Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2020-2021 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

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Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Plane Waves

Time domain (TD)

Spectral domains

Phasor Domain (PD)

Fourier Domain (FD)

Dispersive media: attenuation, distortion, phase velocity and group velocity

General expression of plane waves (PD)

Incidence

Plane Waves

Spectral domains

Plane Waves (Spectral Domains)

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \qquad \frac{d^2E_x}{dz^2} + k^2E_x = 0$$

$$E_x = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$k = \omega\sqrt{\mu\varepsilon}$$
$$k = \beta - j\alpha$$

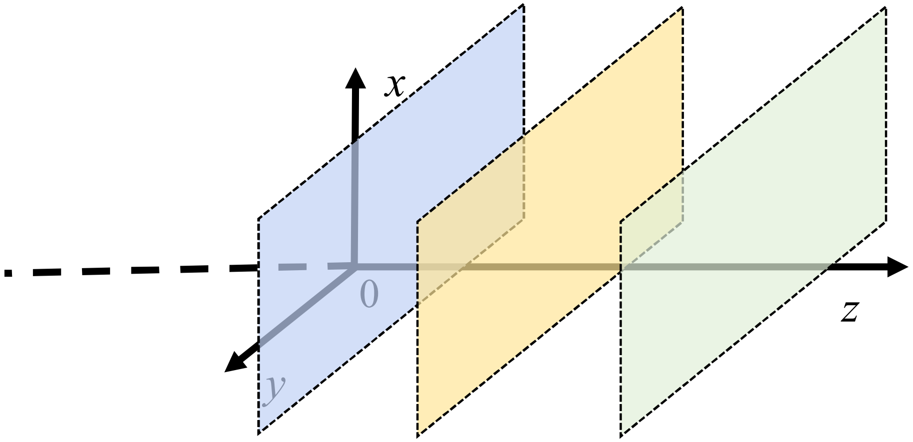
$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{E_y, H_x\}$

$$\begin{cases} \dfrac{dE_y}{dz} = j\omega\mu H_x \\ \dfrac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases} \qquad \frac{d^2E_y}{dz^2} + k^2E_y = 0$$

$$E_y = E_y^+ e^{-jkz} + E_y^- e^{jkz}$$

$$-\zeta H_x = E_y^+ e^{-jkz} - E_y^- e^{jkz}$$



Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$ **Independent each other**

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$k(\omega_0)=\omega_0\sqrt{\mu(\omega_0)\varepsilon(\omega_0)}=\beta(\omega_0)-j\alpha(\omega_0)$$

$$\lambda=\frac{2\pi}{\beta}=\frac{v_p}{f_0}$$

$$E_x^+(z)=E^+e^{-jkz}$$
$$\zeta H_y^+(z)=E^+e^{-jkz}$$

$$v_p=\frac{\omega_0}{\beta}$$

$$\zeta(\omega_0)=\sqrt{\frac{\mu(\omega_0)}{\varepsilon(\omega_0)}}$$

$$\omega_o=2\pi f_0$$

$$E^+e^{-jkz}\quad\rightarrow\quad e^+(z,t)=e^{-\alpha z}\left|E^+\right|\cos\left(\omega_o t-\beta z+\varphi^+\right)$$

Time dispersive (lossy)

$$\begin{cases}\varepsilon(\omega_0)=\varepsilon_1(\omega_0)-j\varepsilon_2(\omega_0)\\ \mu(\omega_0)=\mu_1(\omega_0)-j\mu_2(\omega_0)\\ \sigma:\ real\end{cases}$$

$$E_x^+(z)=E^+e^{-j\beta z}$$
$$\zeta H_y^+(z)=E^+e^{-j\beta z}$$

$$k(\omega_0)=\omega_0\sqrt{\mu\varepsilon}=\beta(\omega_0)$$

$$\omega_o=2\pi f_0$$

$$v_p=\frac{\omega_0}{\beta}\qquad \zeta=\sqrt{\frac{\mu}{\varepsilon}}\qquad \lambda=\frac{2\pi}{\beta}=\frac{v_p}{f_0}$$

Time nondispersive & lossless

$$\begin{cases}\varepsilon:real\\ \mu:real\\ \sigma=0\end{cases}$$

$$c=\frac{1}{\sqrt{\mu\varepsilon}}$$

$$E^+e^{-j\beta z}\rightarrow e_x^+(z,t)=\left|E^+\right|\cos\left(\omega_o t-\beta z+\varphi^+\right)=e_x^+(z-v_p t)$$

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$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

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$$E_x^+(z) = E^+ e^{-jkz}$$

$$\zeta H_y^+(z) = E^+ e^{-jkz}$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\mathrm{Re}\left\{\omega_0\sqrt{\mu(\omega_0)\varepsilon(\omega_0)}\right\}} = \frac{1}{\mathrm{Re}\left\{\sqrt{\mu(\omega_0)\varepsilon(\omega_0)}\right\}} = v_p(\omega_0)$$

$$E^+ e^{-jkz} \quad \rightarrow \quad e^+(z,t) = e^{-\alpha z}\left|E^+\right|\cos\left(\omega_o t - \beta z + \varphi^+\right)$$

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma : real \end{cases}$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$\boxed{c = \frac{1}{\sqrt{\mu\varepsilon}}}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = \left|E^+\right|\cos\left(\omega_o t - \beta z + \varphi^+\right) = e_x^+\left(z - v_p t\right)$$

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$\{E_y, H_x\}$

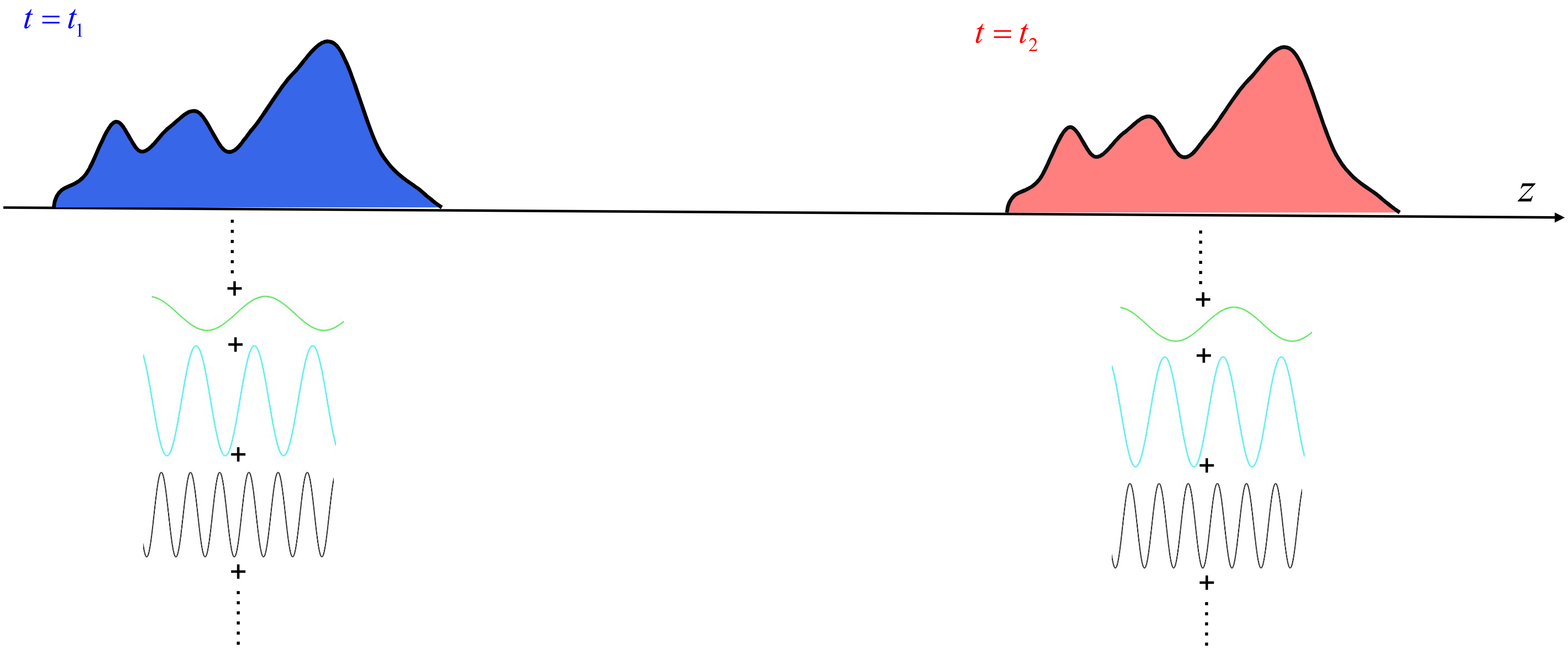
$\{E_x, H_y\}$

Independent each other

Plane Waves

Time nondispersive & lossless medium

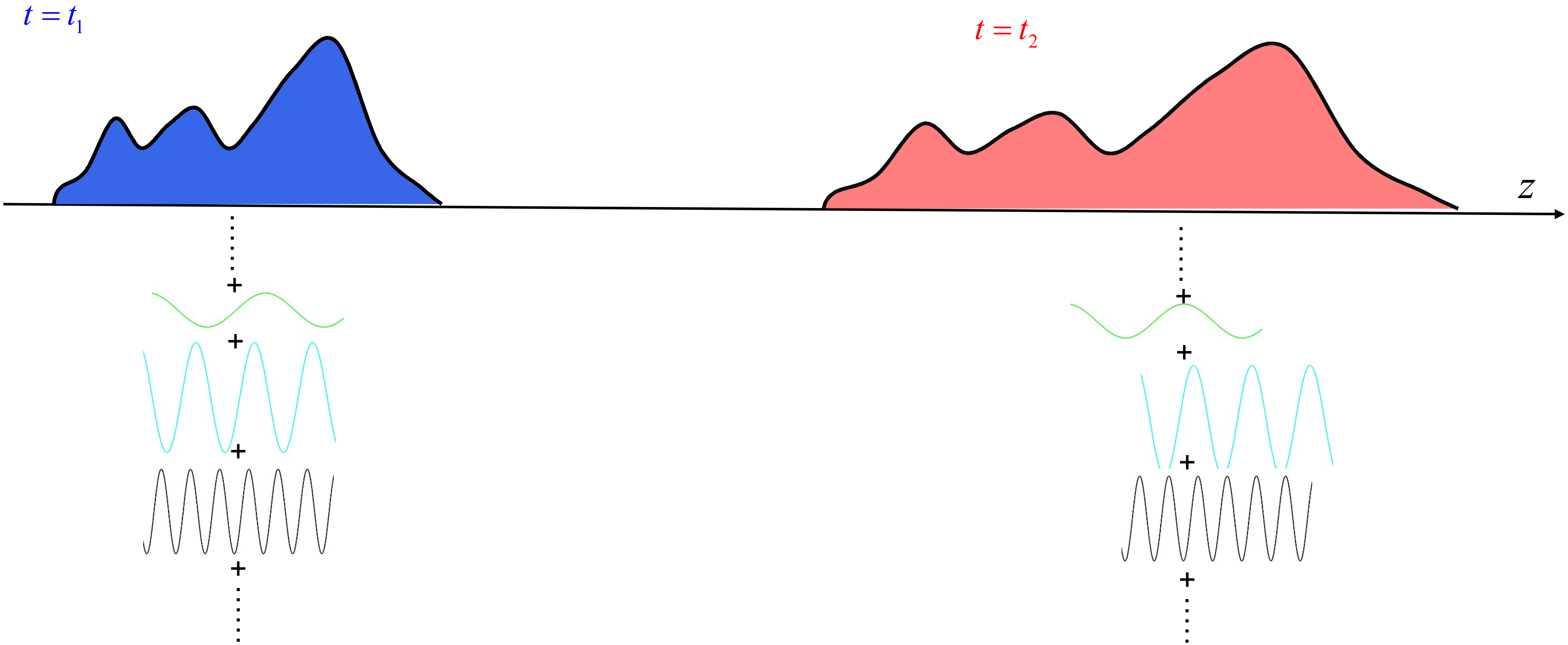
$$\boxed{v_p = c}$$



Plane Waves

Time dispersive medium

$$\boxed{v_p = v_p(\omega_0)}$$



Plane Waves (Phasor Domain)

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta} = c$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma : \ real \end{cases}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0)\varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

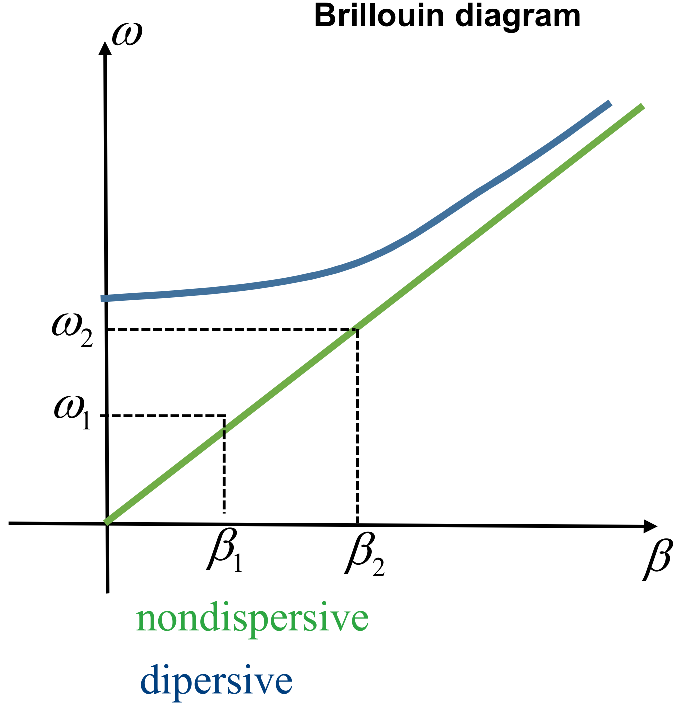
Attenuation

$$\alpha \neq 0$$

Distortion

$$v_p = \frac{\omega_0}{\beta} = v_p(\omega_0)$$

Brillouin diagram



$$v_p = \frac{\omega_0}{\beta} = v_p(\omega_0)$$

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Plane Waves (Phasor Domain)

Time nondispersive & lossless

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Attenuation

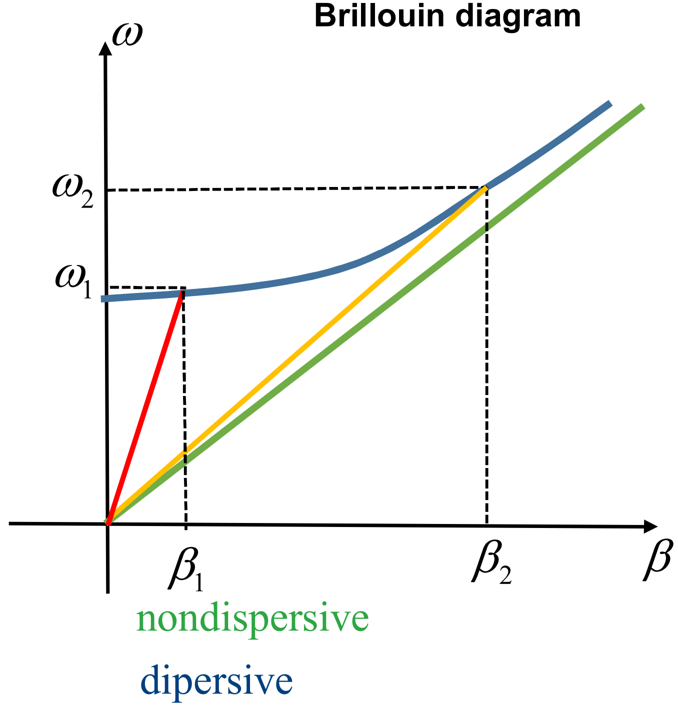
$$\alpha \neq 0$$

Distortion

$$v_p = \frac{\omega_0}{\beta} = v_p(\omega_0)$$

$$k(\omega_0) = \omega_0\sqrt{\mu(\omega_0)\varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

Brillouin diagram



$$v_p = \frac{\omega_0}{\beta} = v_p(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta} = c$$

Plane Waves (Fourier Domain)

$$k=\omega\sqrt{\mu\varepsilon}$$
$$k=\beta-j\alpha$$

$$\zeta=\sqrt{\frac{\mu}{\varepsilon}}$$

$$\left\{E_x,H_y\right\}$$

$$\begin{cases}\dfrac{dE_x}{dz}=-j\omega\mu H_y\\ \dfrac{dH_y}{dz}=-j\omega\varepsilon E_x\end{cases}$$

$$\frac{d^2E_x}{dz^2}+k^2E_x=0$$

$$E_x^+(z,\omega)=E^+(\omega)e^{-jkz}$$

$$\zeta H_y^+(z,\omega)=E^+(\omega)e^{-jkz}$$

$$\begin{cases}\varepsilon(\omega)=\varepsilon_1(\omega)-j\varepsilon_2(\omega)\\ \mu(\omega)=\mu_1(\omega)-j\mu_2(\omega)\end{cases}$$

$$k(\omega)=\omega\sqrt{\mu(\omega)\varepsilon(\omega)}$$
$$k(\omega)=\beta(\omega)-j\alpha(\omega)$$

Fourier Domain

Source-free

Medium

- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
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- ~~**Lossless**~~

$$\frac{\partial}{\partial x}=\frac{\partial}{\partial y}=0$$

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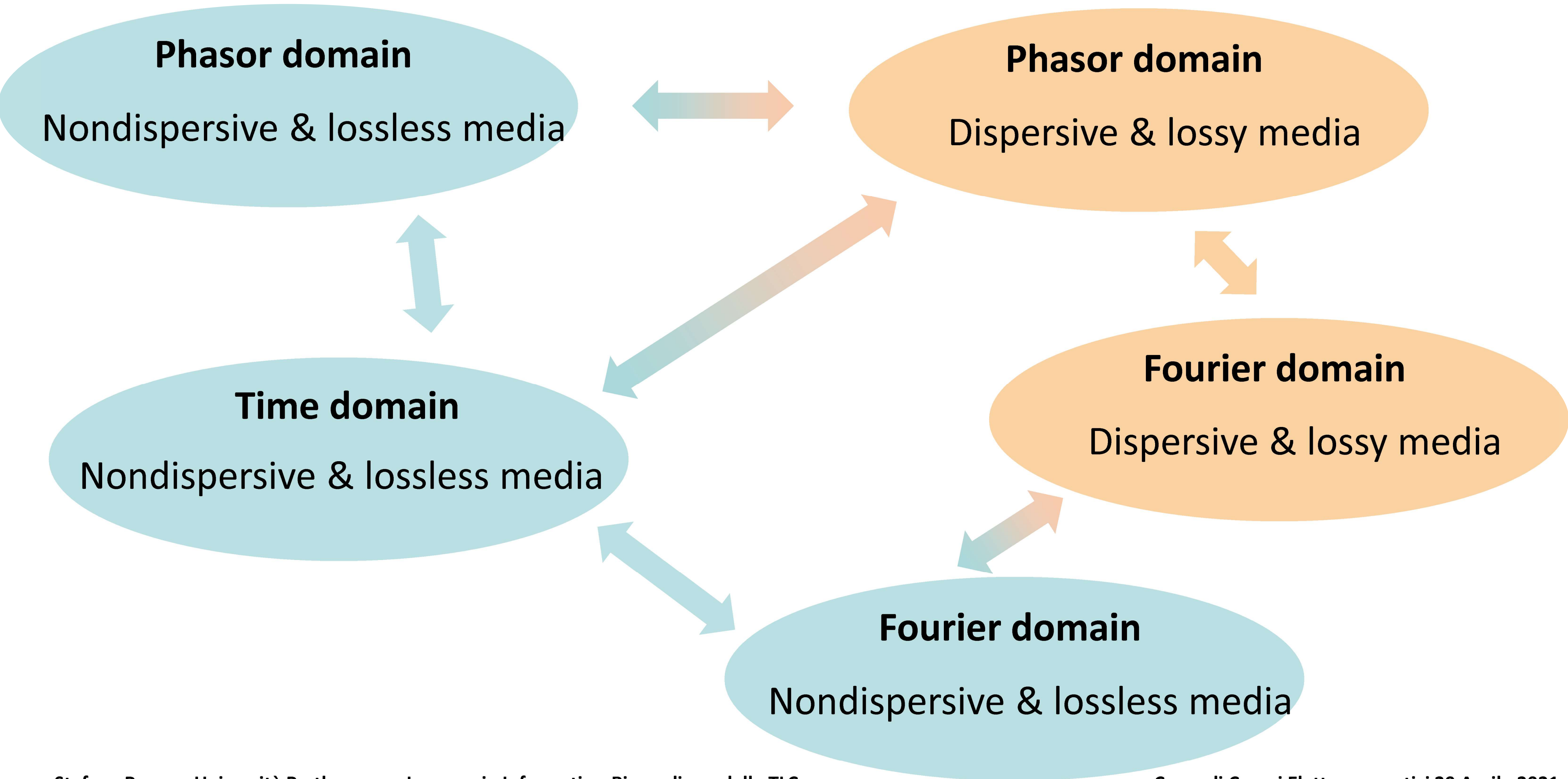
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}(z,t)$$

$$E_z=H_z=0$$

$$\left\{E_y,H_x\right\}$$
$$\left\{E_x,H_y\right\}$$

Independent each other

Rationale



Phasor domain
Nondispersive & lossless media
Phasor domain
Dispersive & lossy media
Time domain
Nondispersive & lossless media
Fourier domain
Dispersive & lossy media
Fourier domain
Nondispersive & lossless media

Plane Waves (Fourier Domain)

$\{E_x, H_y\}$

$$E_x^+(z,\omega) = E^+(\omega)e^{-j\beta z}$$

$$\zeta H_y^+(z,\omega) = E^+(\omega)e^{-j\beta z}$$

$$k(\omega) = \omega\sqrt{\mu\varepsilon} = \beta(\omega)$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\beta = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{c}$$

$$E_x^+(z,\omega) = E^+(\omega)e^{-j\beta z}$$

$$e_x^+(z,t) = \frac{1}{2\pi}\int E_x^+(z,\omega)e^{j\omega t}d\omega = \frac{1}{2\pi}\int E^+(\omega)e^{-j\beta z}e^{j\omega t}d\omega$$

$$= \frac{1}{2\pi}\int E^+(\omega)e^{-j\frac{\omega}{c}z}e^{j\omega t}d\omega = \frac{1}{2\pi}\int E^+(\omega)e^{j\omega\left(t-\frac{z}{c}\right)}d\omega$$

$$= f\left(t-\frac{z}{c}\right) = f\left[-\frac{1}{c}(z-ct)\right]$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

Progressive plane wave

Source-free

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$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

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$$\Downarrow$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

Plane Waves (Fourier Domain)

$\{E_x, H_y\}$

$k(\omega) = \omega\sqrt{\mu\varepsilon} = \beta(\omega)$

$E_x^+(z,\omega) = E^+(\omega)e^{-j\beta z}$

$\zeta H_y^+(z,\omega) = E^+(\omega)e^{-j\beta z}$

$\zeta = \sqrt{\dfrac{\mu}{\varepsilon}}$

$\beta = \omega\sqrt{\mu\varepsilon} = \dfrac{\omega}{c}$

$E_x^+(z,\omega) = E^+(\omega)e^{-j\beta z}$

$$e_x^+(z,t) = \frac{1}{2\pi}\int E_x^+(z,\omega)e^{j\omega t}d\omega \quad \boxed{= \frac{1}{2\pi}\int E^+(\omega)e^{-j\beta z}e^{j\omega t}d\omega}$$

$$\boxed{= \frac{1}{2\pi}\int E^+(\omega)e^{-j\frac{\omega}{c}z}e^{j\omega t}d\omega} = \frac{1}{2\pi}\int E^+(\omega)e^{j\omega\left(t-\frac{z}{c}\right)}d\omega$$

$$= f\left(t-\frac{z}{c}\right) = f\left[-\frac{1}{c}(z-ct)\right]$$

Progressive plane wave

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$c = \dfrac{1}{\sqrt{\mu\varepsilon}}$

Source-free

Medium
- Linear
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$\dfrac{\partial}{\partial x} = \dfrac{\partial}{\partial y} = 0$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$

$E_z = H_z = 0$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

Plane Waves (Fourier Domain)

Source-free

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$$\left\{E_x, H_y\right\}$$

$$k(\omega) = \omega\sqrt{\mu\varepsilon} = \beta(\omega)$$

$$E_x^+(z,\omega) = E^+(\omega)e^{-j\beta z}$$

$$\zeta H_y^+(z,\omega) = E^+(\omega)e^{-j\beta z}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\beta = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{c}$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$$E_x^+(z,\omega) = E^+(\omega)e^{-j\beta z}$$

$$e_x^+(z,t) = \frac{1}{2\pi}\int E_x^+(z,\omega)e^{j\omega t}d\omega \; \boxed{= \frac{1}{2\pi}\int E^+(\omega)e^{-j\beta z}e^{j\omega t}d\omega}$$

$$\boxed{= \frac{1}{2\pi}\int E^+(\omega)e^{-j\frac{\omega}{c}z}e^{j\omega t}d\omega} \; = \frac{1}{2\pi}\int E^+(\omega)e^{j\omega\left(t-\frac{z}{c}\right)}d\omega$$

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Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

Progressive plane wave

$$\left\{E_y, H_x\right\}$$
$$\left\{E_x, H_y\right\}$$

Independent each other

Mathematical tools

$$f(t) \longrightarrow \boxed{\textbf{FT}} \longrightarrow F(\omega)$$

$$f(t)\ \text{real}\ \Rightarrow \boxed{F(\omega) = F^*(-\omega)}$$

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega \quad = \frac{1}{2\pi}\left[\boxed{\int_{-\infty}^{0} F(\omega)e^{j\omega t}d\omega} + \int_{0}^{\infty} F(\omega)e^{j\omega t}d\omega\right] \quad = \frac{1}{2\pi}\left[\int_{0}^{\infty}\left[F(\omega)e^{j\omega t}\right]^* d\omega + \int_{0}^{\infty} F(\omega)e^{j\omega t}d\omega\right]$$

$$\eta = -\omega$$

$$\boxed{\int_{-\infty}^{0} F(\omega)e^{j\omega t}d\omega} = -\int_{\infty}^{0} F(-\eta)e^{-j\eta t}d\eta \quad = \int_{0}^{\infty} F(-\eta)e^{-j\eta t}d\eta \quad = \int_{0}^{\infty} F^*(\eta)\left[e^{j\eta t}\right]^* d\eta = \int_{0}^{\infty}\left[F(\eta)e^{j\eta t}\right]^* d\eta = \boxed{\int_{0}^{\infty}\left[F(\omega)e^{j\omega t}\right]^* d\omega}$$

Mathematical tools

$$f(t) \longrightarrow \boxed{\textbf{FT}} \longrightarrow F(\omega)$$

$$f(t) \text{ real } \Rightarrow F(\omega) = F^*(-\omega)$$

$$\boxed{f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega} = \frac{1}{2\pi}\left[\int_{-\infty}^{0} F(\omega)e^{j\omega t}d\omega + \int_{0}^{\infty} F(\omega)e^{j\omega t}d\omega\right] = \frac{1}{2\pi}\left[\int_{0}^{\infty}\left[F(\omega)e^{j\omega t}\right]^* d\omega + \int_{0}^{\infty} F(\omega)e^{j\omega t}d\omega\right]$$

$$= \frac{1}{2\pi}\int_{0}^{\infty} F(\omega)e^{j\omega t} + \left[F(\omega)e^{j\omega t}\right]^* d\omega = \frac{1}{2\pi}\left[\int_{0}^{\infty} 2\,\mathrm{Re}\left\{F(\omega)e^{j\omega t}\right\}d\omega\right] = \boxed{\mathrm{Re}\left\{\frac{1}{\pi}\int_{0}^{\infty} F(\omega)e^{j\omega t}d\omega\right\}}$$

$$q = a + jb$$

$$q + q^* = (a + jb) + (a - jb) = 2a = 2\,\mathrm{Re}\{q\}$$

Plane Waves : dispersion

Source-free

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$$f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(\omega)e^{j\omega t}d\omega \quad =\mathrm{Re}\left\{\frac{1}{\pi}\int_{0}^{\infty}F(\omega)e^{j\omega t}d\omega\right\}$$

Plane Waves : dispersion

$\{E_x, H_y\}$

$$k(\omega)=\omega\sqrt{\mu(\omega)\varepsilon(\omega)}=\beta(\omega)-j\alpha(\omega)$$

$$E_x^+(z,\omega)=E^+(\omega)e^{-jkz}=E^+(\omega)e^{-j\beta z}e^{-\alpha z}$$

$$\zeta H_y^+(z,\omega)=E^+(\omega)e^{-jkz}=E^+(\omega)e^{-j\beta z}e^{-\alpha z}$$

$$\zeta=\sqrt{\frac{\mu}{\varepsilon}}$$

$$f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(\omega)e^{j\omega t}d\omega \quad =\mathrm{Re}\left\{\frac{1}{\pi}\int_{0}^{\infty}F(\omega)e^{j\omega t}d\omega\right\}$$

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$$\zeta=\sqrt{\frac{\mu}{\varepsilon}}$$

$$E_x^+(z,\omega)=E^+(\omega)e^{-jkz}=E^+(\omega)e^{-j\beta(\omega)z}e^{-\alpha(\omega)z}$$

$$e_x^+(z,t)=\frac{1}{2\pi}\int E_x^+(z,\omega)e^{j\omega t}d\omega=\mathrm{Re}\left\{\frac{1}{\pi}\int_0^\infty E_x^+(z,\omega)e^{j\omega t}d\omega\right\}$$

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Plane Waves : dispersion

$\{E_x, H_y\}$

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$$\boxed{\zeta = \sqrt{\frac{\mu}{\varepsilon}}}$$

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- ~~Lossy~~

$\alpha(\omega) \approx 0$

$$\begin{cases} \varepsilon(\omega) = \varepsilon_1(\omega) \\ \mu(\omega) = \mu_1(\omega) \\ \sigma = 0 \end{cases}$$

Plane Waves : dispersion

$\{E_x, H_y\}$

$$k(\omega)=\omega\sqrt{\mu(\omega)\varepsilon(\omega)}=\beta(\omega)-j\cancel{\alpha(\omega)}$$

$$E_x^+(z,\omega)=E^+(\omega)e^{-jkz}=E^+(\omega)e^{-j\beta z}\cancel{e^{-\alpha z}}$$

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Plane Waves : dispersion

Source-free

Medium

- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- Lossless

$\alpha(\omega) \approx 0$

$$\begin{cases} \varepsilon(\omega) = \varepsilon_1(\omega) \\ \mu(\omega) = \mu_1(\omega) \\ \sigma = 0 \end{cases}$$

$\{E_x, H_y\}$

$$k(\omega) = \omega\sqrt{\mu(\omega)\varepsilon(\omega)} = \beta(\omega)$$

$$E_x^+(z,\omega) = E^+(\omega)e^{-jkz} = E^+(\omega)e^{-j\beta z}$$

$$\zeta H_y^+(z,\omega) = E^+(\omega)e^{-jkz} = E^+(\omega)e^{-j\beta z}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E_x^+(z,\omega) = E^+(\omega)e^{-jkz} = E^+(\omega)e^{-j\beta(\omega)z}e^{-\alpha(\omega)z} \approx E^+(\omega)e^{-j\beta(\omega)z}$$

$$e_x^+(z,t) = \frac{1}{2\pi}\int E_x^+(z,\omega)e^{j\omega t}d\omega = \mathrm{Re}\left\{\frac{1}{\pi}\int_0^\infty E_x^+(z,\omega)e^{j\omega t}d\omega\right\}$$

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$$\frac{1}{\pi}\int_0^\infty E^+(\omega)e^{-j\beta(\omega)z}e^{j\omega t}d\omega$$

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$\left\{E_x, H_y\right\}$

$$E_x^+(z,\omega) = E^+(\omega)e^{-j\beta(\omega)z}$$

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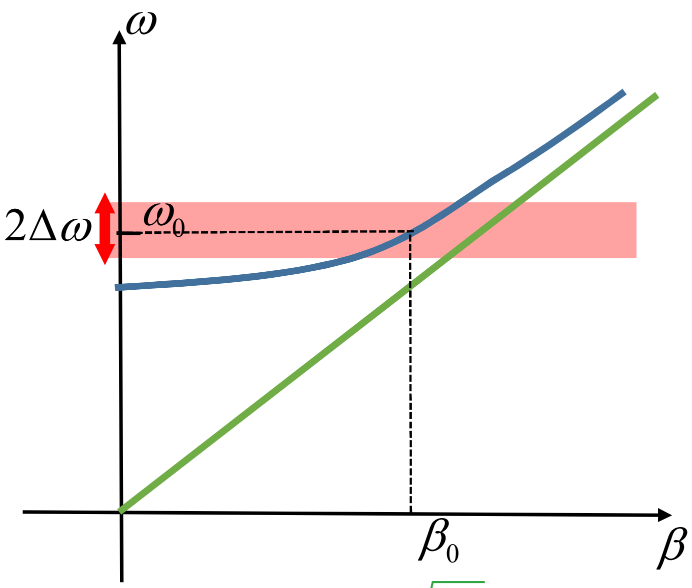

nondispersive : $\beta = \omega\sqrt{\mu\varepsilon}$

dipersive : $\beta = \omega\sqrt{\mu(\omega)\varepsilon(\omega)}$

$$\frac{1}{\pi}\int_0^{\infty} E^+(\omega)e^{-j\beta(\omega)z}e^{j\omega t}d\omega \approx \frac{1}{\pi}\int_{\omega_0-\Delta\omega}^{\omega_0+\Delta\omega} E^+(\omega)e^{-j\beta(\omega)z}e^{j\omega t}d\omega$$

$$\beta(\omega) \approx \beta(\omega_0) + \beta'(\omega_0)(\omega-\omega_0) + \cancel{\frac{1}{2}\beta''(\omega_0)(\omega-\omega_0)^2} + \cdots$$

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$$\beta_0 = \beta(\omega_0)$$

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Plane Waves : dispersion

$$\frac{1}{\pi}\int_0^{\infty} E^+(\omega)e^{-j\beta(\omega)z}e^{j\omega t}d\omega \approx \frac{1}{\pi}\int_{\omega_0-\Delta\omega}^{\omega_0+\Delta\omega} E^+(\omega)e^{-j\beta(\omega)z}e^{j\omega t}d\omega \approx \frac{1}{\pi}\int_{\omega_0-\Delta\omega}^{\omega_0+\Delta\omega} E^+(\omega)e^{-j\left[\beta_0+\frac{(\omega-\omega_0)}{v_g}\right]z}e^{j\omega t}d\omega$$

$$=\frac{1}{\pi}\int_{-\Delta\omega}^{\Delta\omega} E^+(\eta+\omega_0)e^{-j\left[\beta_0+\frac{\eta}{v_g}\right]z}e^{j(\eta+\omega_0)t}d\eta = \frac{1}{\pi}\boxed{e^{-j\beta_0 z}e^{j\omega_0 t}}\int_{-\Delta\omega}^{\Delta\omega} E^+(\omega_0+\eta)\boxed{e^{-j\frac{\eta}{v_g}z}e^{j\eta t}}d\eta$$

$$=\frac{1}{\pi}e^{j\omega_0\left(t-\frac{z}{v_p}\right)}\int_{-\Delta\omega}^{\Delta\omega} E^+(\omega_0+\eta)e^{j\left(t-\frac{z}{v_g}\right)\eta}d\eta$$

$$\eta = \omega - \omega_0$$

$$\boxed{e^{-j\beta_0 z}e^{j\omega_0 t}} = e^{j\omega_0\left(t-\frac{\beta_0}{\omega_0}z\right)} = \boxed{e^{j\omega_0\left(t-\frac{z}{v_p}\right)}}$$

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Plane Waves : dispersion

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$$\frac{1}{\pi}\int_0^\infty E^+(\omega)e^{-j\beta(\omega)z}e^{j\omega t}d\omega = \frac{1}{\pi}e^{j\omega_0\left(t-\frac{z}{v_p}\right)}\int_{-\Delta\omega}^{\Delta\omega} E^+(\omega_0+\eta)e^{j\left(t-\frac{z}{v_g}\right)\eta}d\eta$$

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$$\boxed{e_x^+(z,t)=\mathrm{Re}\left\{\frac{1}{\pi}e^{j\omega_0\left(t-\frac{z}{v_p}\right)}\int_{-\Delta\omega}^{\Delta\omega}E^+(\omega_0+\eta)e^{j\left(t-\frac{z}{v_g}\right)\eta}d\eta\right\}}$$

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$$e_x^+(z=0,t) = \mathrm{Re}\left\{\frac{1}{\pi}e^{j\omega_0 t}\underbrace{\int_{-\Delta\omega}^{\Delta\omega}E^+(\omega_0+\eta)e^{j\eta t}d\eta}_{f(t)}\right\} = \mathrm{Re}\left\{\frac{1}{\pi}e^{j\omega_0 t}f(t)\right\} = \frac{1}{\pi}f(t)\cos[\omega_0 t]$$

$$e_x^+(z>0,t) = \mathrm{Re}\left\{\frac{1}{\pi}e^{j\omega_0\left(t-\frac{z}{v_p}\right)}\underbrace{\int_{-\Delta\omega}^{\Delta\omega}E^+(\omega_0+\eta)e^{j\left(t-\frac{z}{v_g}\right)\eta}d\eta}_{f\left(t-\frac{z}{v_g}\right)}\right\} = \mathrm{Re}\left\{\frac{1}{\pi}e^{j\omega_0\left(t-\frac{z}{v_p}\right)}f\left(t-\frac{z}{v_g}\right)\right\}$$

$$= \frac{1}{\pi}f\left(t-\frac{z}{v_g}\right)\cos\left[\omega_0\left(t-\frac{z}{v_p}\right)\right]$$

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$$e_x^+(z=0,t) \qquad = \frac{1}{\pi}f(t)\cos[\omega_0 t]$$

$$e_x^+(z>0,t)$$

$$= \frac{1}{\pi}f\left(t-\frac{z}{v_g}\right)\cos\left[\omega_0\left(t-\frac{z}{v_p}\right)\right]$$

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$$e_x^+(z=0,t) = \frac{1}{\pi}f(t)\cos[\omega_0 t]$$

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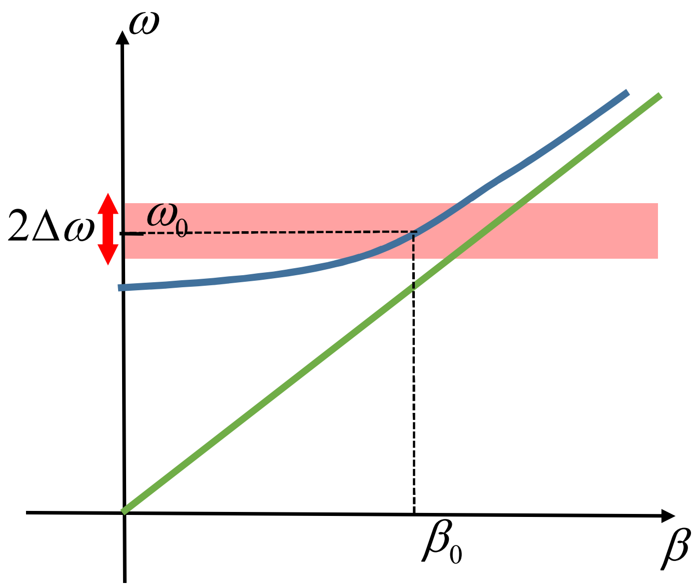

nondispersive : $\beta=\omega\sqrt{\mu\varepsilon}$

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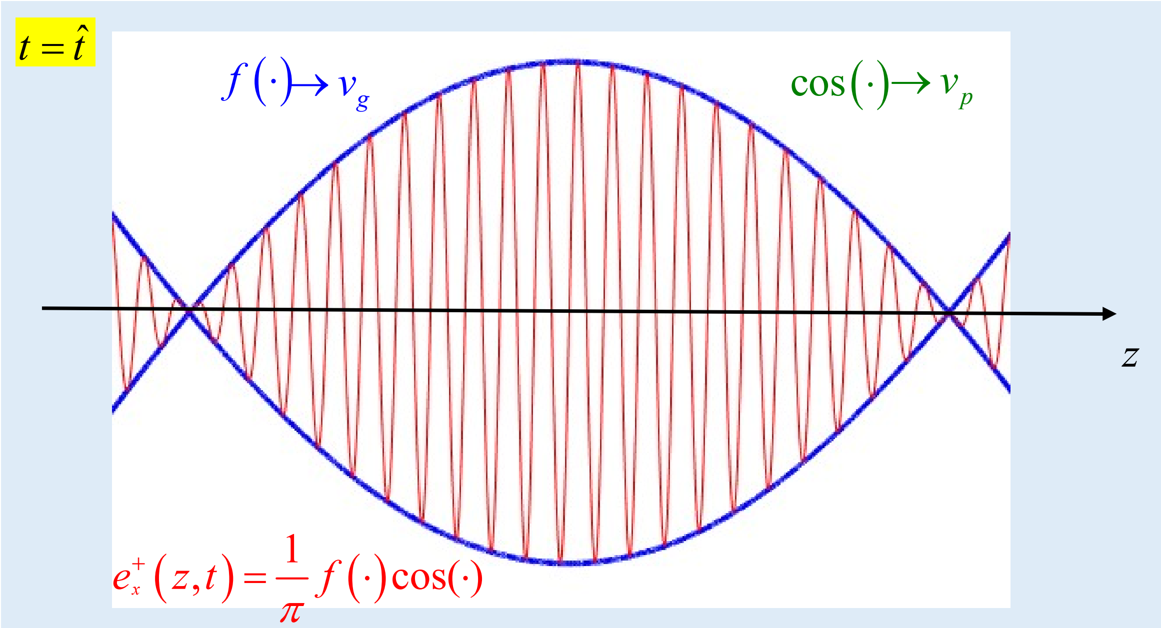
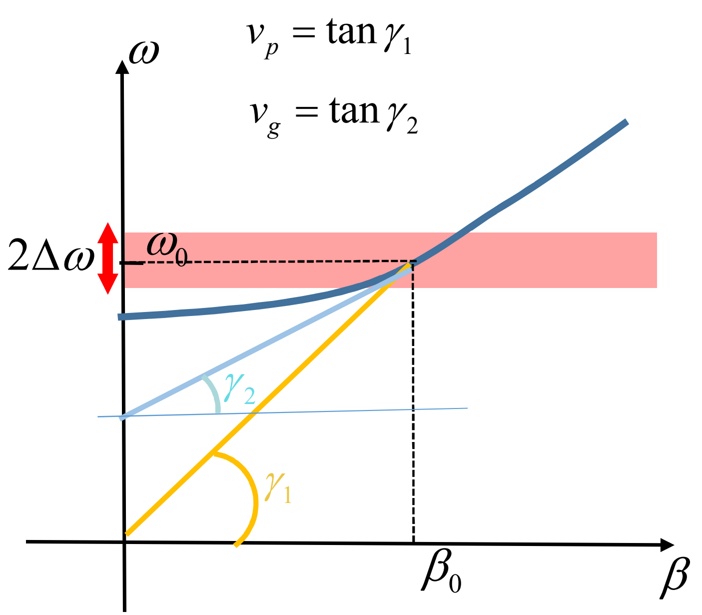


Plane Waves : dispersion



$$e_x^+(z,t) = \frac{1}{\pi} f\left(t - \frac{z}{v_g}\right)\cos\left[\omega_0\left(t - \frac{z}{v_p}\right)\right]$$

$$\beta_0 = \beta(\omega_0)$$
$$v_g = \frac{1}{\beta'(\omega_0)} = \omega'(\beta_0)$$
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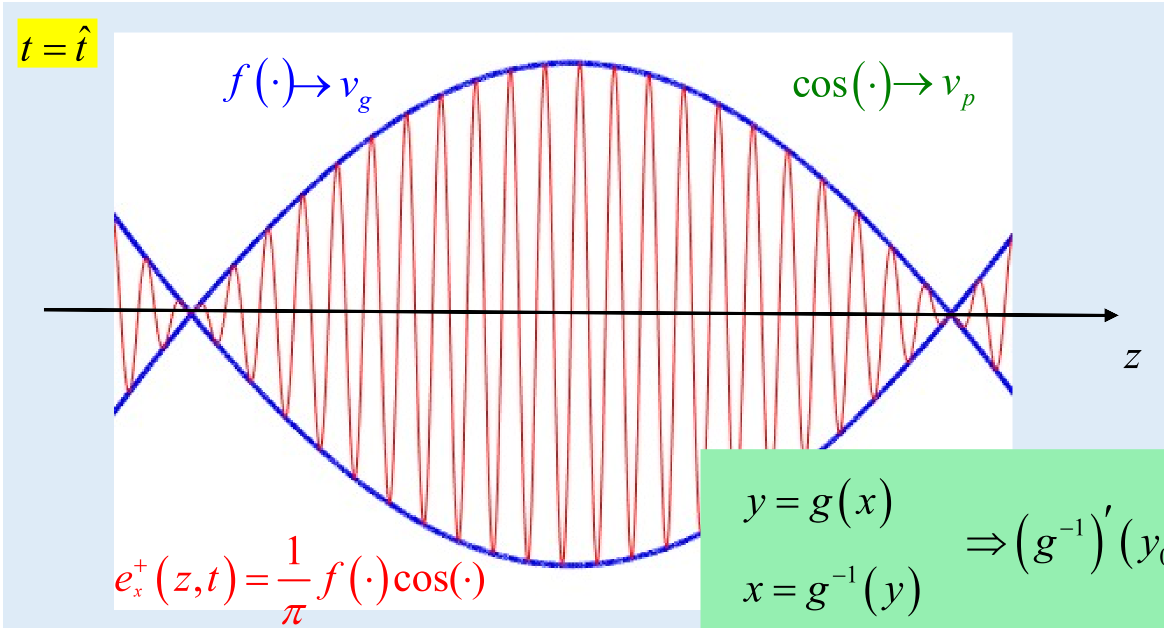
$$\text{dipersive}: \beta = \omega\sqrt{\mu(\omega)\varepsilon(\omega)}$$



$$y = g(x) \qquad x = g^{-1}(y) \qquad \Rightarrow \left(g^{-1}\right)'(y_0) = \frac{1}{g'(x_0)}$$

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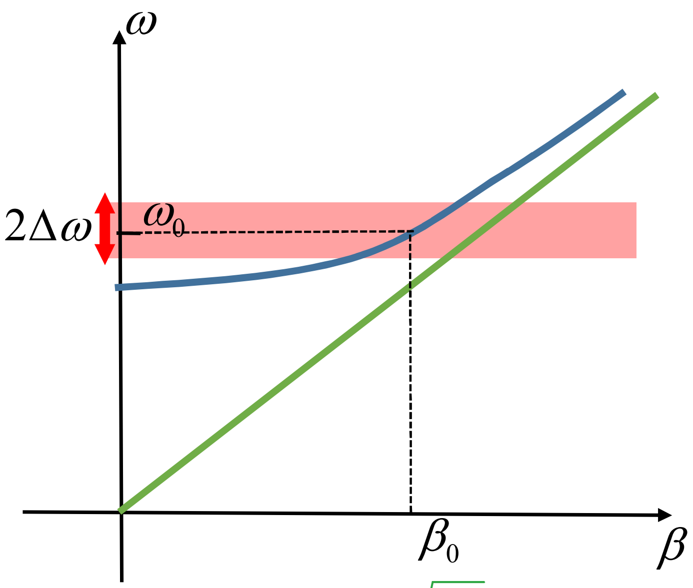

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$$\beta(\omega)\approx\beta(\omega_0)+\beta'(\omega_0)(\omega-\omega_0)+\cancel{\frac{1}{2}\beta''(\omega_0)(\omega-\omega_0)^2}+\cdots$$

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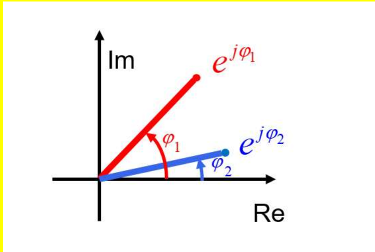
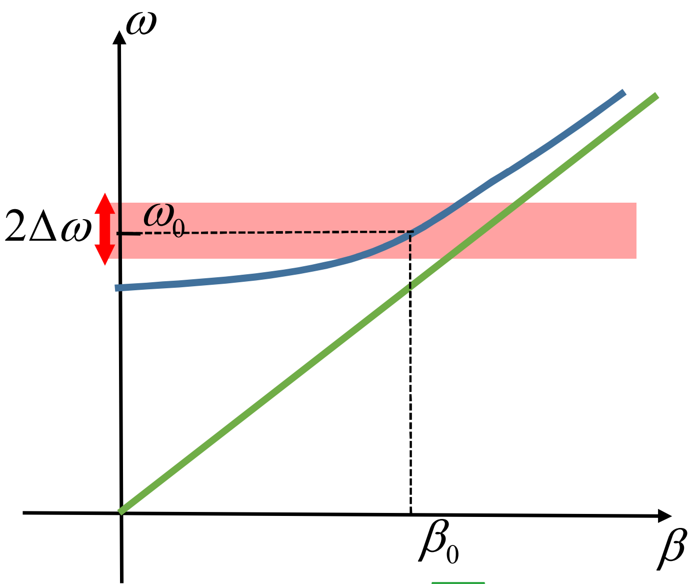
$$e^{-j\frac{\beta''(\omega_0)}{2}(\omega-\omega_0)^2 z} \approx 1$$



$$\frac{1}{2}\boxed{\beta''(\omega_0)}\Delta\omega^2 z << 2\pi$$

Channel &
carrier frequency



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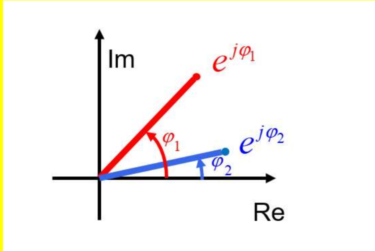
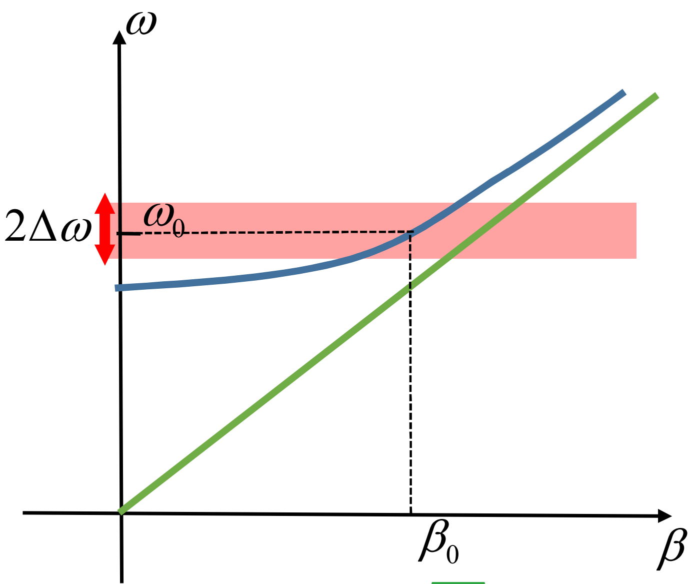


$$\frac{1}{2}\beta''(\omega_0)\boxed{\Delta\omega^2}z << 2\pi$$

Channel & carrier frequency

Bandwidth



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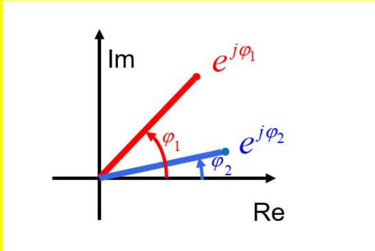
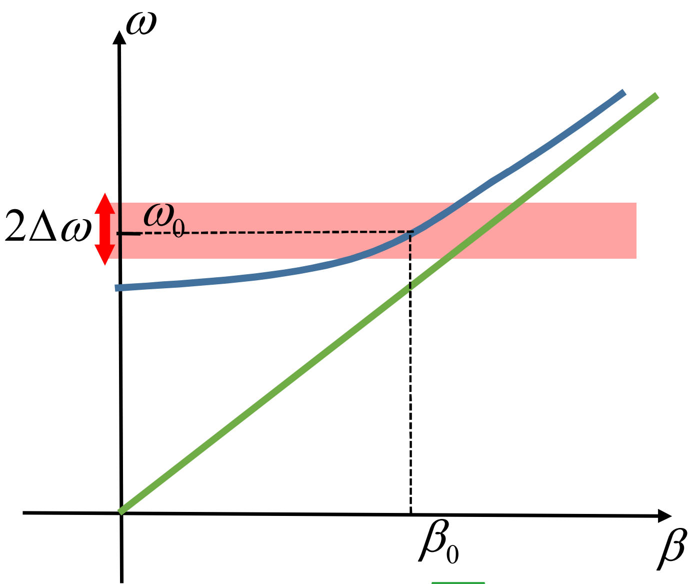


$$\frac{1}{2}\beta''(\omega_0)\Delta\omega^2 \boxed{z} << 2\pi$$

Channel & carrier frequency

Bandwidth

Distance



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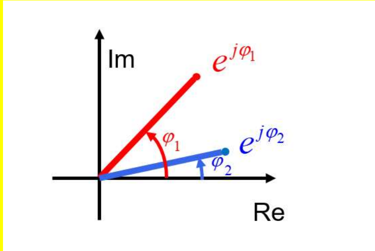
Plane Waves : dispersion

$$e^{-j\beta(\omega)z} = e^{-j\beta(\omega_0)z} e^{-j\beta'(\omega_0)(\omega-\omega_0)z} \cancel{e^{-j\frac{\beta''(\omega_0)}{2}(\omega-\omega_0)^2 z}} \cdots$$

$$e^{-j\frac{\beta''(\omega_0)}{2}(\omega-\omega_0)^2 z} \approx 1$$

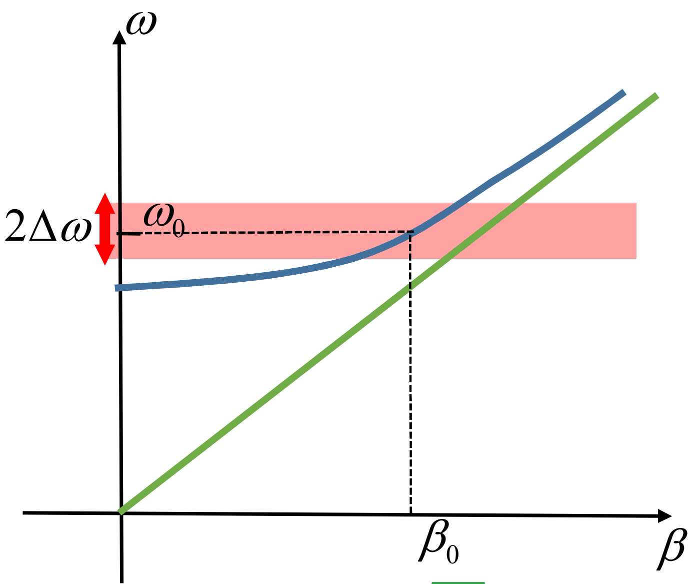


$$\frac{1}{2}\beta''(\omega_0)\Delta\omega^2 z << 2\pi$$

Channel & carrier frequency

Bandwidth

Distance



nondispersive : $\beta = \omega\sqrt{\mu\varepsilon}$

dipersive : $\beta = \omega\sqrt{\mu(\omega)\varepsilon(\omega)}$

$$\frac{1}{\pi}\int_0^\infty E^+(\omega)e^{-j\beta(\omega)z}e^{j\omega t}d\omega \approx \frac{1}{\pi}\int_{\omega_0-\Delta\omega}^{\omega_0+\Delta\omega} E^+(\omega)e^{-j\beta(\omega)z}e^{j\omega t}d\omega$$

$$\beta(\omega) \approx \beta(\omega_0) + \beta'(\omega_0)(\omega-\omega_0) + \cancel{\frac{1}{2}\beta''(\omega_0)(\omega-\omega_0)^2} + \cdots$$