Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2020-2021 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

Stefano Perna

Color legend

New formulas, important considerations,
important formulas, important concepts

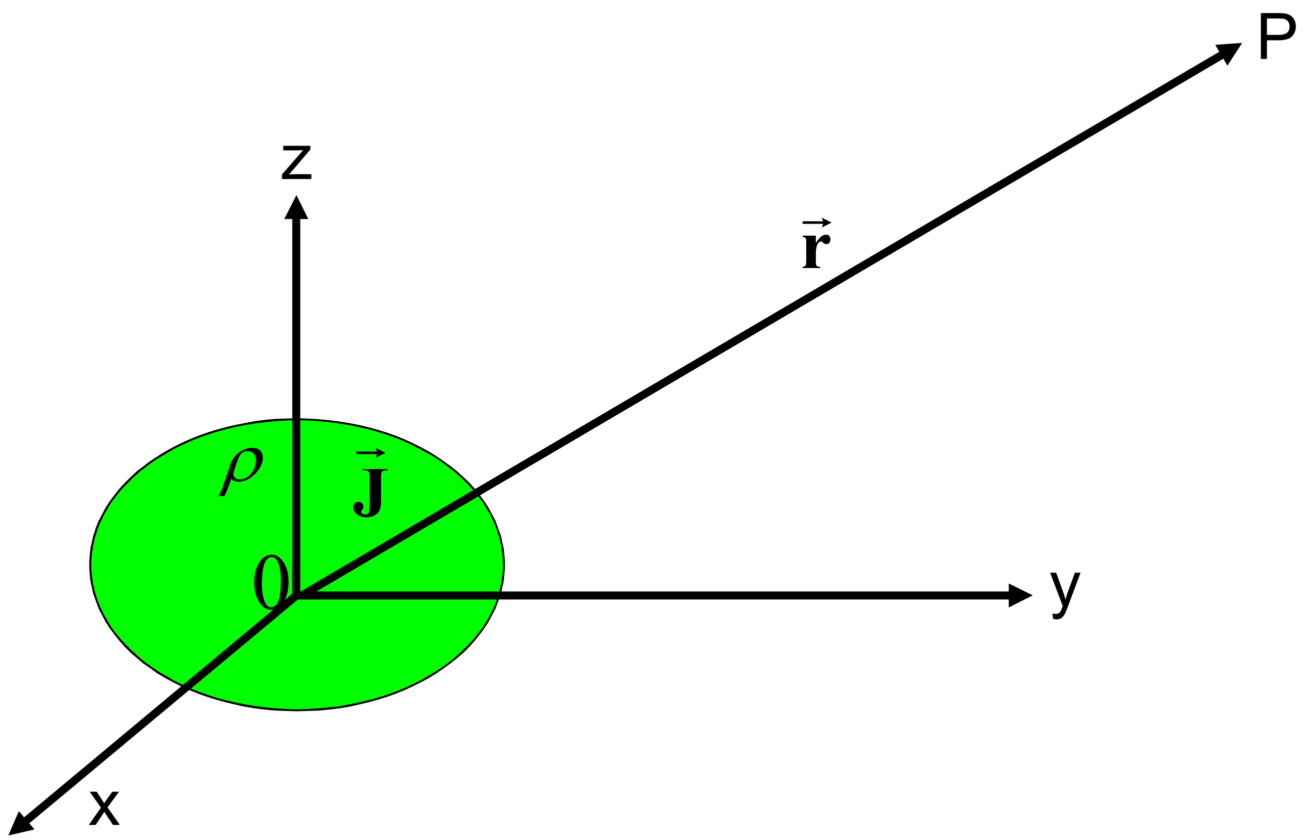
Very important for the discussion

Memo

Mathematical tools to be exploited

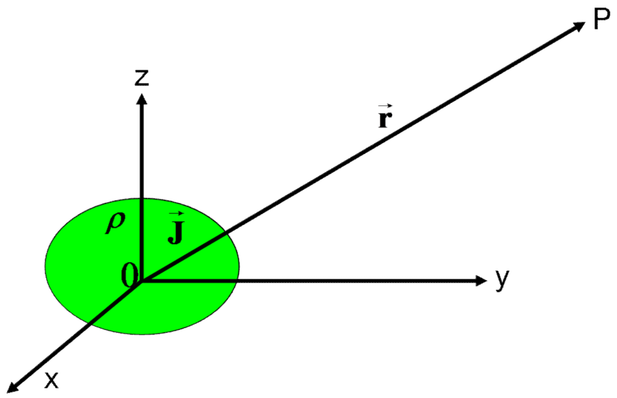
Mathematics

Radiation problem



$\vec{\mathbf{E}}(\vec{\mathbf{r}})$
$\vec{\mathbf{H}}(\vec{\mathbf{r}})$

An antenna (or aerial) is an electrical device which converts electric power into radio waves, and vice versa.



Potentials

$\boldsymbol{J}(\boldsymbol{r})$

$$\nabla^2 \boldsymbol{A} + k^2 \boldsymbol{A} = -\mu \boldsymbol{J}$$

Vector Helmholtz equation

$$\mathbf{A}(\mathbf{r}) = \frac{\mu}{4\pi} \int \mathbf{J}(\mathbf{r}') \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r}'$$

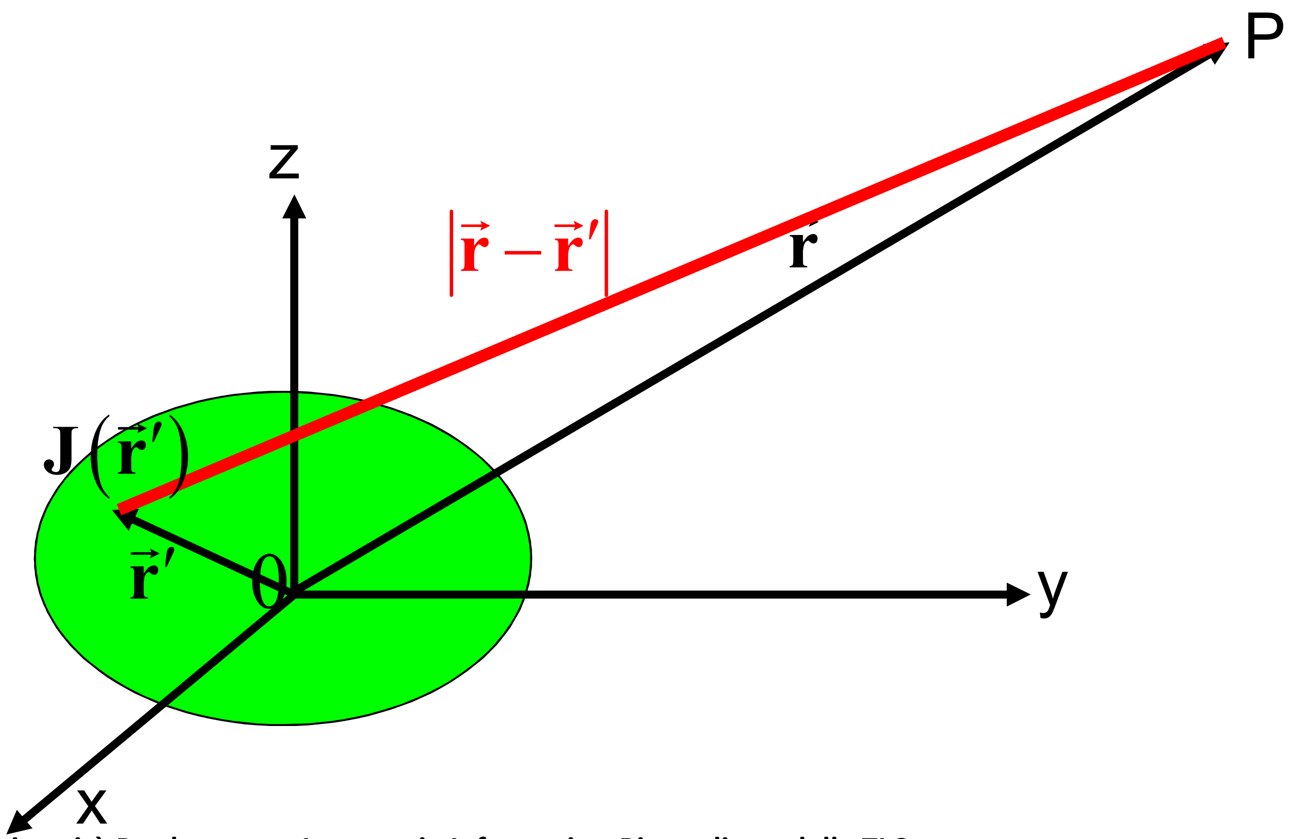
$$\boldsymbol{H} = \frac{1}{\mu} \nabla \times \boldsymbol{A} \quad \text{and} \quad \boldsymbol{E} = -j\omega \boldsymbol{A} + \frac{\nabla\nabla \cdot \boldsymbol{A}}{j\omega\varepsilon\mu}$$

$\boldsymbol{E}(\boldsymbol{r})$

$\boldsymbol{H}(\boldsymbol{r})$

Potentials

$$\mathbf{A}(\vec{\mathbf{r}}) = \frac{\mu}{4\pi}\int \mathbf{J}(\vec{\mathbf{r}}')\frac{e^{-jk|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}d\vec{\mathbf{r}}'$$

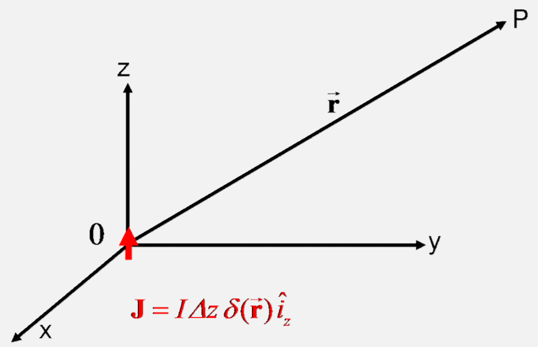


Elementary electrical dipole vs. small loop antenna

Elementary electrical dipole

$$\mathbf{J} = I\Delta z\,\delta(\vec{\mathbf{r}})\,\hat{i}_z$$

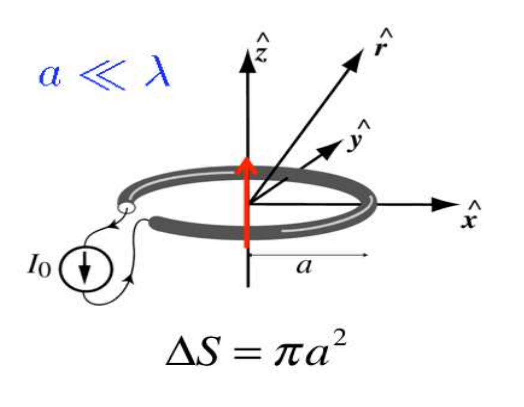
$$\begin{cases} E_r = \zeta \dfrac{I\Delta z}{2\pi}\left(\dfrac{1}{r^2}+\dfrac{1}{j\beta r^3}\right)\cos\vartheta\exp(-j\beta r) \\ E_\vartheta = \zeta \dfrac{I\Delta z}{4\pi}\left(\dfrac{j\beta}{r}+\dfrac{1}{r^2}+\dfrac{1}{j\beta r^3}\right)\sin\vartheta\exp(-j\beta r) \\ H_\varphi = \dfrac{I\Delta z}{4\pi}\left(\dfrac{j\beta}{r}+\dfrac{1}{r^2}\right)\sin\vartheta\exp(-j\beta r) \end{cases}$$



Small loop antenna

$$\mathbf{J} = I\delta(z)\,\delta(r-a)\,\hat{i}'_\varphi$$

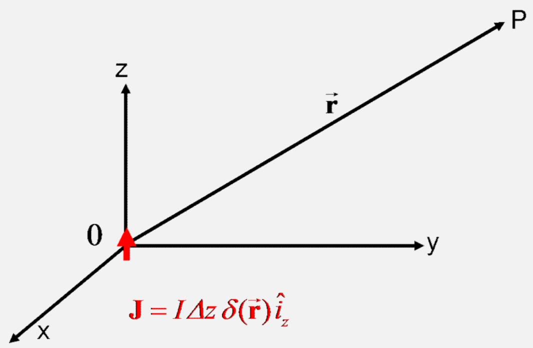
$$\begin{cases} H_r = \dfrac{I\Delta S}{2\pi}j\beta\left(\dfrac{1}{r^2}+\dfrac{1}{j\beta r^3}\right)\cos\vartheta\exp(-j\beta r) \\ H_\vartheta = \dfrac{I\Delta S}{4\pi}j\beta\left(\dfrac{j\beta}{r}+\dfrac{1}{r^2}+\dfrac{1}{j\beta r^3}\right)\sin\vartheta\exp(-j\beta r) \\ E_\varphi = -\dfrac{\zeta I\Delta S}{4\pi}j\beta\left(\dfrac{j\beta}{r}+\dfrac{1}{r^2}\right)\sin\vartheta\exp(-j\beta r) \end{cases}$$



Elementary electrical and magnetic dipoles

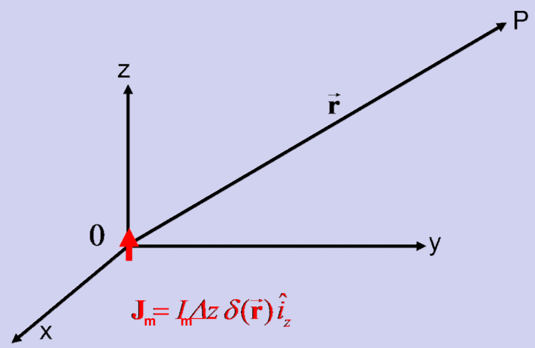
Elementary electrical dipole

$$\mathbf{J} = I\,\Delta z\,\delta(\vec{\mathbf{r}})\,\hat{i}_z$$



Elementary magnetic dipole

$$\mathbf{J}_m = I_m\Delta z\,\delta(\vec{\mathbf{r}})\,\hat{i}_z$$



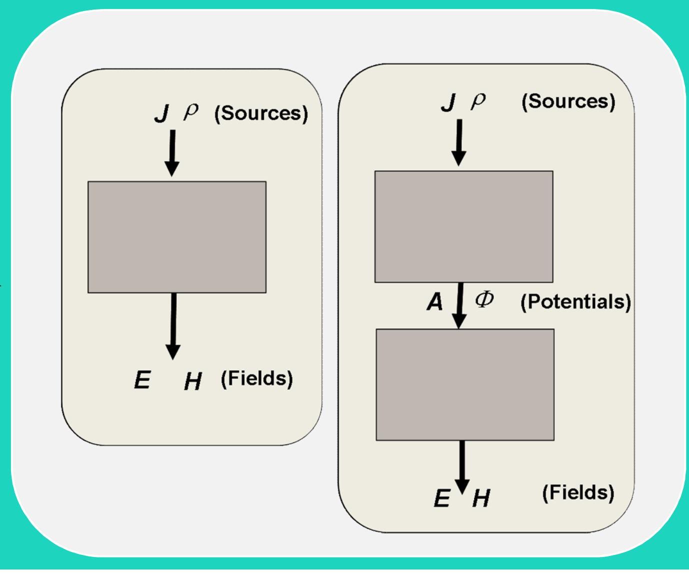
Duality Theorem

$$\mathbf{J}\ \rho \;\longrightarrow\; \mathbf{E}, \mathbf{H}$$

$$\Longrightarrow \quad \mathbf{J}_m = -\zeta \mathbf{J} \quad \rho_m = -\zeta \rho \;\longrightarrow\; \mathbf{E}' = \zeta \mathbf{H} \;, \mathbf{H}' = -\frac{1}{\zeta}\mathbf{E}$$

$$\mathbf{J}_m \quad \rho_m$$

$$\mathbf{J} = -\frac{1}{\zeta}\mathbf{J}_m \qquad \rho = -\frac{1}{\zeta}\rho_m$$



$$\mathbf{E}' = \zeta \mathbf{H} \qquad \mathbf{H}' = -\frac{1}{\zeta}\mathbf{E}$$

Extended antennas

$J(r)$

$$\nabla^2 \boldsymbol{A} + k^2 \boldsymbol{A} = -\mu \boldsymbol{J}$$ Vector Helmholtz equation

$$\mathbf{A}(\mathbf{r}) = \frac{\mu}{4\pi} \int \mathbf{J}(\mathbf{r}') \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r}'$$

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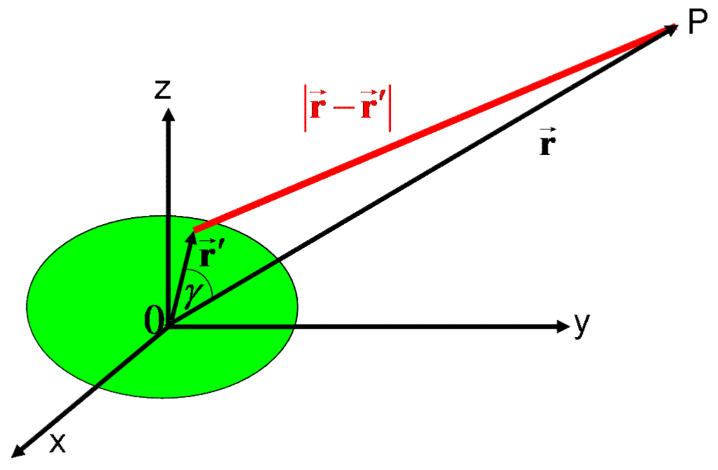
$E(r)$

$H(r)$

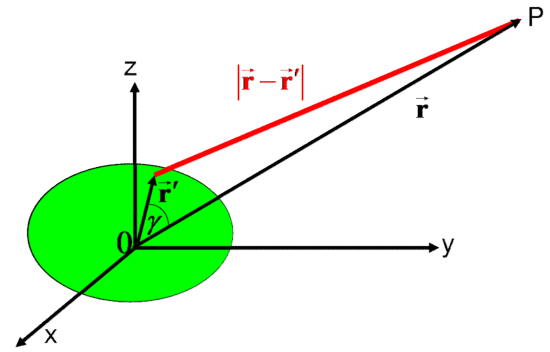


Is it possible to simplify the expressions of the fields, possibly via proper approximation of the vector potential A?

Extended antennas

$$\mathbf{A}(\vec{\mathbf{r}}) = \frac{\mu}{4\pi}\int \mathbf{J}(\vec{\mathbf{r}}')\frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}d\,\vec{\mathbf{r}}'$$

$$|\vec{\mathbf{r}}-\vec{\mathbf{r}}'| = r - r'\cos\gamma + \frac{(r')^2}{2r}\sin^2\gamma + \cdots$$



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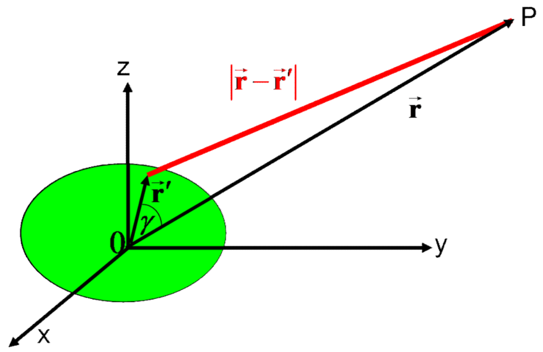

$$\frac{1}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} = \frac{1}{r - r'\cos\gamma + \frac{(r')^2}{2r}\sin^2\gamma + \cdots} \approx \frac{1}{r} \quad \textit{if} \quad r' << r$$

$$e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} = e^{-j\beta r}e^{j\beta r'\cos\gamma}e^{-j\beta\frac{(r')^2}{2r}\sin^2\gamma}\cdots \approx e^{-j\beta r} \quad \textit{if} \quad r' << \lambda$$

$$e^{j\beta r'\cos\gamma} \approx 1 \Rightarrow \frac{2\pi}{\lambda}r' << 2\pi \Rightarrow r' << \lambda$$

Extended antennas

$$\mathbf{A}(\vec{\mathbf{r}}) = \frac{\mu}{4\pi}\int \mathbf{J}(\vec{\mathbf{r}}')\frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}d\vec{\mathbf{r}}'$$

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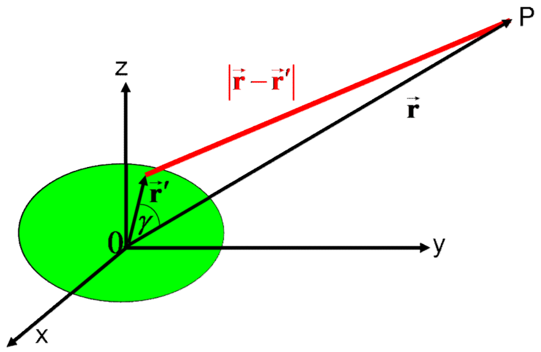
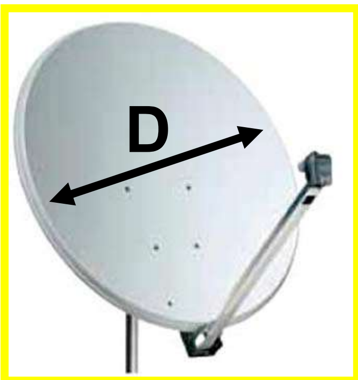


$$\frac{1}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} = \frac{1}{r - \xcancel{r'\cos\gamma} + \xcancel{\frac{(r')^2}{2r}\sin^2\gamma + \cdots}} \approx \frac{1}{r} \quad \text{if} \quad D << r$$

$$e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} = e^{-j\beta r}\,\xcancel{e^{j\beta r'\cos\gamma}}\,\xcancel{e^{-j\beta\frac{(r')^2}{2r}\sin^2\gamma}}\cdots \approx e^{-j\beta r} \quad \text{if} \quad D << \lambda$$



Extended antennas

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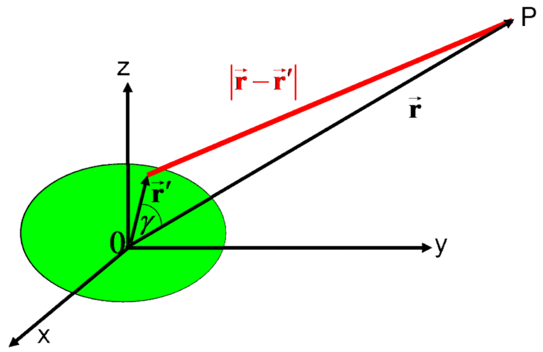

$$\frac{1}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} = \frac{1}{r - r'\cos\gamma + \frac{(r')^2}{2r}\sin^2\gamma + \cdots} \approx \frac{1}{r} \quad \textit{if} \quad D << r$$

$$e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} = e^{-j\beta r}e^{j\beta r'\cos\gamma}e^{-j\beta\frac{(r')^2}{2r}\sin^2\gamma}\cdots \approx e^{-j\beta r} \quad \textit{if} \quad D << \lambda$$

When the antennas are **small** with respect to the wavelength **and** to the distance from the observation point

$$\frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} \approx \frac{e^{-j\beta r}}{r} \quad \Rightarrow \quad \mathbf{A}(\vec{\mathbf{r}}) \approx \frac{\mu}{4\pi}\frac{e^{-j\beta r}}{r}\int \mathbf{J}(\vec{\mathbf{r}}')d\vec{\mathbf{r}}'$$

Extended antennas

$$\mathbf{A}(\vec{\mathbf{r}}) = \frac{\mu}{4\pi}\int \mathbf{J}(\vec{\mathbf{r}}')\frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}\,d\,\vec{\mathbf{r}}'$$

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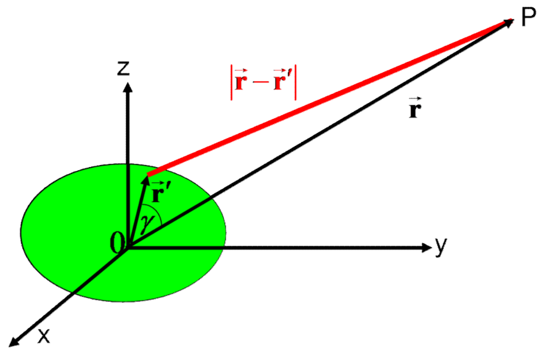

$$\frac{1}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} = \frac{1}{r - r'\cos\gamma + \frac{(r')^2}{2r}\sin^2\gamma + \cdots} \approx \frac{1}{r} \quad \textit{if} \quad D << r$$

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$$\mathbf{A}(\vec{\mathbf{r}}) = \frac{\mu}{4\pi}\int \mathbf{J}(\vec{\mathbf{r}}')\frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}d\vec{\mathbf{r}}'$$

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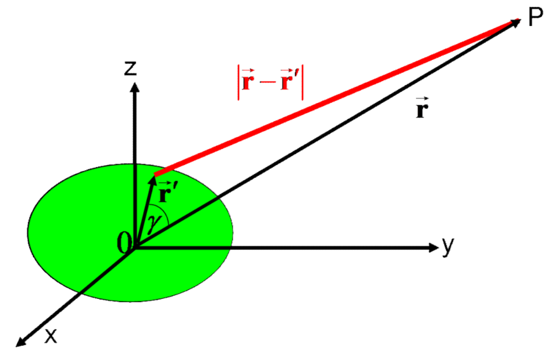

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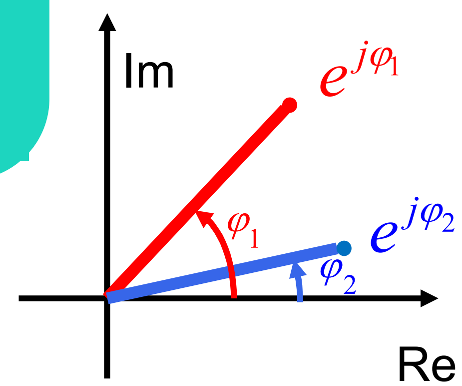

$$e^{-j\beta\frac{(r')^2}{2r}\sin^2\gamma} \approx 1 \Rightarrow \beta\frac{(r')^2}{2r}\sin^2\gamma << 2\pi \Rightarrow \frac{2\pi}{\lambda}\left(\frac{D}{2}\right)^2\frac{1}{2r} < \frac{\pi}{8} \Rightarrow r > \frac{2D^2}{\lambda}$$

Extended antennas

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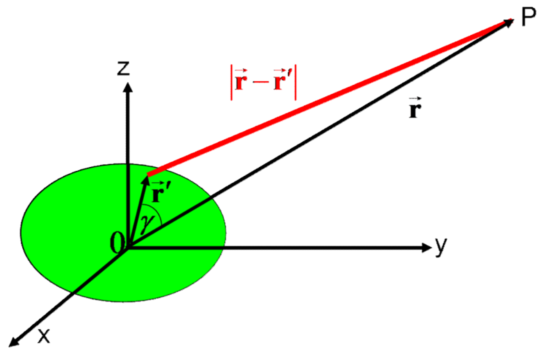

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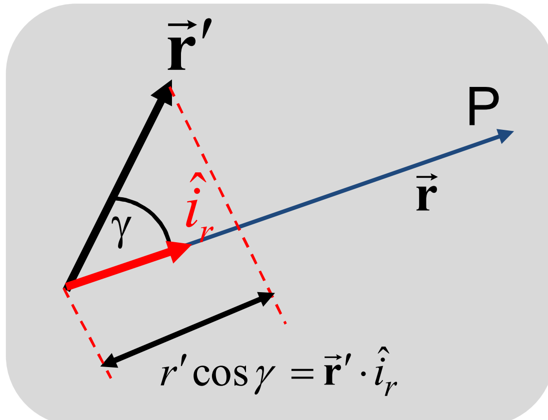

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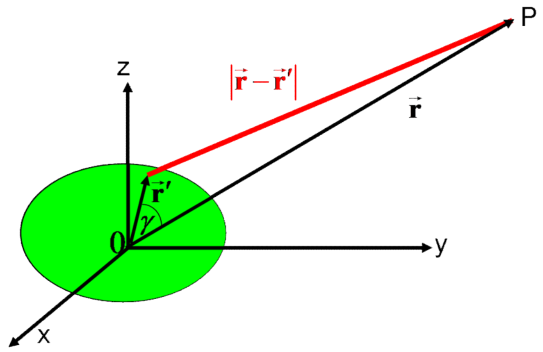

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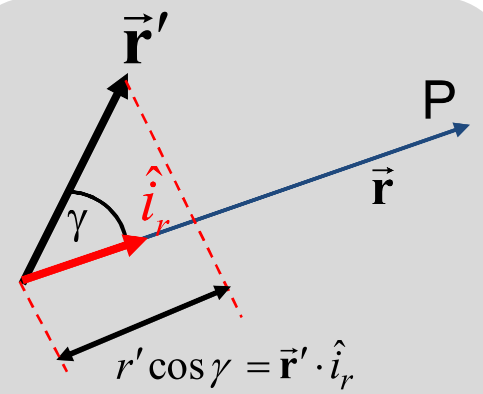

$$e^{-j\beta\frac{(r')^2}{2r}\sin^2\gamma} \approx 1 \Rightarrow \beta\frac{(r')^2}{2r}\sin^2\gamma << 2\pi \Rightarrow \frac{2\pi}{\lambda}\left(\frac{D}{2}\right)^2\frac{1}{2r} < \frac{\pi}{8} \Rightarrow r > \frac{2D^2}{\lambda}$$

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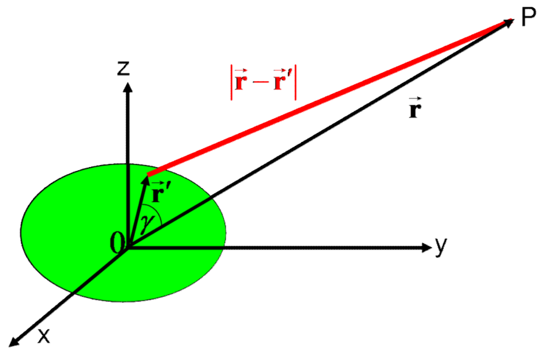

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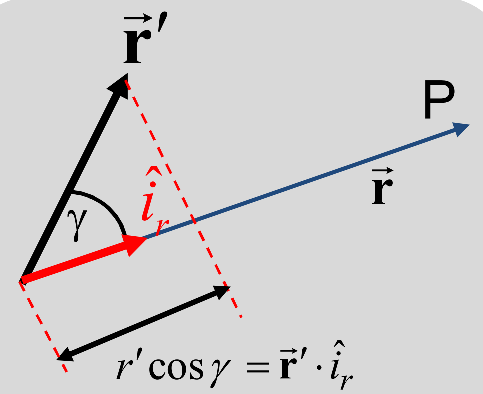

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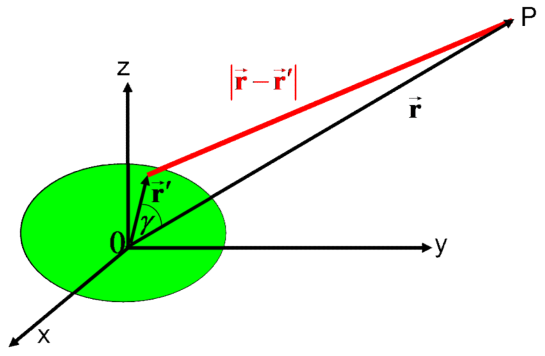

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For **<u>all</u>** the antennas, if the distance from the observation point **is sufficiently large**

$$\frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} \approx \frac{e^{-j\beta r}e^{j\beta\vec{\mathbf{r}}'\cdot\hat{i}_r}}{r} \quad \Rightarrow \quad \mathbf{A}(\vec{\mathbf{r}}) \approx \frac{\mu}{4\pi}\frac{e^{-j\beta r}}{r}\int \mathbf{J}(\vec{\mathbf{r}}')e^{j\beta\vec{\mathbf{r}}'\cdot\hat{i}_r}d\,\vec{\mathbf{r}}'$$

Extended antennas

$$\mathbf{A}(\vec{\mathbf{r}}) = \frac{\mu}{4\pi} \int \mathbf{J}(\vec{\mathbf{r}}') \frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} d\vec{\mathbf{r}}'$$

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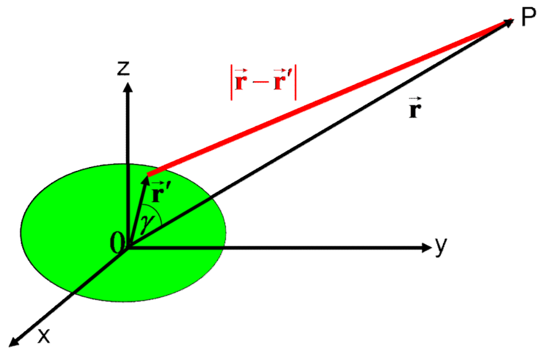

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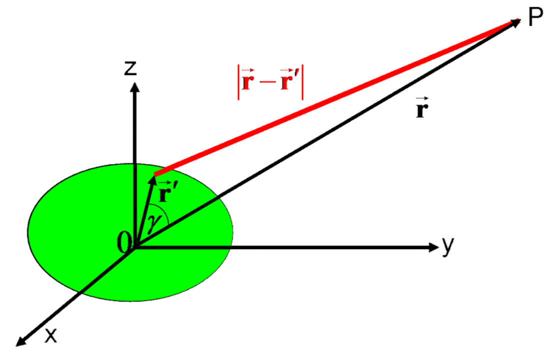
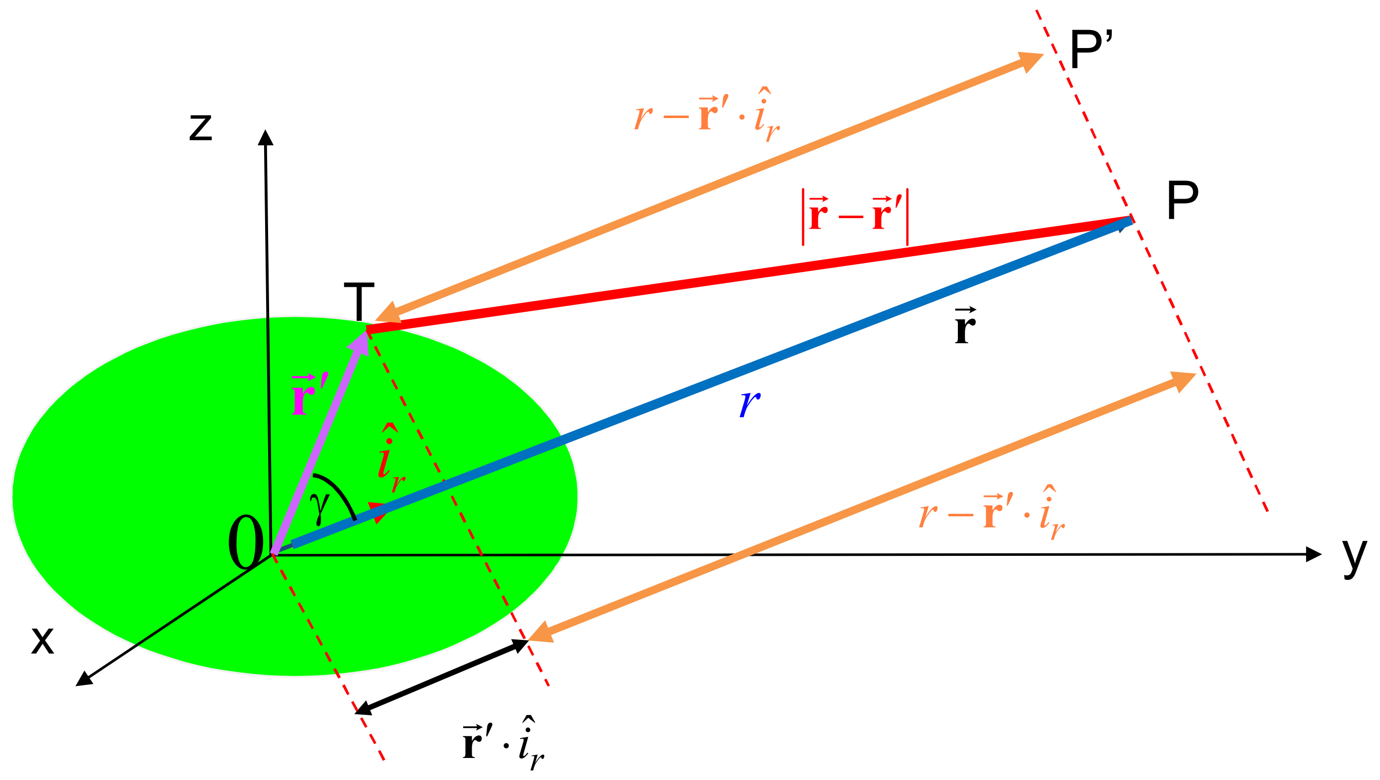
$$\mathbf{A}(\vec{\mathbf{r}}) = \frac{\mu}{4\pi}\int \mathbf{J}(\vec{\mathbf{r}}')\frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}d\vec{\mathbf{r}}'$$

$$|\vec{\mathbf{r}}-\vec{\mathbf{r}}'| = r - \vec{\mathbf{r}}'\cdot\hat{i}_r + \frac{(r')^2}{2r}\sin^2\gamma + \cdots$$

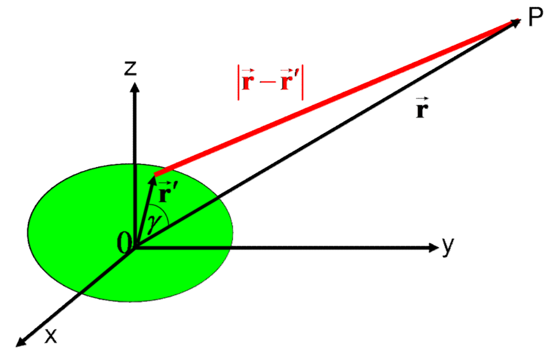


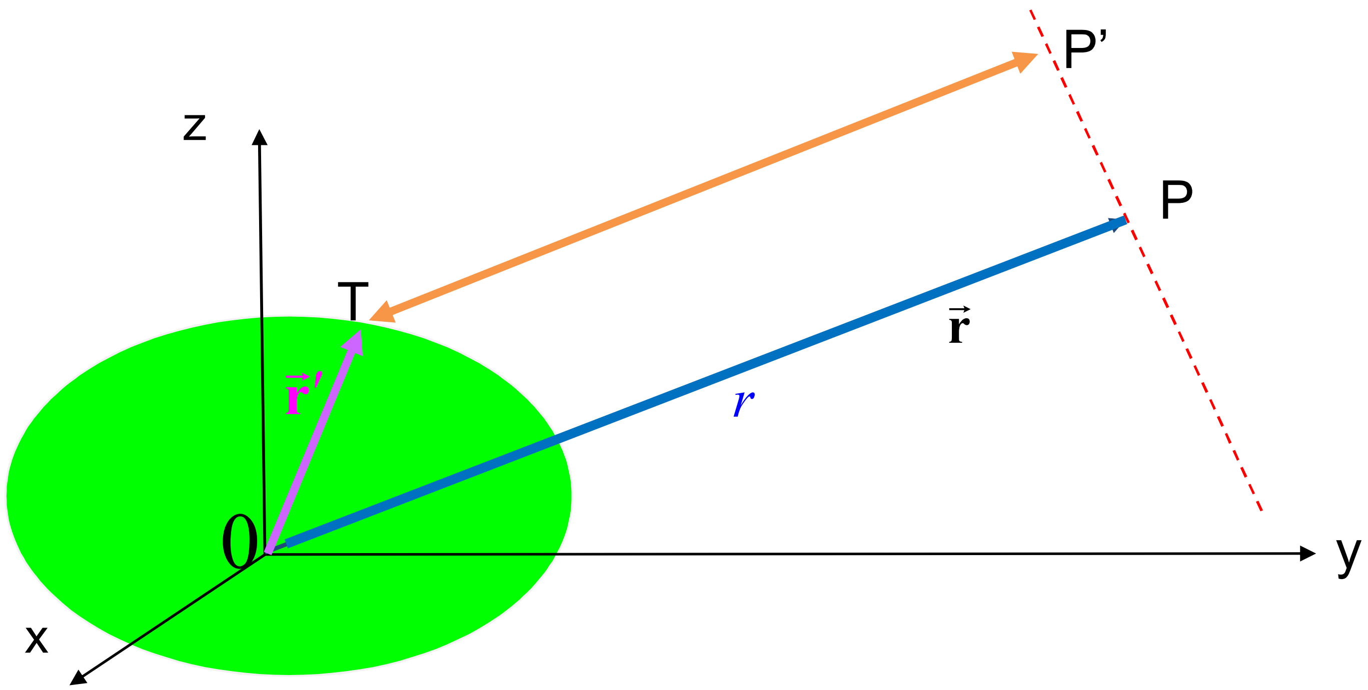


Extended antennas

$$\mathbf{A}(\vec{\mathbf{r}}) = \frac{\mu}{4\pi}\int \mathbf{J}(\vec{\mathbf{r}}')\frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}d\vec{\mathbf{r}}'$$

$$|\vec{\mathbf{r}}-\vec{\mathbf{r}}'| = r - \vec{\mathbf{r}}'\cdot\hat{i}_r + \frac{(r')^2}{2r}\sin^2\gamma + \cdots$$

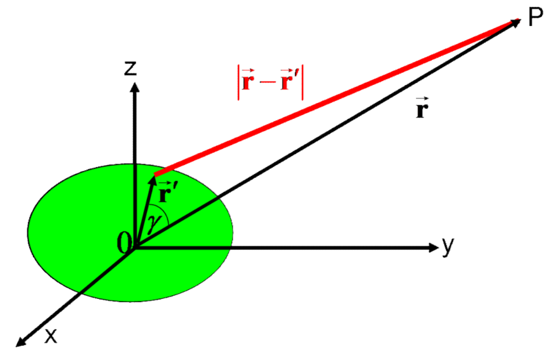


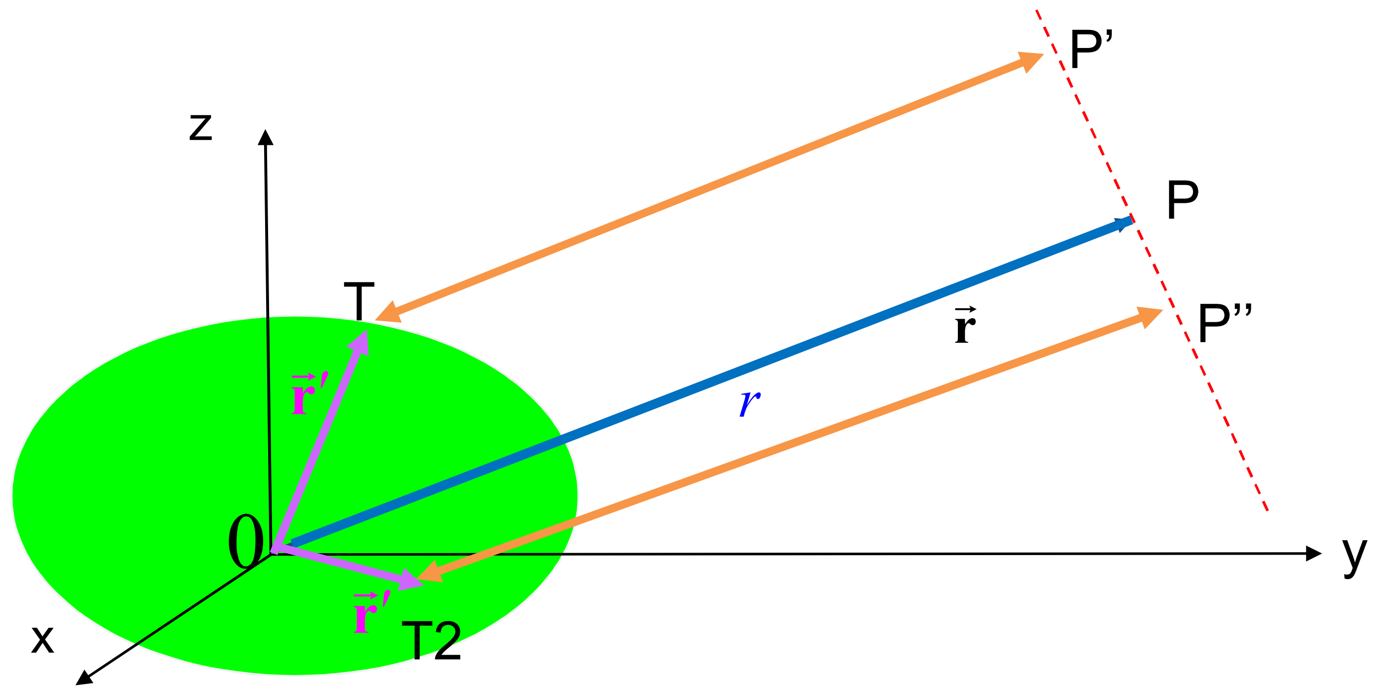


Extended antennas

$$\mathbf{A}(\vec{\mathbf{r}}) = \frac{\mu}{4\pi}\int \mathbf{J}(\vec{\mathbf{r}}') \frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} d\,\vec{\mathbf{r}}'$$

$$|\vec{\mathbf{r}}-\vec{\mathbf{r}}'| = r - \vec{\mathbf{r}}' \cdot \hat{i}_r + \frac{(r')^2}{2r}\sin^2\gamma + \cdots$$

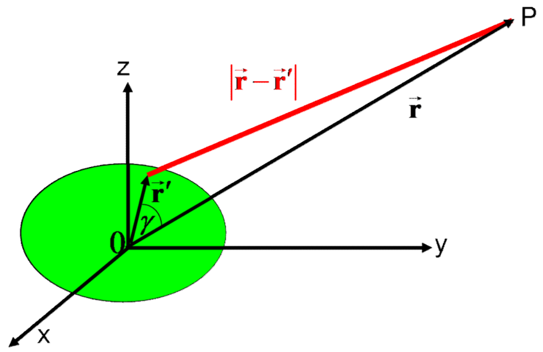




Extended antennas

$$\mathbf{A}(\vec{\mathbf{r}}) = \frac{\mu}{4\pi}\int \mathbf{J}(\vec{\mathbf{r}}')\frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}d\,\vec{\mathbf{r}}'$$

$$|\vec{\mathbf{r}}-\vec{\mathbf{r}}'| = r - \vec{\mathbf{r}}'\cdot\hat{i}_r + \frac{(r')^2}{2r}\sin^2\gamma + \cdots$$



$$\frac{1}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} = \frac{1}{r - r'\cos\gamma + \frac{(r')^2}{2r}\sin^2\gamma + \cdots} \approx \frac{1}{r} \quad \text{if} \quad D << r$$

$$e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} = e^{-j\beta r}e^{j\beta r'\cos\gamma}e^{-j\beta\frac{(r')^2}{2r}\sin^2\gamma}\cdots \approx e^{-j\beta r}e^{j\beta\vec{\mathbf{r}}'\cdot\hat{i}_r} \quad \text{if} \quad r > \frac{2D^2}{\lambda}$$

For **all** the antennas, if the distance from the observation point **is sufficiently large**

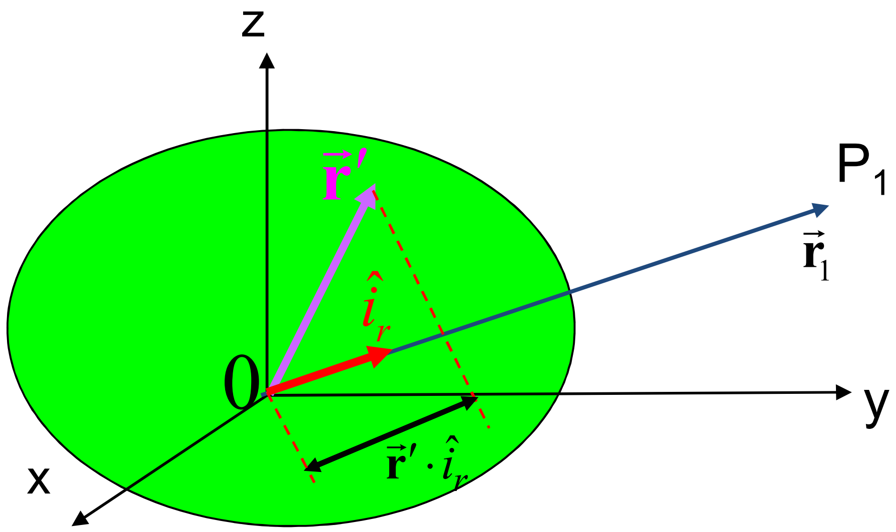
$$\frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} \approx \frac{e^{-j\beta r}e^{j\beta\vec{\mathbf{r}}'\cdot\hat{i}_r}}{r} \Rightarrow \mathbf{A}(\vec{\mathbf{r}}) \approx \frac{\mu}{4\pi}\frac{e^{-j\beta r}}{r}\int \mathbf{J}(\vec{\mathbf{r}}')e^{j\beta\vec{\mathbf{r}}'\cdot\hat{i}_r}d\,\vec{\mathbf{r}}'$$

Extended antennas

$$r >> D$$

$$r > \frac{2D^2}{\lambda}$$

$$\int \mathbf{J}(\vec{\mathbf{r}}') e^{j\beta \vec{\mathbf{r}}' \cdot \hat{i}_r} d\,\vec{\mathbf{r}}' =$$



For **all** the antennas, if the distance from the observation point **is sufficiently large**

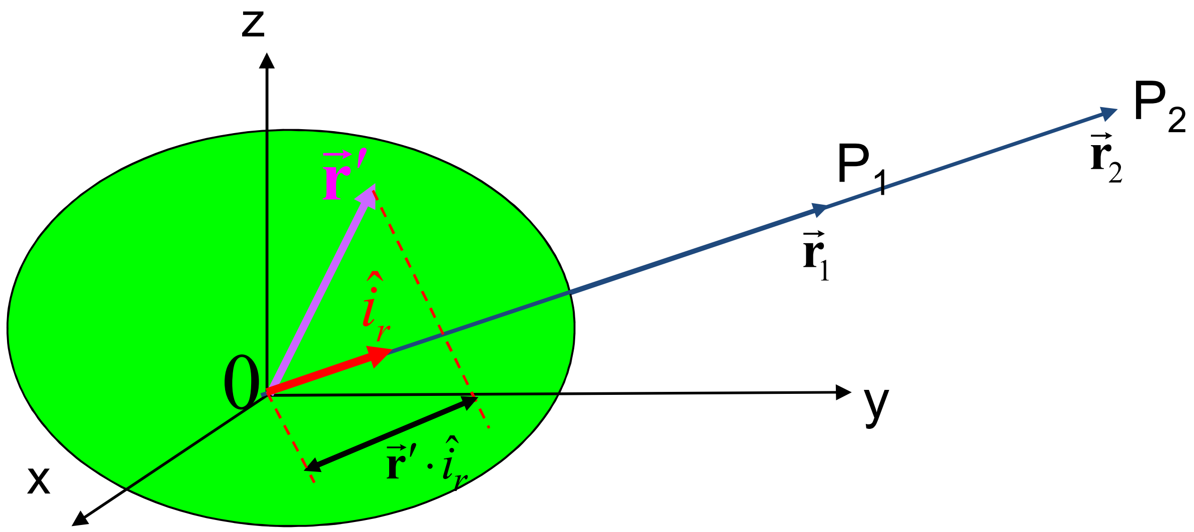
$$\frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} \approx \frac{e^{-j\beta r} e^{j\beta \vec{\mathbf{r}}' \cdot \hat{i}_r}}{r} \quad \Rightarrow \quad \mathbf{A}(\vec{\mathbf{r}}) \approx \frac{\mu}{4\pi} \frac{e^{-j\beta r}}{r} \int \mathbf{J}(\vec{\mathbf{r}}') e^{j\beta \vec{\mathbf{r}}' \cdot \hat{i}_r} d\,\vec{\mathbf{r}}'$$

Extended antennas

$$r >> D$$

$$r > \frac{2D^2}{\lambda}$$

$$\int \mathbf{J}(\vec{\mathbf{r}}')\, e^{j\beta \vec{\mathbf{r}}'\cdot\hat{i}_r}\, d\,\vec{\mathbf{r}}' =$$



For **all** the antennas, if the distance from the observation point **is sufficiently large**

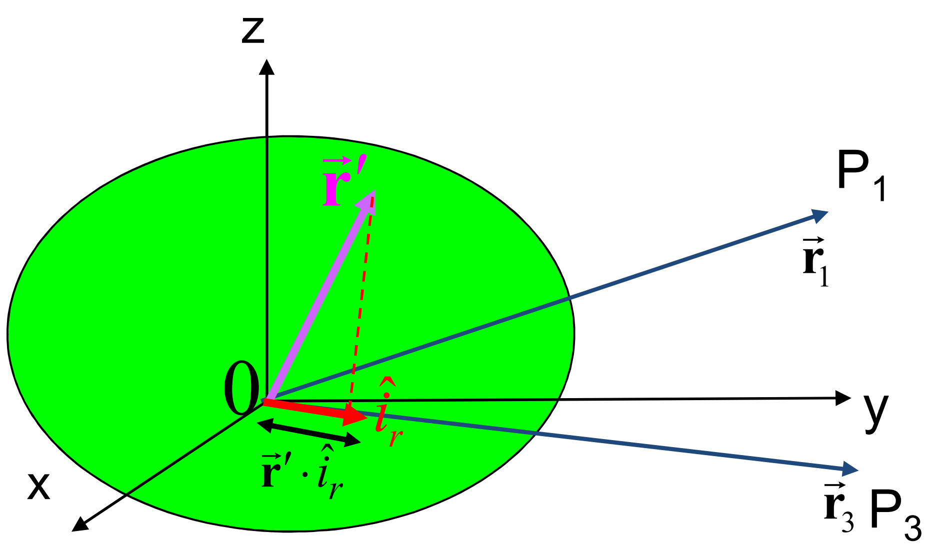
$$\frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} \approx \frac{e^{-j\beta r} e^{j\beta \vec{\mathbf{r}}'\cdot\hat{i}_r}}{r} \quad \Rightarrow \quad \mathbf{A}(\vec{\mathbf{r}}) \approx \frac{\mu}{4\pi}\frac{e^{-j\beta r}}{r}\int \mathbf{J}(\vec{\mathbf{r}}')\, e^{j\beta \vec{\mathbf{r}}'\cdot\hat{i}_r}\, d\,\vec{\mathbf{r}}'$$

Extended antennas

$$r >> D$$

$$r > \frac{2D^2}{\lambda}$$

$$\int \mathbf{J}(\vec{\mathbf{r}}') e^{j\beta \vec{\mathbf{r}}' \cdot \hat{i}_r} d\,\vec{\mathbf{r}}' = \mathbf{M}(\vartheta, \varphi) \quad \Rightarrow \quad \mathbf{A}(\vec{\mathbf{r}}) \approx \frac{\mu}{4\pi} \frac{e^{-j\beta r}}{r} \mathbf{M}(\vartheta, \varphi)$$



For **all** the antennas, if the distance from the observation point **is sufficiently large**

$$\frac{e^{-j\beta|\vec{\mathbf{r}} - \vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}} - \vec{\mathbf{r}}'|} \approx \frac{e^{-j\beta r} e^{j\beta \vec{\mathbf{r}}' \cdot \hat{i}_r}}{r} \quad \Rightarrow \quad \boxed{\mathbf{A}(\vec{\mathbf{r}}) \approx \frac{\mu}{4\pi} \frac{e^{-j\beta r}}{r} \int \mathbf{J}(\vec{\mathbf{r}}') e^{j\beta \vec{\mathbf{r}}' \cdot \hat{i}_r} d\,\vec{\mathbf{r}}'}$$

Extended antennas

$r >> D$

$$r > \frac{2D^2}{\lambda}$$

$$\int \mathbf{J}(\vec{\mathbf{r}}') e^{j\beta \vec{\mathbf{r}}' \cdot \hat{i}_r} d\,\vec{\mathbf{r}}' = \mathbf{M}(\vartheta, \varphi) \quad \Longrightarrow \quad \mathbf{A}(\vec{\mathbf{r}}) \approx \frac{\mu}{4\pi} \frac{e^{-j\beta r}}{r} \mathbf{M}(\vartheta, \varphi)$$

Fraunhofer region

$r >> D$

$r > \frac{2D^2}{\lambda}$

$r >> \lambda$

$$\int \mathbf{J}(\vec{\mathbf{r}}') e^{j\beta \vec{\mathbf{r}}' \cdot \hat{i}_r} d\vec{\mathbf{r}}' = \mathbf{M}(\vartheta, \varphi) \Rightarrow \mathbf{A}(\vec{\mathbf{r}}) \approx \frac{\mu}{4\pi} \frac{e^{-j\beta r}}{r} \mathbf{M}(\vartheta, \varphi)$$

$$\Rightarrow \begin{cases} \nabla\nabla \cdot \mathbf{A}(\vec{\mathbf{r}}) \approx -\beta^2 A_r(\vec{\mathbf{r}}) \hat{i}_r \\ \nabla \times \mathbf{A}(\vec{\mathbf{r}}) \approx -j\beta \hat{i}_r \times \mathbf{A}(\vec{\mathbf{r}}) \end{cases}$$

Fraunhofer region

Radiation problem for extended antennas

$\mathbf{J}(\mathbf{r})$

$\nabla^2 \mathbf{A} + k^2 \mathbf{A} = -\mu \mathbf{J}$ **Vector Helmholtz equation**

$$\mathbf{A}(\mathbf{r}) = \frac{\mu}{4\pi}\int \mathbf{J}(\mathbf{r}')\frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}d\mathbf{r}'$$

$$\mathbf{A}(\mathbf{r}) \approx \frac{\mu}{4\pi}\frac{e^{-j\beta r}}{r}\int \mathbf{J}(\vec{\mathbf{r}}')e^{j\beta \vec{\mathbf{r}}'\cdot \hat{i}_r}d\,\vec{\mathbf{r}}'$$

$$\mathbf{H} = \frac{1}{\mu}\nabla\times\mathbf{A} \quad \text{and} \quad \mathbf{E} = -j\omega\mathbf{A} + \frac{\nabla\nabla\cdot\mathbf{A}}{j\omega\varepsilon\mu}$$

$\nabla\times\mathbf{A} \rightarrow -j\beta\,\hat{i}_r\times\mathbf{A}$

$\nabla\nabla\cdot\mathbf{A} \rightarrow -\beta^2 A_r\hat{i}_r$

$\mathbf{E}(\mathbf{r})$

$\mathbf{H}(\mathbf{r})$

$r >> D$

$r > \frac{2D^2}{\lambda}$

$r >> \lambda$

$\int \mathbf{J}(\vec{\mathbf{r}}')e^{j\beta\vec{\mathbf{r}}'\cdot\hat{i}_r}d\vec{\mathbf{r}}' = \mathbf{M}(\vartheta,\varphi) \Longrightarrow \mathbf{A}(\vec{\mathbf{r}}) \approx \frac{\mu}{4\pi}\frac{e^{-j\beta r}}{r}\mathbf{M}(\vartheta,\varphi)$

$\nabla\nabla\cdot\mathbf{A}(\vec{\mathbf{r}}) \approx -\beta^2 A_r(\vec{\mathbf{r}})\hat{i}_r$

$\nabla\times\mathbf{A}(\vec{\mathbf{r}}) \approx -j\beta\hat{i}_r\times\mathbf{A}(\vec{\mathbf{r}})$

$$\nabla\nabla\cdot\mathbf{A}(\vec{\mathbf{r}}) \approx -\beta^2 A_r(\vec{\mathbf{r}})\hat{i}_r$$

$$\mathbf{E} = -j\omega\mathbf{A} + \frac{\nabla\nabla\cdot\mathbf{A}}{j\omega\varepsilon\mu} \approx -j\omega\mathbf{A} - \frac{\beta^2 A_r\hat{i}_r}{j\omega\varepsilon\mu} = -j\omega\mathbf{A} + j\omega A_r\hat{i}_r = -j\omega\left[\mathbf{A} - A_r\hat{i}_r\right]$$

$$\boldsymbol{H} = \frac{1}{\mu}\nabla\times\boldsymbol{A} \quad \text{and} \quad \boldsymbol{E} = -j\omega\boldsymbol{A} + \frac{\nabla\nabla\cdot\boldsymbol{A}}{j\omega\varepsilon\mu}$$

$$\nabla\nabla\cdot\mathbf{A}(\vec{\mathbf{r}}) \approx -\beta^2 A_r(\vec{\mathbf{r}})\hat{i}_r$$

$$\nabla\times\mathbf{A}(\vec{\mathbf{r}}) \approx -j\beta\hat{i}_r\times\mathbf{A}(\vec{\mathbf{r}})$$

$$\mathbf{E}(\vec{\mathbf{r}}) = -j\omega\left[\mathbf{A}(\vec{\mathbf{r}}) - A_r(\vec{\mathbf{r}})\hat{i}_r\right]$$

$r >> D$

$r > \frac{2D^2}{\lambda}$

$r >> \lambda$

$$\int \mathbf{J}(\vec{\mathbf{r}}') e^{j\beta\vec{\mathbf{r}}'\cdot\hat{i}_r} d\vec{\mathbf{r}}' = \mathbf{M}(\vartheta,\varphi) \Longrightarrow \mathbf{A}(\vec{\mathbf{r}}) \approx \frac{\mu}{4\pi}\frac{e^{-j\beta r}}{r}\mathbf{M}(\vartheta,\varphi)$$

$$\nabla\nabla\cdot\mathbf{A}(\vec{\mathbf{r}}) \approx -\beta^2 A_r(\vec{\mathbf{r}})\hat{i}_r$$

$$\nabla\times\mathbf{A}(\vec{\mathbf{r}}) \approx -j\beta\hat{i}_r\times\mathbf{A}(\vec{\mathbf{r}})$$

$$\nabla\nabla\cdot\mathbf{A}(\vec{\mathbf{r}}) \approx -\beta^2 A_r(\vec{\mathbf{r}})\hat{i}_r$$

$$\mathbf{E} = -j\omega\mathbf{A} + \frac{\nabla\nabla\cdot\mathbf{A}}{j\omega\varepsilon\mu} \approx -j\omega\mathbf{A} - \frac{\beta^2 A_r\hat{i}_r}{j\omega\varepsilon\mu} = -j\omega\mathbf{A} + j\omega A_r\hat{i}_r = -j\omega\left[\mathbf{A} - A_r\hat{i}_r\right]$$

$$\boldsymbol{H} = \frac{1}{\mu}\nabla\times\boldsymbol{A} \quad \text{and} \quad \boldsymbol{E} = -j\omega\boldsymbol{A} + \frac{\nabla\nabla\cdot\boldsymbol{A}}{j\omega\varepsilon\mu}$$

$$\nabla\nabla\cdot\mathbf{A}(\vec{\mathbf{r}}) \approx -\beta^2 A_r(\vec{\mathbf{r}})\hat{i}_r$$

$$\nabla\times\mathbf{A}(\vec{\mathbf{r}}) \approx -j\beta\hat{i}_r\times\mathbf{A}(\vec{\mathbf{r}})$$

$$\mathbf{E}(\vec{\mathbf{r}}) = -j\omega\left[\mathbf{A}(\vec{\mathbf{r}}) - A_r(\vec{\mathbf{r}})\hat{i}_r\right]$$

$r >> D$

$r > \frac{2D^2}{\lambda}$

$r >> \lambda$

$$\int \mathbf{J}(\vec{\mathbf{r}}') e^{j\beta \vec{\mathbf{r}}' \cdot \hat{i}_r} d\vec{\mathbf{r}}' = \mathbf{M}(\vartheta,\varphi) \Rightarrow \mathbf{A}(\vec{\mathbf{r}}) \approx \frac{\mu}{4\pi}\frac{e^{-j\beta r}}{r}\mathbf{M}(\vartheta,\varphi)$$

$$\nabla\nabla\cdot\mathbf{A}(\vec{\mathbf{r}}) \approx -\beta^2 A_r(\vec{\mathbf{r}})\hat{i}_r$$

$$\nabla\times\mathbf{A}(\vec{\mathbf{r}}) \approx -j\beta\hat{i}_r\times\mathbf{A}(\vec{\mathbf{r}})$$

$$\nabla\times\mathbf{A}(\vec{\mathbf{r}}) \approx -j\beta\hat{i}_r\times\mathbf{A}(\vec{\mathbf{r}})$$

$$\zeta\mathbf{H} = \frac{\zeta}{\mu}\nabla\times\mathbf{A} \approx \frac{\zeta}{\mu}(-j\beta)\hat{i}_r\times\mathbf{A} = \frac{\zeta\sqrt{\mu}\sqrt{\varepsilon}}{\mu}\hat{i}_r\times[-j\omega\mathbf{A}] = \hat{i}_r\times[-j\omega\mathbf{A}] = i_r\times\left[-j\omega\left(\mathbf{A} - A_r\hat{i}_r\right)\right] = \hat{i}_r\times\mathbf{E}$$

$$\boldsymbol{H} = \frac{1}{\mu}\nabla\times\boldsymbol{A} \quad \text{and} \quad \boldsymbol{E} = -j\omega\boldsymbol{A} + \frac{\nabla\nabla\cdot\boldsymbol{A}}{j\omega\varepsilon\mu}$$

$$\nabla\nabla\cdot\mathbf{A}(\vec{\mathbf{r}}) \approx -\beta^2 A_r(\vec{\mathbf{r}})\hat{i}_r$$

$$\nabla\times\mathbf{A}(\vec{\mathbf{r}}) \approx -j\beta\hat{i}_r\times\mathbf{A}(\vec{\mathbf{r}})$$

$$\mathbf{E}(\vec{\mathbf{r}}) = -j\omega\left[\mathbf{A}(\vec{\mathbf{r}}) - A_r(\vec{\mathbf{r}})\hat{i}_r\right]$$

$$\zeta\mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

$r >> D$

$r > \frac{2D^2}{\lambda}$

$r >> \lambda$

$$\int \mathbf{J}(\vec{\mathbf{r}}')e^{j\beta\vec{\mathbf{r}}'\cdot\hat{i}_r} d\vec{\mathbf{r}}' = \mathbf{M}(\vartheta,\varphi) \Longrightarrow \mathbf{A}(\vec{\mathbf{r}}) \approx \frac{\mu}{4\pi}\frac{e^{-j\beta r}}{r}\mathbf{M}(\vartheta,\varphi)$$

$$\nabla\nabla\cdot\mathbf{A}(\vec{\mathbf{r}}) \approx -\beta^2 A_r(\vec{\mathbf{r}})\hat{i}_r$$

$$\nabla\times\mathbf{A}(\vec{\mathbf{r}}) \approx -j\beta\hat{i}_r \times \mathbf{A}(\vec{\mathbf{r}})$$

$$\nabla\times\mathbf{A}(\vec{\mathbf{r}}) \approx -j\beta\hat{i}_r \times \mathbf{A}(\vec{\mathbf{r}})$$

$$\Downarrow$$

$$\zeta\mathbf{H} = \frac{\zeta}{\mu}\nabla\times\mathbf{A} \approx \frac{\zeta}{\mu}(-j\beta)\hat{i}_r\times\mathbf{A} = \frac{\zeta\sqrt{\mu}\sqrt{\varepsilon}}{\mu}\hat{i}_r\times[-j\omega\mathbf{A}] = \hat{i}_r\times[-j\omega\mathbf{A}] = i_r\times\left[-j\omega\left(\mathbf{A}-A_r\hat{i}_r\right)\right] = \hat{i}_r\times\mathbf{E}$$

$$\boldsymbol{H} = \frac{1}{\mu}\nabla\times\boldsymbol{A} \quad \text{and} \quad \boldsymbol{E} = -j\omega\boldsymbol{A} + \frac{\nabla\nabla\cdot\boldsymbol{A}}{j\omega\varepsilon\mu}$$

$$\nabla\nabla\cdot\mathbf{A}(\vec{\mathbf{r}}) \approx -\beta^2 A_r(\vec{\mathbf{r}})\hat{i}_r$$

$$\nabla\times\mathbf{A}(\vec{\mathbf{r}}) \approx -j\beta\hat{i}_r \times \mathbf{A}(\vec{\mathbf{r}})$$

Fraunhofer region

$r >> D$

$r > \frac{2D^2}{\lambda}$

$r >> \lambda$

$$\int \mathbf{J}(\vec{\mathbf{r}}') e^{j\beta \vec{\mathbf{r}}' \cdot \hat{i}_r} d\vec{\mathbf{r}}' = \mathbf{M}(\vartheta, \varphi) \Rightarrow \mathbf{A}(\vec{\mathbf{r}}) \approx \frac{\mu}{4\pi} \frac{e^{-j\beta r}}{r} \mathbf{M}(\vartheta, \varphi)$$

$$\begin{cases} \nabla\nabla \cdot \mathbf{A}(\vec{\mathbf{r}}) \approx -\beta^2 A_r(\vec{\mathbf{r}}) \hat{i}_r \\ \nabla \times \mathbf{A}(\vec{\mathbf{r}}) \approx -j\beta \hat{i}_r \times \mathbf{A}(\vec{\mathbf{r}}) \end{cases}$$

$$\mathbf{A}(\vec{\mathbf{r}}) = A_r(\vec{\mathbf{r}}) \hat{i}_r + A_\vartheta(\vec{\mathbf{r}}) \hat{i}_\vartheta + A_\varphi(\vec{\mathbf{r}}) \hat{i}_\varphi$$

Fraunhofer region

$$\mathbf{E}(\vec{\mathbf{r}}) = -j\omega\left[\mathbf{A}(\vec{\mathbf{r}}) - A_r(\vec{\mathbf{r}}) \hat{i}_r\right] = -j\omega\left[\frac{\mu}{4\pi} \frac{e^{-j\beta r}}{r} \mathbf{M}(\vartheta, \varphi) - \frac{\mu}{4\pi} \frac{e^{-j\beta r}}{r} M_r(\vartheta, \varphi) \hat{i}_r\right] = -\frac{j\omega\mu}{4\pi} \frac{e^{-j\beta r}}{r}\left[\mathbf{M}(\vartheta, \varphi) - M_r(\vartheta, \varphi) \hat{i}_r\right]$$

$$\zeta \mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

Fraunhofer region

$$r >> D$$

$$r > \frac{2D^2}{\lambda}$$

$$r >> \lambda$$

$$\int \mathbf{J}(\vec{\mathbf{r}}') e^{j\beta \vec{\mathbf{r}}' \cdot \hat{i}_r} d\,\vec{\mathbf{r}}' = \mathbf{M}(\vartheta, \varphi)$$

Fraunhofer region

$$\mathbf{E}(\vec{\mathbf{r}}) = \;\; = -\frac{j\omega\mu}{4\pi} \frac{e^{-j\beta r}}{r} \left[\mathbf{M}(\vartheta, \varphi) - M_r(\vartheta, \varphi) \hat{i}_r \right]$$

$$\zeta \mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

Fraunhofer region

$r >> D$

$r > \frac{2D^2}{\lambda}$

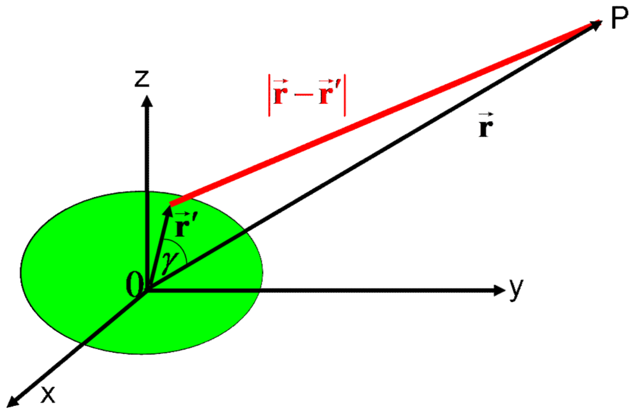
$r >> \lambda$

Fraunhofer region

$$\mathbf{E}(\vec{\mathbf{r}}) = \mathbf{E}(r,\vartheta,\varphi) = -\frac{j\omega\mu}{4\pi}\frac{e^{-j\beta r}}{r}\left[\mathbf{M}(\vartheta,\varphi) - M_r(\vartheta,\varphi)\hat{i}_r\right]$$

$$\zeta\mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

$$\mathbf{M}(\vartheta,\varphi) = \int \mathbf{J}(\vec{\mathbf{r}}')e^{j\beta\vec{\mathbf{r}}'\cdot\hat{i}_r}\,d\,\vec{\mathbf{r}}'$$



- the e.m. field propagates along $\hat{i}_r$
- the e.m. field lies on the plane orthogonal to the propagation direction
- |E| and |H| exhibit the decaying factor 1/r
- |E| and |H| are proportional through ζ

Field regions

Far-field (Fraunhofer) region is defined as "that region of the field of an antenna where the angular field distribution is essentially independent of the distance from the antenna. If the antenna has a maximum overall dimension D (D>λ), the far-field region is commonly taken to exist at distances greater than $2D^2/\lambda$ from the antenna, λ being the wavelength".

In this region, the field components are essentially transverse

Fraunhofer region

$r >> D$

$r > \frac{2D^2}{\lambda}$

$r >> \lambda$

Fraunhofer region

$$\mathbf{E}(\vec{\mathbf{r}}) = \mathbf{E}(r,\vartheta,\varphi) = -\frac{j\omega\mu}{4\pi}\frac{e^{-j\beta r}}{r}\left[\mathbf{M}(\vartheta,\varphi) - M_r(\vartheta,\varphi)\hat{i}_r\right]$$

$$\zeta\mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

$$\mathbf{M}(\vartheta,\varphi) = \int \mathbf{J}(\vec{\mathbf{r}}')e^{j\beta\vec{\mathbf{r}}'\cdot\hat{i}_r}d\,\vec{\mathbf{r}}'$$

- the e.m. field propagates along $\hat{i}_r$
- the e.m. field lies on the plane orthogonal to the propagation direction
- |E| and |H| exhibit the decaying factor 1/r
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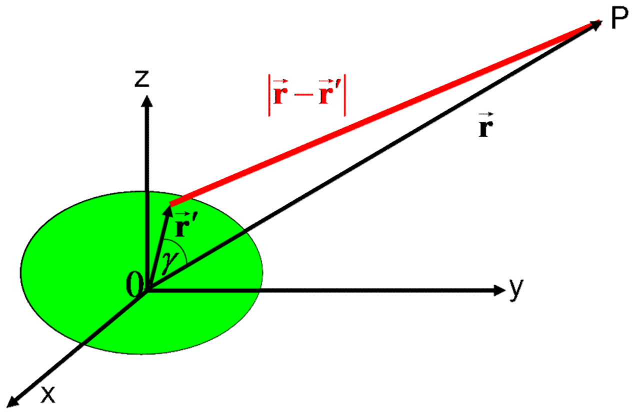


The radiation condition

$r >> D$

$$r > \frac{2D^2}{\lambda}$$

$r >> \lambda$

Fraunhofer region

$$\mathbf{E}(\vec{\mathbf{r}}) = \mathbf{E}(r,\vartheta,\varphi) = -\frac{j\omega\mu}{4\pi}\frac{e^{-j\beta r}}{r}\left[\mathbf{M}(\vartheta,\varphi) - M_r(\vartheta,\varphi)\hat{i}_r\right]$$

$$\zeta\mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

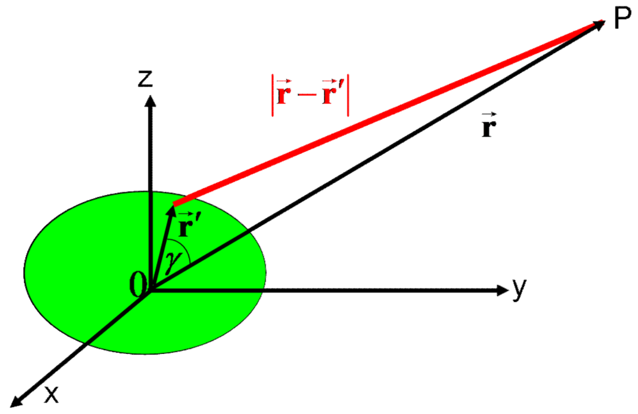
$$\mathbf{M}(\vartheta,\varphi) = \int \mathbf{J}(\vec{\mathbf{r}}')e^{j\beta\vec{\mathbf{r}}'\cdot\hat{i}_r}d\,\vec{\mathbf{r}}'$$

$$\mathbf{E} \sim O\left(\frac{1}{r}\right)$$

$$\mathbf{H} \sim O\left(\frac{1}{r}\right)$$

$$\zeta\mathbf{H} - \hat{i}_r \times \mathbf{E} \sim o\left(\frac{1}{r}\right)$$

$as \quad r \to \infty$



Fraunhofer region

$r >> D$

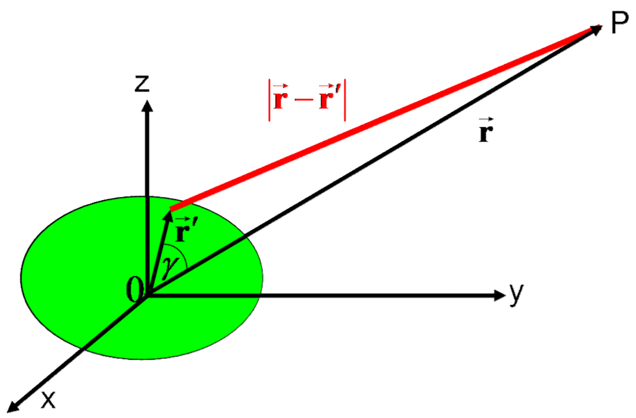
$r > \frac{2D^2}{\lambda}$

$r >> \lambda$

$$\mathbf{E}(\vec{\mathbf{r}}) = \mathbf{E}(r,\vartheta,\varphi) = -\frac{j\omega\mu}{4\pi}\frac{e^{-j\beta r}}{r}\left[\mathbf{M}(\vartheta,\varphi) - M_r(\vartheta,\varphi)\hat{i}_r\right] = -\frac{j\omega\mu}{4\pi}\frac{e^{-j\beta r}}{r}\left[M_\vartheta(\vartheta,\varphi)\hat{i}_\vartheta + M_\varphi(\vartheta,\varphi)\hat{i}_\varphi\right]$$

$$\zeta\mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

$$\mathbf{M}(\vartheta,\varphi) = \int \mathbf{J}(\vec{\mathbf{r}}')e^{j\beta\vec{\mathbf{r}}'\cdot\hat{i}_r}d\vec{\mathbf{r}}'$$



Fraunhofer region

$$\vec{\mathbf{S}} = \frac{1}{2}\vec{\mathbf{E}}\times\vec{\mathbf{H}}^* = \frac{1}{2\zeta}\vec{\mathbf{E}}\times\left(\hat{i}_r\times\vec{\mathbf{E}}\right)^* = \frac{1}{2\zeta}\vec{\mathbf{E}}\times\left(\hat{i}_r\times\vec{\mathbf{E}}^*\right) = \frac{1}{2\zeta}\left[\left|\vec{\mathbf{E}}\right|^2\hat{i}_r - \left(\hat{i}_r\cdot\vec{\mathbf{E}}\right)\vec{\mathbf{E}}^*\right] = \frac{1}{2\zeta}\left|\vec{\mathbf{E}}\right|^2\hat{i}_r$$

$$\vec{\mathbf{S}} = \frac{1}{2}\vec{\mathbf{E}}\times\vec{\mathbf{H}}^* = \frac{\zeta}{2}\left|\vec{\mathbf{H}}\right|^2\hat{i}_r$$

$$\vec{\mathbf{A}}\times\left(\vec{\mathbf{B}}\times\vec{\mathbf{C}}\right) = \vec{\mathbf{B}}\left(\vec{\mathbf{A}}\cdot\vec{\mathbf{C}}\right) - \vec{\mathbf{C}}\left(\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}\right)$$

Fraunhofer region

$r >> D$

$r > \frac{2D^2}{\lambda}$

$r >> \lambda$

Fraunhofer region

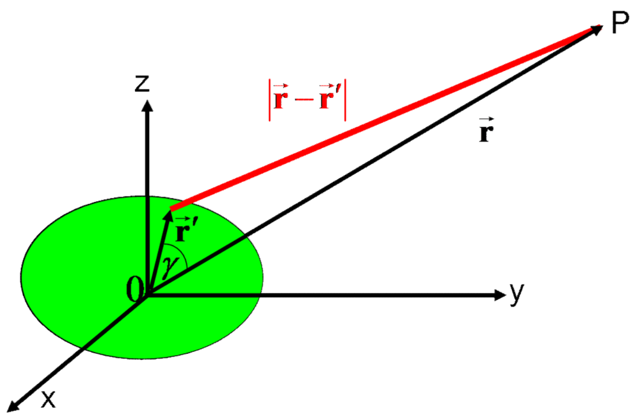
$$\mathbf{E}(\vec{\mathbf{r}}) = \mathbf{E}(r,\vartheta,\varphi) = -\frac{j\omega\mu}{4\pi}\frac{e^{-j\beta r}}{r}\left[\mathbf{M}(\vartheta,\varphi) - M_r(\vartheta,\varphi)\hat{i}_r\right] = -\frac{j\omega\mu}{4\pi}\frac{e^{-j\beta r}}{r}\left[M_\vartheta(\vartheta,\varphi)\hat{i}_\vartheta + M_\varphi(\vartheta,\varphi)\hat{i}_\varphi\right]$$

$$\zeta\mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

$$\mathbf{M}(\vartheta,\varphi) = \int \mathbf{J}(\vec{\mathbf{r}}')e^{j\beta\vec{\mathbf{r}}'\cdot\hat{i}_r}d\vec{\mathbf{r}}'$$

- the e.m. field propagates along $\hat{i}_r$
- the e.m. field lies on the plane orthogonal to the propagation direction
- |E| and |H| exhibit the decaying factor 1/r
- |E| and |H| are proportional through ζ

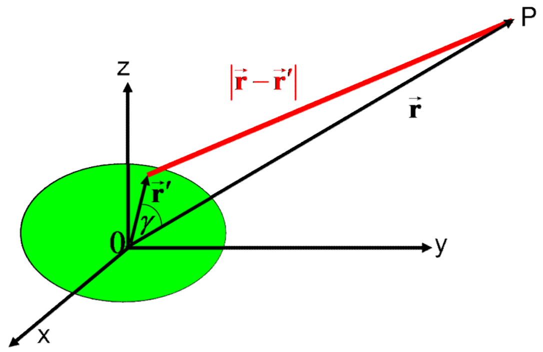
$$\vec{\mathbf{S}} = \frac{1}{2\zeta}\left|\vec{\mathbf{E}}\right|^2\hat{i}_r = \frac{\zeta}{2}\left|\vec{\mathbf{H}}\right|^2\hat{i}_r$$



Fraunhofer region

$$\mathbf{A}(\vec{\mathbf{r}}) = \frac{\mu}{4\pi}\int \mathbf{J}(\vec{\mathbf{r}}')\frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}d\vec{\mathbf{r}}'$$

$$|\vec{\mathbf{r}}-\vec{\mathbf{r}}'| = r - \vec{\mathbf{r}}'\cdot\hat{i}_r + \frac{(r')^2}{2r}\sin^2\gamma + \cdots$$



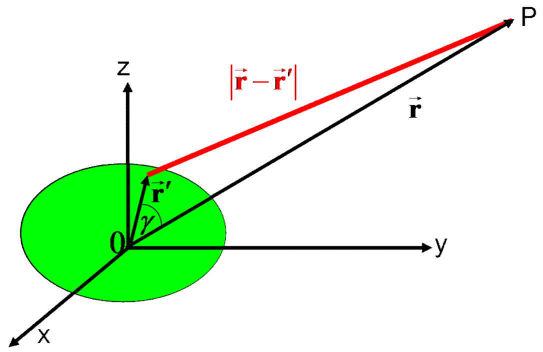
$$\frac{1}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} = \frac{1}{r - r'\cos\gamma + \frac{(r')^2}{2r}\sin^2\gamma + \cdots} \approx \frac{1}{r} \quad if \quad D << r$$

$$e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} = e^{-j\beta r}e^{j\beta\vec{\mathbf{r}}'\cdot\hat{i}_r}e^{-j\beta\frac{(r')^2}{2r}\sin^2\gamma}\cdots \approx e^{-j\beta r}e^{j\beta\vec{\mathbf{r}}'\cdot\hat{i}_r} \quad if \quad r > \frac{2D^2}{\lambda}$$

Fresnel region

$$\mathbf{A}(\vec{\mathbf{r}}) = \frac{\mu}{4\pi}\int \mathbf{J}(\vec{\mathbf{r}}')\frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}d\vec{\mathbf{r}}'$$

$$|\vec{\mathbf{r}}-\vec{\mathbf{r}}'| = r - \vec{\mathbf{r}}'\cdot\hat{i}_r + \frac{(r')^2}{2r}\sin^2\gamma + \cdots$$



$$\frac{1}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} = \frac{1}{r - r'\cos\gamma + \frac{(r')^2}{2r}\sin^2\gamma + \cdots} \approx \frac{1}{r}$$

if $D << r$

$$e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} \approx e^{-j\beta r}e^{j\beta\vec{\mathbf{r}}'\cdot\hat{i}_r}e^{-j\beta\frac{(r')^2}{2r}\sin^2\gamma}\cdots$$

if $r > 0.62\sqrt{\frac{D^3}{\lambda}}$

Fresnel region

Fraunhofer region

$r >> D$

$r > \frac{2D^2}{\lambda}$

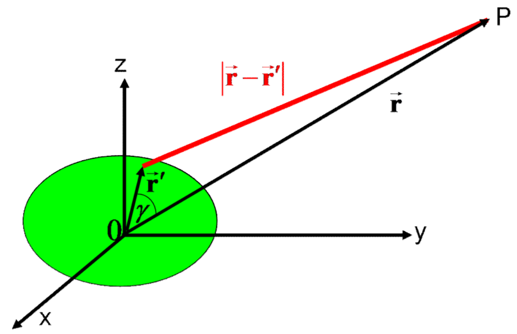
$r >> \lambda$

$$\mathbf{E}(\vec{\mathbf{r}}) = \mathbf{E}(r,\vartheta,\varphi) = -\frac{j\omega\mu}{4\pi}\frac{e^{-j\beta r}}{r}\left[\mathbf{M}(\vartheta,\varphi) - M_r(\vartheta,\varphi)\hat{i}_r\right] = -\frac{j\omega\mu}{4\pi}\frac{e^{-j\beta r}}{r}\left[M_\vartheta(\vartheta,\varphi)\hat{i}_\vartheta + M_\varphi(\vartheta,\varphi)\hat{i}_\varphi\right]$$

$$\zeta\mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

$$\vec{\mathbf{S}} = \frac{1}{2\zeta}\left|\vec{\mathbf{E}}\right|^2 \hat{i}_r = \frac{\zeta}{2}\left|\vec{\mathbf{H}}\right|^2 \hat{i}_r$$

$$\mathbf{M}(\vartheta,\varphi) = \int \mathbf{J}(\vec{\mathbf{r}}')e^{j\beta\vec{\mathbf{r}}'\cdot\hat{i}_r}d\vec{\mathbf{r}}'$$



Fraunhofer region

$$\frac{j\omega\mu}{4\pi} = \frac{j\omega\sqrt{\mu}\sqrt{\mu}}{4\pi}\frac{\sqrt{\varepsilon}}{\sqrt{\varepsilon}} = \frac{j\zeta}{4\pi}\beta = \frac{j\zeta}{4\pi}\frac{2\pi}{\lambda} = \frac{j\zeta}{2\lambda}$$

$$\omega\sqrt{\mu\varepsilon} = \beta = \frac{2\pi}{\lambda}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

Fraunhofer region

$$r >> D$$

$$r > \frac{2D^2}{\lambda}$$

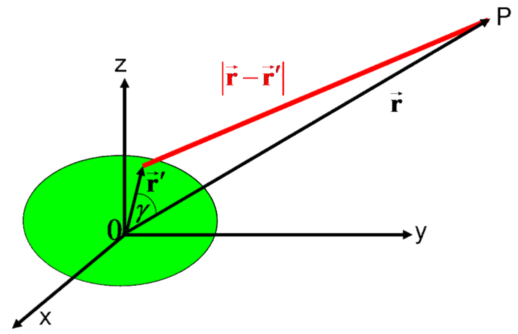
$$r >> \lambda$$

$$\mathbf{E}(\vec{\mathbf{r}}) = \mathbf{E}(r,\vartheta,\varphi) = -\frac{j\omega\mu}{4\pi}\frac{e^{-j\beta r}}{r}\left[\mathbf{M}(\vartheta,\varphi) - M_r(\vartheta,\varphi)\hat{i}_r\right] = -\frac{j\zeta}{2\lambda}\frac{e^{-j\beta r}}{r}\left[M_\vartheta(\vartheta,\varphi)\hat{i}_\vartheta + M_\varphi(\vartheta,\varphi)\hat{i}_\varphi\right]$$

$$\zeta\mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

$$\vec{\mathbf{S}} = \frac{1}{2\zeta}\left|\vec{\mathbf{E}}\right|^2\hat{i}_r = \frac{\zeta}{2}\left|\vec{\mathbf{H}}\right|^2\hat{i}_r$$

$$\mathbf{M}(\vartheta,\varphi) = \int \mathbf{J}(\vec{\mathbf{r}}')\, e^{j\beta\vec{\mathbf{r}}'\cdot\hat{i}_r}\, d\,\vec{\mathbf{r}}'$$



Fraunhofer region

$$\frac{j\omega\mu}{4\pi} = \frac{j\omega\sqrt{\mu}\sqrt{\mu}}{4\pi}\frac{\sqrt{\varepsilon}}{\sqrt{\varepsilon}} = \frac{j\zeta}{4\pi}\beta = \frac{j\zeta}{4\pi}\frac{2\pi}{\lambda} = \frac{j\zeta}{2\lambda}$$

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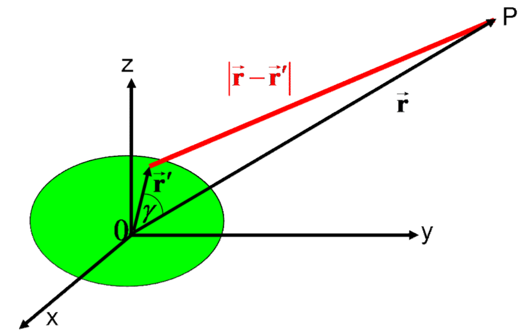
$$\zeta\mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

$$\vec{\mathbf{S}} = \frac{1}{2\zeta}\left|\vec{\mathbf{E}}\right|^2\hat{i}_r = \frac{\zeta}{2}\left|\vec{\mathbf{H}}\right|^2\hat{i}_r$$

$$\mathbf{M}(\vartheta,\varphi) = \int \mathbf{J}(\vec{\mathbf{r}}')e^{j\beta\vec{\mathbf{r}}'\cdot\hat{i}_r}\,d\,\vec{\mathbf{r}}'$$



Fraunhofer region

Fraunhofer region

$r >> D$

$r > \frac{2D^2}{\lambda}$

$r >> \lambda$

$$\mathbf{E}(\vec{\mathbf{r}}) = \mathbf{E}(r, \vartheta, \varphi) = -\frac{j\omega\mu}{4\pi}\frac{e^{-j\beta r}}{r}\left[\mathbf{M}(\vartheta,\varphi) - M_r(\vartheta,\varphi)\hat{i}_r\right] = \frac{j\zeta}{2\lambda}\frac{e^{-j\beta r}}{r}\left[-M_\vartheta(\vartheta,\varphi)\hat{i}_\vartheta - M_\varphi(\vartheta,\varphi)\hat{i}_\varphi\right]$$

$$\zeta\mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

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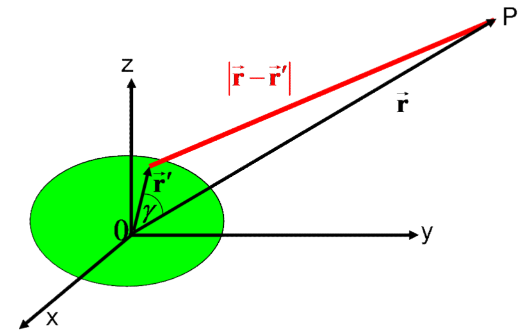
$$\mathbf{M}(\vartheta,\varphi) = \int \mathbf{J}(\vec{\mathbf{r}}')e^{j\beta\vec{\mathbf{r}}'\cdot\hat{i}_r}d\,\vec{\mathbf{r}}'$$



Fraunhofer region

Fraunhofer region

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Fraunhofer region

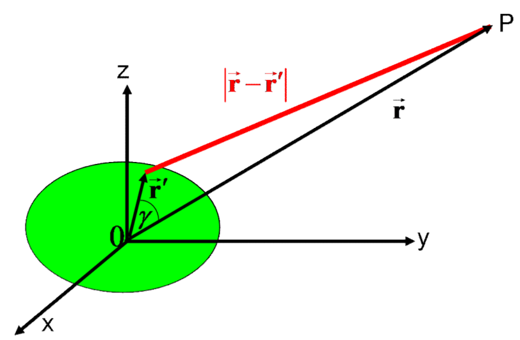
$$\mathbf{E}(\vec{\mathbf{r}}) = \mathbf{E}(r,\vartheta,\varphi) = -\frac{j\omega\mu}{4\pi}\frac{e^{-j\beta r}}{r}\left[\mathbf{M}(\vartheta,\varphi) - M_r(\vartheta,\varphi)\hat{i}_r\right] = \frac{j\zeta}{2\lambda}\frac{e^{-j\beta r}}{r} I\left[\frac{-M_\vartheta(\vartheta,\varphi)\hat{i}_\vartheta - M_\varphi(\vartheta,\varphi)\hat{i}_\varphi}{I}\right]$$

$$\zeta\mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

I is the phasor associated to the input current at the antenna input terminals

$$\vec{\mathbf{S}} = \frac{1}{2\zeta}\left|\vec{\mathbf{E}}\right|^2 \hat{i}_r = \frac{\zeta}{2}\left|\vec{\mathbf{H}}\right|^2 \hat{i}_r$$

$$\mathbf{M}(\vartheta,\varphi) = \int \mathbf{J}(\vec{\mathbf{r}}') e^{j\beta\vec{\mathbf{r}}'\cdot\hat{i}_r} d\,\vec{\mathbf{r}}'$$



Fraunhofer region

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$$r > \frac{2D^2}{\lambda}$$

$$r >> \lambda$$

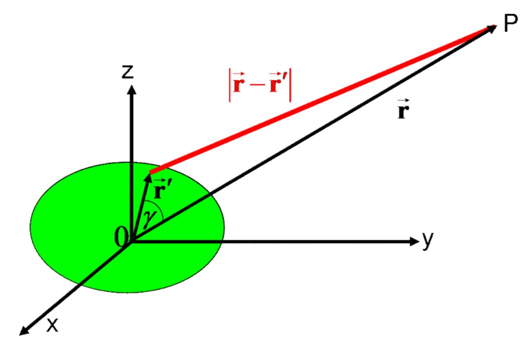
$$\mathbf{E}(\vec{\mathbf{r}}) = \mathbf{E}(r,\vartheta,\varphi) = = \frac{j\zeta}{2\lambda}\frac{e^{-j\beta r}}{r} I \left[\frac{-M_\vartheta(\vartheta,\varphi)\hat{i}_\vartheta - M_\varphi(\vartheta,\varphi)\hat{i}_\varphi}{I} \right]$$

$$\zeta \mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

I is the phasor associated to the input current at the antenna input terminals

$$\vec{\mathbf{S}} = \frac{1}{2\zeta}\left|\vec{\mathbf{E}}\right|^2 \hat{i}_r = \frac{\zeta}{2}\left|\vec{\mathbf{H}}\right|^2 \hat{i}_r$$

$$\mathbf{M}(\vartheta,\varphi) = \int \mathbf{J}(\vec{\mathbf{r}}') e^{j\beta \vec{\mathbf{r}}'\cdot\hat{i}_r} d\,\vec{\mathbf{r}}'$$



Fraunhofer region

Fraunhofer region

$r >> D$

$r > \frac{2D^2}{\lambda}$

$r >> \lambda$

Fraunhofer region

$$\mathbf{E}(\vec{\mathbf{r}}) = \mathbf{E}(r,\vartheta,\varphi) = \frac{j\zeta I}{2\lambda}\frac{e^{-j\beta r}}{r}\left[\frac{-M_\vartheta(\vartheta,\varphi)\hat{i}_\vartheta - M_\varphi(\vartheta,\varphi)\hat{i}_\varphi}{I}\right] = \frac{j\zeta I}{2\lambda}\frac{e^{-j\beta r}}{r}\mathbf{l}(\vartheta,\varphi)$$

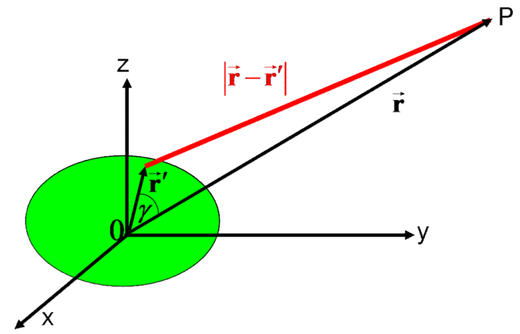
$$\zeta\mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

I is the phasor associated to the input current at the antenna input terminals

$$\vec{\mathbf{S}} = \frac{1}{2\zeta}\left|\vec{\mathbf{E}}\right|^2 \hat{i}_r = \frac{\zeta}{2}\left|\vec{\mathbf{H}}\right|^2 \hat{i}_r$$

$$\mathbf{M}(\vartheta,\varphi) = \int \mathbf{J}(\vec{\mathbf{r}}') e^{j\beta\vec{\mathbf{r}}'\cdot\hat{i}_r} d\,\vec{\mathbf{r}}'$$

$$[\mathbf{M}] = \frac{Ampere}{m^2} \times m^3 = Ampere \times m$$



$$\frac{-M_\vartheta(\vartheta,\varphi)\hat{i}_\vartheta - M_\varphi(\vartheta,\varphi)\hat{i}_\varphi}{I} = \mathbf{l}(\vartheta,\varphi) = l_\vartheta(\vartheta,\varphi)\hat{i}_\vartheta + l_\varphi(\vartheta,\varphi)\hat{i}_\varphi$$

is said **effective length** of the antenna

Fraunhofer region

$$r >> D$$

$$r > \frac{2D^2}{\lambda}$$

$$r >> \lambda$$

$$\mathbf{E}(\vec{\mathbf{r}}) = \mathbf{E}(r,\vartheta,\varphi) = \frac{j\zeta I}{2\lambda}\frac{e^{-j\beta r}}{r}\mathbf{l}(\vartheta,\varphi)$$

$$\zeta\mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

$$\mathbf{l}(\vartheta,\varphi) = l_\vartheta(\vartheta,\varphi)\hat{i}_\vartheta + l_\varphi(\vartheta,\varphi)\hat{i}_\varphi$$

$$\vec{\mathbf{S}} = \frac{1}{2\zeta}\left|\vec{\mathbf{E}}\right|^2\hat{i}_r = \frac{\zeta}{2}\left|\vec{\mathbf{H}}\right|^2\hat{i}_r$$