Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2020-2021 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

Stefano Perna

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Plane Waves

Time domain (TD)

Spectral domains

- Phasor Domain (PD)
- Fourier Domain (FD)
- Dispersive media: attenuation, distortion, phase velocity and group velocity

General expression of plane waves (PD)

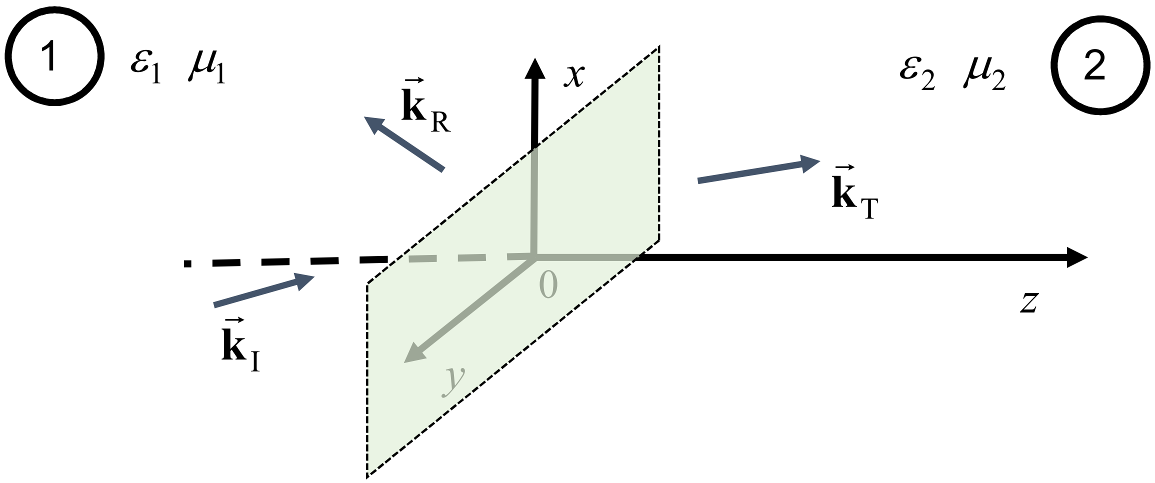
Incidence (PD)

Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_{\mathrm{I}x}x} e^{-jk_{\mathrm{I}y}y} e^{-jk_{\mathrm{I}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{I}} = k_{\mathrm{I}x}\hat{i}_x + k_{\mathrm{I}y}\hat{i}_y + k_{\mathrm{I}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{k}}_{\mathrm{I}} = k_{\mathrm{I}x}^2 + k_{\mathrm{Iy}}^2 + k_{\mathrm{Iz}}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_{\mathrm{R}x}x} e^{-jk_{\mathrm{R}y}y} e^{-jk_{\mathrm{R}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{R}} = k_{\mathrm{R}x}\hat{i}_x + k_{\mathrm{R}y}\hat{i}_y + k_{\mathrm{R}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{k}}_{\mathrm{R}} = k_{\mathrm{R}x}^2 + k_{\mathrm{Ry}}^2 + k_{\mathrm{Rz}}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

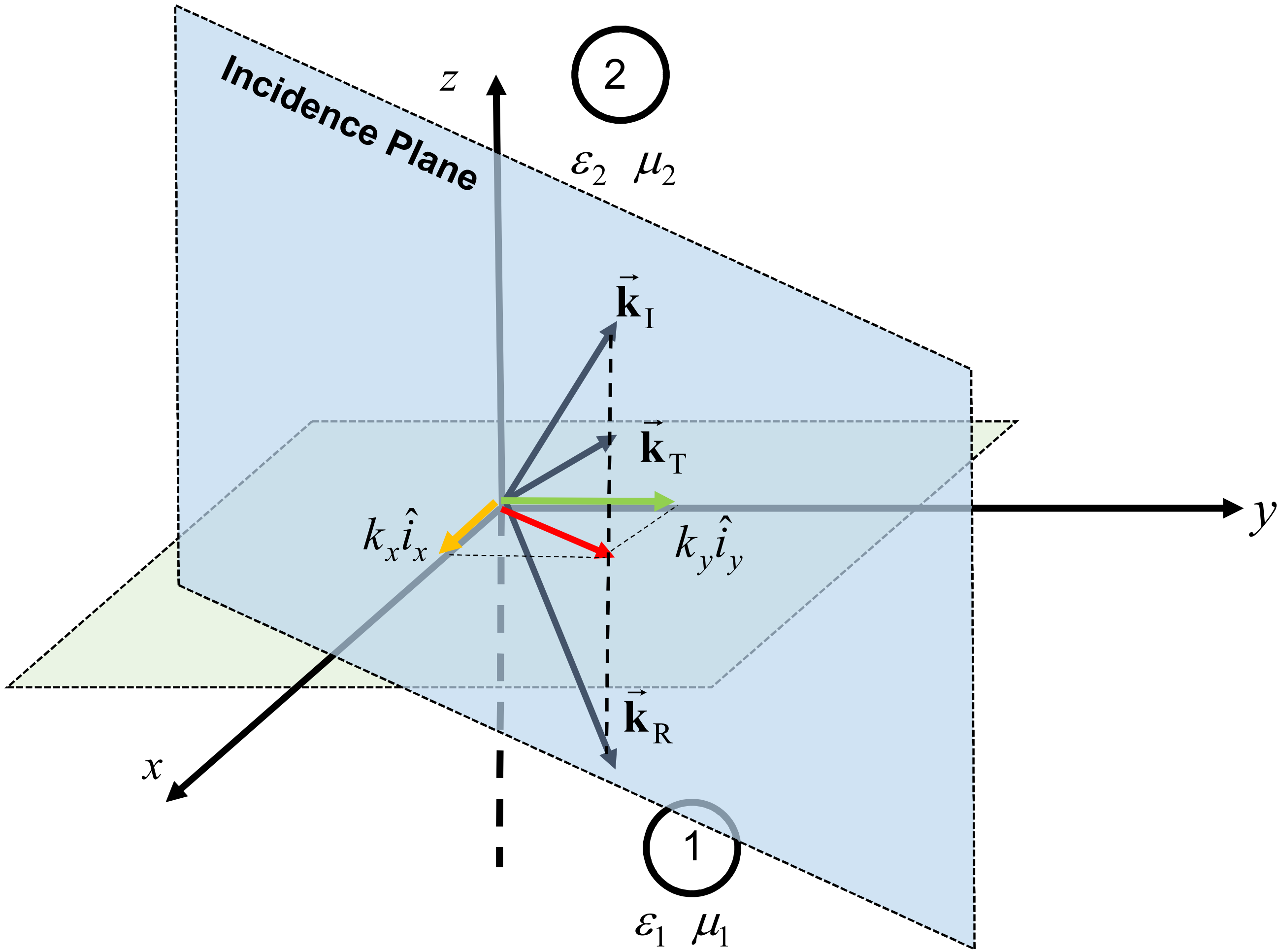
$$\vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_{\mathrm{T}x}x} e^{-jk_{\mathrm{T}y}y} e^{-jk_{\mathrm{T}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{T}} = k_{\mathrm{T}x}\hat{i}_x + k_{\mathrm{T}y}\hat{i}_y + k_{\mathrm{T}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{k}}_{\mathrm{T}} = k_{\mathrm{T}x}^2 + k_{\mathrm{Ty}}^2 + k_{\mathrm{Tz}}^2 = \omega^2\mu_2\varepsilon_2 = k_2^2$$



$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}}$$

Incidence on a dielectric half-space

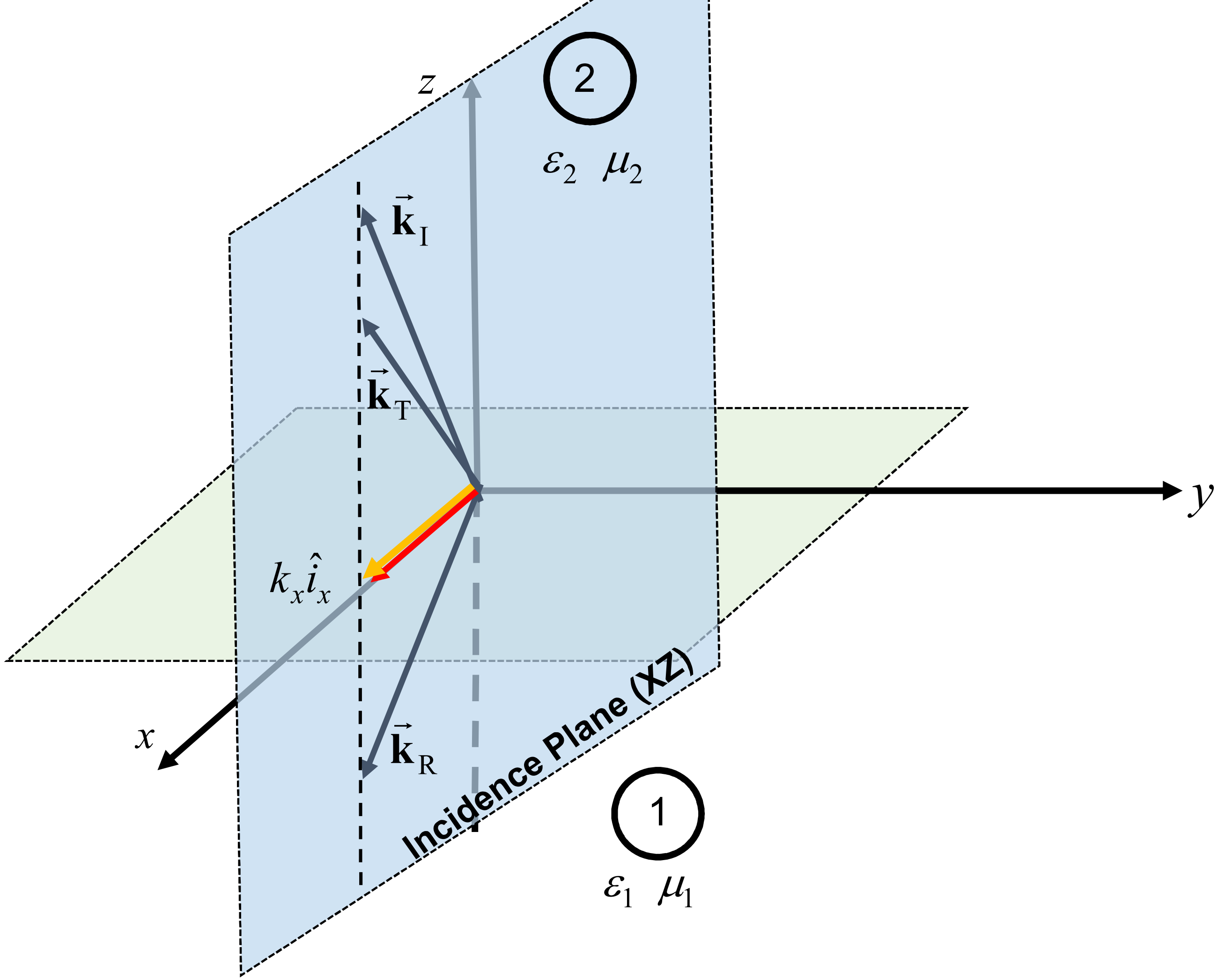


$$\vec{\mathbf{k}}_{\mathrm{I}} = k_x\hat{i}_x + k_y\hat{i}_y + k_{\mathrm{I}z}\hat{i}_z$$

$$\vec{\mathbf{k}}_{\mathrm{R}} = k_x\hat{i}_x + k_y\hat{i}_y + k_{\mathrm{R}z}\hat{i}_z$$

$$\vec{\mathbf{k}}_{\mathrm{T}} = k_x\hat{i}_x + k_y\hat{i}_y + k_{\mathrm{T}z}\hat{i}_z$$

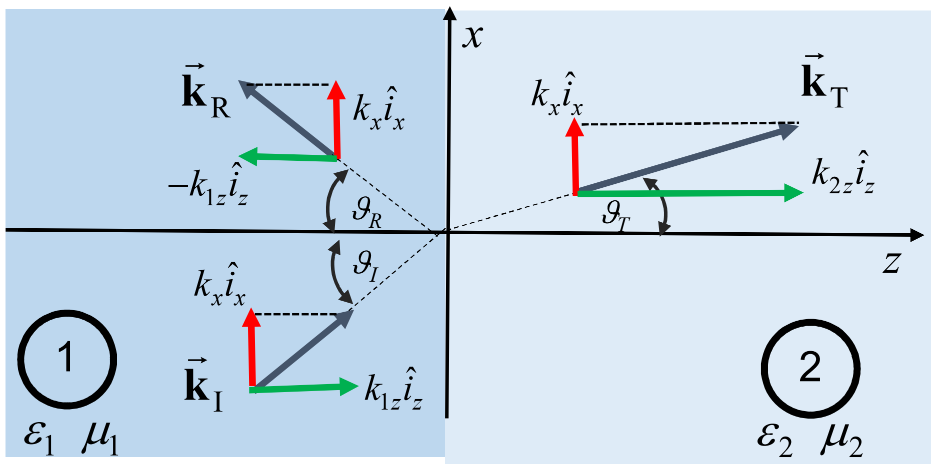
Incidence on a dielectric half-space



$$\vec{\mathbf{k}}_{\mathrm{I}} = k_x\hat{i}_x + k_{\mathrm{I}z}\hat{i}_z$$

$$\vec{\mathbf{k}}_{\mathrm{R}} = k_x\hat{i}_x + k_{\mathrm{R}z}\hat{i}_z$$

$$\vec{\mathbf{k}}_{\mathrm{T}} = k_x\hat{i}_x + k_{\mathrm{T}z}\hat{i}_z$$



Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2 = \vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_x x} e^{-jk_{1z} z}$$

$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_x x} e^{jk_{1z} z}$$

$$\vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_x x} e^{-jk_{2z} z}$$

$$k_1 = \omega\sqrt{\mu_1 \varepsilon_1}$$

$$k_2 = \omega\sqrt{\mu_2 \varepsilon_2}$$

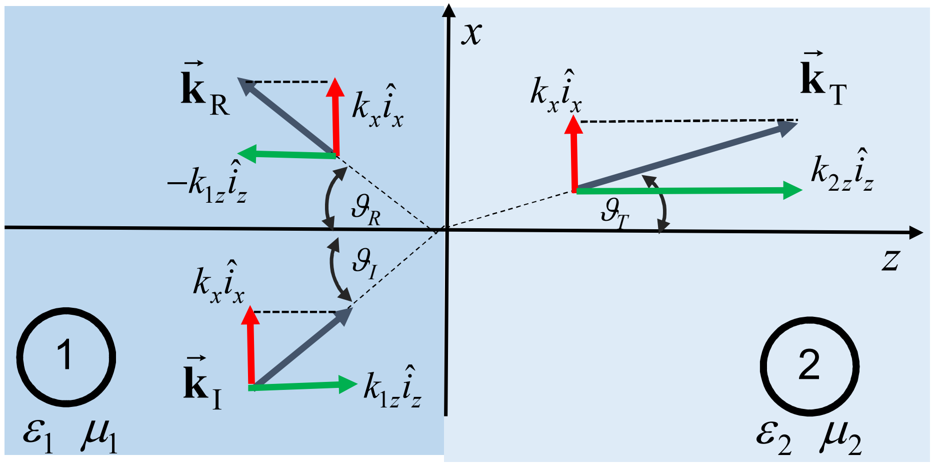
$$k_x = k_1 \sin\vartheta_{\mathrm{I}}$$

$$k_{1z} = k_1 \cos\vartheta_{\mathrm{I}}$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin\vartheta_{\mathrm{I}} = k_2 \sin\vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_{\mathrm{I}}}$$



Incidence on a dielectric half-space

$$\begin{cases} \nabla \times \vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}} \\ \nabla \times \vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}} \end{cases}$$

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2 = \vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_I e^{-jk_x x} e^{-jk_{1z} z}$$

$$\vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_R e^{-jk_x x} e^{jk_{1z} z}$$

$$\vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_T e^{-jk_x x} e^{-jk_{2z} z}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$

$$k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_I$$

$$k_{1z} = k_1 \cos\vartheta_I$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin\vartheta_I = k_2 \sin\vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_I}$$

$[H_x, E_y, H_z]$ **Perpendicular Polarization** $\perp$

$$-\frac{\partial E_y}{\partial z} = -j\omega\mu H_x$$

$$\frac{\partial H_x}{\partial z} + jk_x H_z = j\omega\varepsilon E_y$$

$$-jk_x E_y = -j\omega\mu H_z$$

$[E_x, H_y, E_z]$ **Parallel Polarization** $\parallel$

$$-\frac{\partial H_y}{\partial z} = j\omega\varepsilon E_x$$

$$\frac{\partial E_x}{\partial z} + jk_x E_z = -j\omega\mu H_y$$

$$-jk_x H_y = j\omega\varepsilon E_z$$

Incidence on a dielectric half-space: ⊥ polarization

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases} \qquad \Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega\sqrt{\mu_1 \varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2 \varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_I \qquad k_{1z} = k_1 \cos\vartheta_I$$

$$\vartheta_I = \vartheta_R \qquad k_1 \sin\vartheta_I = k_2 \sin\vartheta_T \qquad k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_I}$$

$[H_x, E_y, H_z]$ **Perpendicular Polarization** ⊥

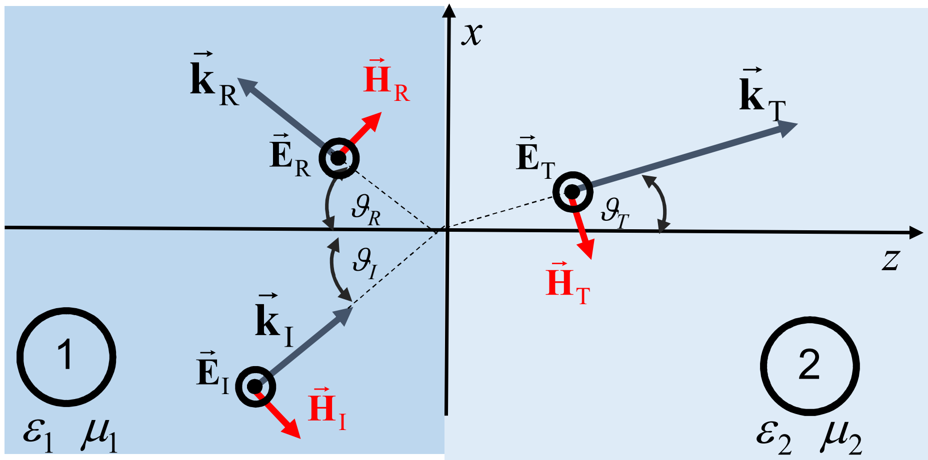
$$E_{1y}(z,x) = \left[E_I e^{-jk_{1z}z} + E_R e^{jk_{1z}z}\right] e^{-jk_x x} \quad (1)$$

$$E_{2y}(z,x) = E_T e^{-jk_{2z}z} e^{-jk_x x} \quad (2)$$

$$\Gamma_\perp \triangleq \frac{E_R}{E_I} \qquad Z_1 = \frac{\omega\mu_1}{k_{1z}}$$

$$\mathrm{T}_\perp \triangleq \frac{E_T}{E_I} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

$$\Gamma_\perp = \frac{\cos\vartheta_I - (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}{\cos\vartheta_I + (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}$$



Incidence on a dielectric half-space: ‖ polarization

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_\mathrm{I} \qquad k_{1z} = k_1 \cos\vartheta_\mathrm{I}$$

$$\vartheta_I = \vartheta_R \qquad k_1 \sin\vartheta_\mathrm{I} = k_2 \sin\vartheta_T \qquad k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_\mathrm{I}}$$

$[E_x, H_y, E_z]$ **Parallel Polarization** ‖

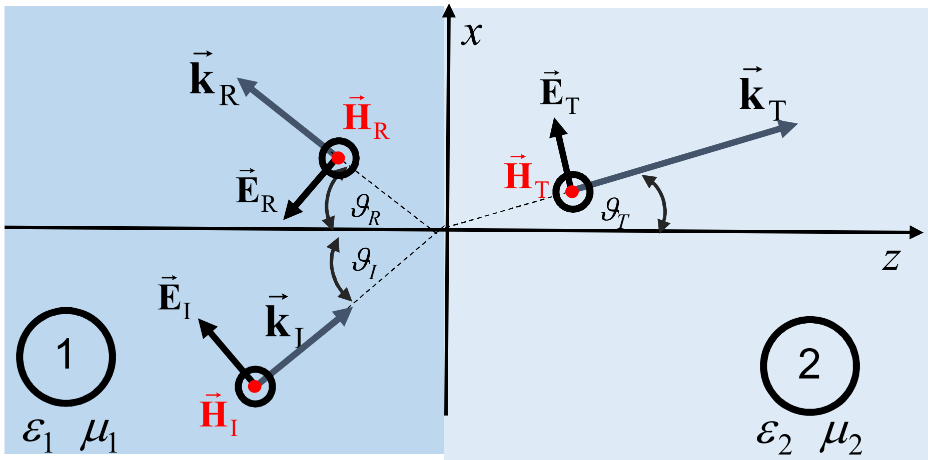
$$H_{1y}(z,x) = \left[H_\mathrm{I} e^{-jk_{1z}z} + H_\mathrm{R} e^{jk_{1z}z}\right] e^{-jk_x x} \quad (1)$$

$$H_{2y}(z,x) = H_\mathrm{T} e^{-jk_{2z}z} e^{-jk_x x} \quad (2)$$

$$\Gamma_\parallel \triangleq -\frac{H_\mathrm{R}}{H_\mathrm{I}} \qquad Z_1 = \frac{k_{1z}}{\omega\varepsilon_1}$$

$$\mathrm{T}_\parallel \triangleq \frac{Z_2 H_\mathrm{T}}{Z_1 H_\mathrm{I}} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$\Gamma_\parallel = -\frac{\cos\vartheta_\mathrm{I} - (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}{\cos\vartheta_\mathrm{I} + (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}$$



Incidence on a dielectric half-space

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_\mathrm{I} \qquad k_{1z} = k_1 \cos\vartheta_\mathrm{I}$$

$$\vartheta_I = \vartheta_R \qquad k_1 \sin\vartheta_\mathrm{I} = k_2 \sin\vartheta_T \qquad k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_\mathrm{I}}$$

$\left[H_x, E_y, H_z\right]$ Perpendicular Polarization $\perp$

$$E_{1y}(z,x) = \left[E_\mathrm{I} e^{-jk_{1z}z} + E_\mathrm{R} e^{jk_{1z}z}\right] e^{-jk_x x} \quad (1)$$

$$E_{2y}(z,x) = E_\mathrm{T} e^{-jk_{2z}z} e^{-jk_x x} \quad (2)$$

$$\Gamma_\perp \triangleq \frac{E_\mathrm{R}}{E_\mathrm{I}} \qquad Z_1 = \frac{\omega\mu_1}{k_{1z}}$$

$$\mathrm{T}_\perp \triangleq \frac{E_\mathrm{T}}{E_\mathrm{I}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

$$\Gamma_\perp = \frac{\cos\vartheta_\mathrm{I} - (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}{\cos\vartheta_\mathrm{I} + (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}$$

$\left[E_x, H_y, E_z\right]$ Parallel Polarization $\|$

$$H_{1y}(z,x) = \left[H_\mathrm{I} e^{-jk_{1z}z} + H_\mathrm{R} e^{jk_{1z}z}\right] e^{-jk_x x} \quad (1)$$

$$H_{2y}(z,x) = H_\mathrm{T} e^{-jk_{2z}z} e^{-jk_x x} \quad (2)$$

$$\Gamma_\| \triangleq -\frac{H_\mathrm{R}}{H_\mathrm{I}} \qquad Z_1 = \frac{k_{1z}}{\omega\varepsilon_1}$$

$$\mathrm{T}_\| \triangleq \frac{Z_2 H_\mathrm{T}}{Z_1 H_\mathrm{I}} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$\Gamma_\| = -\frac{\cos\vartheta_\mathrm{I} - (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}{\cos\vartheta_\mathrm{I} + (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}$$

Incidence on a dielectric half-space

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega\sqrt{\mu_1 \varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2 \varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_\mathrm{I} \qquad k_{1z} = k_1 \cos\vartheta_\mathrm{I}$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin\vartheta_\mathrm{I} = k_2 \sin\vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_\mathrm{I}}$$

$\left[H_x, E_y, H_z\right]$ **Perpendicular Polarization** $\perp$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

$$\Gamma_\perp = \frac{\cos\vartheta_\mathrm{I} - (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}{\cos\vartheta_\mathrm{I} + (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}$$

$\left[E_x, H_y, E_z\right]$ **Parallel Polarization** $\parallel$

$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$\Gamma_\parallel = -\frac{\cos\vartheta_\mathrm{I} - (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}{\cos\vartheta_\mathrm{I} + (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}$$

Incidence on a dielectric half-space

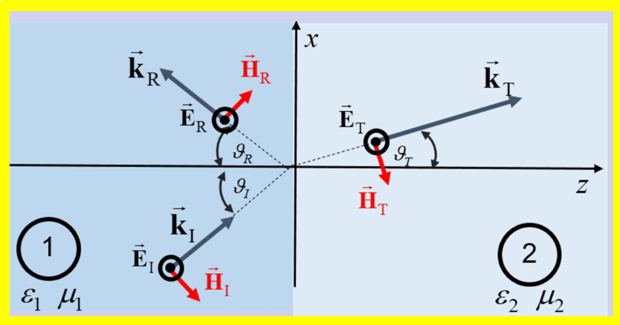
$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases} \qquad \Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_I \qquad k_{1z} = k_1 \cos\vartheta_I$$

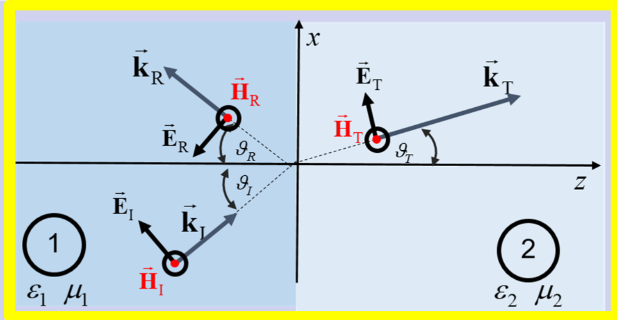
$$\vartheta_I = \vartheta_R \qquad k_1 \sin\vartheta_I = k_2 \sin\vartheta_T \qquad k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_I}$$

$$\left[H_x, E_y, H_z\right] \qquad Z_1 = \frac{\omega\mu_1}{k_{1z}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

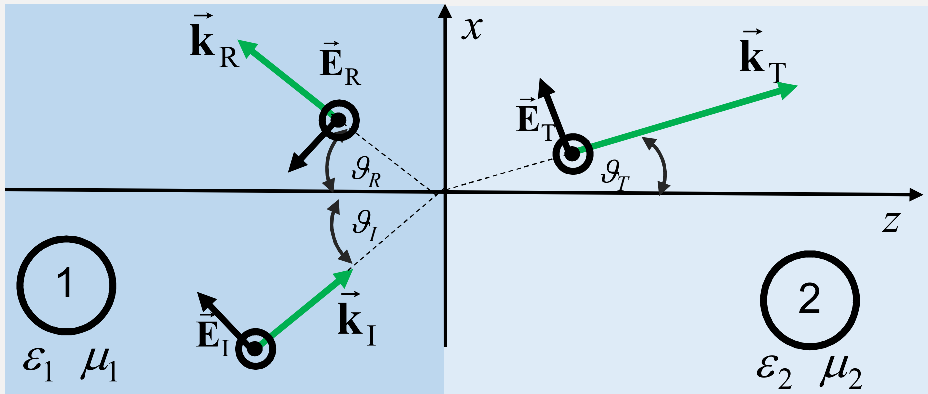


$\perp$

$$\left[E_x, H_y, E_z\right] \qquad Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

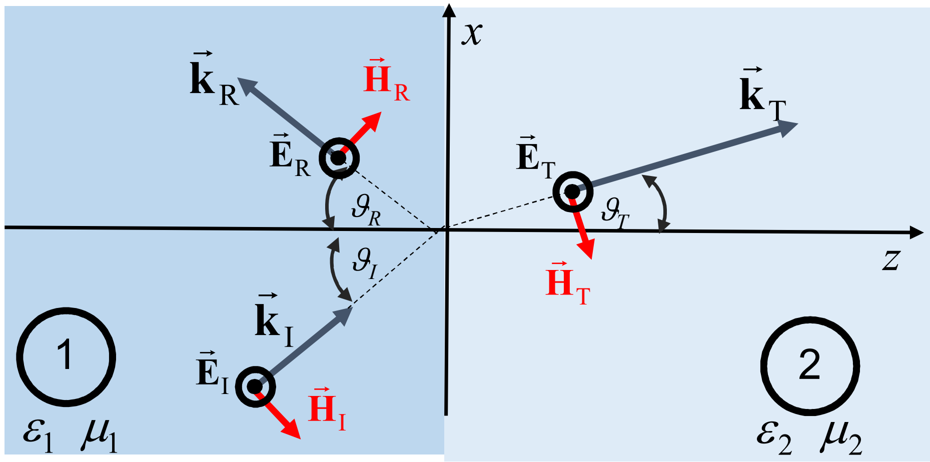


$\parallel$

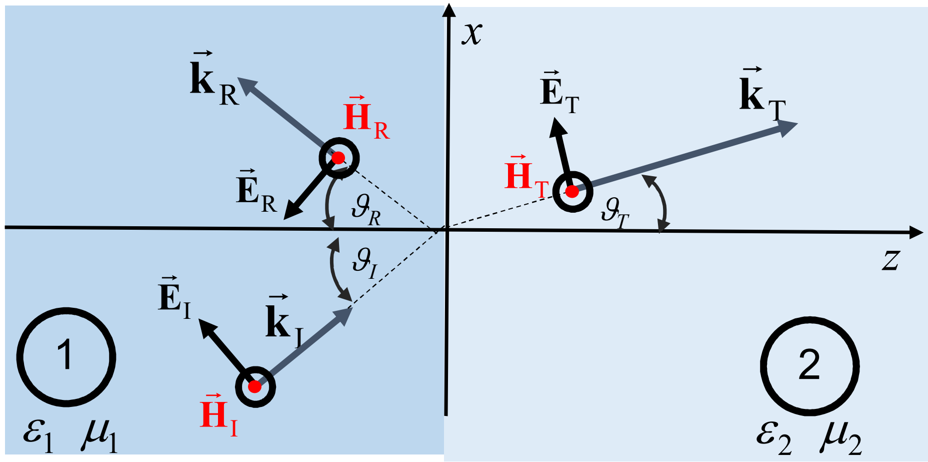


Incidence on a dielectric half-space

Perpendicular Polarization $\perp$

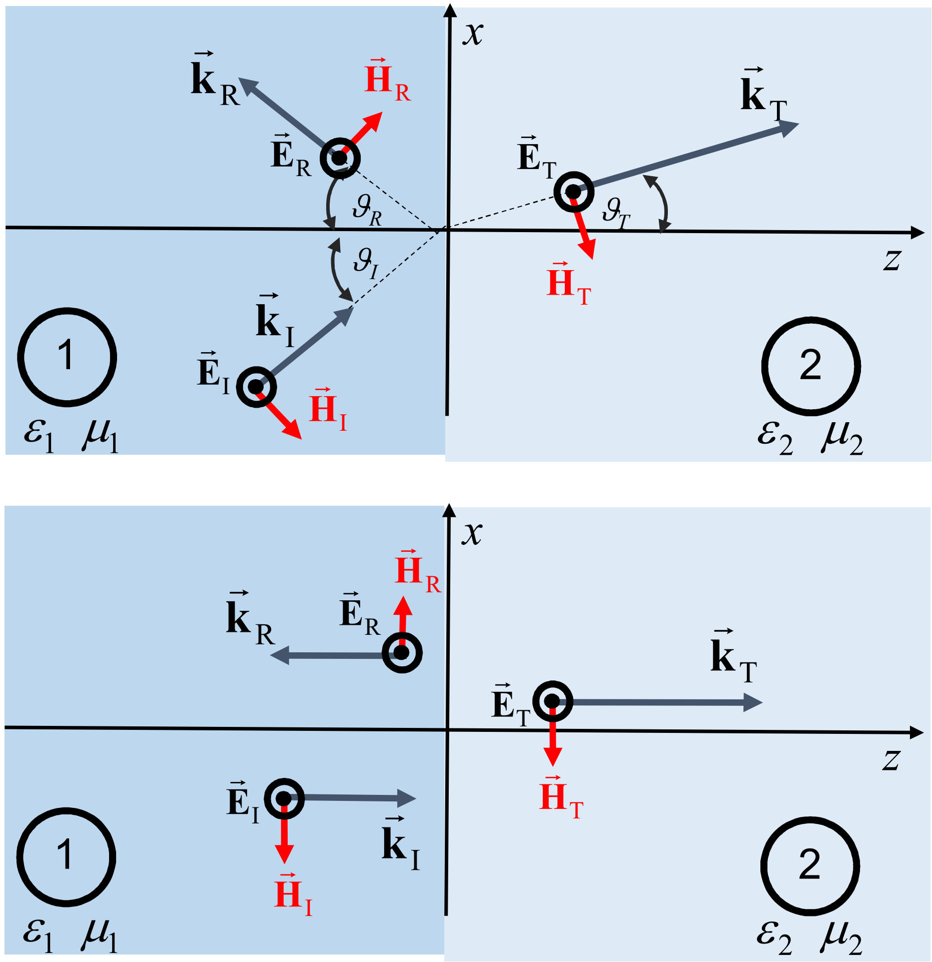


Parallel Polarization $\parallel$

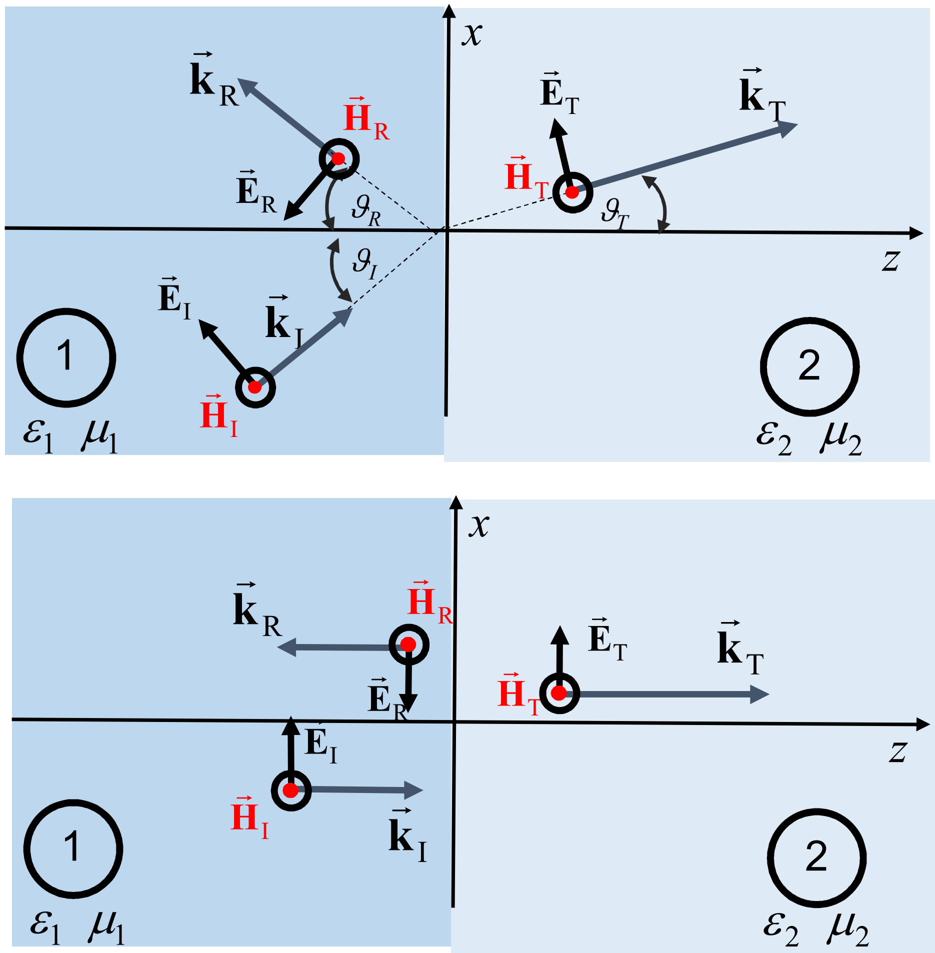


Incidence on a dielectric half-space

Perpendicular Polarization $\perp$



Parallel Polarization $\parallel$

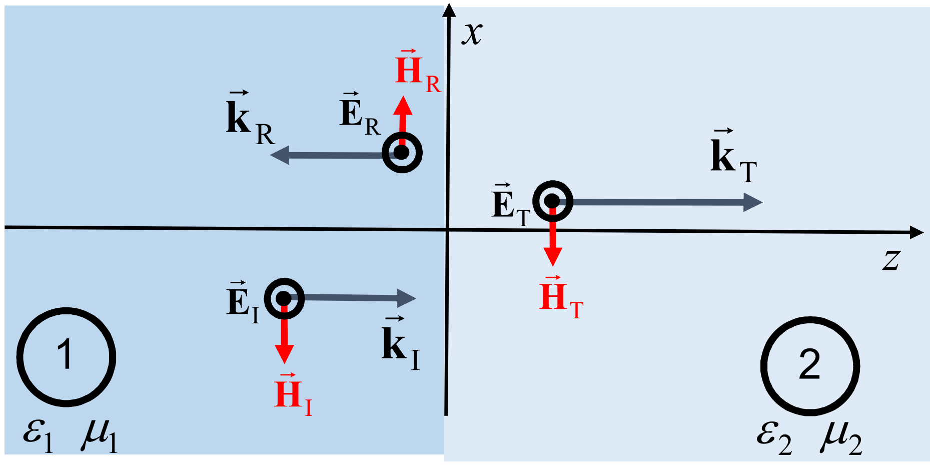


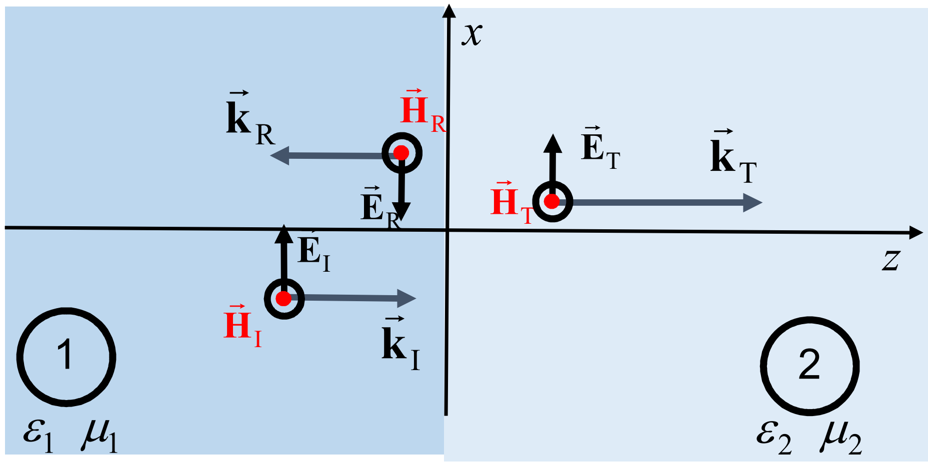
Normal Incidence

Perpendicular Polarization $\perp$

Parallel Polarization $\parallel$

In the case of normal incidence, perpendicular and parallel polarizations behave the same





Incidence: Limit and Brewster angles

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_I \qquad k_{1z} = k_1 \cos\vartheta_I$$

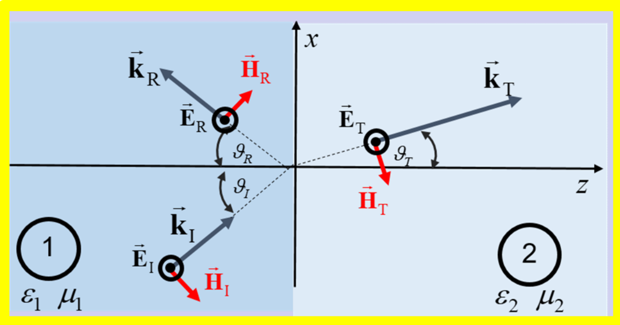
$$\vartheta_I = \vartheta_R$$

$$k_1 \sin\vartheta_I = k_2 \sin\vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_I}$$

$$\left[H_x, E_y, H_z\right]$$

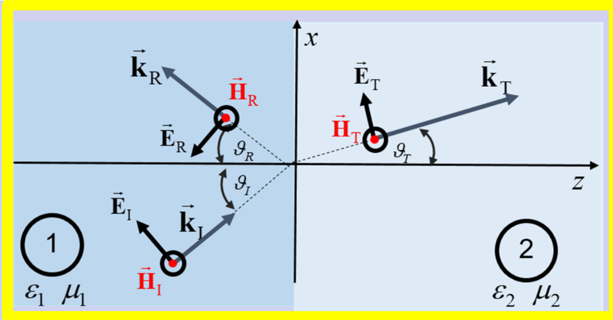
$$Z_1 = \frac{\omega\mu_1}{k_{1z}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$



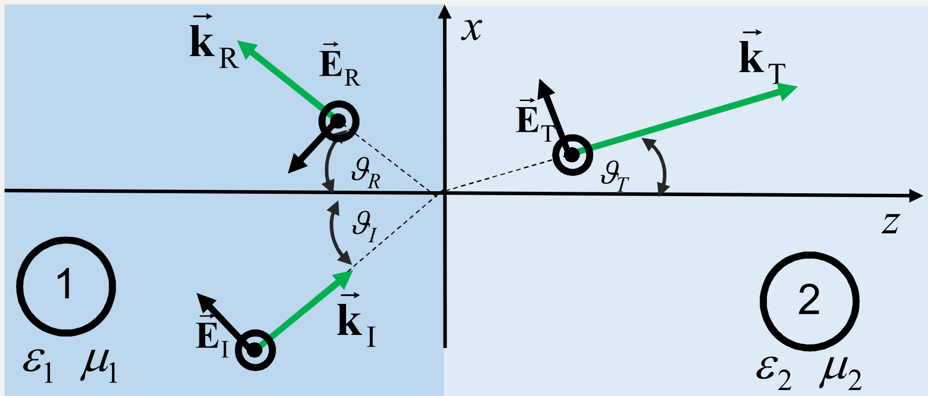
$\perp$

$$\left[E_x, H_y, E_z\right]$$

$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$



$\parallel$



Incidence: Brewster angle

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases} \qquad \Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

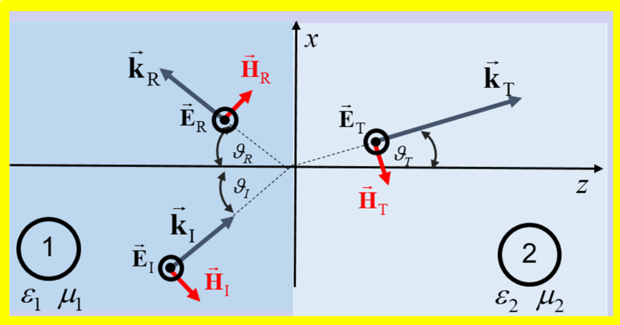
$$k_1 = \omega\sqrt{\mu_1\varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_{\mathrm{I}} \qquad k_{1z} = k_1 \cos\vartheta_{\mathrm{I}}$$

$$\vartheta_I = \vartheta_R \qquad k_1 \sin\vartheta_{\mathrm{I}} = k_2 \sin\vartheta_T \qquad k_{2z} = \sqrt{k_2^2 - k_1^2\sin^2\vartheta_{\mathrm{I}}}$$

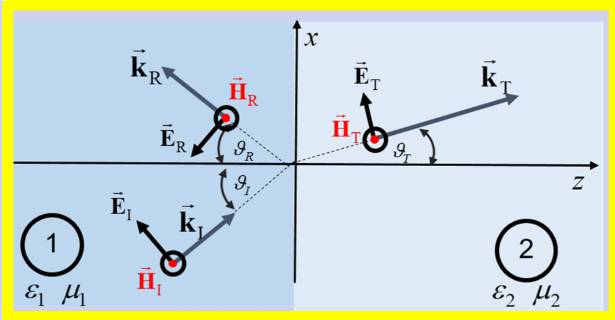
$[H_x, E_y, H_z]$ $\perp$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$



$[E_x, H_y, E_z]$ $\|$

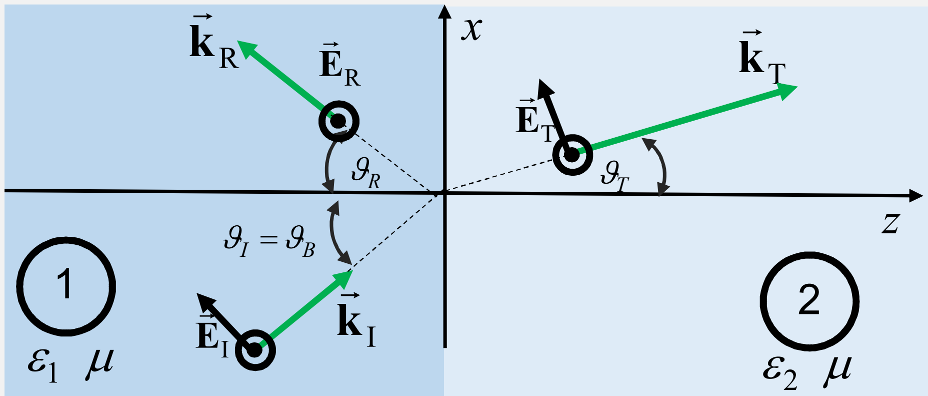
$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$



if $\mu_1 = \mu_2$ *and* $\varepsilon_1 \neq \varepsilon_2$

An angle ϑ_{B} exists, referred to as **Brewster angle**, such that an unpolarized plane wave incident at angle $\vartheta_I = \vartheta_{\mathrm{B}}$ is reflected with perpendicular polarization

$$\sin^2\vartheta_B = \frac{\varepsilon_2}{\varepsilon_1 + \varepsilon_2}$$



Incidence: Limit angle

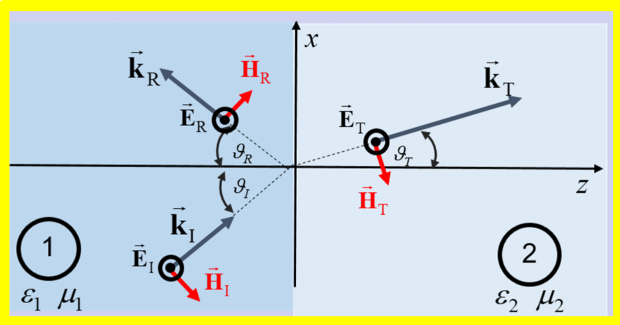
$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases} \qquad \Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega\sqrt{\mu_1 \varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2 \varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_I \qquad k_{1z} = k_1 \cos\vartheta_I$$

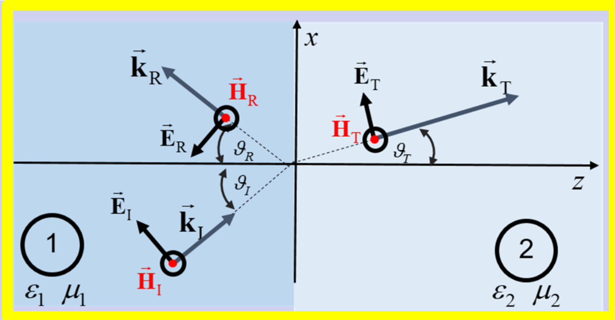
$$\vartheta_I = \vartheta_R \qquad k_1 \sin\vartheta_I = k_2 \sin\vartheta_T \qquad k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_I}$$

$$[H_x, E_y, H_z] \qquad Z_1 = \frac{\omega\mu_1}{k_{1z}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$



$\perp$

$$[E_x, H_y, E_z] \qquad Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

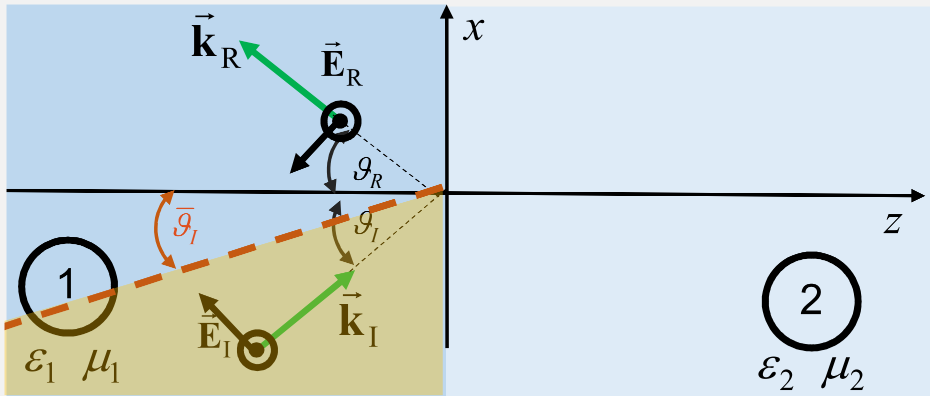


$\parallel$

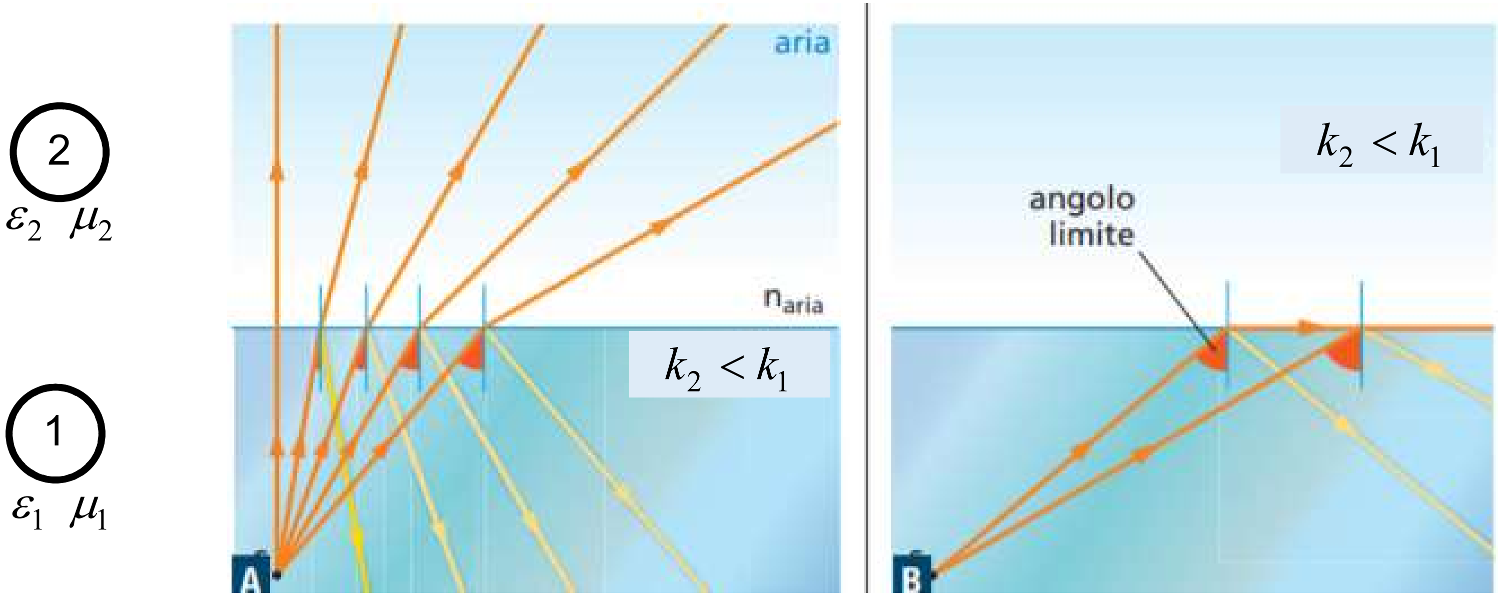
$$if \ \frac{k_2}{k_1} < 1$$

An angle $\overline{\vartheta}_I$ exists, referred to as **limit angle**, such that for $\vartheta_I \geq \overline{\vartheta}_I$ no propagation occurs in the second half-space

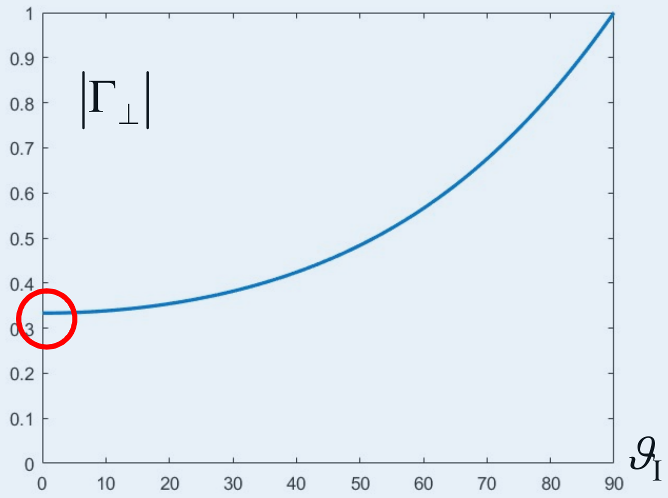
$$\sin\overline{\vartheta}_I = \frac{k_2}{k_1}$$



Incidence: Limit angle



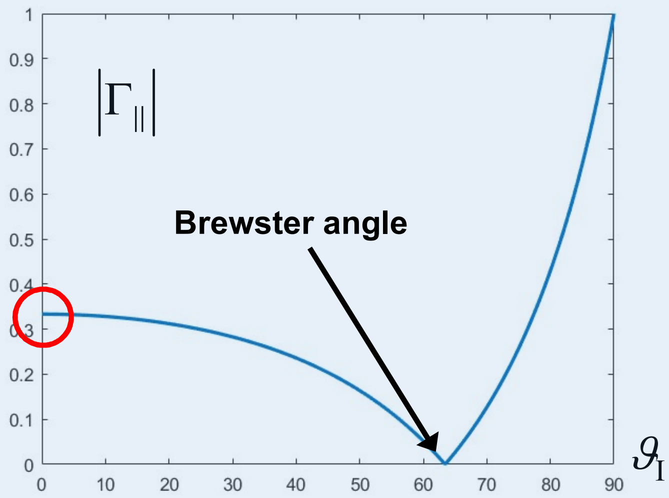
Fresnel coefficients



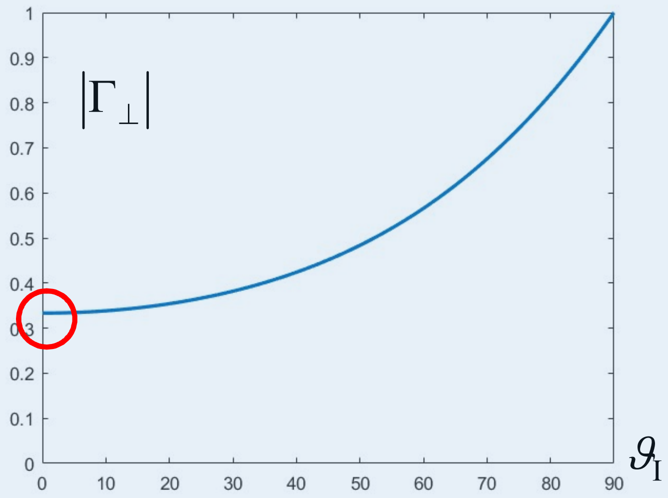
$$\mu_1 = \mu_2$$

$$\varepsilon_2 = 4\varepsilon_1$$

$$\frac{k_2}{k_1} > 1$$



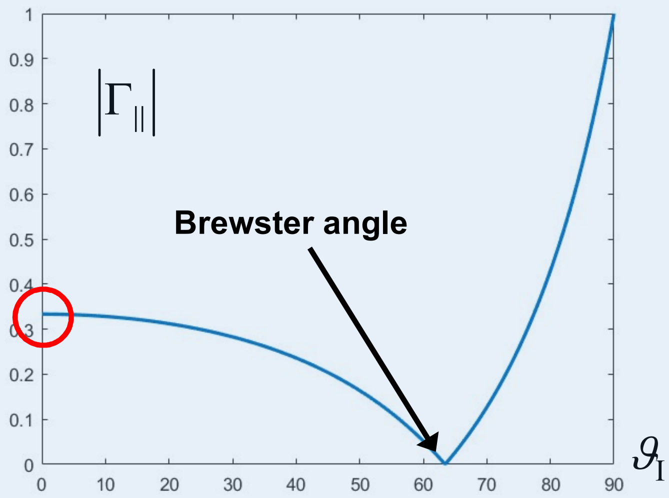
Fresnel coefficients

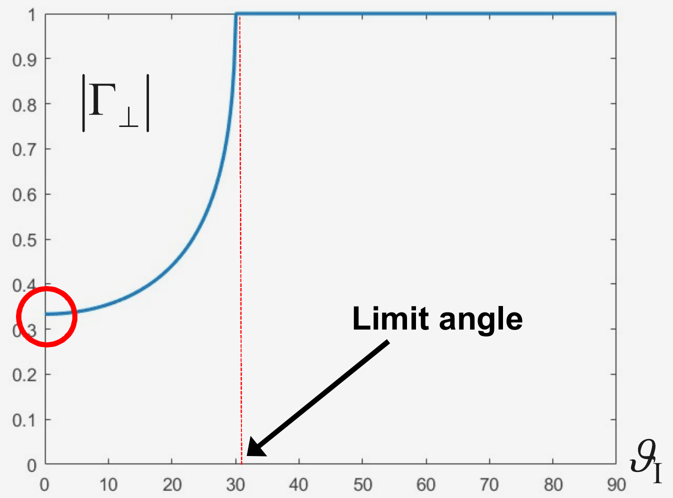


$\mu_1 = \mu_2$

$\varepsilon_2 = 4\varepsilon_1$

$\frac{k_2}{k_1} > 1$





$\mu_1 = \mu_2$

$4\varepsilon_2 = \varepsilon_1$

$\frac{k_2}{k_1} < 1$