

# **Campi Elettromagnetici**

**Corso di Laurea in Ingegneria Informatica,  
Biomedica e delle Telecomunicazioni**

**a.a. 2020-2021 - Laurea “Triennale” – Secondo semestre - Secondo anno**

**Università degli Studi di Napoli “Parthenope”**

**Stefano Perna**

# Color legend

New formulas, important considerations,  
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

# Plane Waves

Time domain (TD)

Spectral domains

Phasor Domain (PD)

Fourier Domain (FD)

Dispersive media: attenuation, distortion, phase velocity and group velocity

General expression of plane waves (PD)

Incidence (PD)

# General expression of plane waves (PD)

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^+ e^{-j\vec{\mathbf{k}} \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}^+ e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}^+ e^{-j\vec{\mathbf{k}} \cdot \vec{\mathbf{r}}} = \vec{\mathbf{H}}^+ e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

$$\vec{\mathbf{k}} = k_x \hat{i}_x + k_y \hat{i}_y + k_z \hat{i}_z$$

$$\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} = k_x x + k_y y + k_z z$$

$$\vec{\mathbf{r}} = x \hat{i}_x + y \hat{i}_y + z \hat{i}_z$$

Source-free

$$\vec{\mathbf{k}} \times \vec{\mathbf{E}} = \omega \mu \vec{\mathbf{H}}$$

$$\vec{\mathbf{k}} \times \vec{\mathbf{H}} = -\omega \epsilon \vec{\mathbf{E}}$$

$$\vec{\mathbf{k}} \cdot \vec{\mathbf{E}} = 0$$

$$\vec{\mathbf{k}} \cdot \vec{\mathbf{H}} = 0$$

$$\vec{\mathbf{k}} \cdot \vec{\mathbf{k}} = \omega^2 \mu \epsilon$$

$\vec{\mathbf{k}}$  : propagation vector

Medium

- Linear - Isotropic - Space nondispersive - Time dispersive - Lossy

- Homogeneous (Time-invariant & Space-invariant)

$$k = \omega \sqrt{\mu \epsilon} = \beta - j\alpha$$

$$\vec{\mathbf{k}} \cdot \vec{\mathbf{k}} = \omega^2 \mu \epsilon = (\beta - j\alpha)^2 \quad \Rightarrow \quad \vec{\mathbf{k}} = \vec{\mathbf{k}}_1 - j\vec{\mathbf{k}}_2$$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^+ e^{-j\vec{\mathbf{k}} \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}^+ e^{-j(\vec{\mathbf{k}}_1 - j\vec{\mathbf{k}}_2) \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}^+ e^{-j\vec{\mathbf{k}}_1 \cdot \vec{\mathbf{r}}} e^{-\vec{\mathbf{k}}_2 \cdot \vec{\mathbf{r}}}$$



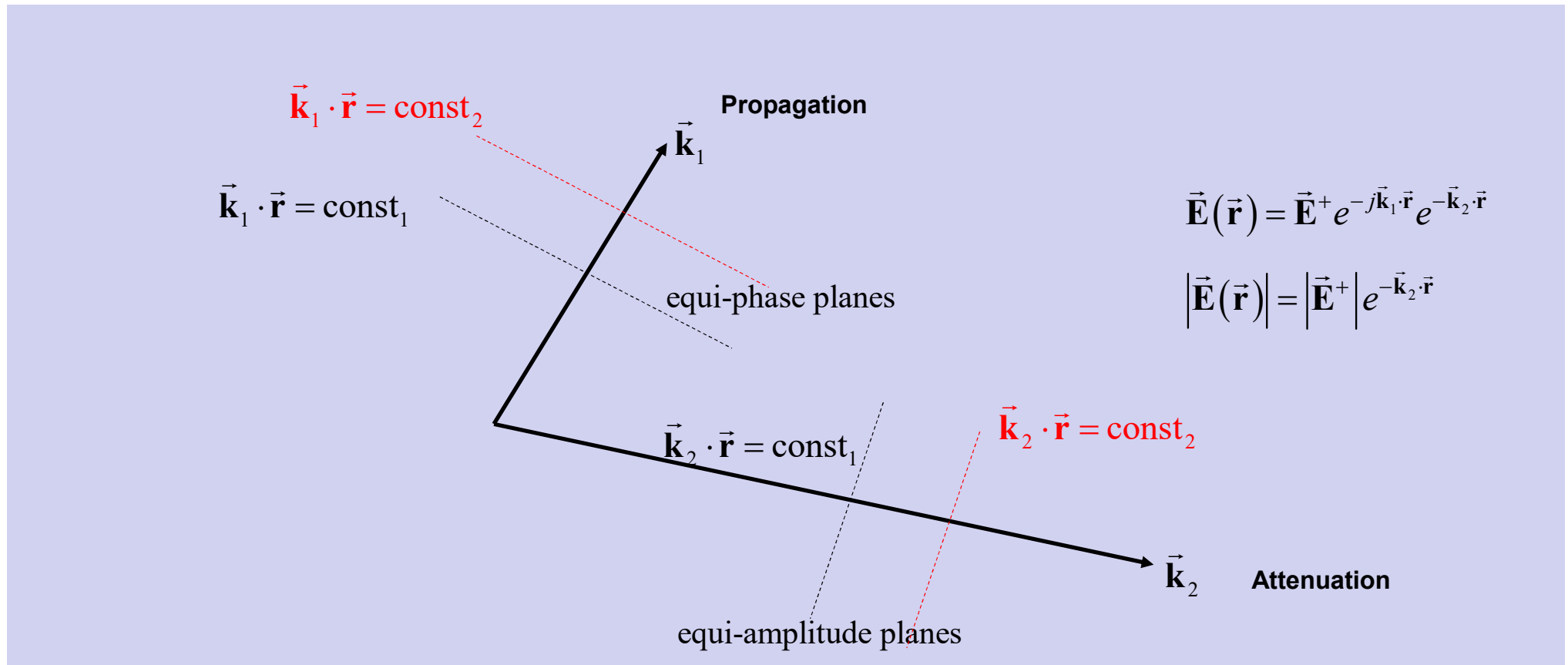
The wave propagates along  $\vec{\mathbf{k}}_1$

The wave attenuates along  $\vec{\mathbf{k}}_2$

$$\vec{\mathbf{k}}_1 \cdot \vec{\mathbf{r}} = \text{const} \Rightarrow \text{equi-phase planes}$$

$$\vec{\mathbf{k}}_2 \cdot \vec{\mathbf{r}} = \text{const} \Rightarrow \text{equi-amplitude planes}$$

# General expression of plane waves (PD)



When  $\vec{k}_1$  and  $\vec{k}_2$  are proportional, equi-amplitude and equi-phase planes become coincident: the plane wave is said **HOMOGENEOUS**

More generally, equi-amplitude and equi-phase planes may be not coincident: in this the plane wave is said **NOT-HOMOGENEOUS**

# Plane Waves

## Incidence on a dielectric half-space

# Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_I e^{-jk_{Ix}x} e^{-jk_{Iy}y} e^{-jk_{Iz}z}$$

$$\vec{\mathbf{k}}_I = k_{Ix}\hat{i}_x + k_{Iy}\hat{i}_y + k_{Iz}\hat{i}_z$$

$$\vec{\mathbf{k}}_I \cdot \vec{\mathbf{k}}_I = k_{Ix}^2 + k_{Iy}^2 + k_{Iz}^2 = \omega^2 \mu_1 \epsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_R e^{-jk_{Rx}x} e^{-jk_{Ry}y} e^{-jk_{Rz}z}$$

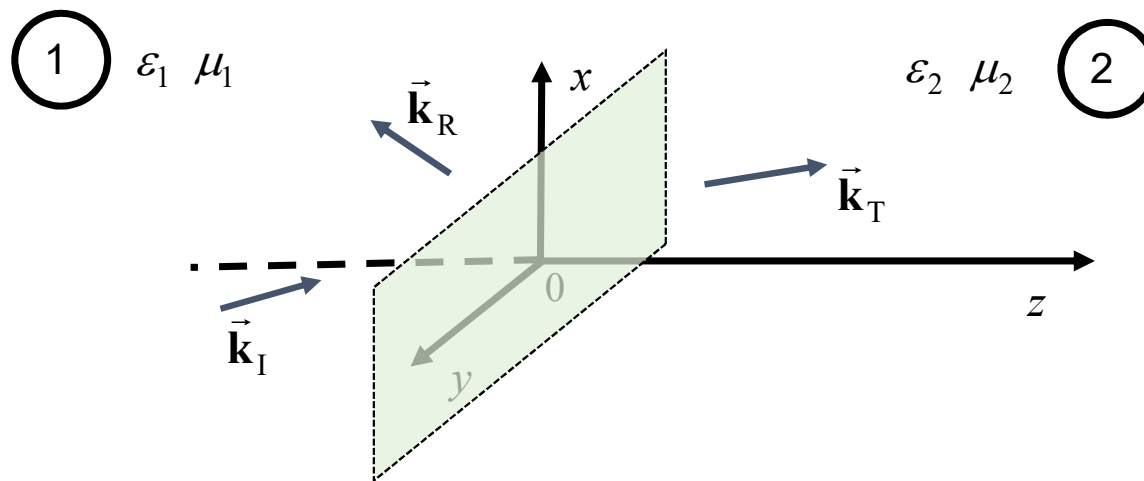
$$\vec{\mathbf{k}}_R = k_{Rx}\hat{i}_x + k_{Ry}\hat{i}_y + k_{Rz}\hat{i}_z$$

$$\vec{\mathbf{k}}_R \cdot \vec{\mathbf{k}}_R = k_{Rx}^2 + k_{Ry}^2 + k_{Rz}^2 = \omega^2 \mu_1 \epsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_T e^{-jk_{Tx}x} e^{-jk_{Ty}y} e^{-jk_{Tz}z}$$

$$\vec{\mathbf{k}}_T = k_{Tx}\hat{i}_x + k_{Ty}\hat{i}_y + k_{Tz}\hat{i}_z$$

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$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}}$$

# Incidence on a dielectric half-space

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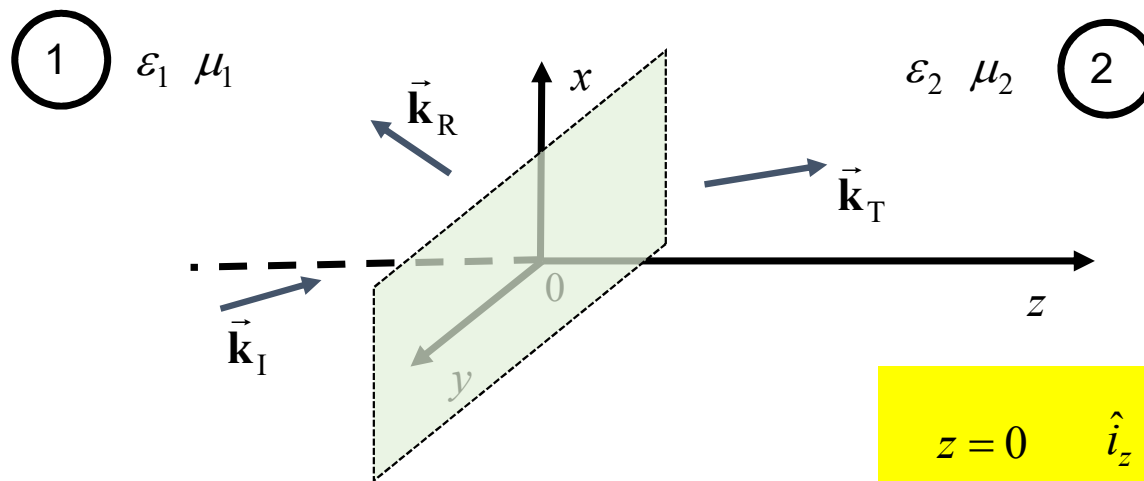
$$\vec{\mathbf{k}}_R = k_{Rx}\hat{i}_x + k_{Ry}\hat{i}_y + k_{Rz}\hat{i}_z$$

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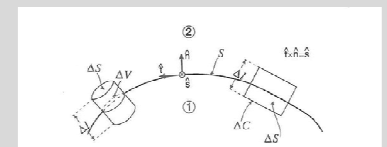
$$\vec{\mathbf{k}}_T \cdot \vec{\mathbf{k}}_T = k_{Tx}^2 + k_{Ty}^2 + k_{Tz}^2 = \omega^2 \mu_2 \epsilon_2 = k_2^2$$



## MEMO

### Fields at boundaries

$$\begin{aligned} \hat{\mathbf{n}} \times (\vec{\mathbf{c}}_2 - \vec{\mathbf{c}}_1) &= 0 \\ \hat{\mathbf{n}} \times (\vec{\mathbf{h}}_2 - \vec{\mathbf{h}}_1) &= \mathbf{j} \\ (\vec{\mathbf{d}}_2 - \vec{\mathbf{d}}_1) \cdot \hat{\mathbf{n}} &= \rho_s \\ (\vec{\mathbf{b}}_2 - \vec{\mathbf{b}}_1) \cdot \hat{\mathbf{n}} &= 0 \\ (\hat{\mathbf{j}}_2 - \hat{\mathbf{j}}_1) \cdot \hat{\mathbf{n}} &= -\frac{\partial \rho_s}{\partial t} \end{aligned}$$



$$z = 0 \quad \hat{i}_z \times (\vec{\mathbf{E}}_2 - \vec{\mathbf{E}}_1) = 0 \quad \hat{i}_z \times \vec{\mathbf{E}}_1 = \hat{i}_z \times \vec{\mathbf{E}}_2$$



# Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_I e^{-jk_{Ix}x} e^{-jk_{Iy}y} e^{-jk_{Iz}z}$$

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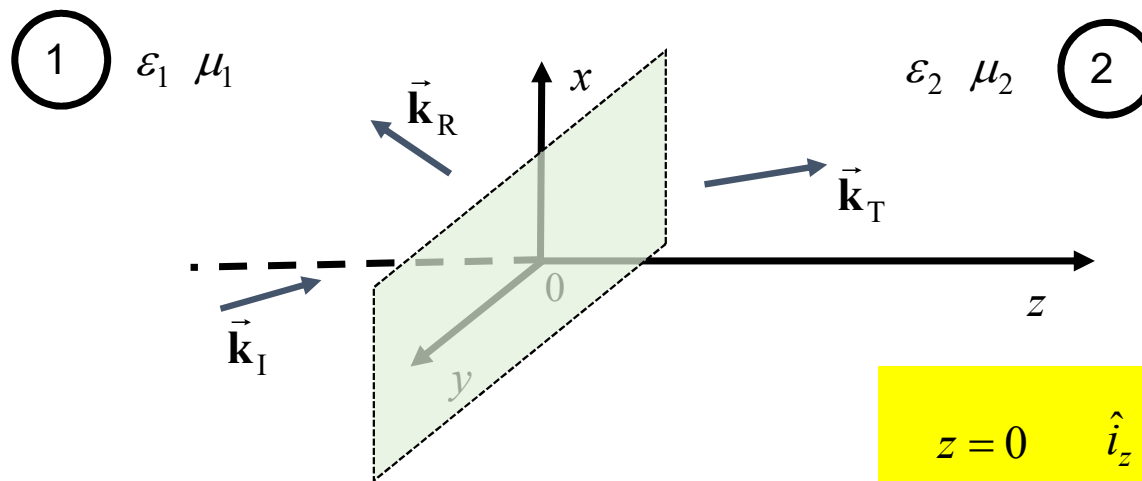
$$\vec{\mathbf{k}}_R = k_{Rx}\hat{i}_x + k_{Ry}\hat{i}_y + k_{Rz}\hat{i}_z$$

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$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}}$$

$$z = 0 \quad \hat{i}_z \times (\vec{\mathbf{E}}_2 - \vec{\mathbf{E}}_1) = \mathbf{0} \quad \hat{i}_z \times \vec{\mathbf{E}}_1 = \hat{i}_z \times \vec{\mathbf{E}}_2$$

# Incidence on a dielectric half-space

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$$\vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_T e^{-jk_{Tx}x} e^{-jk_{Ty}y} e^{-jk_{Tz}z}$$

$$z = 0 \quad \hat{\mathbf{i}}_z \times \vec{\mathbf{E}}_1 = \hat{\mathbf{i}}_z \times \vec{\mathbf{E}}_2$$

$$z = 0 \quad y = 0$$

$$\vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_I e^{-jk_{Ix}x}$$

$$\vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_R e^{-jk_{Rx}x}$$

$$\vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_T e^{-jk_{Tx}x}$$

$$\hat{\mathbf{i}}_z \times \vec{\mathbf{E}}_I e^{-jk_{Ix}x} + \hat{\mathbf{i}}_z \times \vec{\mathbf{E}}_R e^{-jk_{Rx}x} = \hat{\mathbf{i}}_z \times \vec{\mathbf{E}}_T e^{-jk_{Tx}x}$$



$$k_{Ix} = k_{Rx} = k_{Tx} = k_x$$

$$z = 0 \quad x = 0$$

$$\vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_I e^{-jk_{Iy}y}$$

$$\vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_R e^{-jk_{Ry}y}$$

$$\vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_T e^{-jk_{Ty}y}$$

$$\hat{\mathbf{i}}_z \times \vec{\mathbf{E}}_I e^{-jk_{Iy}y} + \hat{\mathbf{i}}_z \times \vec{\mathbf{E}}_R e^{-jk_{Ry}y} = \hat{\mathbf{i}}_z \times \vec{\mathbf{E}}_T e^{-jk_{Ty}y}$$



$$k_{Iy} = k_{Ry} = k_{Ty} = k_y$$

# Incidence on a dielectric half-space

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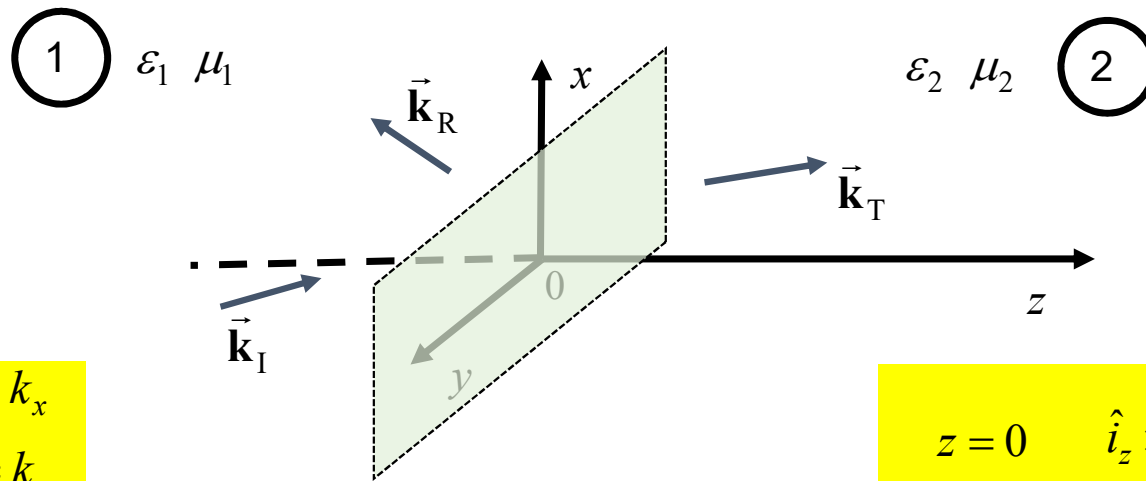
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# Incidence on a dielectric half-space

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$$k_{Ix} = k_{Rx} = k_{Tx} = k_x$$

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$$\vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_I e^{-jk_x x} e^{-jk_y y} e^{-jk_{Iz} z}$$

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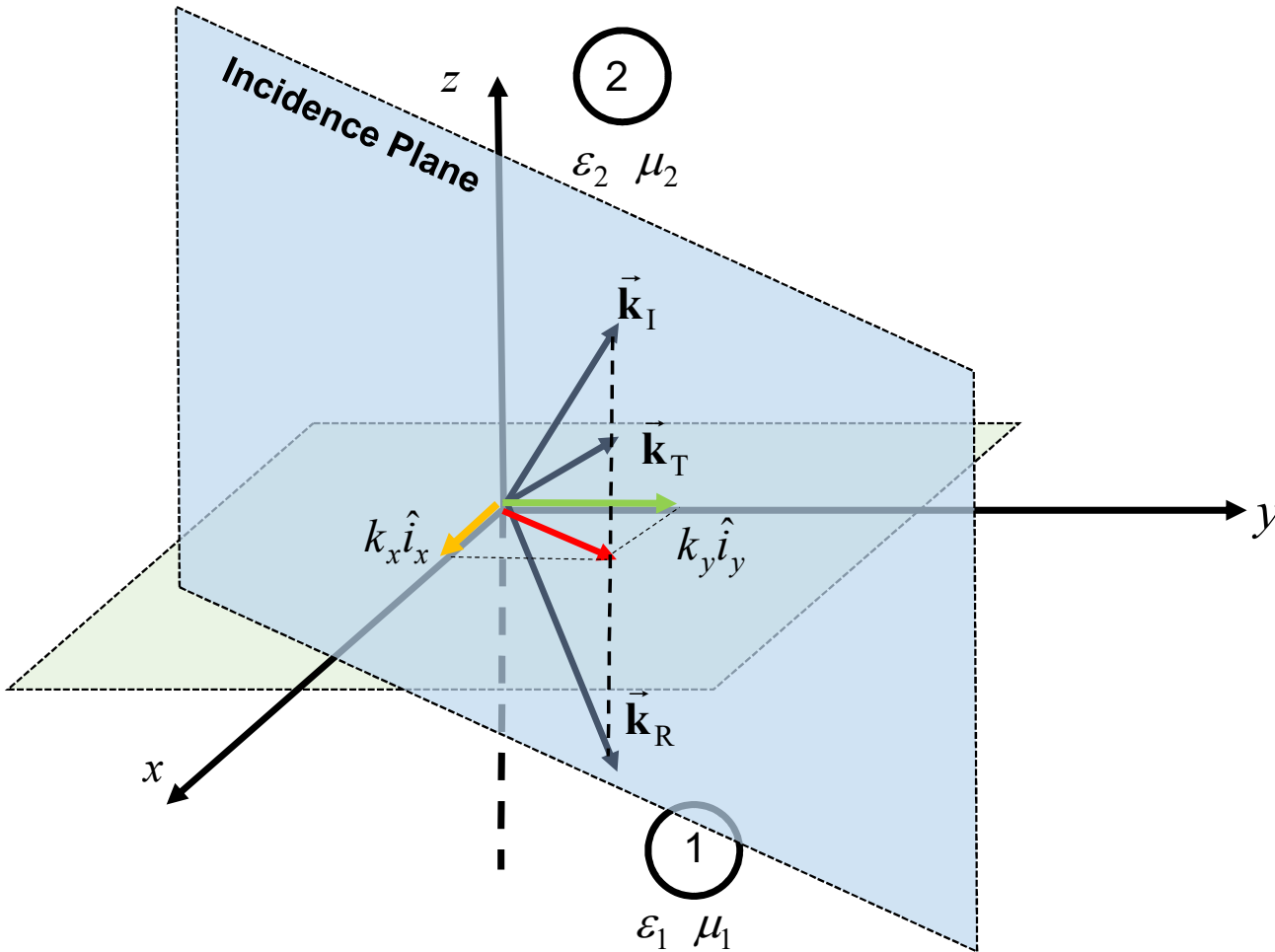
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$$\vec{\mathbf{k}}_T = k_x \hat{i}_x + k_y \hat{i}_y + k_{Tz} \hat{i}_z$$

$$\vec{\mathbf{k}}_T \cdot \vec{\mathbf{k}}_T = k_x^2 + k_y^2 + k_{Tz}^2 = \omega^2 \mu_2 \epsilon_2 = k_2^2$$

# Incidence on a dielectric half-space

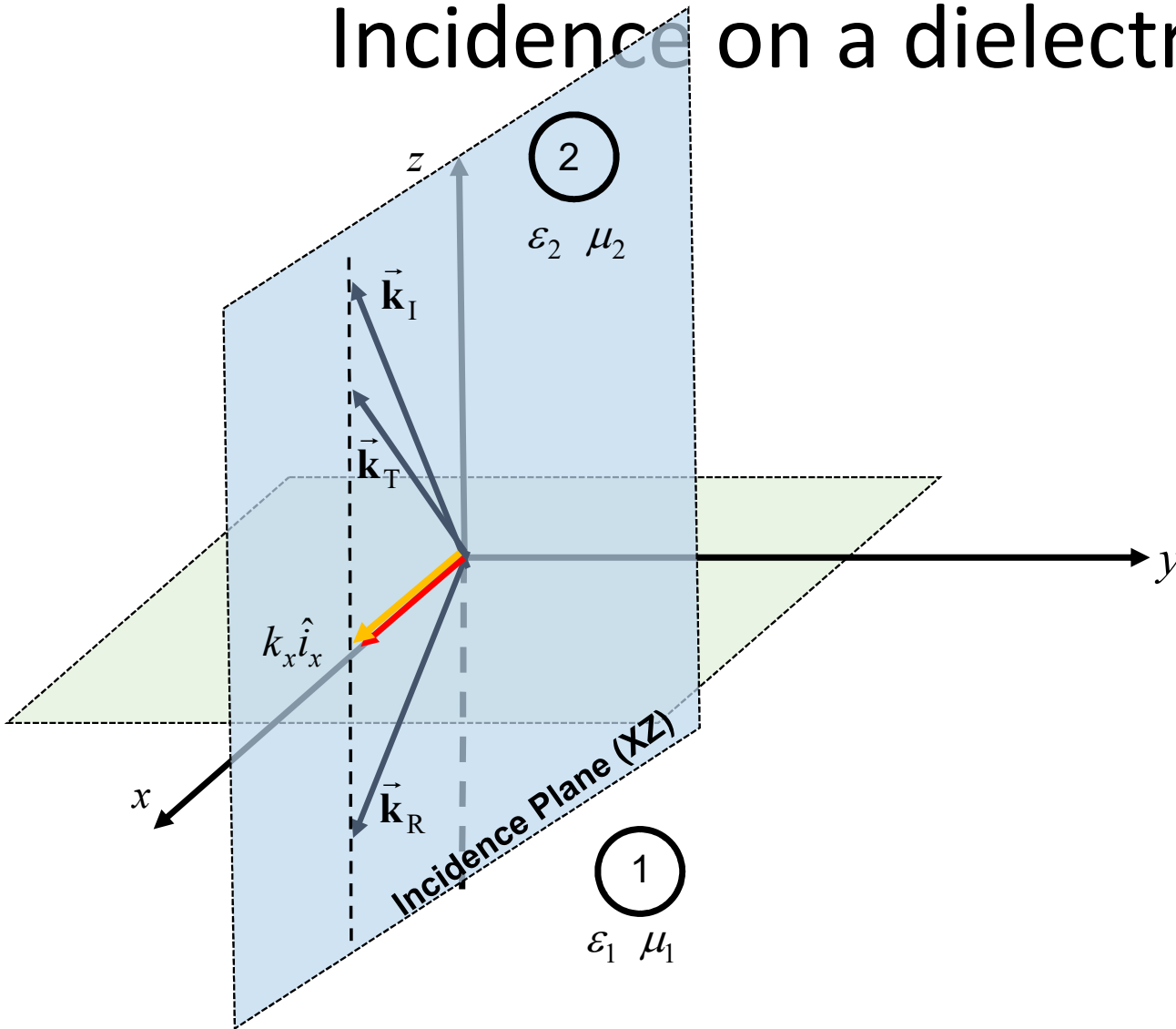


$$\vec{k}_I = k_x \hat{i}_x + k_y \hat{i}_y + k_{Iz} \hat{i}_z$$

$$\vec{k}_R = k_x \hat{i}_x + k_y \hat{i}_y + k_{Rz} \hat{i}_z$$

$$\vec{k}_T = k_x \hat{i}_x + k_y \hat{i}_y + k_{Tz} \hat{i}_z$$

# Incidence on a dielectric half-space

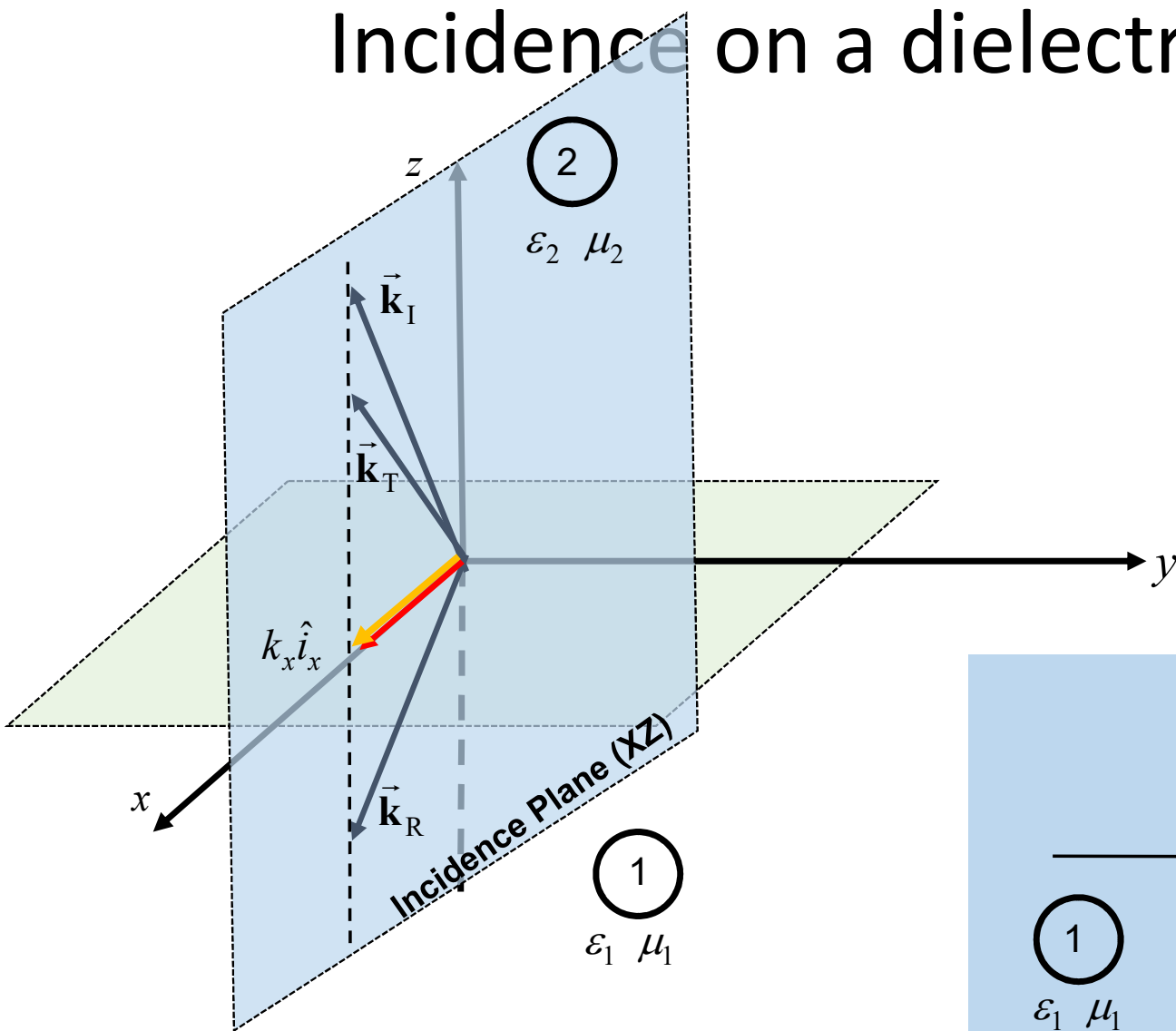


$$\vec{k}_I = k_x \hat{i}_x + k_y \hat{i}_y + k_{Iz} \hat{i}_z$$

$$\vec{k}_R = k_x \hat{i}_x + k_y \hat{i}_y + k_{Rz} \hat{i}_z$$

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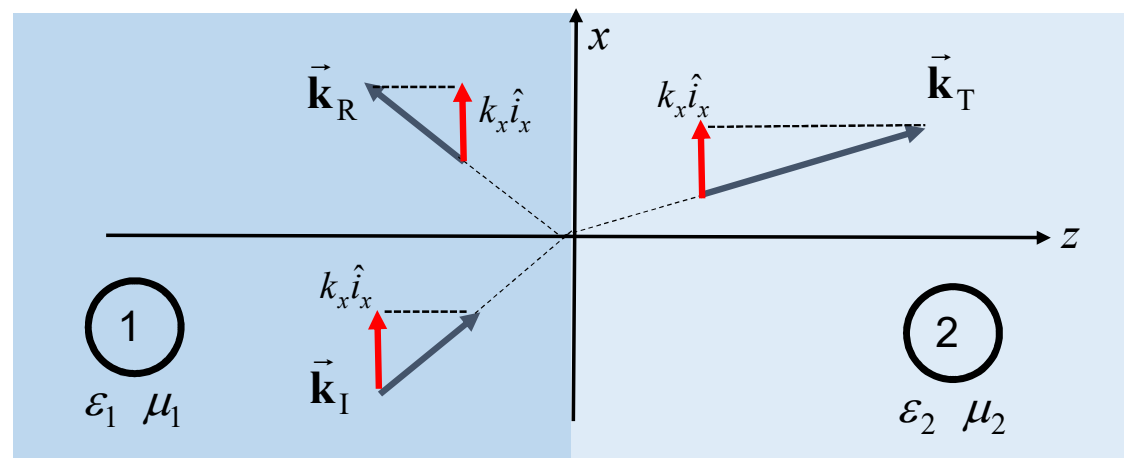
# Incidence on a dielectric half-space



$$\vec{k}_I = k_x \hat{i}_x + k_z \hat{i}_z$$

$$\vec{k}_R = k_x \hat{i}_x + k_{Rz} \hat{i}_z$$

$$\vec{k}_T = k_x \hat{i}_x + k_{Tz} \hat{i}_z$$



# Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_I e^{-jk_x x} e^{-jk_{Iz} z}$$

$$\vec{\mathbf{k}}_I = k_x \hat{i}_x + k_{Iz} \hat{i}_z$$

$$\vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_R e^{-jk_x x} e^{-jk_{Rz} z}$$

$$\vec{\mathbf{k}}_R = k_x \hat{i}_x + k_{Rz} \hat{i}_z$$

$$\vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_T e^{-jk_x x} e^{-jk_{Tz} z}$$

$$\vec{\mathbf{k}}_T = k_x \hat{i}_x + k_{Tz} \hat{i}_z$$

$$\vec{\mathbf{k}}_I \cdot \vec{\mathbf{k}}_I = k_x^2 + k_{Iz}^2 = \omega^2 \mu_1 \epsilon_1 = k_1^2$$

$$\vec{\mathbf{k}}_R \cdot \vec{\mathbf{k}}_R = k_x^2 + k_{Rz}^2 = \omega^2 \mu_1 \epsilon_1 = k_1^2$$

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$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}} \quad \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}}$$

$$k_x = k_1 \sin \vartheta_I$$

$$k_x = k_1 \sin \vartheta_R$$

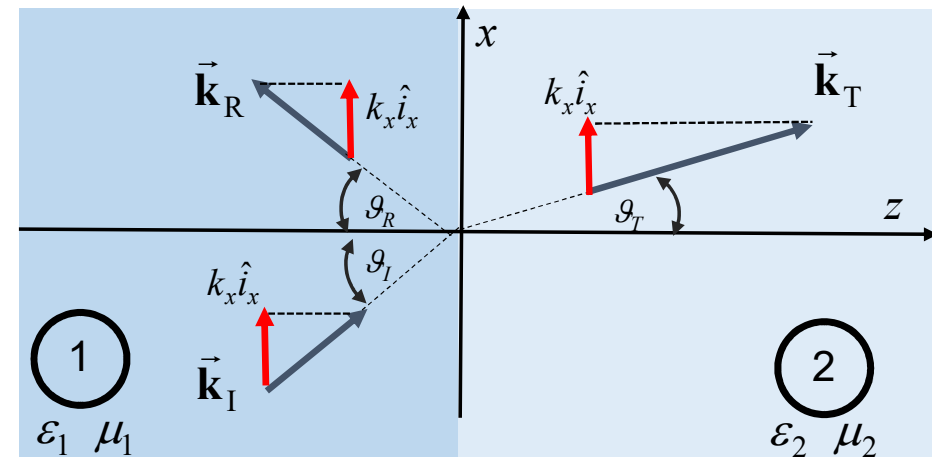
$$k_x = k_2 \sin \vartheta_T$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin \vartheta_I = k_2 \sin \vartheta_T$$

Snellius Law

$$\sin \vartheta_I = \frac{k_2}{k_1} \sin \vartheta_T \quad \sin \vartheta_I = \frac{\omega \sqrt{\mu_2 \epsilon_2}}{\omega \sqrt{\mu_1 \epsilon_1}} \sin \vartheta_T \quad n = \frac{\sqrt{\mu_2 \epsilon_2}}{\sqrt{\mu_1 \epsilon_1}}$$





# Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_I e^{-jk_x x} e^{-jk_{Iz} z}$$

$$\vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_R e^{-jk_x x} e^{-jk_{Rz} z}$$

$$\vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_T e^{-jk_x x} e^{-jk_{Tz} z}$$

$$\vec{\mathbf{k}}_I = k_x \hat{i}_x + k_{Iz} \hat{i}_z$$

$$\vec{\mathbf{k}}_R = k_x \hat{i}_x + k_{Rz} \hat{i}_z$$

$$\vec{\mathbf{k}}_T = k_x \hat{i}_x + k_{Tz} \hat{i}_z$$

$$\vec{\mathbf{k}}_I \cdot \vec{\mathbf{k}}_I = k_x^2 + k_{Iz}^2 = \omega^2 \mu_1 \varepsilon_1 = k_1^2$$

$$\vec{\mathbf{k}}_R \cdot \vec{\mathbf{k}}_R = k_x^2 + k_{Rz}^2 = \omega^2 \mu_1 \varepsilon_1 = k_1^2$$

$$\vec{\mathbf{k}}_T \cdot \vec{\mathbf{k}}_T = k_x^2 + k_{Tz}^2 = \omega^2 \mu_2 \varepsilon_2 = k_2^2$$

$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}}$$

$$k_x = k_1 \sin \vartheta_I$$

$$k_x = k_1 \sin \vartheta_R$$

$$k_x = k_2 \sin \vartheta_T$$

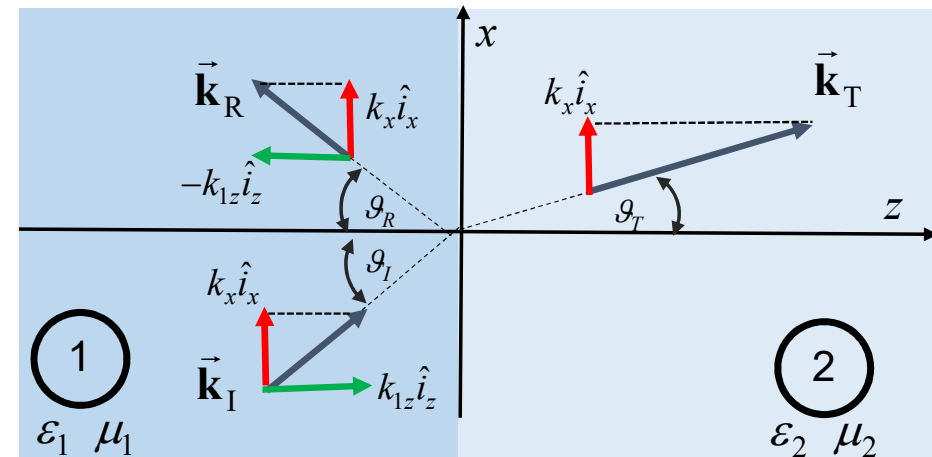
$$\vartheta_I = \vartheta_R$$

$$k_1 \sin \vartheta_I = k_2 \sin \vartheta_T$$

$$k_{Iz}^2 = k_{Rz}^2 \implies k_{Iz} = -k_{Rz} = k_{1z}$$

$$\vec{\mathbf{k}}_I = k_x \hat{i}_x + k_{1z} \hat{i}_z$$

$$\vec{\mathbf{k}}_R = k_x \hat{i}_x - k_{1z} \hat{i}_z$$



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$$\vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_T e^{-jk_x x} e^{-jk_{2z} z}$$

$$\vec{\mathbf{k}}_I = k_x \hat{i}_x + k_{1z} \hat{i}_z$$

$$\vec{\mathbf{k}}_R = k_x \hat{i}_x - k_{1z} \hat{i}_z$$

$$\vec{\mathbf{k}}_T = k_x \hat{i}_x + k_{2z} \hat{i}_z$$

$$\vec{\mathbf{k}}_I \cdot \vec{\mathbf{k}}_I = k_x^2 + k_{1z}^2 = \omega^2 \mu_1 \epsilon_1 = k_1^2$$

$$\vec{\mathbf{k}}_R \cdot \vec{\mathbf{k}}_R = k_x^2 + k_{1z}^2 = \omega^2 \mu_1 \epsilon_1 = k_1^2$$

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$$k_x = k_1 \sin \vartheta_R$$

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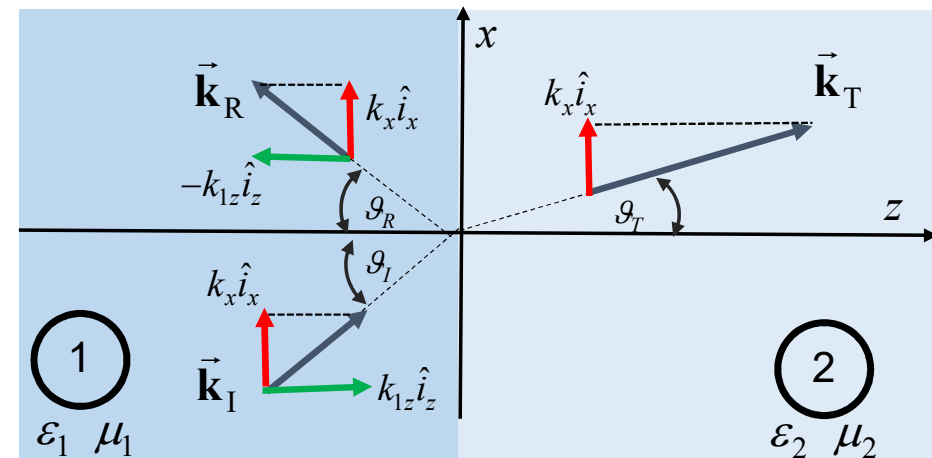
$$\vartheta_I = \vartheta_R$$

$$k_1 \sin \vartheta_I = k_2 \sin \vartheta_T$$

$$k_{1z}^2 = k_{Rz}^2 \implies k_{1z} = -k_{Rz} = k_{1z}$$

$$\vec{\mathbf{k}}_I = k_x \hat{i}_x + k_{1z} \hat{i}_z$$

$$\vec{\mathbf{k}}_R = k_x \hat{i}_x - k_{1z} \hat{i}_z$$



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$$\vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_I e^{-jk_x x} e^{-jk_{1z} z}$$

$$\vec{\mathbf{k}}_I = k_x \hat{i}_x + k_{1z} \hat{i}_z$$

$$\vec{\mathbf{k}}_I \cdot \vec{\mathbf{k}}_I = k_x^2 + k_{1z}^2 = \omega^2 \mu_1 \epsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_R e^{-jk_x x} e^{jk_{1z} z}$$

$$\vec{\mathbf{k}}_R = k_x \hat{i}_x - k_{1z} \hat{i}_z$$

$$\vec{\mathbf{k}}_R \cdot \vec{\mathbf{k}}_R = k_x^2 + k_{1z}^2 = \omega^2 \mu_1 \epsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_T e^{-jk_x x} e^{-jk_{2z} z}$$

$$\vec{\mathbf{k}}_T = k_x \hat{i}_x + k_{2z} \hat{i}_z$$

$$\vec{\mathbf{k}}_T \cdot \vec{\mathbf{k}}_T = k_x^2 + k_{2z}^2 = \omega^2 \mu_2 \epsilon_2 = k_2^2$$

$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}}$$

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$$k_x = k_1 \sin \vartheta_I$$

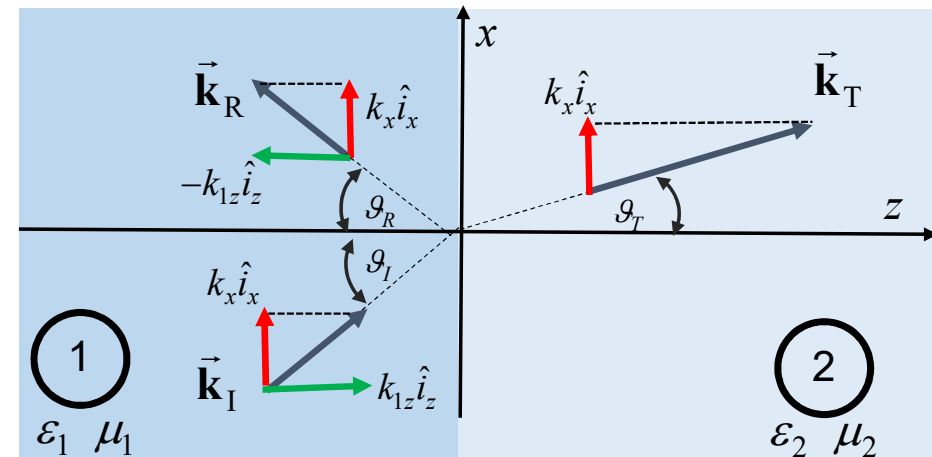
$$k_{1z} = k_1 \cos \vartheta_I$$

$$\vartheta_I = \vartheta_R$$

$$k_x = k_1 \sin \vartheta_R$$

$$k_x = k_2 \sin \vartheta_T$$

$$k_1 \sin \vartheta_I = k_2 \sin \vartheta_T$$



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$$\vec{\mathbf{k}}_I = k_x \hat{i}_x + k_{1z} \hat{i}_z$$

$$\vec{\mathbf{k}}_I \cdot \vec{\mathbf{k}}_I = k_x^2 + k_{1z}^2 = \omega^2 \mu_1 \epsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_R e^{-jk_x x} e^{jk_{1z} z}$$

$$\vec{\mathbf{k}}_R = k_x \hat{i}_x - k_{1z} \hat{i}_z$$

$$\vec{\mathbf{k}}_R \cdot \vec{\mathbf{k}}_R = k_x^2 + k_{1z}^2 = \omega^2 \mu_1 \epsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_T e^{-jk_x x} e^{-jk_{2z} z}$$

$$\vec{\mathbf{k}}_T = k_x \hat{i}_x + k_{2z} \hat{i}_z$$

$$\vec{\mathbf{k}}_T \cdot \vec{\mathbf{k}}_T = k_x^2 + k_{2z}^2 = \omega^2 \mu_2 \epsilon_2 = k_2^2$$

$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}}$$

$$k_x = k_1 \sin \vartheta_I$$

$$k_x = k_1 \sin \vartheta_R$$

$$k_x = k_2 \sin \vartheta_T$$

$$k_{1z} = k_1 \cos \vartheta_I$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin \vartheta_I = k_2 \sin \vartheta_T$$

$$k_{2z} = k_2 \cos \vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_x^2} = \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_I}$$

