Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2020-2021 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

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Color legend

New formulas, important considerations,
important formulas, important concepts

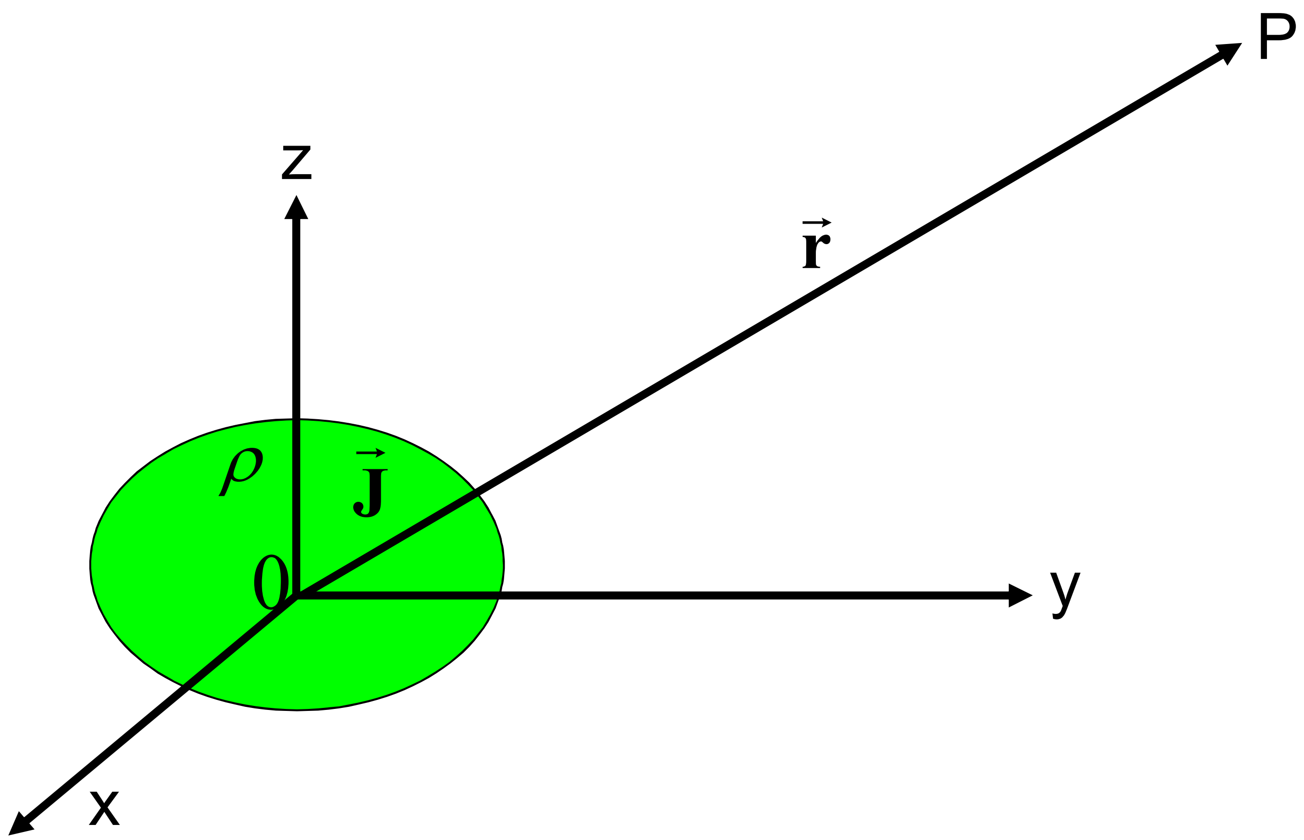
Very important for the discussion

Memo

Mathematical tools to be exploited

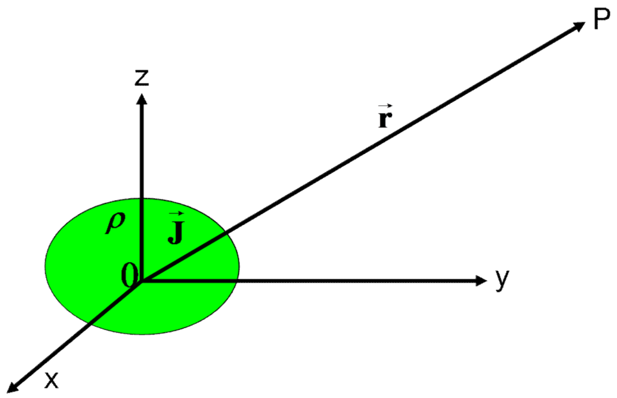
Mathematics

Radiation problem



$\vec{\mathbf{E}}(\vec{\mathbf{r}})$
$\vec{\mathbf{H}}(\vec{\mathbf{r}})$

An antenna (or aerial) is an electrical device which converts electric power into radio waves, and vice versa.



Radiation problem

- We approach the radiation problem of an antenna assuming that the sources are known and the media in which propagation occurs is simple.
- Mathematically we exploit the potentials solution.

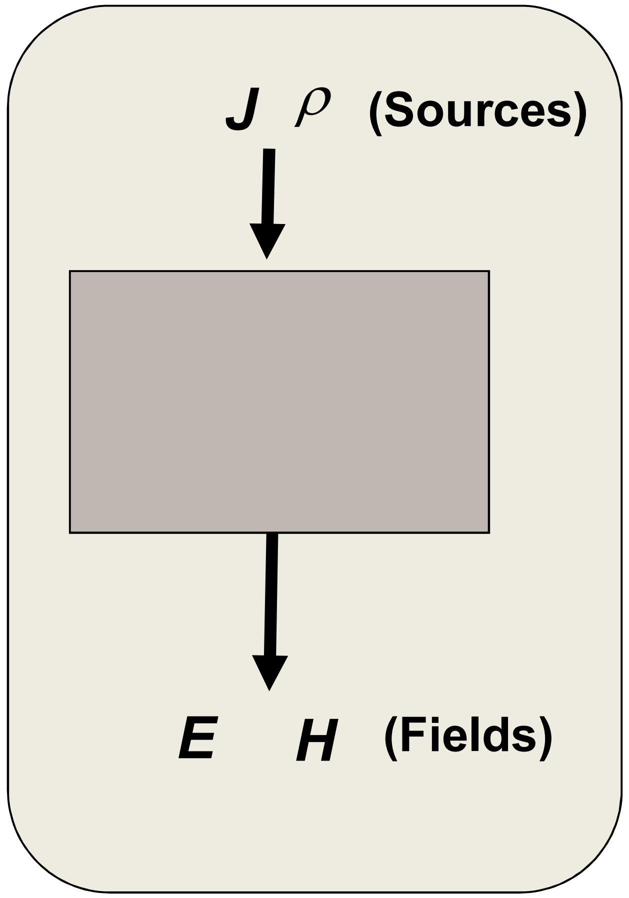
Medium: linear, isotropic, homogeneous (time-invariant and space-invariant), non-dispersive in space and time

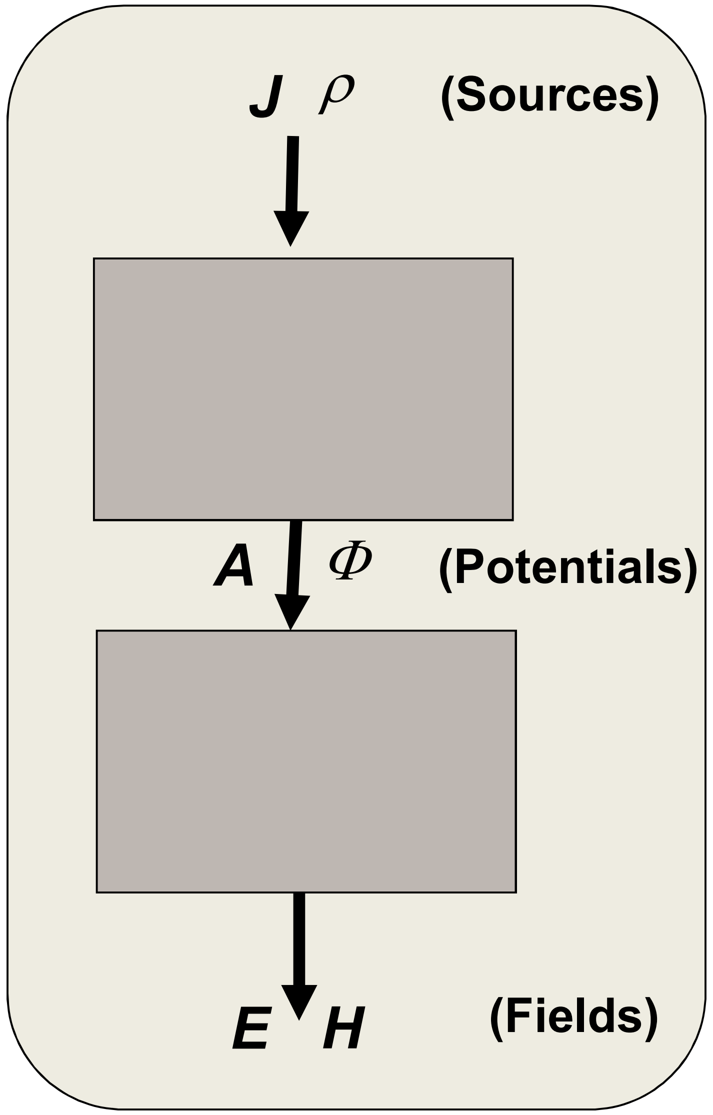
Phasor domain

$$\begin{cases} \nabla \times \boldsymbol{E} = -j\omega\mu \boldsymbol{H} \\ \nabla \times \boldsymbol{H} = j\omega\varepsilon \boldsymbol{E} + \boldsymbol{J} \\ \nabla \cdot \varepsilon \boldsymbol{E} = \rho \\ \nabla \cdot \mu \boldsymbol{H} = 0 \end{cases}$$

Radiation problem & potentials

$$
\begin{cases}
\nabla \times \boldsymbol{E} = -j\omega\mu \boldsymbol{H} \\
\nabla \times \boldsymbol{H} = j\omega\varepsilon \boldsymbol{E} + \boldsymbol{J} \\
\nabla \cdot \varepsilon \boldsymbol{E} = \rho \\
\nabla \cdot \mu \boldsymbol{H} = 0
\end{cases}
$$



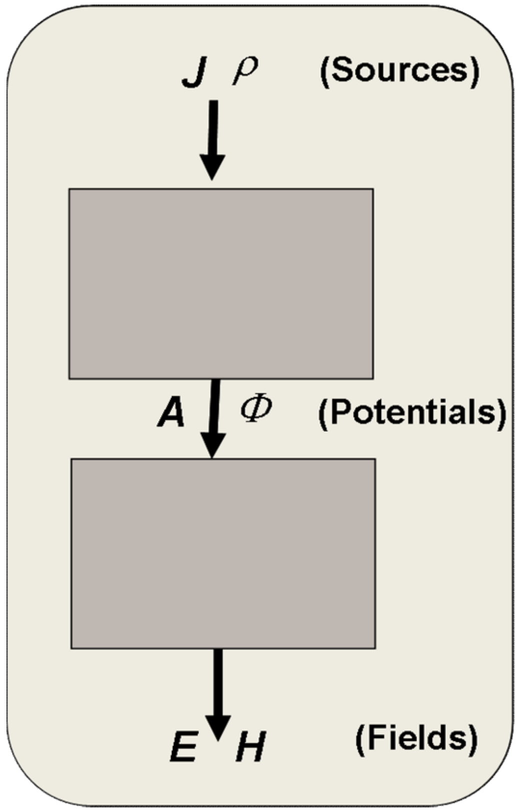


… mathematical tools that we will exploit today…

$$\nabla \times \mathbf{C} = \mathbf{0} \quad \Rightarrow \quad \exists\, \Phi \quad : \quad \mathbf{C} = \nabla \Phi$$

$$\nabla \cdot \mathbf{C} = 0 \quad \Rightarrow \quad \exists\, \mathbf{A} \quad : \quad \mathbf{C} = \nabla \times \mathbf{A}$$

Potentials



$\boldsymbol{J}$ ↓

$$\nabla^2 \boldsymbol{A} + k^2 \boldsymbol{A} = -\mu \boldsymbol{J}$$ **Vector Helmholtz equation**

↓ $\boldsymbol{A}$

$$\boldsymbol{H} = \frac{1}{\mu} \nabla \times \boldsymbol{A} \quad \text{and} \quad \boldsymbol{E} = -j\omega \boldsymbol{A} + \frac{\nabla\nabla \cdot \boldsymbol{A}}{j\omega\varepsilon\mu}$$

↓

$\boldsymbol{E}$ $\boldsymbol{H}$

Potentials

J

$$\nabla^2 \boldsymbol{A} + k^2 \boldsymbol{A} = -\mu \boldsymbol{J}$$ **Vector Helmholtz equation**

A

$$\begin{cases} \nabla^2 A_x + k^2 A_x = -\mu J_x \\ \nabla^2 A_y + k^2 A_y = -\mu J_y \\ \nabla^2 A_z + k^2 A_z = -\mu J_z \end{cases}$$

Potentials

$$\begin{cases} \nabla^2 A_x + k^2 A_x = -\mu J_x \\ \nabla^2 A_y + k^2 A_y = -\mu J_y \\ \nabla^2 A_z + k^2 A_z = -\mu J_z \end{cases}$$

Let us address the solution of the following scalar Helmholtz equation

$$\nabla^2 A_x + k^2 A_x = -\mu J_x$$

Potentials

$$\nabla^2 A_x + k^2 A_x = -\mu J_x$$

$$-\mu J_x(\vec{\mathbf{r}}) \longrightarrow \boxed{\quad g(\vec{\mathbf{r}}) \quad} \longrightarrow A_x(\vec{\mathbf{r}})$$

$$\delta(\vec{\mathbf{r}}) \longrightarrow \boxed{\qquad\qquad} \longrightarrow g(\vec{\mathbf{r}})$$

$$\delta(\vec{\mathbf{r}} - \vec{\mathbf{r}}') \longrightarrow \boxed{\qquad\qquad} \longrightarrow g(\vec{\mathbf{r}} - \vec{\mathbf{r}}')$$

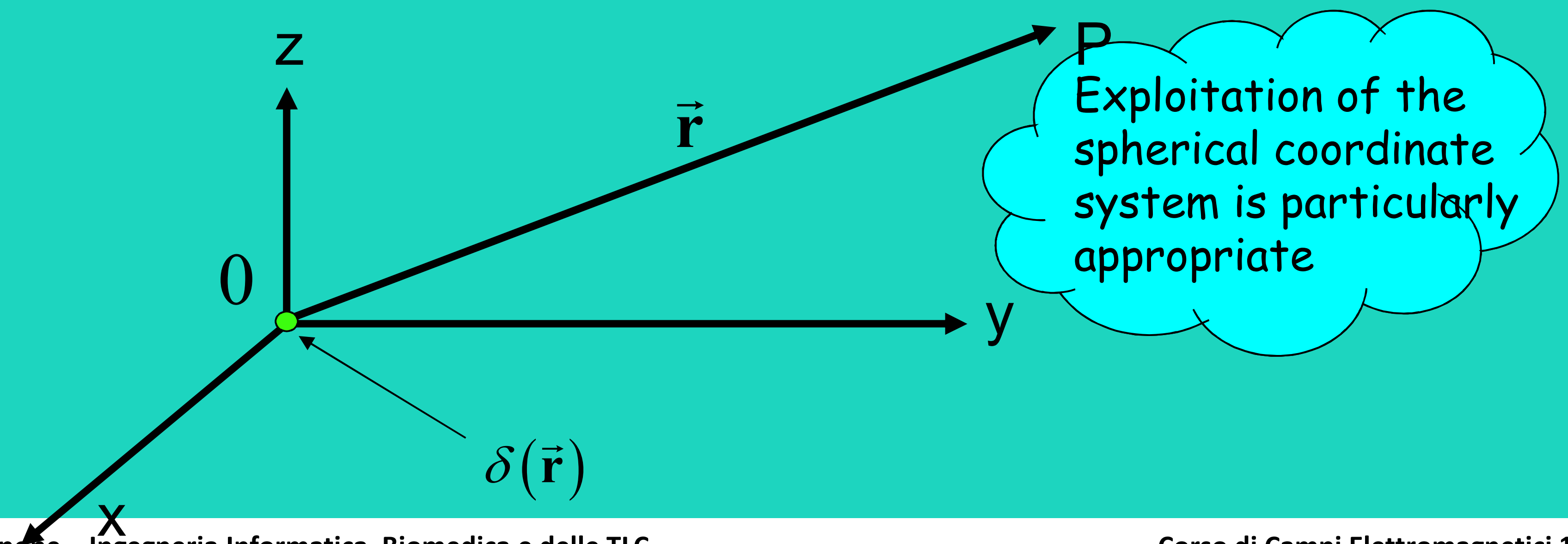
$$A_x(\vec{\mathbf{r}}) = \int -\mu J_x(\vec{\mathbf{r}}')\, g(\vec{\mathbf{r}} - \vec{\mathbf{r}}')\, d\vec{\mathbf{r}}'$$

Potentials

$$\nabla^2 A_x + k^2 A_x = -\mu J_x$$

$$\delta(\vec{\mathbf{r}}) \longrightarrow \boxed{} \longrightarrow g(\vec{\mathbf{r}})$$

$$\nabla^2 g(\vec{\mathbf{r}}) + k^2 g(\vec{\mathbf{r}}) = \delta(\vec{\mathbf{r}})$$



Potentials

$$\nabla^2 A_x + k^2 A_x = -\mu J_x$$

$$-\mu J_x(\vec{\mathbf{r}}) \longrightarrow \boxed{g(\vec{\mathbf{r}})} \longrightarrow A_x(\vec{\mathbf{r}})$$

$$\delta(\vec{\mathbf{r}}) \longrightarrow \boxed{\phantom{g(\vec{\mathbf{r}})}} \longrightarrow g(\vec{\mathbf{r}}) = -\frac{1}{4\pi}\frac{e^{-jkr}}{r} = -\frac{1}{4\pi}\frac{e^{-jk|\vec{\mathbf{r}}|}}{|\vec{\mathbf{r}}|}$$

$$\delta(\vec{\mathbf{r}}-\vec{\mathbf{r}}') \longrightarrow \boxed{\phantom{g(\vec{\mathbf{r}})}} \longrightarrow = -\frac{1}{4\pi}\frac{e^{-jk|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} = g(\vec{\mathbf{r}}-\vec{\mathbf{r}}')$$

$$A_x(\vec{\mathbf{r}}) = \int -\mu J_x(\vec{\mathbf{r}}')\, g(\vec{\mathbf{r}}-\vec{\mathbf{r}}')\, d\vec{\mathbf{r}}' = \frac{\mu}{4\pi}\int J_x(\vec{\mathbf{r}}')\frac{e^{-jk|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}\, d\vec{\mathbf{r}}'$$

Potentials

$$\downarrow \boldsymbol{J}(\boldsymbol{r})$$

$$\nabla^2 \boldsymbol{A} + k^2 \boldsymbol{A} = -\mu \boldsymbol{J} \qquad \textbf{Vector Helmholtz equation}$$

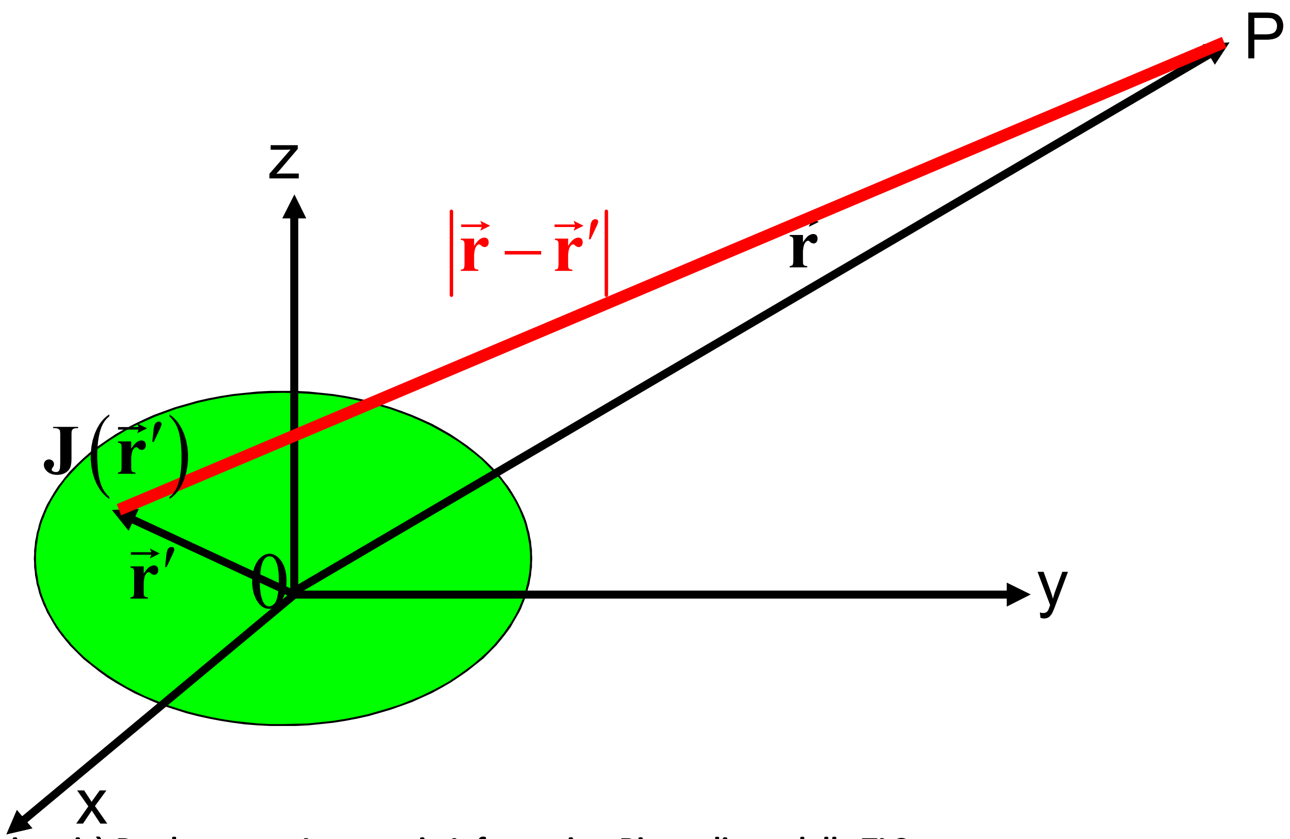
$$\downarrow \quad \mathbf{A}(\mathbf{r}) = \frac{\mu}{4\pi} \int \mathbf{J}(\mathbf{r}') \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r}'$$

$$\boldsymbol{H} = \frac{1}{\mu} \nabla \times \boldsymbol{A} \quad \text{and} \quad \boldsymbol{E} = -j\omega \boldsymbol{A} + \frac{\nabla\nabla \cdot \boldsymbol{A}}{j\omega\varepsilon\mu}$$

$$\downarrow \begin{array}{l} \boldsymbol{E}(\boldsymbol{r}) \\ \boldsymbol{H}(\boldsymbol{r}) \end{array}$$

Potentials

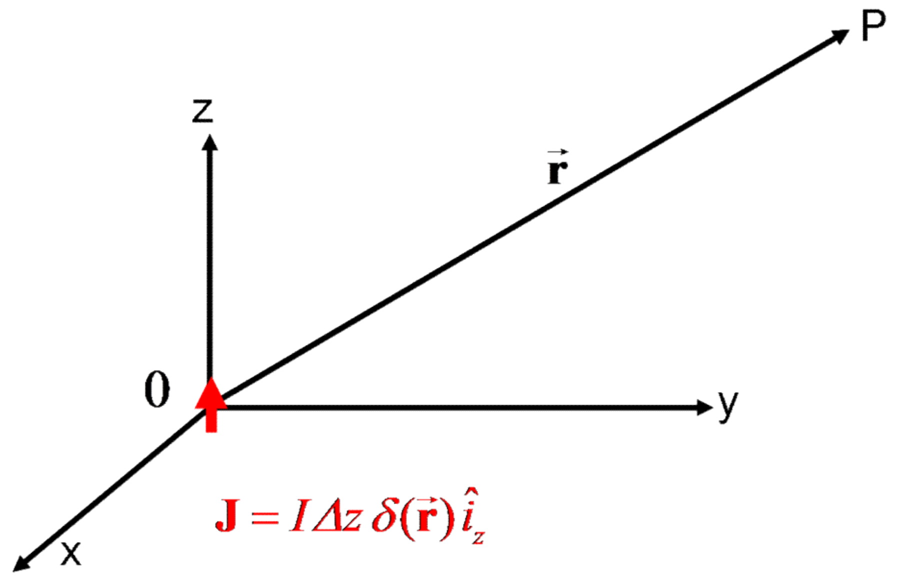
$$\mathbf{A}(\vec{\mathbf{r}}) = \frac{\mu}{4\pi}\int \mathbf{J}(\vec{\mathbf{r}}')\frac{e^{-jk|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}d\,\vec{\mathbf{r}}'$$



Elementary electrical dipole

- A δ-source radiating element is also known as elementary electrical dipole.

$$\mathbf{J} = I\Delta z\,\delta(\vec{\mathbf{r}})\,\hat{i}_z = I\Delta z\,\delta(x)\delta(y)\delta(z)\,\hat{i}_z$$



- Why are we interested in such a radiating element?
- Why is such a radiating element referred to as elementary electrical dipole?
- How can we physically approximate an elementary electrical dipole?

Elementary electrical dipole

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$$\mathbf{J} = I\Delta z\,\delta(\vec{\mathbf{r}})\hat{i}_z = I\Delta z\,\delta(x)\delta(y)\delta(z)\hat{i}_z$$

Elementary electrical dipole

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$$\mathbf{J} = I\Delta z\,\delta(\vec{\mathbf{r}})\hat{i}_z = I\Delta z\,\delta(x)\delta(y)\delta(z)\hat{i}_z$$

$$\boldsymbol{J} \downarrow$$

$$\nabla^2 \boldsymbol{A} + k^2 \boldsymbol{A} = -\mu \boldsymbol{J}$$ **Vector Helmholtz equation**

$$\downarrow \boldsymbol{A}$$

$$\begin{cases} \nabla^2 A_x + k^2 A_x = -\mu J_x \\ \nabla^2 A_y + k^2 A_y = -\mu J_y \\ \nabla^2 A_z + k^2 A_z = -\mu J_z \end{cases}$$

Elementary electrical dipole

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$$\mathbf{J} = I\Delta z\,\delta(\vec{\mathbf{r}})\,\hat{i}_z = I\Delta z\,\delta(x)\delta(y)\delta(z)\,\hat{i}_z$$

$\boldsymbol{J}$ ↓

$\nabla^2 \boldsymbol{A} + k^2 \boldsymbol{A} = -\mu \boldsymbol{J}$ **Vector Helmholtz equation**

↓ $\boldsymbol{A}$

$$\begin{cases} \nabla^2 A_x + k^2 A_x = 0 \\ \nabla^2 A_y + k^2 A_y = 0 \\ \nabla^2 A_z + k^2 A_z = -\mu J_z = -\mu I \Delta z\,\delta(x)\delta(y)\delta(z) = -\mu I \Delta z\,\delta(\vec{\mathbf{r}}) \end{cases}$$

Elementary electrical dipole

- A δ-source radiating element is also known as elementary electrical dipole.

$$\mathbf{J} = I\Delta z\,\delta(\vec{\mathbf{r}})\hat{i}_z = I\Delta z\,\delta(x)\delta(y)\delta(z)\hat{i}_z$$

J

$$\nabla^2 \boldsymbol{A} + k^2 \boldsymbol{A} = -\mu \boldsymbol{J}$$ **Vector Helmholtz equation**

A

$$\begin{cases} \nabla^2 A_x + k^2 A_x = 0 & \Rightarrow A_x = 0 \\ \nabla^2 A_y + k^2 A_y = 0 & \Rightarrow A_y = 0 \\ \nabla^2 A_z + k^2 A_z = -\mu J_z = -\mu I \Delta z\,\delta(x)\delta(y)\delta(z) = -\mu I \Delta z\,\delta(\vec{\mathbf{r}}) & \end{cases}$$

Elementary electrical dipole

- A δ-source radiating element is also known as elementary electrical dipole.

$$\mathbf{J} = I\Delta z\,\delta(\vec{\mathbf{r}})\,\hat{i}_z = I\Delta z\,\delta(x)\delta(y)\delta(z)\,\hat{i}_z$$

J

↓

$\nabla^2 \boldsymbol{A} + k^2 \boldsymbol{A} = -\mu \boldsymbol{J}$ **Vector Helmholtz equation**

↓

A

$$\begin{cases} \nabla^2 A_x + k^2 A_x = 0 & \Rightarrow A_x = 0 \\ \nabla^2 A_y + k^2 A_y = 0 & \Rightarrow A_y = 0 \\ \boxed{\nabla^2 A_z + k^2 A_z = -\mu J_z = -\mu I \Delta z\,\delta(x)\delta(y)\delta(z) = -\mu I \Delta z\,\delta(\vec{\mathbf{r}})} \end{cases}$$

Elementary electrical dipole

- A δ-source radiating element is also known as elementary electrical dipole.

$$\mathbf{J} = I\Delta z\,\delta(\vec{\mathbf{r}})\hat{i}_z = I\Delta z\,\delta(x)\delta(y)\delta(z)\hat{i}_z$$

$\mathbf{J}$ ↓

$\nabla^2 \boldsymbol{A} + k^2 \boldsymbol{A} = -\mu \boldsymbol{J}$ **Vector Helmholtz equation**

↓ $\mathbf{A}$

$$\begin{cases} \nabla^2 A_x + k^2 A_x = 0 & \Rightarrow A_x = 0 \\ \nabla^2 A_y + k^2 A_y = 0 & \Rightarrow A_y = 0 \\ \boxed{\nabla^2 A_z + k^2 A_z = -\mu J_z = -\mu I \Delta z\,\delta(x)\delta(y)\delta(z) = -\mu I \Delta z\,\delta(\vec{\mathbf{r}})} \end{cases}$$

$$\nabla^2 g(\vec{\mathbf{r}}) + k^2 g(\vec{\mathbf{r}}) = -\mu I \Delta z\ \delta(\vec{\mathbf{r}})$$

$$-\mu I \Delta z\ \delta(\vec{\mathbf{r}}) \longrightarrow \boxed{} \longrightarrow g(\vec{\mathbf{r}}) = (-\mu I \Delta z) - \frac{1}{4\pi}\frac{e^{-jkr}}{r} = \frac{\mu}{4\pi} I \Delta z \frac{e^{-jkr}}{r}$$

Elementary electrical dipole

- A δ-source radiating element is also known as elementary electrical dipole.

$$\mathbf{J} = I\Delta z\,\delta(\vec{\mathbf{r}})\,\hat{i}_z = I\Delta z\,\delta(x)\delta(y)\delta(z)\,\hat{i}_z$$

$\downarrow$ *J*

$$\nabla^2 \boldsymbol{A} + k^2 \boldsymbol{A} = -\mu \boldsymbol{J}$$ **Vector Helmholtz equation**

$$\downarrow \quad \mathbf{A} = \frac{\mu}{4\pi} I\Delta z \frac{e^{-jkr}}{r}\hat{i}_z$$

$$\boldsymbol{H} = \frac{1}{\mu}\nabla \times \boldsymbol{A} \quad \text{and} \quad \boldsymbol{E} = -j\omega \boldsymbol{A} + \frac{\nabla\nabla\cdot \boldsymbol{A}}{j\omega\varepsilon\mu}$$

$\downarrow$ *E*, *H*

Elementary electrical dipole

- A δ-source radiating element is also known as elementary electrical dipole.

$$\mathbf{J} = I\Delta z\,\delta(\vec{\mathbf{r}})\hat{i}_z = I\Delta z\,\delta(x)\delta(y)\delta(z)\hat{i}_z$$

The E.M. field radiated by the elementary electrical dipole

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathrm{E}_r(r,\vartheta)\hat{i}_r + \mathrm{E}_\vartheta(r,\vartheta)\hat{i}_\vartheta$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \mathrm{H}_\varphi(r,\vartheta)\hat{i}_\varphi$$

$$\begin{cases} E_r = \zeta \dfrac{I\Delta z}{2\pi}\left(\dfrac{1}{r^2} + \dfrac{1}{j\beta r^3}\right)\cos\vartheta\exp(-j\beta r) \\ E_\vartheta = \zeta \dfrac{I\Delta z}{4\pi}\left(\dfrac{j\beta}{r} + \dfrac{1}{r^2} + \dfrac{1}{j\beta r^3}\right)\sin\vartheta\exp(-j\beta r) \\ H_\varphi = \dfrac{I\Delta z}{4\pi}\left(\dfrac{j\beta}{r} + \dfrac{1}{r^2}\right)\sin\vartheta\exp(-j\beta r) \end{cases}$$

Because of the problem symmetry there is no dependence on the azimuth angle φ.

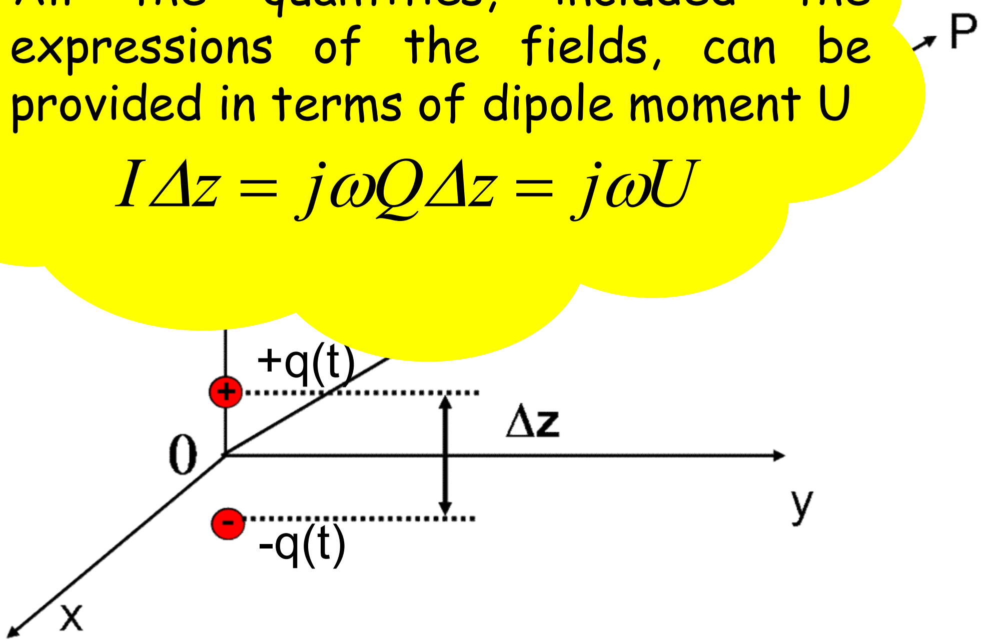
Elementary electrical dipole

- A δ-source radiating element is also known as elementary electrical dipole.

$$\mathbf{J} = I\Delta z\,\delta(\vec{\mathbf{r}})\hat{i}_z = I\Delta z\,\delta(x)\delta(y)\delta(z)\hat{i}_z$$

All the quantities, included the expressions of the fields, can be provided in terms of dipole moment U

$$I\Delta z = j\omega Q\Delta z = j\omega U$$



It can be easily shown that this electric current density is the same as that of an electrical dipole such that:

1) the two charges, of opposite sign, have equal time variation;

2) in the spectral domain, the relation between *I* and the time-varying charge *Q* is:

$$j\omega Q = I$$

Elementary electrical dipole

The E.M. field radiated by the elementary electrical dipole

$$\begin{cases} E_r = \zeta \dfrac{I\Delta z}{2\pi}\left(\dfrac{1}{r^2}+\dfrac{1}{j\beta r^3}\right)\cos\vartheta \exp(-j\beta r) \\ E_\vartheta = \zeta \dfrac{I\Delta z}{4\pi}\left(\dfrac{j\beta}{r}+\dfrac{1}{r^2}+\dfrac{1}{j\beta r^3}\right)\sin\vartheta \exp(-j\beta r) \\ H_\varphi = \dfrac{I\Delta z}{4\pi}\left(\dfrac{j\beta}{r}+\dfrac{1}{r^2}\right)\sin\vartheta \exp(-j\beta r) \end{cases}$$

$$j\omega Q = I$$

Elementary electrical dipole: static case

The E.M. field radiated by the elementary electrical dipole

$$\begin{cases} E_r = \zeta \dfrac{j\omega \mathrm{Q}\Delta z}{2\pi}\left(\dfrac{1}{r^2}+\dfrac{1}{j\beta r^3}\right)\cos\vartheta \exp(-j\beta r) \\ E_\vartheta = \zeta \dfrac{j\omega \mathrm{Q}\Delta z}{4\pi}\left(\dfrac{j\beta}{r}+\dfrac{1}{r^2}+\dfrac{1}{j\beta r^3}\right)\sin\vartheta \exp(-j\beta r) \\ H_\varphi = \dfrac{j\omega \mathrm{Q}\Delta z}{4\pi}\left(\dfrac{j\beta}{r}+\dfrac{1}{r^2}\right)\sin\vartheta \exp(-j\beta r) \end{cases}$$

$$j\omega Q = I$$

… for $\omega=0$ simplifies as

Elementary electrical dipole: static case

The E.M. field radiated by the elementary electrical dipole

$$\begin{cases} E_r = \zeta \dfrac{jQ\Delta z}{2\pi}\left(\cancel{\dfrac{\omega}{r^2}} + \dfrac{\omega}{j\beta r^3}\right)\cos\vartheta \boxed{\exp(-j\beta r)} \\ E_\vartheta = \zeta \dfrac{jQ\Delta z}{4\pi}\left(\cancel{\dfrac{j\omega\beta}{r}} + \cancel{\dfrac{\omega}{r^2}} + \dfrac{\omega}{j\beta r^3}\right)\sin\vartheta \boxed{\exp(-j\beta r)} \\ H_\varphi = \dfrac{jQ\Delta z}{4\pi}\left(\cancel{\dfrac{j\omega\beta}{r}} + \cancel{\dfrac{\omega}{r^2}}\right)\sin\vartheta \boxed{\exp(-j\beta r)} \end{cases} \rightarrow 1$$

$$j\omega Q = I$$

… for ω=0 simplifies as

$$\begin{cases} E_r = \dfrac{\zeta}{\sqrt{\mu\varepsilon}}\dfrac{Q\Delta z}{2\pi}\dfrac{1}{r^3}\cos\vartheta \\ E_\vartheta = \dfrac{\zeta}{\sqrt{\mu\varepsilon}}\dfrac{Q\Delta z}{4\pi}\dfrac{1}{r^3}\sin\vartheta \\ H_\varphi = 0 \end{cases}$$

$$\beta = \omega\sqrt{\mu\varepsilon} \rightarrow 0$$

$$\Rightarrow \omega\beta = \omega^2\sqrt{\mu\varepsilon} \rightarrow 0$$

$$\Rightarrow \omega/\beta = 1/\sqrt{\mu\varepsilon}$$

Elementary electrical dipole: static case

The E.M. field radiated by the elementary electrical dipole

$$\begin{cases} E_r = \zeta \dfrac{jQ\Delta z}{2\pi}\left(\cancel{\dfrac{\omega}{r^2}} + \dfrac{\omega}{j\beta r^3}\right)\cos\vartheta\,\boxed{\exp(-j\beta r)} \\ E_\vartheta = \zeta \dfrac{jQ\Delta z}{4\pi}\left(\cancel{\dfrac{j\omega\beta}{r}} + \cancel{\dfrac{\omega}{r^2}} + \dfrac{\omega}{j\beta r^3}\right)\sin\vartheta\,\boxed{\exp(-j\beta r)} \\ H_\varphi = \dfrac{jQ\Delta z}{4\pi}\left(\cancel{\dfrac{j\omega\beta}{r}} + \cancel{\dfrac{\omega}{r^2}}\right)\sin\vartheta\,\boxed{\exp(-j\beta r)} \end{cases} \rightarrow 1$$

$$j\omega Q = I$$

... for ω=0 simplifies as

$$\begin{cases} E_r = \dfrac{\zeta}{\sqrt{\mu\varepsilon}}\dfrac{Q\Delta z}{2\pi}\dfrac{1}{r^3}\cos\vartheta \\ E_\vartheta = \dfrac{\zeta}{\sqrt{\mu\varepsilon}}\dfrac{Q\Delta z}{4\pi}\dfrac{1}{r^3}\sin\vartheta \\ H_\varphi = 0 \end{cases}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\Rightarrow \frac{\zeta}{\sqrt{\mu\varepsilon}} = \frac{1}{\varepsilon}$$

Elementary electrical dipole: static case

The E.M. field radiated by the elementary electrical dipole

$$\begin{cases} E_r = \zeta \dfrac{jQ\Delta z}{2\pi}\left(\cancel{\dfrac{\omega}{r^2}} + \dfrac{\omega}{j\beta r^3}\right)\cos\vartheta \boxed{\exp(-j\beta r)} \\ E_\vartheta = \zeta \dfrac{jQ\Delta z}{4\pi}\left(\cancel{\dfrac{j\omega\beta}{r}} + \cancel{\dfrac{\omega}{r^2}} + \dfrac{\omega}{j\beta r^3}\right)\sin\vartheta \boxed{\exp(-j\beta r)} \\ H_\varphi = \dfrac{jQ\Delta z}{4\pi}\left(\cancel{\dfrac{j\omega\beta}{r}} + \cancel{\dfrac{\omega}{r^2}}\right)\sin\vartheta \boxed{\exp(-j\beta r)} \end{cases} \rightarrow 1$$

$$j\omega Q = I$$

... for ω=0 simplifies as

$$\begin{cases} E_r = \dfrac{\zeta}{\sqrt{\mu\varepsilon}}\dfrac{Q\Delta z}{2\pi}\dfrac{1}{r^3}\cos\vartheta = \dfrac{Q\Delta z}{2\pi}\dfrac{1}{\varepsilon r^3}\cos\vartheta \\ E_\vartheta = \dfrac{\zeta}{\sqrt{\mu\varepsilon}}\dfrac{Q\Delta z}{4\pi}\dfrac{1}{r^3}\sin\vartheta = \dfrac{Q\Delta z}{4\pi}\dfrac{1}{\varepsilon r^3}\sin\vartheta \\ H_\varphi = 0 \end{cases}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\Rightarrow \quad \frac{\zeta}{\sqrt{\mu\varepsilon}} = \frac{1}{\varepsilon}$$

Elementary electrical dipole: far field

The E.M. field radiated by the elementary electrical dipole

$$
\begin{cases}
E_r = \zeta \dfrac{I\Delta z}{2\pi} \boxed{\left(\dfrac{1}{r^2} + \dfrac{1}{j\beta r^3}\right)} \cos\vartheta \exp(-j\beta r) & = \zeta \dfrac{I\Delta z}{2\pi} \dfrac{j\beta}{r} \left(\dfrac{1}{j\beta r} + \dfrac{1}{[j\beta r]^2}\right) \cos\vartheta \exp(-j\beta r) \\
E_\vartheta = \zeta \dfrac{I\Delta z}{4\pi} \boxed{\left(\dfrac{j\beta}{r} + \dfrac{1}{r^2} + \dfrac{1}{j\beta r^3}\right)} \sin\vartheta \exp(-j\beta r) & = \zeta \dfrac{I\Delta z}{4\pi} \dfrac{j\beta}{r} \left(1 + \dfrac{1}{j\beta r} + \dfrac{1}{[j\beta r]^2}\right) \sin\vartheta \exp(-j\beta r) \\
H_\varphi = \dfrac{I\Delta z}{4\pi} \boxed{\left(\dfrac{j\beta}{r} + \dfrac{1}{r^2}\right)} \sin\vartheta \exp(-j\beta r) & = \dfrac{I\Delta z}{4\pi} \dfrac{j\beta}{r} \left(1 + \dfrac{1}{j\beta r}\right) \sin\vartheta \exp(-j\beta r)
\end{cases}
$$

$$
\left(\frac{j\beta}{r} + \frac{1}{r^2} + \frac{1}{j\beta r^3}\right) = \frac{j\beta}{r}\left(1 + \frac{r}{j\beta}\frac{1}{r^2} + \frac{r}{j\beta}\frac{1}{j\beta r^3}\right) = \frac{j\beta}{r}\left(1 + \frac{1}{j\beta r} + \frac{1}{[j\beta r]^2}\right)
$$

Elementary electrical dipole: far field

The E.M. field radiated by the elementary electrical dipole

$$\begin{cases} E_r = \zeta \dfrac{I\Delta z}{2\pi}\left(\dfrac{1}{r^2}+\dfrac{1}{j\beta r^3}\right)\cos\vartheta\exp(-j\beta r) & = \zeta \dfrac{I\Delta z}{2\pi}\dfrac{j\beta}{r}\left(\dfrac{1}{j\beta r}+\dfrac{1}{\left[j\beta r\right]^2}\right)\cos\vartheta\exp(-j\beta r) \\ E_\vartheta = \zeta \dfrac{I\Delta z}{4\pi}\left(\dfrac{j\beta}{r}+\dfrac{1}{r^2}+\dfrac{1}{j\beta r^3}\right)\sin\vartheta\exp(-j\beta r) & = \zeta \dfrac{I\Delta z}{4\pi}\dfrac{j\beta}{r}\left(1+\dfrac{1}{j\beta r}+\dfrac{1}{\left[j\beta r\right]^2}\right)\sin\vartheta\exp(-j\beta r) \\ H_\varphi = \dfrac{I\Delta z}{4\pi}\left(\dfrac{j\beta}{r}+\dfrac{1}{r^2}\right)\sin\vartheta\exp(-j\beta r) & = \dfrac{I\Delta z}{4\pi}\dfrac{j\beta}{r}\left(1+\dfrac{1}{j\beta r}\right)\sin\vartheta\exp(-j\beta r) \end{cases}$$

Elementary electrical dipole: far field

The E.M. field radiated by the elementary electrical dipole

$$\begin{cases} E_r = \zeta \dfrac{I\Delta z}{2\pi}\left(\dfrac{1}{r^2}+\dfrac{1}{j\beta r^3}\right)\cos\vartheta \exp(-j\beta r) & = \zeta \dfrac{I\Delta z}{2\pi}\dfrac{j\beta}{r}\left(\cancel{\dfrac{1}{j\beta r}}+\cancel{\dfrac{1}{[j\beta r]^2}}\right)\cos\vartheta\exp(-j\beta r) \\ E_\vartheta = \zeta \dfrac{I\Delta z}{4\pi}\left(\dfrac{j\beta}{r}+\dfrac{1}{r^2}+\dfrac{1}{j\beta r^3}\right)\sin\vartheta\exp(-j\beta r) & = \zeta \dfrac{I\Delta z}{4\pi}\dfrac{j\beta}{r}\left(1+\cancel{\dfrac{1}{j\beta r}}+\cancel{\dfrac{1}{[j\beta r]^2}}\right)\sin\vartheta\exp(-j\beta r) \\ H_\varphi = \dfrac{I\Delta z}{4\pi}\left(\dfrac{j\beta}{r}+\dfrac{1}{r^2}\right)\sin\vartheta\exp(-j\beta r) & = \dfrac{I\Delta z}{4\pi}\dfrac{j\beta}{r}\left(1+\cancel{\dfrac{1}{j\beta r}}\right)\sin\vartheta\exp(-j\beta r) \end{cases}$$

$$\beta = \omega\sqrt{\mu\varepsilon} = \frac{2\pi f}{c} = \frac{2\pi}{\lambda}$$

… for $\beta r >> 1$ simplifies as

$$\begin{cases} E_r = 0 \\ E_\vartheta = j\zeta \dfrac{I\Delta z}{r}\boxed{\dfrac{\beta}{4\pi}}\sin\vartheta\exp(-j\beta r) = j\zeta\dfrac{I\Delta z}{2\lambda r}\sin\vartheta\exp(-j\beta r) \\ H_\varphi = j\dfrac{I\Delta z}{r}\dfrac{\beta}{4\pi}\sin\vartheta\exp(-j\beta r) = \dfrac{E_\vartheta}{\zeta} \end{cases}$$

$$\frac{\beta}{4\pi} = \frac{2\pi}{\lambda}\frac{1}{4\pi} = \frac{1}{2\lambda}$$

$$\beta = \omega\sqrt{\mu\varepsilon}$$

$$2\pi f = \omega$$

$$c = 1/\sqrt{\mu\varepsilon}$$

$$\lambda = c/f$$

Elementary electrical dipole: far field

The E.M. field radiated by the elementary electrical dipole

$$\begin{cases} E_r = \zeta \dfrac{I\Delta z}{2\pi}\left(\dfrac{1}{r^2}+\dfrac{1}{j\beta r^3}\right)\cos\vartheta\exp(-j\beta r) & = \zeta \dfrac{I\Delta z}{2\pi}\dfrac{j\beta}{r}\left(\cancel{\dfrac{1}{j\beta r}}+\cancel{\dfrac{1}{[j\beta r]^2}}\right)\cos\vartheta\exp(-j\beta r) \\ E_\vartheta = \zeta \dfrac{I\Delta z}{4\pi}\left(\boxed{\dfrac{j\beta}{r}}+\dfrac{1}{r^2}+\dfrac{1}{j\beta r^3}\right)\sin\vartheta\exp(-j\beta r) & = \zeta \dfrac{I\Delta z}{4\pi}\dfrac{j\beta}{r}\left(1+\cancel{\dfrac{1}{j\beta r}}+\cancel{\dfrac{1}{[j\beta r]^2}}\right)\sin\vartheta\exp(-j\beta r) \\ H_\varphi = \dfrac{I\Delta z}{4\pi}\left(\boxed{\dfrac{j\beta}{r}}+\dfrac{1}{r^2}\right)\sin\vartheta\exp(-j\beta r) & = \dfrac{I\Delta z}{4\pi}\dfrac{j\beta}{r}\left(1+\cancel{\dfrac{1}{j\beta r}}\right)\sin\vartheta\exp(-j\beta r) \end{cases}$$

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$$\beta r >> 1 \Rightarrow \frac{2\pi}{\lambda} r >> 1 \Rightarrow r >> \lambda$$

$$\beta = \omega\sqrt{\mu\varepsilon}$$

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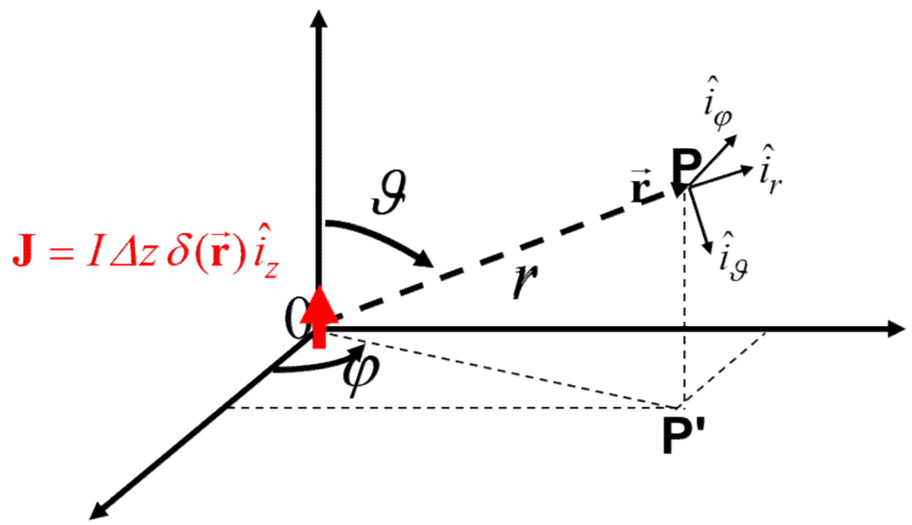
In the far-field case (r>>λ) the elementary electrical dipole behaves as follows

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathrm{E}_{\vartheta}(r,\vartheta)\hat{i}_{\vartheta}$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \mathrm{H}_{\varphi}(r,\vartheta)\hat{i}_{\varphi}$$

$$\begin{cases} E_{\vartheta} = j\zeta \dfrac{I\Delta z}{2\lambda r}\sin\vartheta \exp(-j\beta r) \\ H_{\varphi} = \dfrac{E_{\vartheta}}{\zeta} \end{cases}$$



- the e.m. field propagates along $\hat{i}_r$
- the e.m. field lies on the plane orthogonal to the propagation direction
- |E| and |H| exhibit the decaying factor 1/r
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$$\begin{cases} E_{\vartheta} = j\zeta \dfrac{I\Delta z}{2\lambda r}\sin\vartheta \exp(-j\beta r) \\ \boxed{H_{\varphi} = \dfrac{E_{\vartheta}}{\zeta}} \end{cases}$$

$$\zeta\vec{\mathbf{H}} = \zeta \mathrm{H}_{\varphi}\hat{i}_{\varphi} = E_{\vartheta}\hat{i}_{\varphi}$$

$$\hat{i}_r \times \vec{\mathbf{E}} = \hat{i}_r \times E_{\vartheta}\hat{i}_{\vartheta} = E_{\vartheta}\hat{i}_{\varphi}$$

$$\Rightarrow \quad \zeta\mathbf{H} = \hat{i}_r \times \mathbf{E}$$

$$\hat{i}_{\varphi} = \hat{i}_r \times \hat{i}_{\vartheta}$$
$$\hat{i}_{\vartheta} = \hat{i}_{\varphi} \times \hat{i}_r$$
$$\hat{i}_r = \hat{i}_{\vartheta} \times \hat{i}_{\varphi}$$

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$$\begin{cases} E_\vartheta = j\zeta \dfrac{I\Delta z}{2\lambda r}\sin\vartheta \exp(-j\beta r) \\ \boxed{H_\varphi = \dfrac{E_\vartheta}{\zeta}} \end{cases}$$

$$\vec{\mathbf{S}} = \frac{1}{2}\vec{\mathbf{E}}\times\vec{\mathbf{H}}^* = \frac{1}{2}E_\vartheta\hat{i}_\vartheta\times\left(H_\varphi\hat{i}_\varphi\right)^* \boxed{= \frac{1}{2}E_\vartheta H_\varphi^*\hat{i}_r}$$

$$\hat{i}_\varphi = \hat{i}_r\times\hat{i}_\vartheta$$

$$\hat{i}_\vartheta = \hat{i}_\varphi\times\hat{i}_r$$

$$\hat{i}_r = \hat{i}_\vartheta\times\hat{i}_\varphi$$

$$\frac{1}{2}E_\vartheta H_\varphi^*\hat{i}_r = \frac{1}{2\zeta}E_\vartheta E_\vartheta^*\hat{i}_r = \frac{1}{2\zeta}\left|E_\vartheta\right|^2\hat{i}_r = \frac{1}{2\zeta}\left|\vec{\mathbf{E}}\right|^2\hat{i}_r$$

$$\frac{1}{2}E_\vartheta H_\varphi^*\hat{i}_r = \frac{1}{2}\zeta H_\varphi H_\varphi^*\hat{i}_r = \frac{\zeta}{2}\left|H_\varphi\right|^2\hat{i}_r = \frac{\zeta}{2}\left|\vec{\mathbf{H}}\right|^2\hat{i}_r$$

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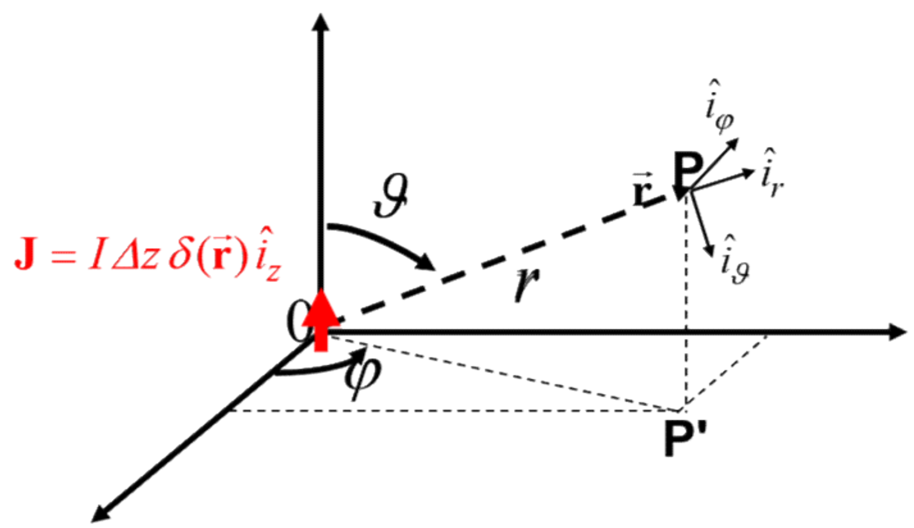
Elementary electrical dipole: far field

In the far-field case (r>>λ) the elementary electrical dipole behaves as follows

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathrm{E}_{\vartheta}(r,\vartheta)\hat{i}_{\vartheta} \qquad \begin{cases} E_{\vartheta} = j\zeta \dfrac{I\Delta z}{2\lambda r}\sin\vartheta \exp(-j\beta r) \\ H_{\varphi} = \dfrac{E_{\vartheta}}{\zeta} \end{cases}$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \mathrm{H}_{\varphi}(r,\vartheta)\hat{i}_{\varphi}$$



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MEMO .....

The radiation condition

$$\vec{\mathbf{E}} \sim O\left(\frac{1}{r}\right) \quad \vec{\mathbf{H}} \sim O\left(\frac{1}{r}\right) \quad \vec{\mathbf{E}} - \zeta\vec{\mathbf{H}}\times\hat{\mathbf{n}} \sim o\left(\frac{1}{r}\right) \quad \left(and\ \zeta\vec{\mathbf{H}} - \hat{\mathbf{n}}\times\vec{\mathbf{E}} \sim o\left(\frac{1}{r}\right)\right) \quad as\ \ r\to\infty$$

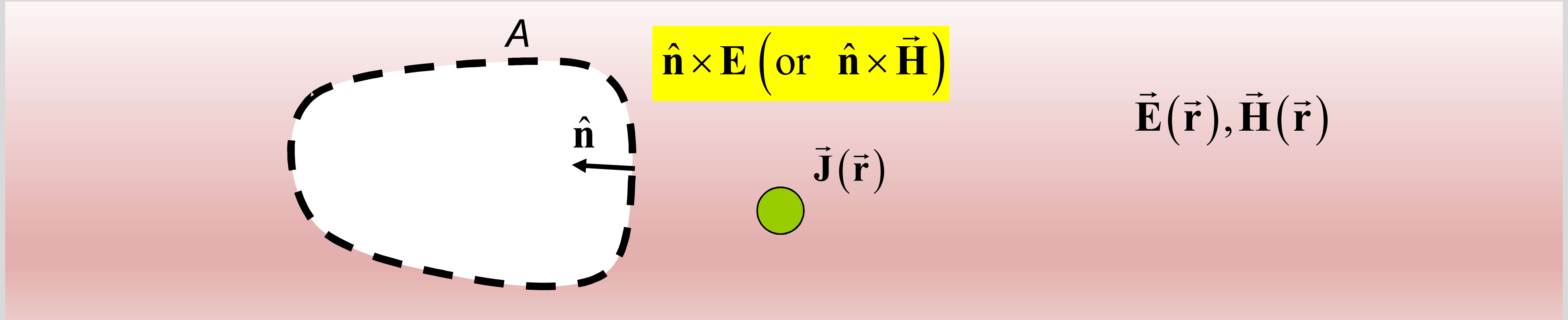
PD

$$\hat{\mathbf{n}}\cdot\vec{\mathbf{E}} = \hat{\mathbf{n}}\cdot\vec{\mathbf{H}} = 0$$

$$\zeta\mathbf{H} = \hat{i}_r \times \mathbf{E}$$

$$\vec{\mathbf{S}} = \frac{1}{2\zeta}\left|\vec{\mathbf{E}}\right|^2\hat{i}_r = \frac{\zeta}{2}\left|\vec{\mathbf{H}}\right|^2\hat{i}_r$$

MEMO…….Uniqueness (PD-Exterior Problem)



I Consider a source distribution $\vec{\mathbf{J}}(\vec{\mathbf{r}})$ with its associated electromagnetic field $\vec{\mathbf{E}}(\vec{\mathbf{r}}), \vec{\mathbf{H}}(\vec{\mathbf{r}})$

II Consider a (smooth) surface *A* with an everywhere defined unit normal $\hat{\mathbf{n}}$

IV Consider the values of the tangential component of the electric (**or** magnetic) field upon the surface A; that is, consider $\hat{\mathbf{n}} \times \mathbf{E} \left(\text{or} \;\; \hat{\mathbf{n}} \times \vec{\mathbf{H}} \right)$ **on the boundary**

The Uniqueness Theorem states that the electromagnetic field produced by the source in (I) within the **infinite volume V outside** the surface A in (II), enforcing **the boundary condition** in (IV) **as well as the radiation condition at infinity** is unique.

Elementary electrical dipole: far field

In the far-field case (r>>λ) the elementary electrical dipole behaves as follows

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathrm{E}_{\vartheta}(r,\vartheta)\hat{i}_{\vartheta} \qquad \begin{cases} E_{\vartheta} = j\zeta \dfrac{I\Delta z}{2\lambda r}\sin\vartheta \exp(-j\beta r) \\ H_{\varphi} = \dfrac{E_{\vartheta}}{\zeta} \end{cases}$$

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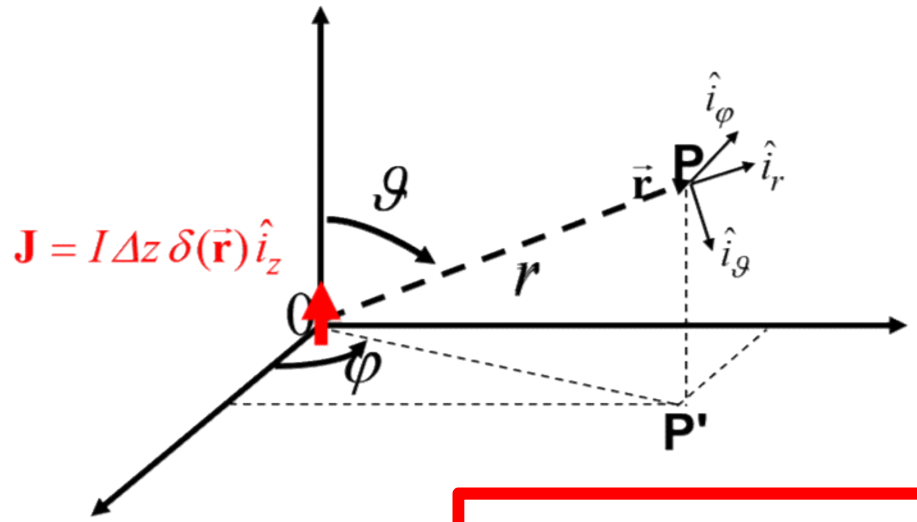

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- |E| and |H| are proportional through ζ

$$\zeta\mathbf{H} = \hat{i}_r \times \mathbf{E}$$

$$\vec{\mathbf{S}} = \frac{1}{2\zeta}\left|\vec{\mathbf{E}}\right|^2 \hat{i}_r = \frac{\zeta}{2}\left|\vec{\mathbf{H}}\right|^2 \hat{i}_r$$

Uniqueness is guaranteed

Elementary electrical dipole: far field

In the far-field case (r>>λ) the elementary electrical dipole behaves as follows

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathrm{E}_\vartheta(r,\vartheta)\hat{i}_\vartheta \qquad \begin{cases} E_\vartheta = j\zeta \dfrac{I\Delta z}{2\lambda r}\sin\vartheta \exp(-j\beta r) \\ H_\varphi = \dfrac{E_\vartheta}{\zeta} \end{cases}$$

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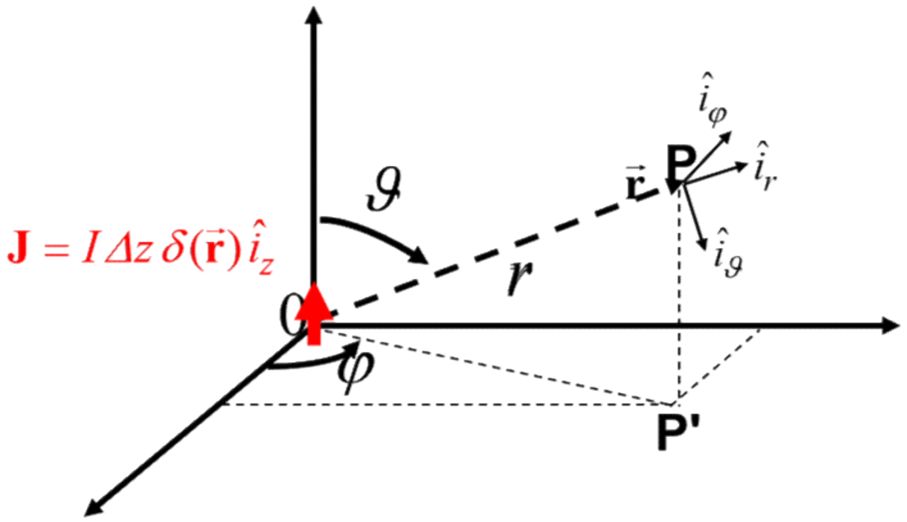

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....MEMO......Plane Waves (TD)

$\{e_x, h_y\}$

$$\frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

$\{e_x^+, h_y^+\}$

$$\vec{\mathbf{s}}^+ = \frac{\left|e_x^+(z-ct)\right|^2}{\zeta}\hat{i}_z$$

$\{e_x^-, h_y^-\}$

$$\vec{\mathbf{s}}^- = -\frac{\left|e_x^-(z+ct)\right|^2}{\zeta}\hat{i}_z$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{e_y, h_x\}$

$$\frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2} = 0$$

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$$\vec{\mathbf{s}}^+ = \frac{\left|e_y^+(z-ct)\right|^2}{\zeta}\hat{i}_z$$

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$$\vec{\mathbf{s}}^- = -\frac{\left|e_y^-(z+ct)\right|^2}{\zeta}\hat{i}_z$$

$\{e_x^+, h_y^+\}$ $\{e_x^-, h_y^-\}$ $\{e_y^+, h_x^+\}$ $\{e_y^-, h_x^-\}$

In all these 4 cases the electric and magnetic fields are related each other through the following relation:

$$\zeta\vec{\mathbf{h}} = \hat{i}_p \times \vec{\mathbf{e}}$$

where $\hat{i}_p$ points to the propagation direction

Source-free

Medium

- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$\Downarrow$$

$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$

$\{e_x, h_y\}$

Independent each other

..MEMO... Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$E_x^+(z) = E^+ e^{-j\beta z}$$
$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c \qquad \zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = |E^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

<u>Progressive</u> plane wave

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \frac{1}{2}\vec{\mathbf{E}}(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^*(\vec{\mathbf{r}})$$
$$\vec{\mathbf{S}}^+(\vec{\mathbf{r}}) = \frac{1}{2}\vec{\mathbf{E}}^+(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^{+*}(\vec{\mathbf{r}}) = \frac{|E_x^+(z)|^2}{2\zeta}\hat{i}_z = \zeta\frac{|H_y^+(z)|^2}{2}\hat{i}_z$$

Source-free

Medium
- **Linear**
- **Time nondispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\varepsilon_{eq} = \varepsilon$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

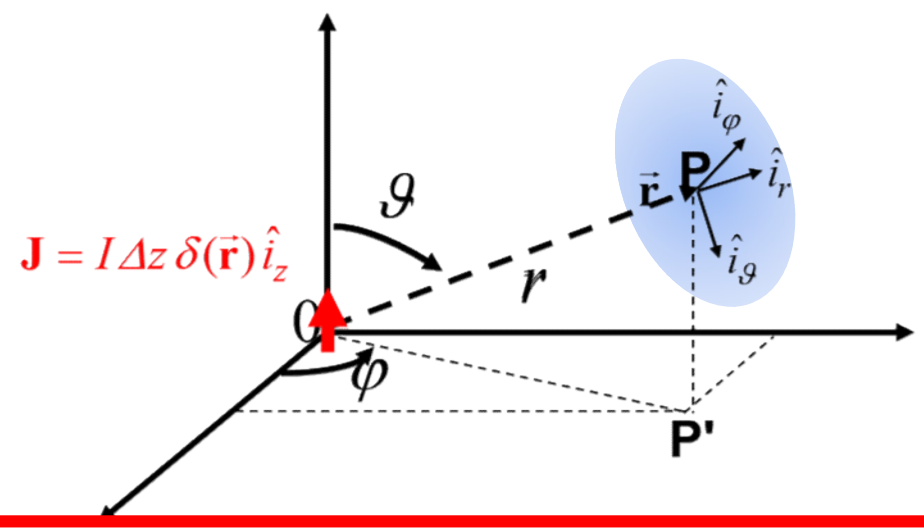
$\{E_y, H_x\}$
$\{E_x, H_y\}$ **Independent each other**

Elementary electrical dipole: far field

In the far-field case (r>>λ) the elementary electrical dipole behaves as follows

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = E_\vartheta(r,\vartheta)\,\hat{i}_\vartheta \qquad \begin{cases} E_\vartheta = j\zeta \dfrac{I\Delta z}{2\lambda r}\sin\vartheta \exp(-j\beta r) \\ H_\varphi = \dfrac{E_\vartheta}{\zeta} \end{cases}$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = H_\varphi(r,\vartheta)\,\hat{i}_\varphi$$



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$$\vec{\mathbf{S}} = \frac{1}{2\zeta}\left|\vec{\mathbf{E}}\right|^2 \hat{i}_r = \frac{\zeta}{2}\left|\vec{\mathbf{H}}\right|^2 \hat{i}_r$$

In the far field case, this field locally behaves as a plane wave