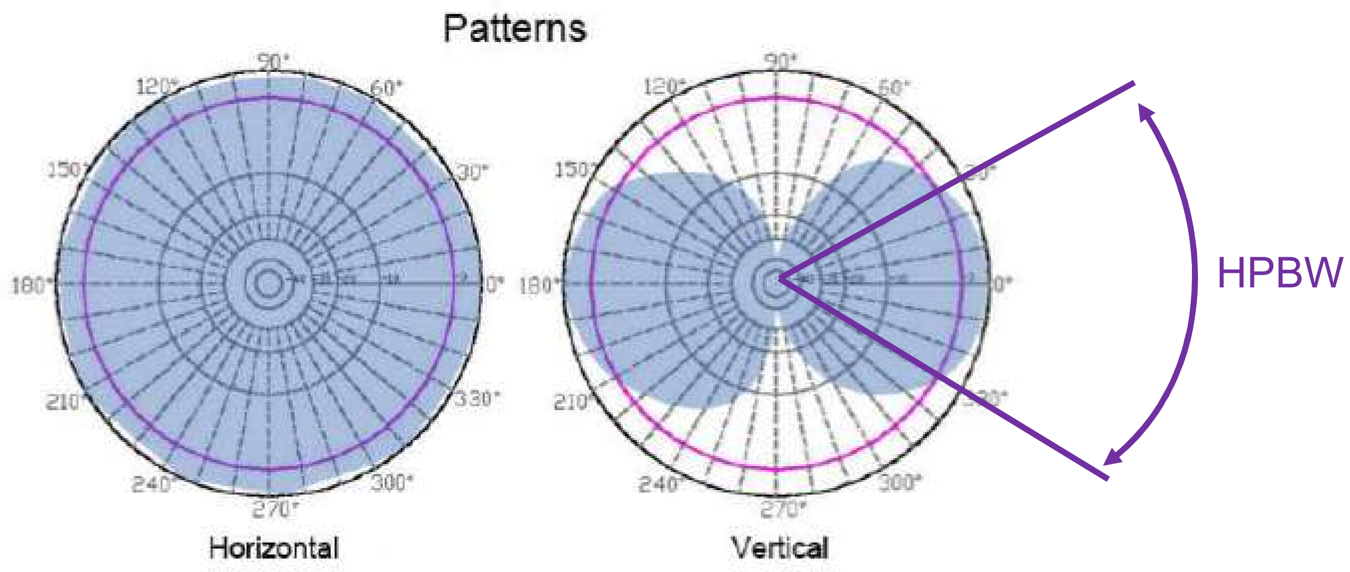
three examples from the real life

Radiation pattern

three examples from the real life

HPBW (vertical) =60°

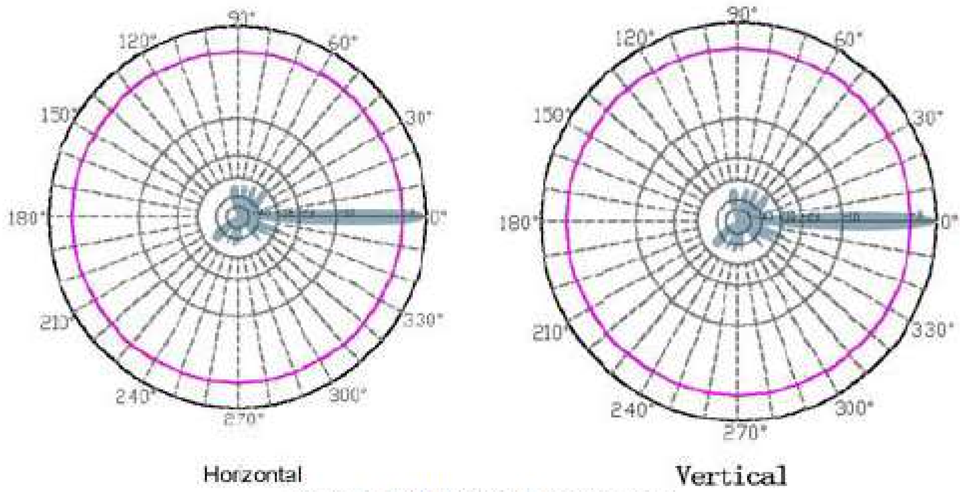
HPBW (horizontal) =360°



ZDAEW1900-3 **Patterns**

Radiation pattern

three examples from the real life



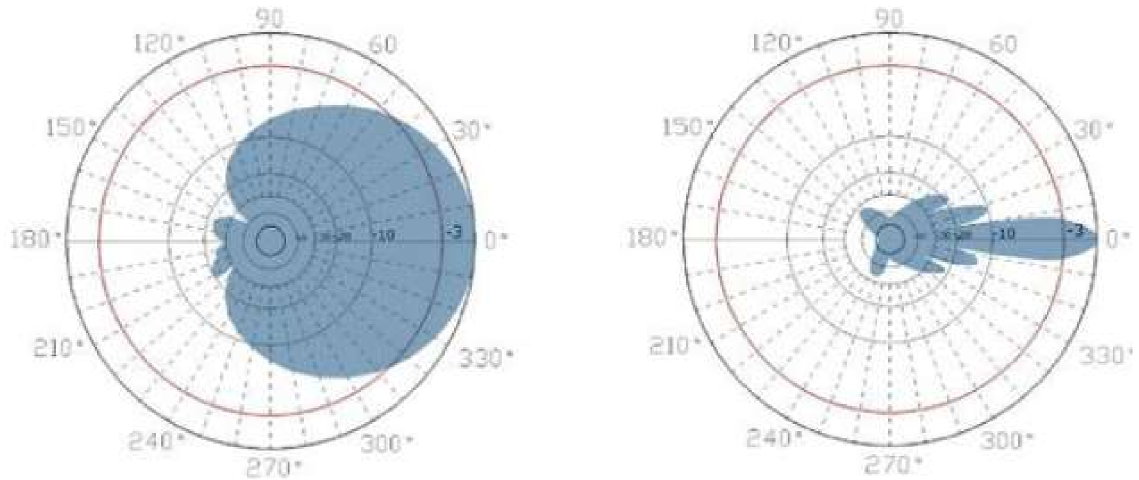
ZDASP5400-29-6 **Patterns**

HPBW (vertical) = 6°

HPBW (horizontal) = 6°

Radiation pattern

three examples from the real life



Horizontal

Vertical

HPBW (vertical) = 14°

HPBW (horizontal) =90°

ZDADJ800-13-90 Patterns

Parameters of the Tx Antenna

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Directivity

$r >> D$

$r > \frac{2D^2}{\lambda}$

$r >> \lambda$

$$\mathbf{E}(\vec{\mathbf{r}}) = \mathbf{E}(r,\vartheta,\varphi) = \frac{j\zeta I}{2\lambda}\frac{e^{-j\beta r}}{r}\mathbf{l}(\vartheta,\varphi)$$

$$\zeta\mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

$$\mathbf{l}(\vartheta,\varphi) = l_\vartheta(\vartheta,\varphi)\hat{i}_\vartheta + l_\varphi(\vartheta,\varphi)\hat{i}_\varphi$$

$$\vec{\mathbf{S}} = \frac{1}{2\zeta}\left|\vec{\mathbf{E}}\right|^2\hat{i}_r = \frac{\zeta}{2}\left|\vec{\mathbf{H}}\right|^2\hat{i}_r$$

The directivity of an antenna is :

$$D(\vartheta,\varphi) = \lim_{r\to\infty}\frac{\frac{1}{2\zeta}\left|\mathbf{E}(r,\vartheta,\varphi)\right|^2}{\frac{1}{4\pi r^2}P_{rad}}$$

The directivity of an isotropic source is equal to 1 (that is, 0 dB)

$r \to \infty$

$$P_{rad} = P_1 = \oiint_A dA\frac{1}{2\zeta}\left|\vec{\mathbf{E}}\right|^2$$

$$\oiint_A dA\vec{\mathbf{S}}\cdot\hat{\mathbf{n}} = P_1 + jP_2$$

Directivity of the elementary electrical dipole

The directivity of an antenna is :

$$D(\vartheta,\varphi) = \lim_{r\to\infty} \frac{\frac{1}{2\zeta}\left|\mathbf{E}\left(r,\vartheta,\varphi\right)\right|^2}{\frac{1}{4\pi r^2} P_{rad}}$$

Directivity of the elementary electrical dipole

Far field expression

$$\mathbf{E}(\mathbf{r}) = \mathrm{E}_{\vartheta}(r,\vartheta)\hat{i}_{\vartheta} = j\zeta \frac{I\Delta z}{2\lambda r}\sin\vartheta \exp(-j\beta r)\hat{i}_{\vartheta}$$

$$P_1 = P_{rad} = \frac{1}{2}\frac{2\pi}{3}\zeta\left(\frac{\Delta z}{\lambda}\right)^2 |I|^2$$

The directivity of an antenna is :

$$D(\vartheta,\varphi) = \lim_{r\to\infty} \frac{\frac{1}{2\zeta}\left|\mathbf{E}(r,\vartheta,\varphi)\right|^2}{\frac{1}{4\pi r^2}P_{rad}}$$

$$|\mathbf{E}|^2 = \zeta^2 \frac{|I|^2 \Delta z^2}{4\lambda^2 r^2}\sin^2\vartheta \qquad \frac{1}{2\zeta}|\mathbf{E}|^2 = \frac{\zeta}{2}\frac{|I|^2 \Delta z^2}{4\lambda^2 r^2}\sin^2\vartheta$$

$$D = D(\vartheta) = \frac{\frac{\zeta}{2}\frac{|I|^2 \Delta z^2}{4\lambda^2 r^2}\sin^2\vartheta}{\frac{1}{4\pi r^2}\frac{1}{2}\frac{2\pi}{3}\zeta\left(\frac{\Delta z}{\lambda}\right)^2 |I|^2} = \frac{3}{2}\sin^2\vartheta$$

$$D_{\max} = 10\log_{10} 1.5 = 1.76 \text{ dB}$$

Directivity of the small loop antenna

Far field expression

$$\mathbf{E}(\mathbf{r}) = \mathrm{E}_\varphi(r,\vartheta)\hat{i}_\varphi = \frac{\zeta\beta\Delta s I}{2\lambda r}\sin\vartheta\exp(-j\beta r)\hat{i}_\varphi$$

$$P_1 = P_{rad} = \frac{1}{2}\frac{2\pi}{3}\zeta\left(\frac{\beta\Delta S}{\lambda}\right)^2|I|^2$$

$$|\mathbf{E}|^2 = \zeta^2\frac{|I|^2(\beta\Delta s)^2}{4\lambda^2 r^2}\sin^2\vartheta$$

$$\frac{1}{2\zeta}|\mathbf{E}|^2 = \frac{\zeta}{2}\frac{|I|^2(\beta\Delta s)^2}{4\lambda^2 r^2}\sin^2\vartheta$$

The directivity of an antenna is :

$$D(\vartheta,\varphi) = \lim_{r\to\infty}\frac{\frac{1}{2\zeta}|\mathbf{E}(r,\vartheta,\varphi)|^2}{\frac{1}{4\pi r^2}P_{rad}}$$

Elementary electric dipole

$$P_1 = P_{rad} = \frac{1}{2}\frac{2\pi}{3}\zeta\left(\frac{\Delta z}{\lambda}\right)^2|I|^2$$

$$\frac{1}{2\zeta}|\mathbf{E}|^2 = \frac{\zeta}{2}\frac{|I|^2\Delta z^2}{4\lambda^2 r^2}\sin^2\vartheta$$

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Directivity of the small loop antenna

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$$\mathbf{E}(\mathbf{r}) = \mathrm{E}_{\varphi}(r,\vartheta)\hat{i}_{\varphi} = \frac{\zeta\beta\Delta s I}{2\lambda r}\sin\vartheta\exp(-j\beta r)\hat{i}_{\varphi}$$

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The directivity of an antenna is :

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$$|\mathbf{E}|^2 = \zeta^2\frac{|I|^2(\beta\Delta s)^2}{4\lambda^2 r^2}\sin^2\vartheta \qquad \frac{1}{2\zeta}|\mathbf{E}|^2 = \frac{\zeta}{2}\frac{|I|^2(\beta\Delta s)^2}{4\lambda^2 r^2}\sin^2\vartheta$$

$$D = D(\vartheta) = \frac{3}{2}\sin^2\vartheta$$

$$D_{\max} = 10\log_{10}1.5 = 1.76\ \mathrm{dB}$$

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- Effective length
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  - Beamwidth
- Directivity
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- Radiation Resistance
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- Input Impedance and Input Resistance

Gain

Directivity

$$D(\vartheta,\varphi) = \lim_{r\to\infty} \frac{\frac{1}{2}\frac{|\mathbf{E}|^2}{\zeta}}{\frac{1}{4\pi r^2}P_{rad}}$$

Gain

$$G(\vartheta,\varphi) = \lim_{r\to\infty} \frac{\frac{1}{2}\frac{|\mathbf{E}|^2}{\zeta}}{\frac{1}{4\pi r^2}P_{in}}$$

***P_{rad}*: radiated power**

***P_{in}*: input power**

If one replace P_{rad} with the input real power to the antenna P_{in} one finds the definition of the *Gain*.

For a lossless antenna, $P_{in}=P_{rad}$ and *G=D.* If losses are present $P_{ing}>P_{rad}$ and *G<D.*

Note that both D and G are dimensionless.

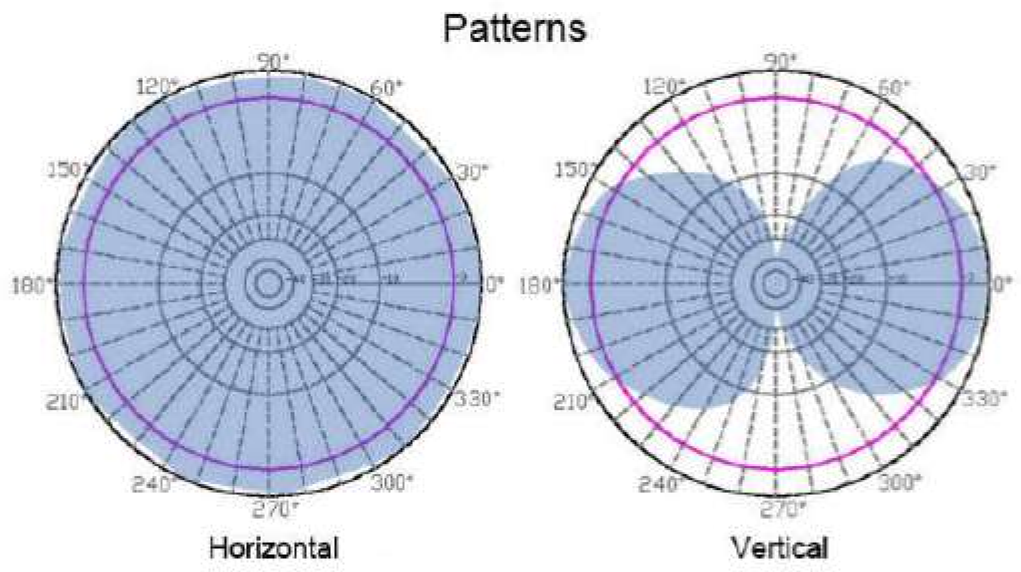
Gain

three examples from the real life

Gain

three examples from the real life

(maximum) Gain = 3 dB

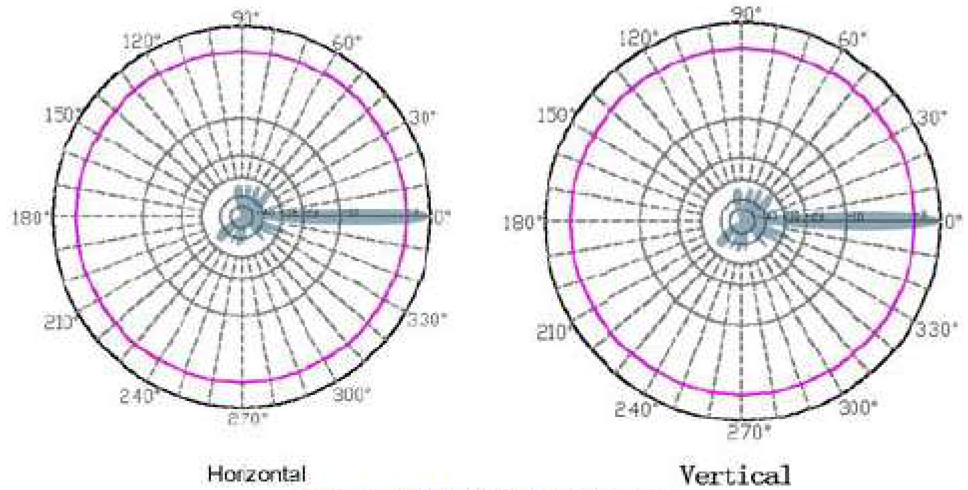


ZDAEW1900-3 **Patterns**

Gain

three examples from the real life

(maximum) Gain = 29 dB

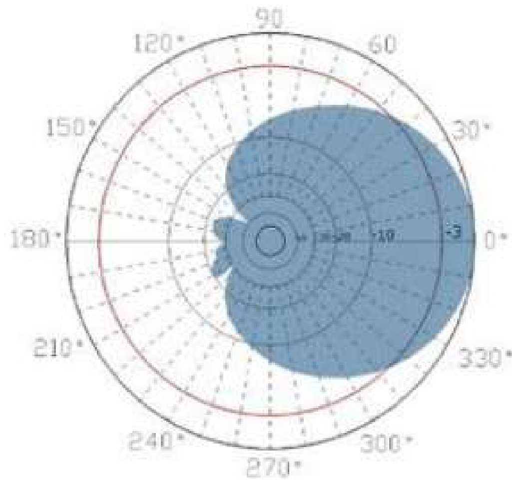


ZDASP5400-29-6 **Patterns**

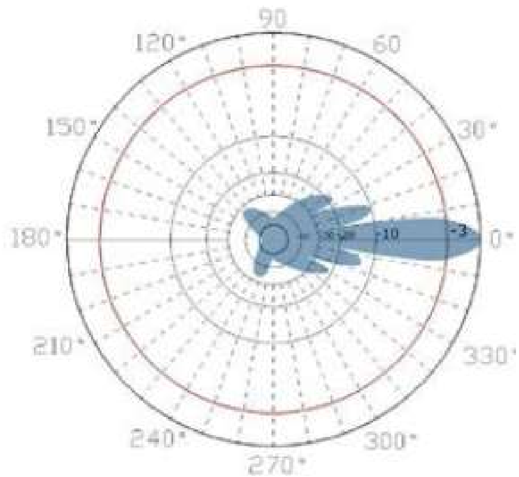
Gain

three examples from the real life

(maximum) Gain = 13 dB



Horizontal



Vertical

ZDADJ800-13-90 Patterns

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  - Radiation pattern lobes
  - Beamwidth
- Directivity
- Gain
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- Input Impedance and Input Resistance

Radiation resistance

Associated to the far-field radiated power one can define the radiation Resistance R_{rad}:

$$P_{rad} = \frac{1}{2} R_{rad} \left| I \right|^2$$

Radiation resistance

Associated to the far-field radiated power one can define the radiation Resistance R_{rad}:

$$P_{rad} = \frac{1}{2} R_{rad} \left| I \right|^2$$

Elementary electrical dipole

$$P_1 = P_{rad} = \frac{1}{2}\frac{2\pi}{3}\zeta\left(\frac{\Delta z}{\lambda}\right)^2 \left| I \right|^2 \quad \Rightarrow \quad R_{rad} = \frac{2\pi}{3}\zeta\left(\frac{\Delta z}{\lambda}\right)^2$$

Radiation resistance

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$$P_1 = P_{rad} = \frac{1}{2}\frac{2\pi}{3}\zeta\left(\frac{\Delta z}{\lambda}\right)^2 |I|^2 \quad \Rightarrow \quad R_{rad} = \frac{2\pi}{3}\zeta\left(\frac{\Delta z}{\lambda}\right)^2$$

Small loop antenna

$$P_1 = P_{rad} = \frac{1}{2}\frac{2\pi}{3}\zeta\left(\frac{\beta\Delta S}{\lambda}\right)^2 |I|^2 \quad \Rightarrow \quad R_{rad} = \frac{2\pi}{3}\zeta\left(\frac{\beta\Delta S}{\lambda}\right)^2$$

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Input impedance

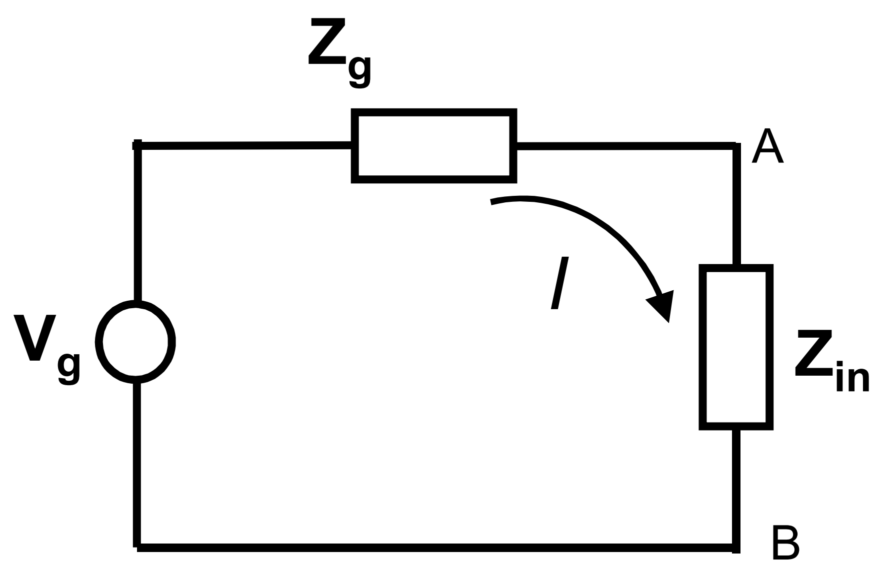
Input impedance is defined as "the impedance presented by an antenna at its terminals or the ratio of the voltage to current at a pair of terminals or the ratio of the appropriate components of the electric to magnetic fields at a point."

The input impedance of an antenna is generally a function of frequency.

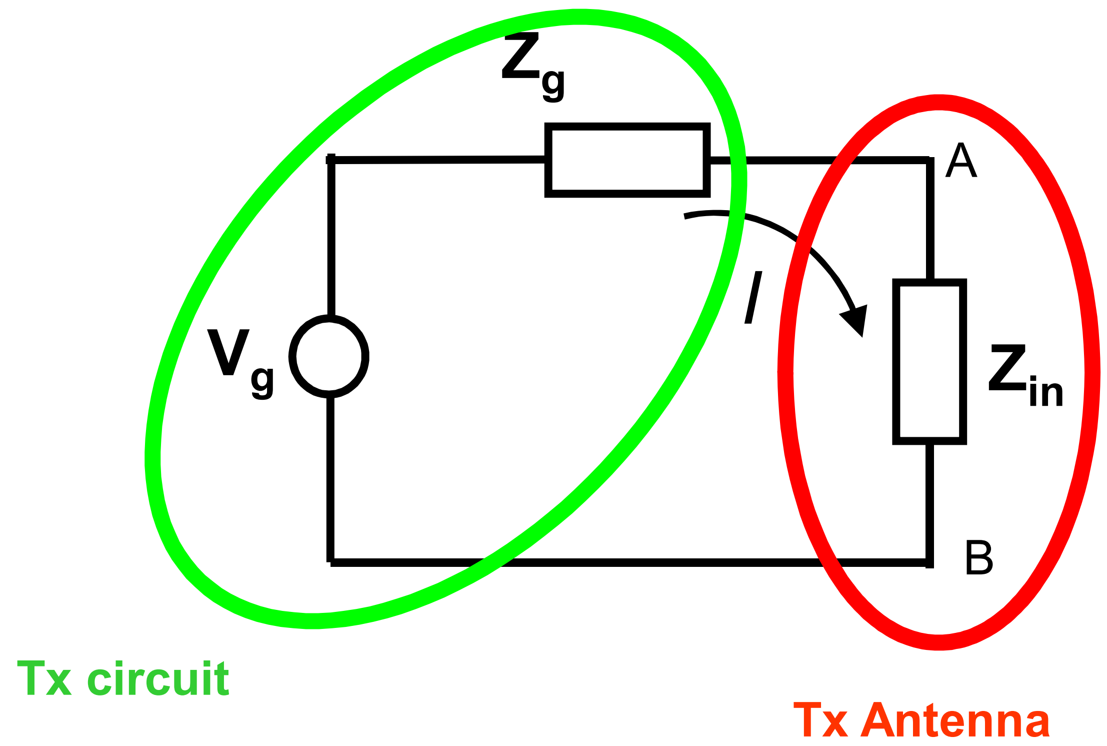
The input impedance of the antenna depends on many factors including its geometry, its method of excitation, and its proximity to surrounding objects.

Because of their complex geometries, only a limited number of practical antennas have been investigated analytically. For many others, the input impedance has been determined experimentally.

Equivalent circuit of the Tx antenna



Equivalent circuit of the Tx antenna



$$Z_{in} = R_{in} + jX_{in}$$

$$P_{in} = \frac{1}{2} R_{in} |I|^2$$

$$P_{rad} = \frac{1}{2} R_{rad} |I|^2$$

$$P_{rad} \leq P_{in}$$

$\Rightarrow$ $R_{rad} \leq R_{in}$ $\Rightarrow$ $R_{in} = R_{rad} + R_{\Omega}$

Radiation efficiency

Directivity

$$D(\vartheta,\phi)=\lim_{r\to\infty}\frac{\frac{1}{2}\frac{|\mathbf{E}|^2}{\zeta}}{\frac{1}{4\pi r^2}P_{rad}}$$

Gain

$$G(\vartheta,\phi)=\lim_{r\to\infty}\frac{\frac{1}{2}\frac{|\mathbf{E}|^2}{\zeta}}{\frac{1}{4\pi r^2}P_{in}}$$

$$P_{rad}=\frac{1}{2}R_{rad}|I|^2$$

$$P_{in}=\frac{1}{2}R_{in}|I|^2=\frac{1}{2}(R_{rad}+R_{\Omega})|I|^2$$

Radiation Efficiency: $\eta=\frac{P_{rad}}{P_{in}}=\frac{R_{rad}}{R_{rad}+R_{\Omega}}=\frac{G}{D}$

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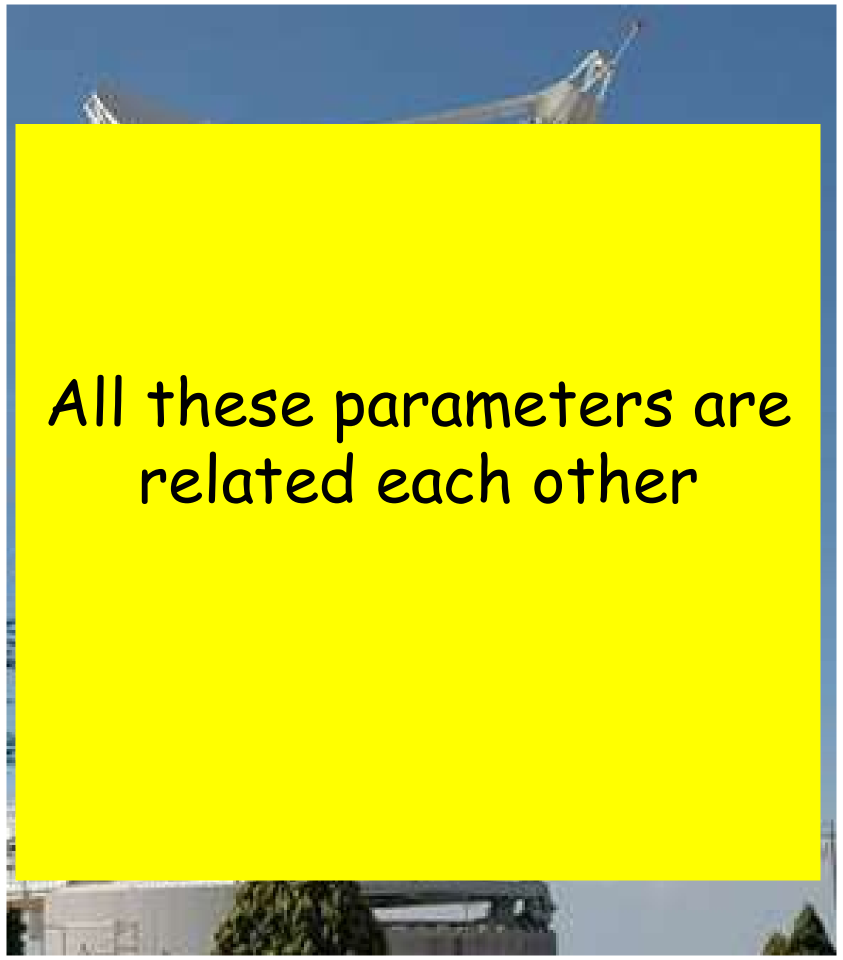

Antenna Parameters

Parameters of the Tx Antenna

Parameters of the Rx Antenna

Parameters of the Tx Antenna

- Effective length
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Antenna Parameters

Parameters of the Tx Antenna

Parameters of the Rx Antenna

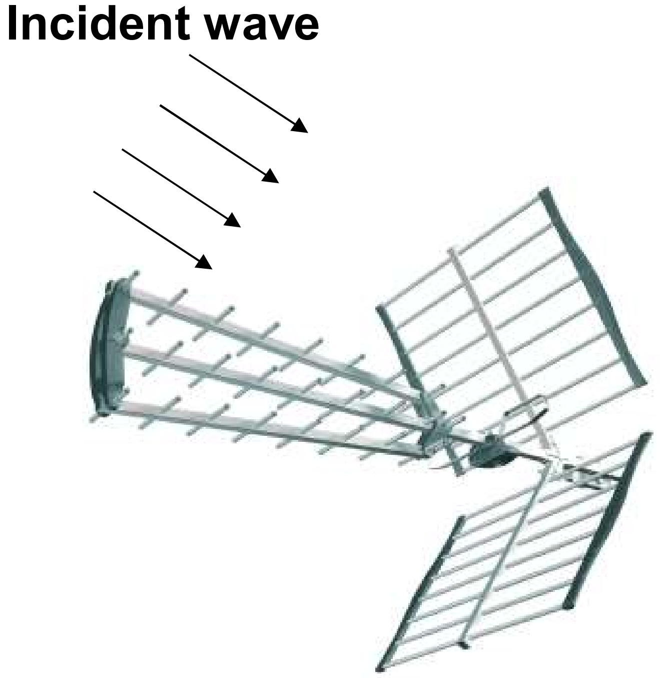
Receiving mode

When an antenna is operating as a receiving antenna, it extracts a certain amount of power from an incident electromagnetic wave.

Since an incident wave comes from a far distance may be thought of as a uniform (local) plane wave being intercepted by the antenna.

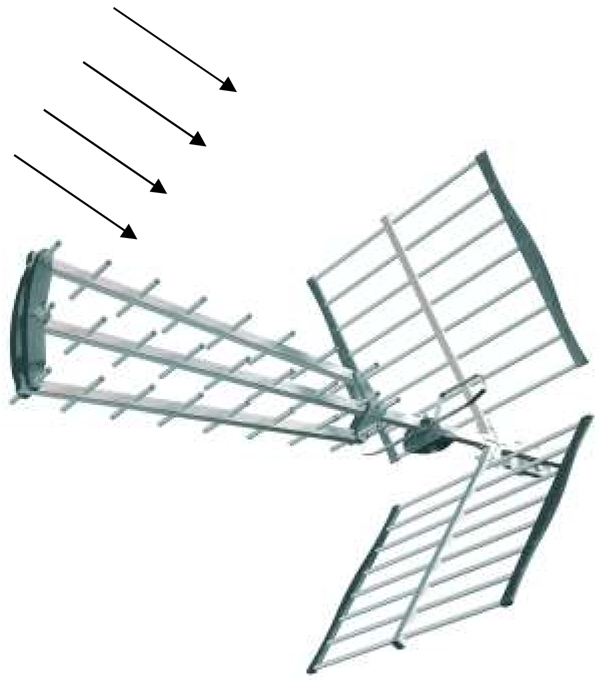
The use of the antenna in the receiving mode is shown in Figure.

The incident wave impinges upon the antenna, and it induces a voltage V_0 at the input terminals .



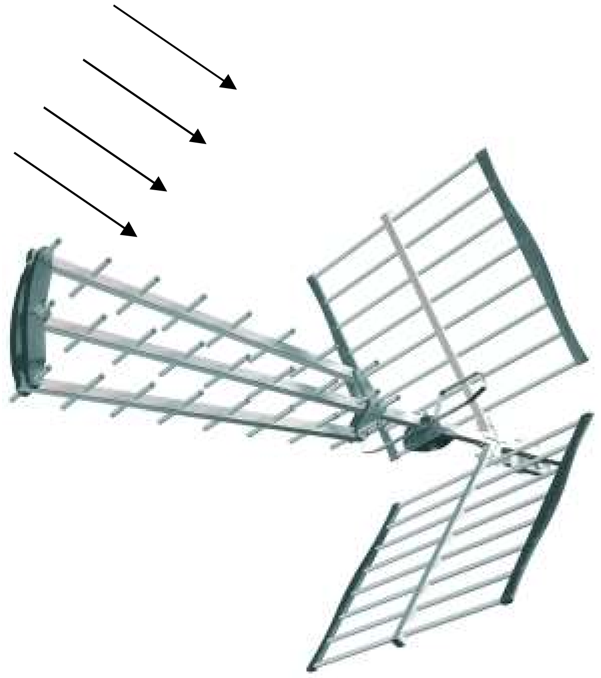
Parameters of the Rx Antenna

- Rx effective length
- Equivalent circuit of the rx antenna
- Effective Area



Parameters of the Rx Antenna

- Rx effective length
- Equivalent circuit of the rx antenna
- Effective Area



Rx effective length

Tx effective length $\mathbf{l}(\vartheta,\varphi) = l_\vartheta(\vartheta,\varphi)\hat{i}_\vartheta + l_\varphi(\vartheta,\varphi)\hat{i}_\varphi$

Fraunhofer region $\mathbf{E}(r,\vartheta,\varphi) = \dfrac{j\zeta I}{2\lambda}\dfrac{e^{-j\beta r}}{r}\mathbf{l}(\vartheta,\varphi)$

Elementary electrical dipole $\mathbf{l}(\vartheta,\varphi) = \Delta z \sin\vartheta\,\hat{i}_\vartheta$

Small loop antenna $\mathbf{l}(\vartheta,\varphi) = -j\beta\Delta S \sin\vartheta\,\hat{i}_\varphi$

It can be shown that **for an elementary electrical dipole or for a small loop antenna**, the following property is valid:

$$|V_0| = |\mathbf{E_i} \cdot \mathbf{l}|$$

$\mathbf{l}$ is the tx antenna effective length

$\boldsymbol{E_i}$ is the incident, locally plane, field

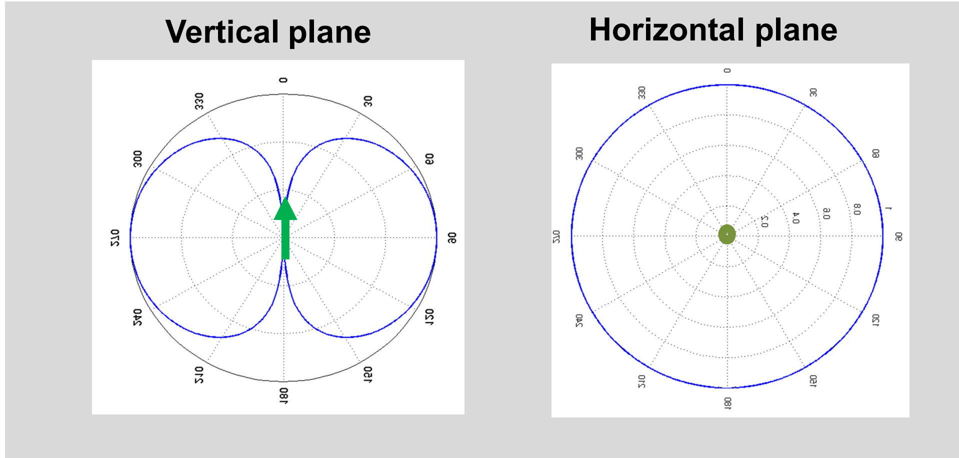
V_0 is the voltage induced at the antenna terminals, which are assumed open-circuited

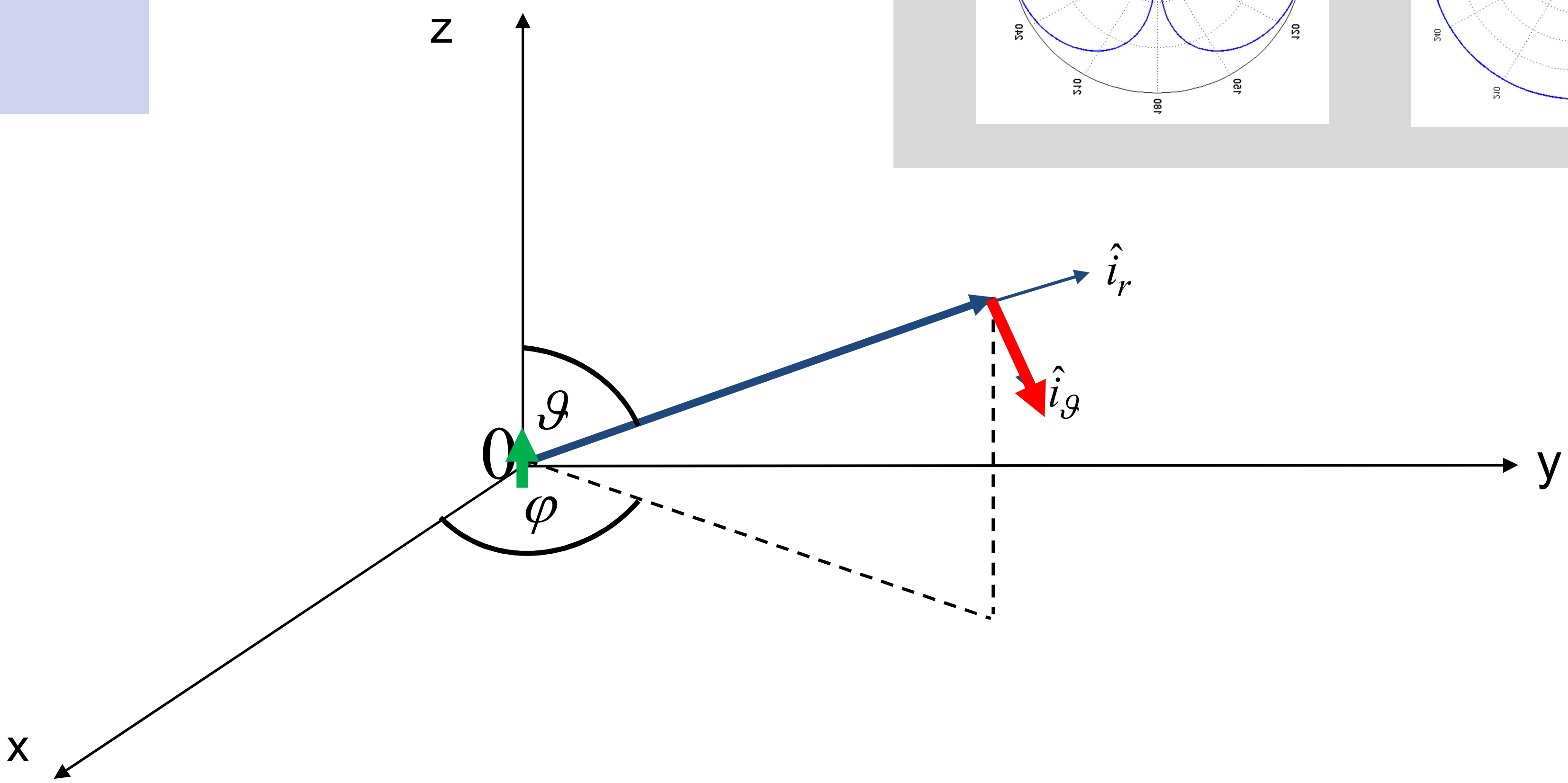
Rx effective length

Elementary electrical dipole

$$\mathbf{l}(\vartheta,\varphi) = \Delta z \sin\vartheta\, \hat{i}_\vartheta$$

$$|V_0| = |\mathbf{E_i} \cdot \mathbf{l}|$$





Rx effective length

Elementary electrical dipole

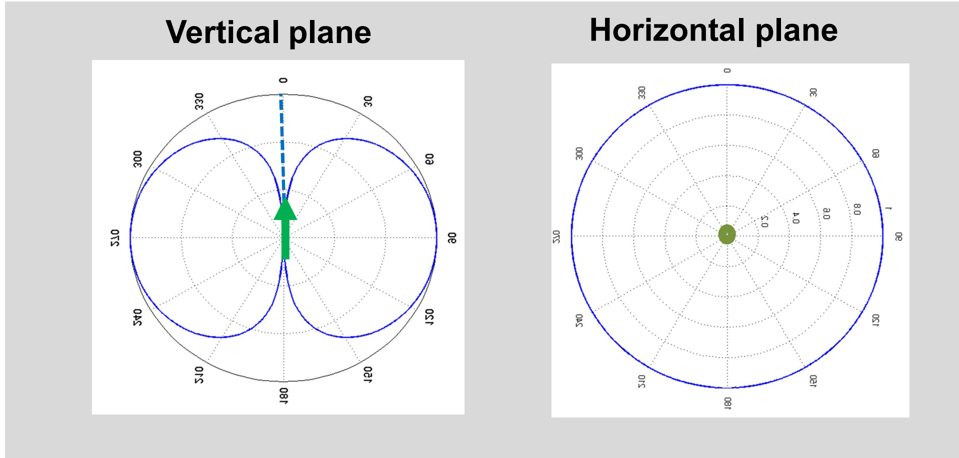
$$\mathbf{l}(\vartheta,\varphi) = \Delta z \sin\vartheta\, \hat{i}_\vartheta$$

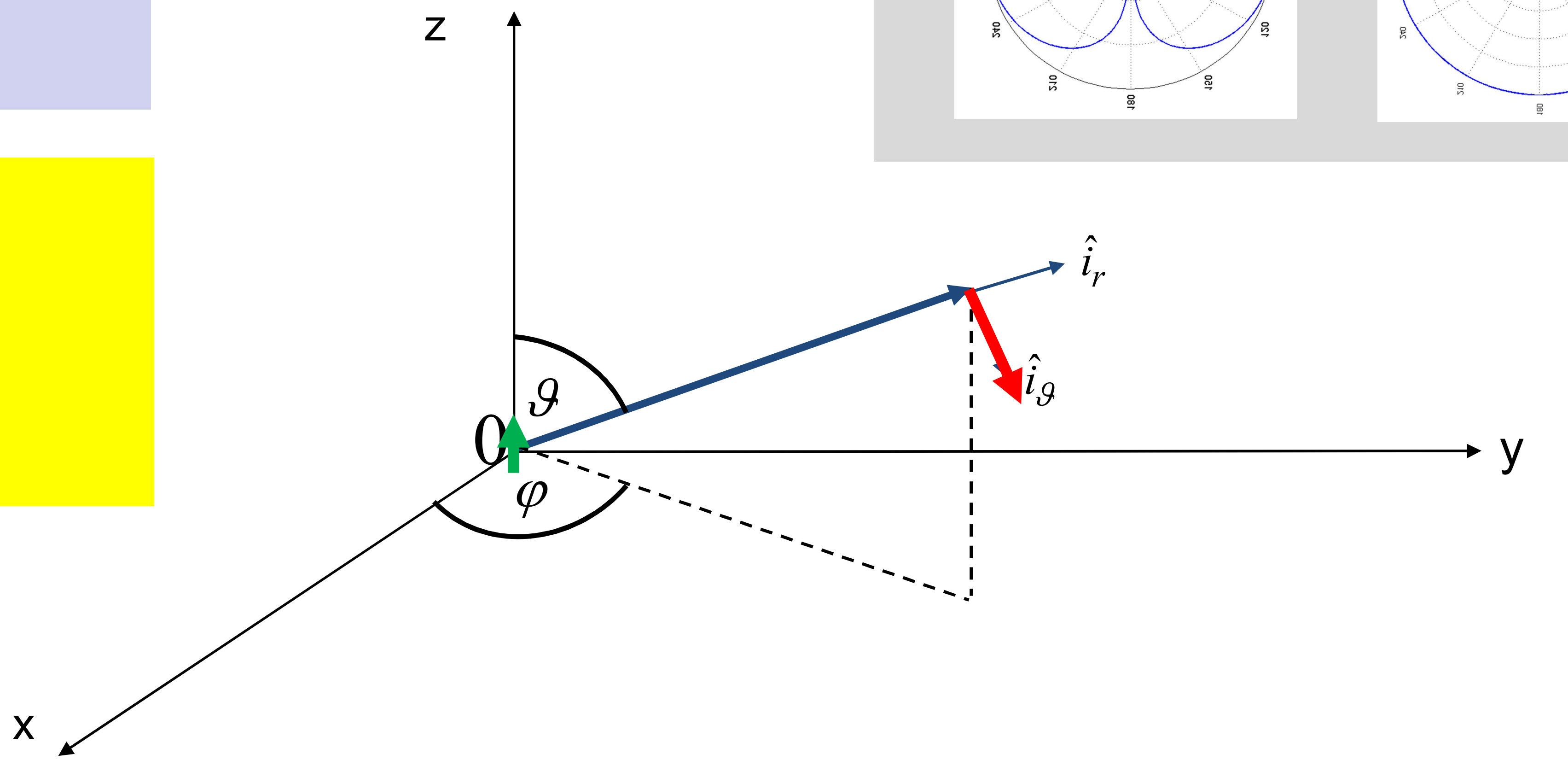
$$|V_0| = |\mathbf{E_i} \cdot \mathbf{l}|$$

First example

$$\vartheta = 0$$

$$|V_0| = 0$$





Rx effective length

Elementary electrical dipole

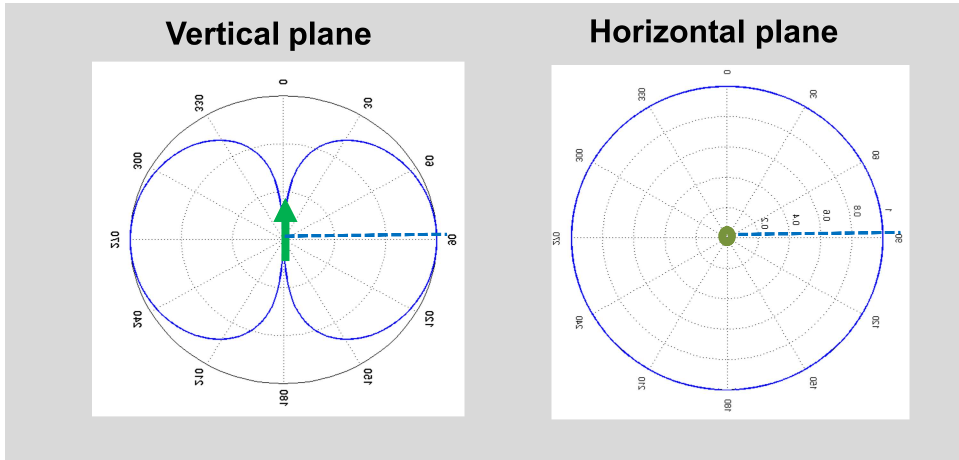
$$\mathbf{l}(\vartheta,\varphi) = \Delta z \sin\vartheta\, \hat{i}_\vartheta$$

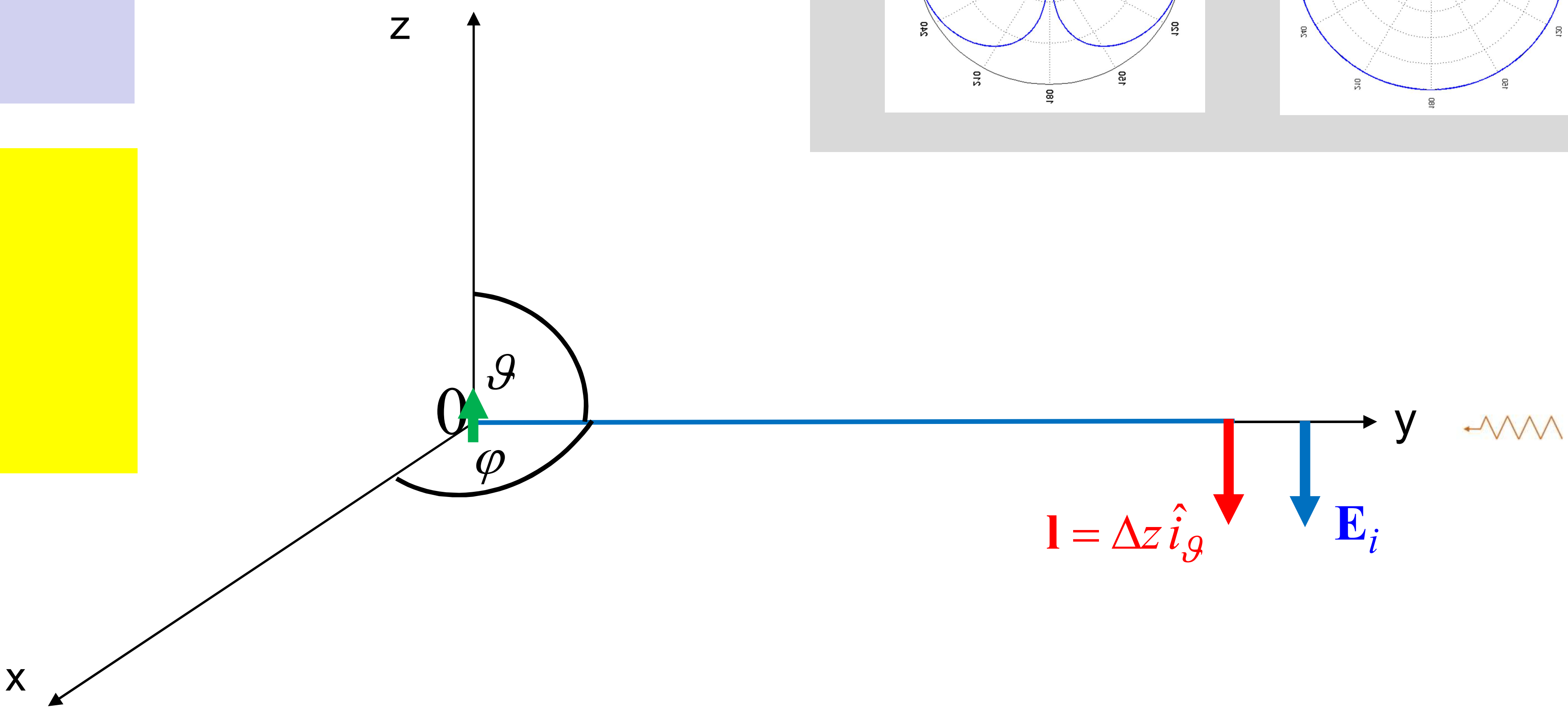
$$|V_0| = |\mathbf{E_i} \cdot \mathbf{l}|$$

Second example

$$\vartheta = \frac{\pi}{2}; \quad \varphi = \frac{\pi}{2}$$

$$|V_0| = |\Delta z \mathbf{E_i}|$$





Rx effective length

Elementary electrical dipole

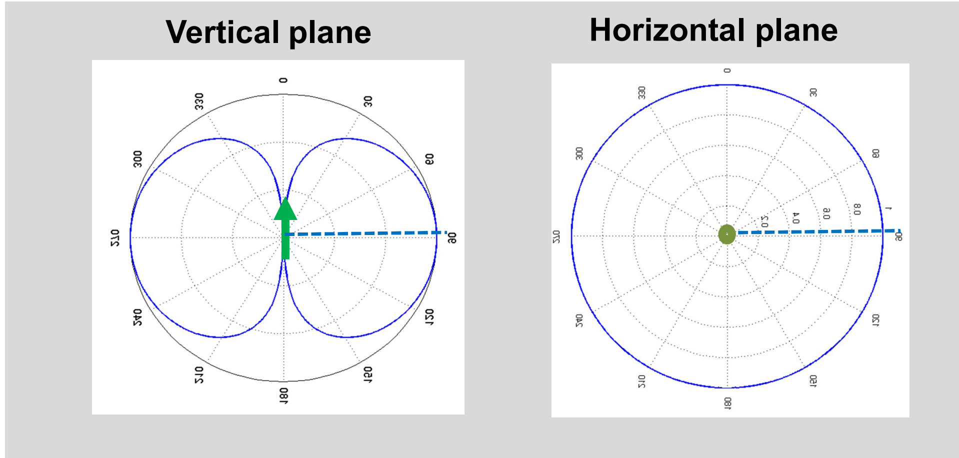
$$\mathbf{l}(\vartheta,\varphi) = \Delta z \sin\vartheta\, \hat{i}_\vartheta$$

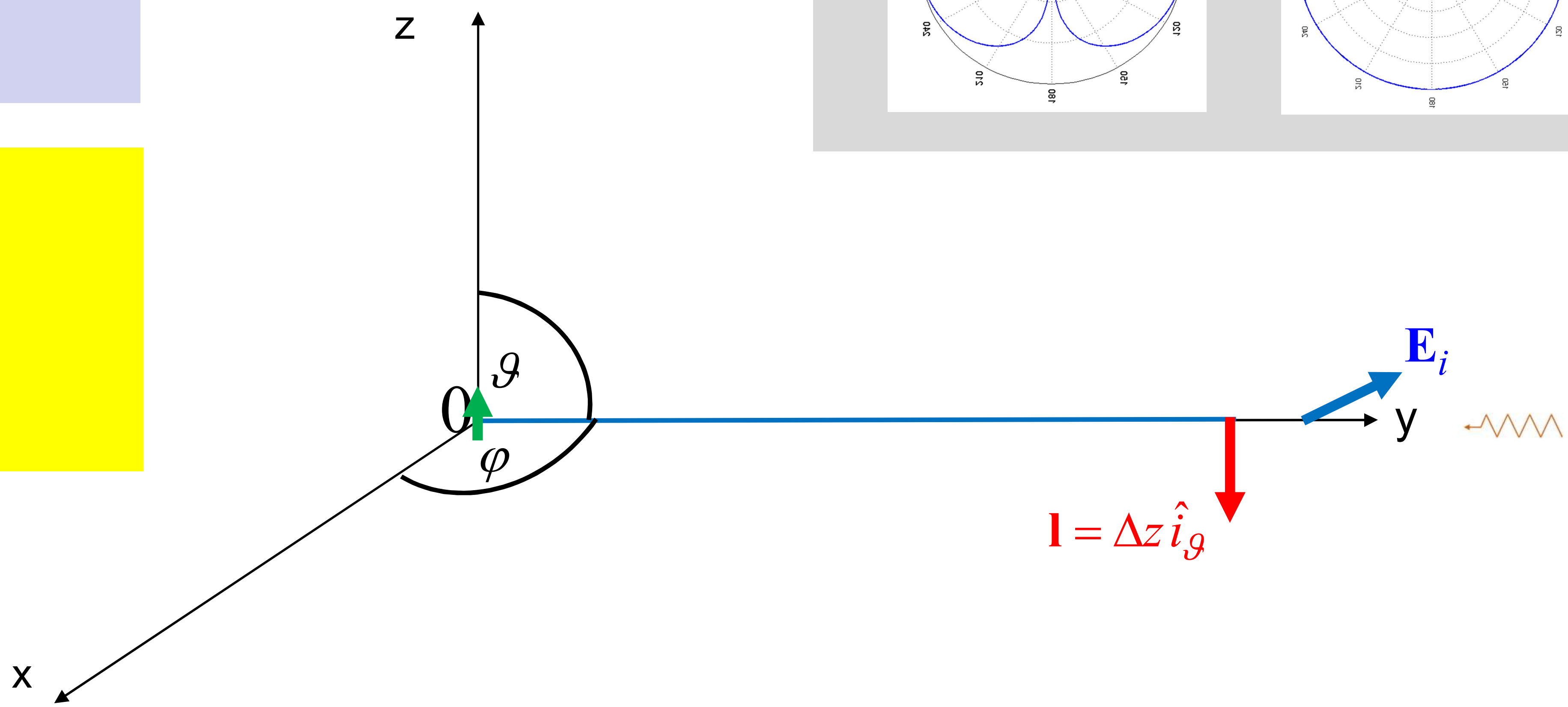
$$|V_0| = |\mathbf{E_i} \cdot \mathbf{l}|$$

Second example

$$\vartheta = \frac{\pi}{2}; \quad \varphi = \frac{\pi}{2}$$

$$|V_0| = 0$$





Rx effective length

Elementary electrical dipole

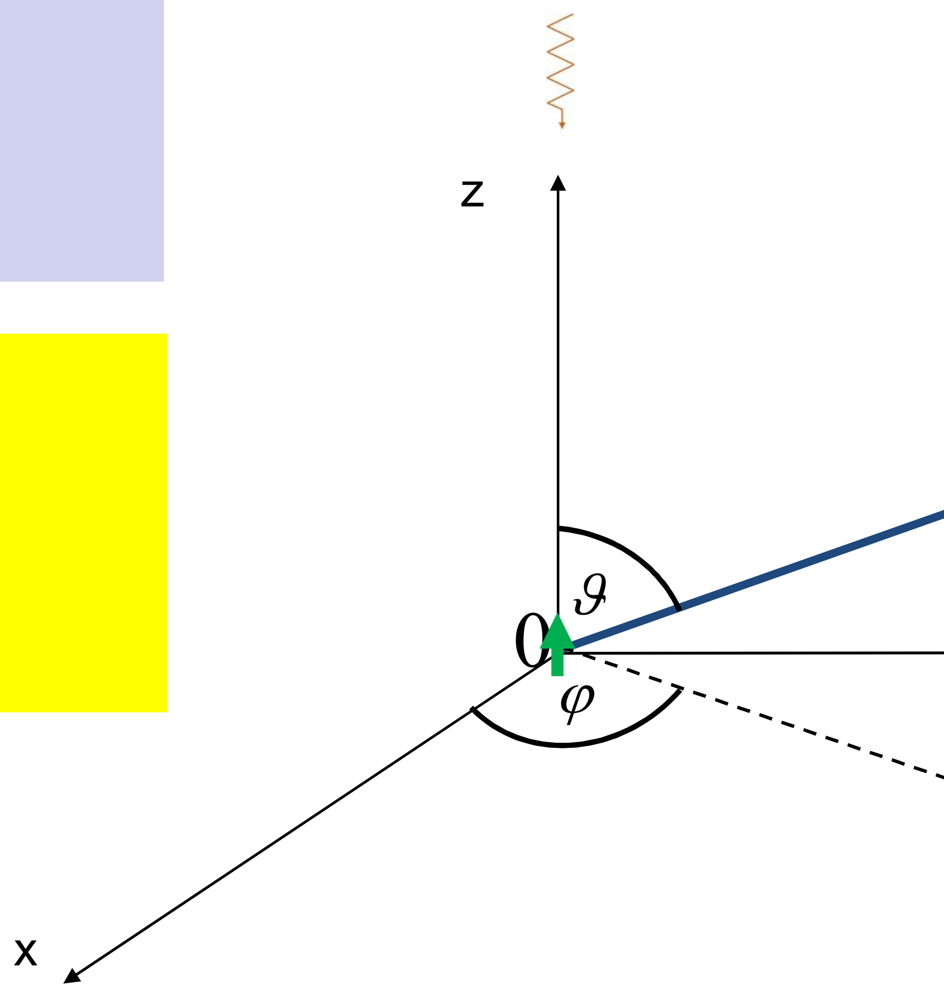
$$\mathbf{l}(\vartheta,\varphi) = \Delta z \sin\vartheta \hat{i}_\vartheta$$

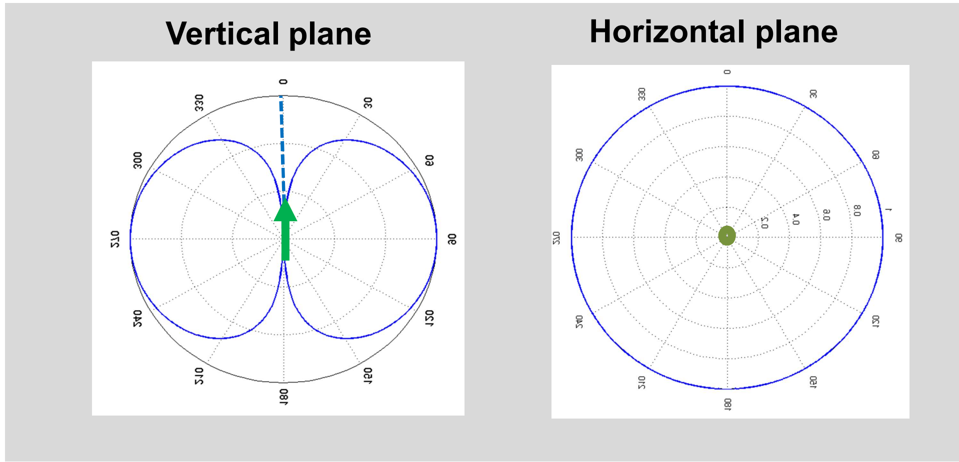
$$|V_0| = |\mathbf{E_i} \cdot \mathbf{l}|$$

First example

$$\vartheta = 0$$

$$|V_0| = 0$$





Rx effective length

Interestingly, this result can be extended to **ALL** the antennas by applying the **RECIPROCITY THEOREM**

It can be shown that **for an elementary electrical dipole or for a small loop antenna**, the following property is valid:

$$|V_0| = |\mathbf{E_i} \cdot \mathbf{l}|$$

$\mathbf{l}$ is the tx antenna effective length

$\boldsymbol{E_i}$ is the incident, locally plane, field

V_0 is the voltage induced at the antenna terminals, which are assumed open-circuited

Rx effective length

$$\left|V_0\right| = \left|\mathbf{E_i} \cdot \mathbf{l}\right|$$

Where

$\boldsymbol{E_i}$ is the incident, locally plane, field

V_0 is the voltage induced at the antenna terminals, which are assumed open-circuited

$\mathbf{l}(\vartheta,\varphi) = l_\vartheta(\vartheta,\varphi)\hat{i}_\vartheta + l_\varphi(\vartheta,\varphi)\hat{i}_\varphi$ can referred to as **receiving effective length** of the antenna (and not only transmitting effective length)

Note that this means that the behavior of an antenna when transmitting and when receiving are related.