Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2020-2021 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

Stefano Perna

Color legend

New formulas, important considerations, important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Plane Waves

Time domain (TD)

Spectral domains

- Phasor Domain (PD)
- Fourier Domain (FD)
- Dispersive media: attenuation, distortion, phase velocity and group velocity

General expression of plane waves (PD)

Incidence

Plane Waves

Time domain

Plane Waves (TD)

$\{e_x, h_y\}$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

$$e_x(z,t) = e_x^+(z-ct) + e_x^-(z+ct)$$

$$\zeta h_y(z,t) = e_x^+(z-ct) - e_x^-(z+ct)$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

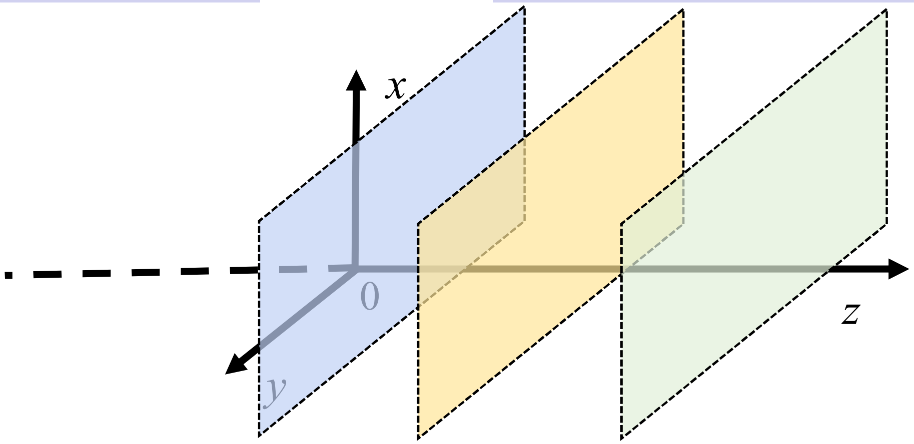
$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{e_y, h_x\}$

$$\begin{cases} \dfrac{\partial e_y}{\partial z} = \mu \dfrac{\partial h_x}{\partial t} \\ \dfrac{\partial h_x}{\partial z} = \varepsilon \dfrac{\partial e_y}{\partial t} \end{cases} \qquad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2} = 0$$

$$e_y(z,t) = e_y^+(z-ct) + e_y^-(z+ct)$$

$$-\zeta h_x(z,t) = e_y^+(z-ct) - e_y^-(z+ct)$$



Source-free

Medium
- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$, $\{e_x, h_y\}$ **Independent each other**

Plane Waves (TD)

$\{e_x^+, h_y^+\}$

$$\begin{cases} e_x^+(z,t) = e_x^+(z-ct) \\ \zeta h_y^+(z,t) = e_x^+(z-ct) \end{cases}$$

$\{e_y^+, h_x^+\}$

$$\begin{cases} e_y^+(z,t) = e_y^+(z-ct) \\ \zeta h_x^+(z,t) = -e_y^+(z-ct) \end{cases}$$

the e.m. field propagates along $\hat{i}_p = \hat{i}_z$

$\{e_x^-, h_y^-\}$

$$\begin{cases} e_x^-(z,t) = e_x^-(z+ct) \\ \zeta h_y^-(z,t) = -e_x^-(z+ct) \end{cases}$$

$\{e_y^-, h_x^-\}$

$$\begin{cases} e_y^-(z,t) = e_y^-(z+ct) \\ \zeta h_x^-(z,t) = e_y^-(z+ct) \end{cases}$$

the e.m. field propagates along $\hat{i}_p = -\hat{i}_z$

- the e.m. field lies on the plane xy orthogonal to the propagation direction
- The Poynting vector is directed along the direction of propagation
- $|\vec{\mathbf{e}}|$ and $|\vec{\mathbf{h}}|$ are proportional through ζ
- $\zeta \vec{\mathbf{h}} = \hat{i}_p \times \vec{\mathbf{e}}$

Source-free

Medium

- **Linear**
- **Time non-dispersive**
- **Space non-dispersive**
- **Isotropic**
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$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

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$\{e_y, h_x\}$, $\{e_x, h_y\}$ **Independent each other**

Plane Waves

Spectral domains

Plane Waves (Spectral Domains)

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$k = \omega\sqrt{\mu\varepsilon}$$
$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{E_y, H_x\}$

$$\begin{cases} \dfrac{dE_y}{dz} = j\omega\mu H_x \\ \dfrac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases} \qquad \frac{d^2 E_y}{dz^2} + k^2 E_y = 0$$

$$E_y = E_y^+ e^{-jkz} + E_y^- e^{jkz}$$

$$-\zeta H_x = E_y^+ e^{-jkz} - E_y^- e^{jkz}$$

Source-free

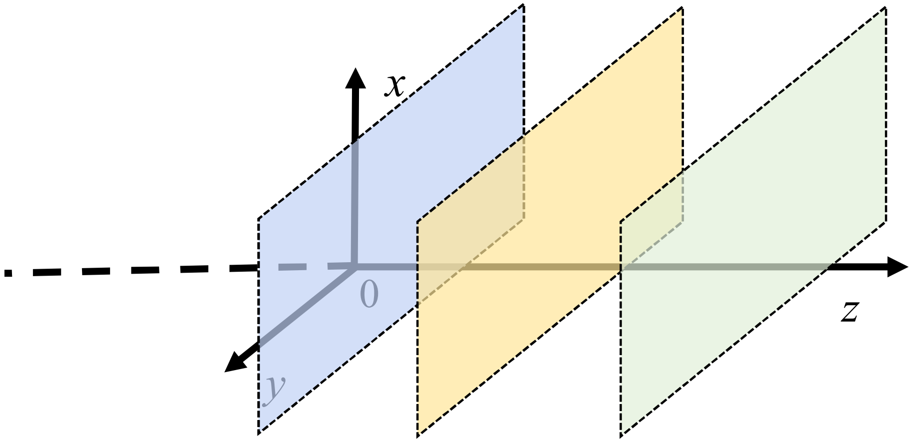
Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{e}}(z, t)$$

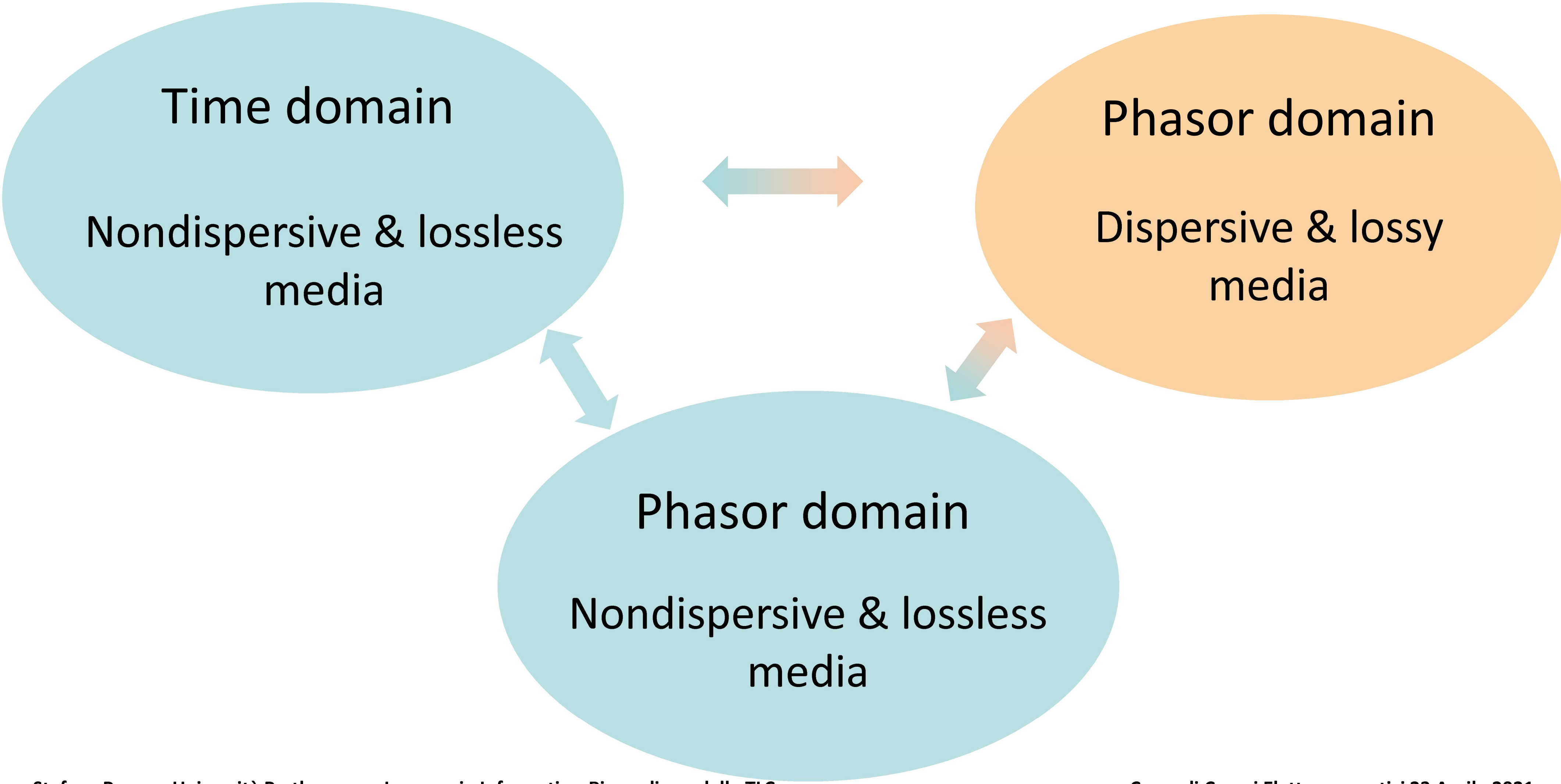
$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{h}}(z, t)$$

$$E_z = H_z = 0$$



$\{E_y, H_x\}$
$\{E_x, H_y\}$ **Independent each other**

Rationale


Time domain
Nondispersive & lossless media
Phasor domain
Dispersive & lossy media
Phasor domain
Nondispersive & lossless media

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$

$$E_x^+(z) = E^+ e^{-j\beta z}$$
$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$E_x^-(z) = E^- e^{j\beta z}$$
$$\zeta H_y^-(z) = -E^- e^{j\beta z}$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = |E^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

$$E^- e^{j\beta z} \rightarrow e_x^-(z,t) = |E^-|\cos(\omega_o t + \beta z + \varphi^-) = e_x^-(z + v_p t)$$

Regressive plane wave

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\varepsilon_{eq} = \varepsilon$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

Source-free

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$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

Plane Waves (Phasor Domain)

Source-free

Medium

- Linear
- Time nondispersive
- Space non-dispersive
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- Homogeneous (TI – SI)
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$\{E_x, H_y\}$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

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$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = |E^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

<u>Progressive</u> plane wave

$$E^- e^{j\beta z} \rightarrow e_x^-(z,t) = |E^-|\cos(\omega_o t + \beta z + \varphi^-) = e_x^-(z + v_p t)$$

<u>Regressive</u> plane wave

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

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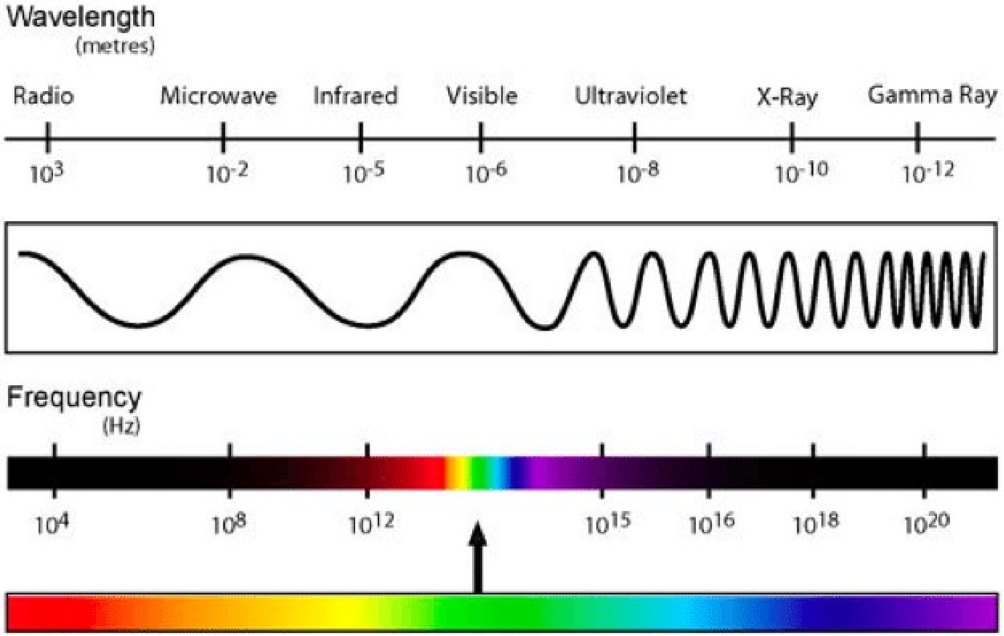
$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

Electromagnetic spectrum



Free space (Linear, isotropic, local, homogeneous)

$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f}$$

f: frequency
λ: wavelength
c: light speed
β: propagation constant

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$\omega_o = 2\pi f_0$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

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Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f_0}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = |E^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

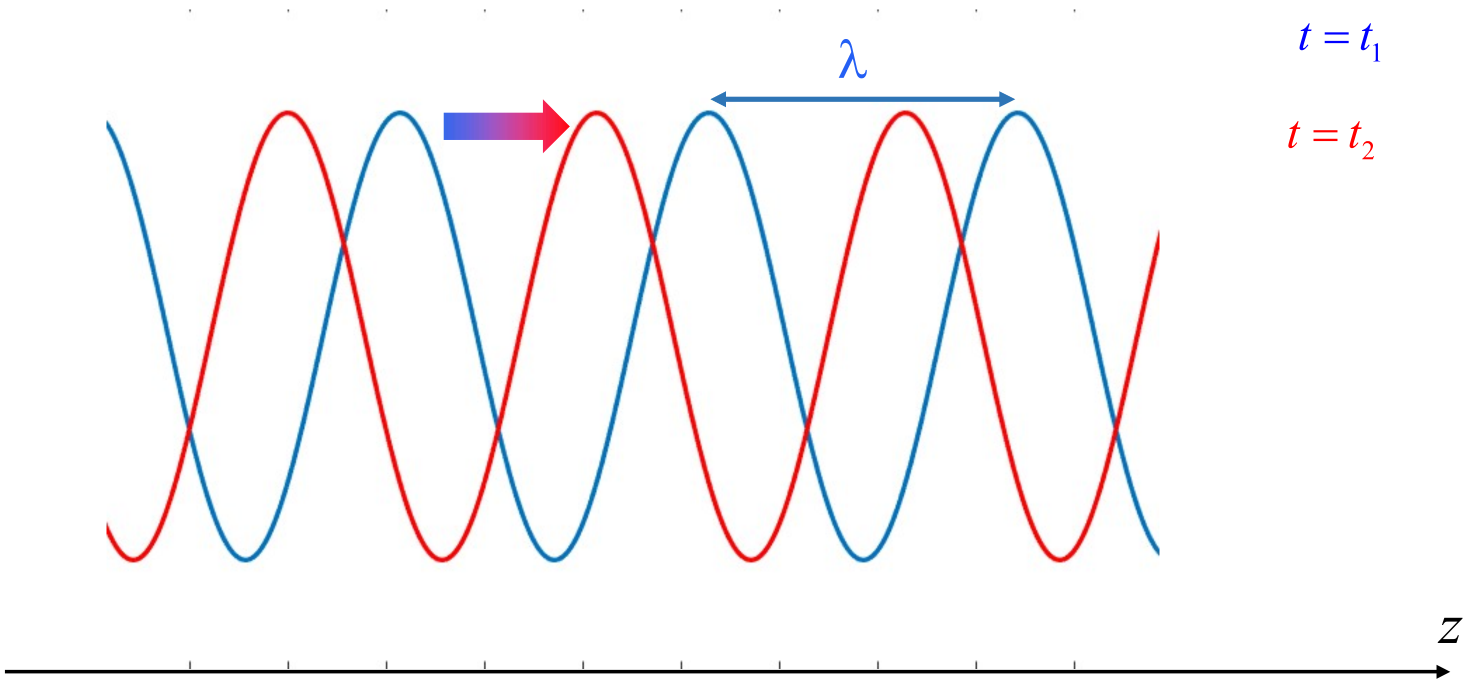
<u>Progressive</u> plane wave

- The term $e^{-j\beta z}$ is related to the propagation along the (positive sense of the) z-axis
- When z is small with respect to λ, the propagation effects become negligible

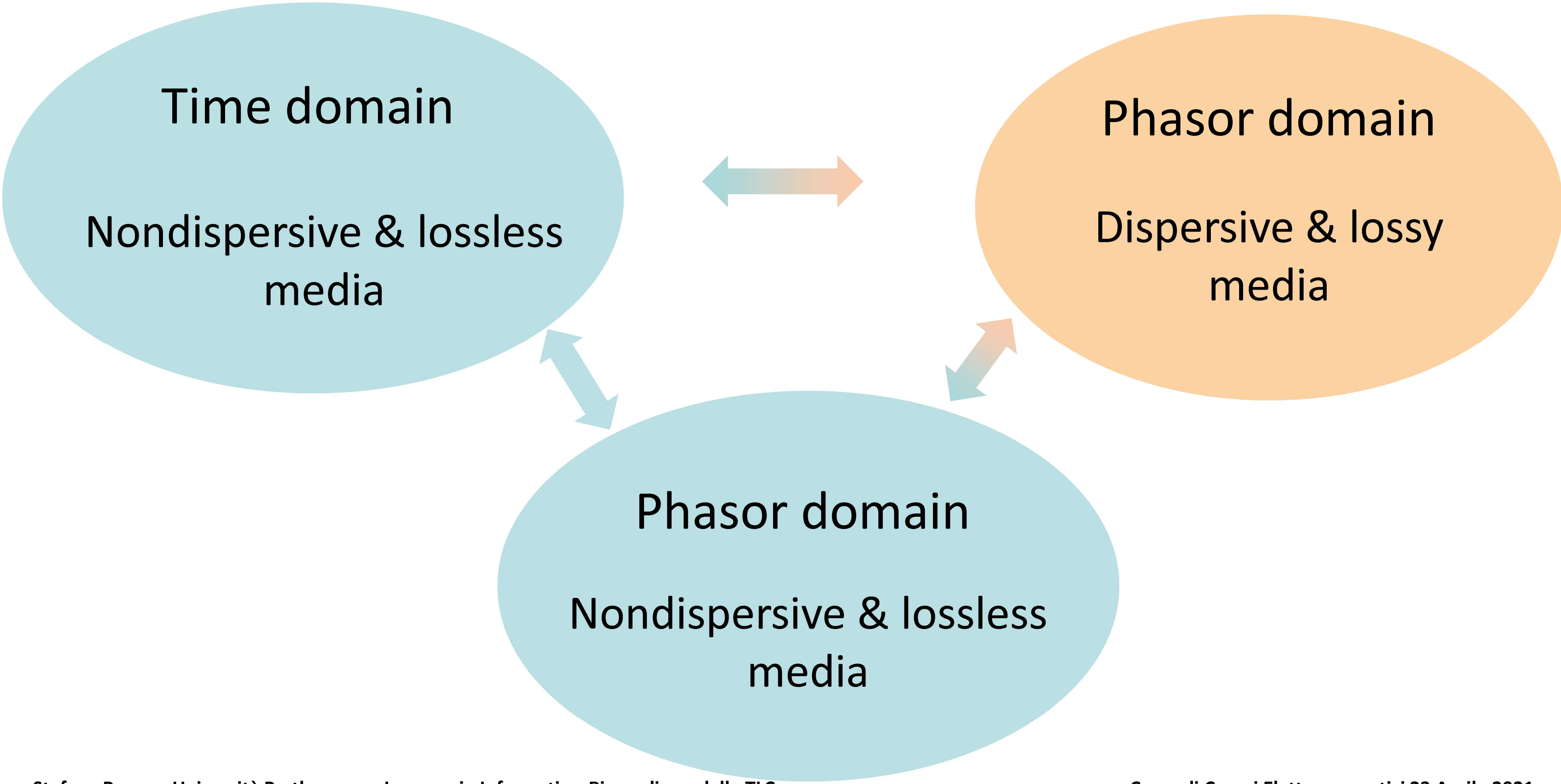
Plane Waves (Phasor Domain)

$$e_x^+(z,t) = \left|E^+\right| \cos\left(\omega_o t - \beta z + \varphi^+\right) = e_x^+\left(z - v_p t\right)$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$



Rationale


Time domain
Nondispersive & lossless media
Phasor domain
Dispersive & lossy media
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Plane Waves (Phasor Domain)

$\left\{E_x, H_y\right\}$

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Time nondispersive & lossless

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Progressive plane wave

Plane Waves (Phasor Domain)

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<u>Progressive</u> plane wave

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

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Plane Waves (Phasor Domain)

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Progressive plane wave

Source-free

Medium
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- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
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$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

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Plane Waves (Phasor Domain)

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Progressive plane wave

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$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

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Plane Waves (Phasor Domain)

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Progressive plane wave

Source-free

Medium
- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- Lossy

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$
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$$E_z = H_z = 0$$

$\{E_y, H_x\}$ $\{E_x, H_y\}$ **Independent each other**

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

Plane Waves (Phasor Domain)

$\left\{E_x, H_y\right\}$

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$$E^+ e^{-j\beta z} \to e_x^+(z,t) = \left|E^+\right|\cos\left(\omega_o t - \beta z + \varphi^+\right) = e_x^+\left(z - v_p t\right)$$

<u>Progressive</u> plane wave

Source-free

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Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma : \ real \end{cases}$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$
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Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$\left\{E_y, H_x\right\}$
$\left\{E_x, H_y\right\}$
Independent each other

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$k(\omega_0) = \omega_0\sqrt{\mu(\omega_0)\varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

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Progressive plane wave

Time dispersive (lossy)

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Time nondispersive & lossless

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$\{E_y, H_x\}$

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Independent each other

Plane Waves (Phasor Domain)

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~~$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$~~

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$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

Plane Waves (Phasor Domain)

$\left\{E_x, H_y\right\}$

$$k(\omega_0) = \omega_0\sqrt{\mu(\omega_0)\varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$
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$$\zeta(\omega_0) = \sqrt{\frac{\mu(\omega_0)}{\varepsilon(\omega_0)}}$$

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$$E^+ e^{-j\beta z} \quad \rightarrow e_x^+(z,t) = \left|E^+\right|\cos\left(\omega_o t - \beta z + \varphi^+\right) = e_x^+\left(z - v_p t\right)$$

Progressive plane wave

Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossy**

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma : \ real \end{cases}$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$
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Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$\left\{E_y, H_x\right\}$

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Independent each other

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Progressive plane wave

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Independent each other

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Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

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$$E^+ e^{-jkz} \quad \rightarrow \quad e^+(z,t) = e^{-\alpha z} |E^+| \cos(\omega_o t - \beta z + \varphi^+)$$

$$E^+ e^{-jkz} = E^+ e^{-j(\beta - j\alpha)z} = E^+ e^{(-j\beta z - \alpha z)} = E^+ e^{-j\beta z} e^{-\alpha z}$$

$$E^+ = |E^+| e^{j\varphi^+} \quad \Rightarrow E^+ e^{-jkz} = |E^+| e^{j\varphi^+} e^{-j\beta z} e^{-\alpha z}$$

$$e_x^+(z,t) = \mathrm{Re}\left\{ |E^+| e^{j\varphi^+} e^{-j\beta z} \underbrace{e^{-\alpha z}} e^{j\omega_0 t} \right\} = e^{-\alpha z} |E^+| \cos(\omega_o t - \beta z + \varphi^+)$$

$$= e^{-\alpha z} |E^+| \cos\left(-\beta \left[z - \frac{\omega_0}{\beta} t \right] + \varphi^+ \right) = e^{-\alpha z} |E^+| \cos\left(-\beta [z - v_p t] + \varphi^+ \right)$$

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma : \ real \end{cases}$$

Time nondispersive & lossless

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<u>Progressive</u> plane wave

Source-free

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$\{E_y, H_x\}$

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Independent each other

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma : \ real \end{cases}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

Plane Waves (Phasor Domain)

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<u>Progressive</u> plane wave

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Time dispersive (lossy)

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Time nondispersive & lossless

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Plane Waves (Phasor Domain)

Time nondispersive & lossless medium

$$e_x^+(z,t) = |E^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

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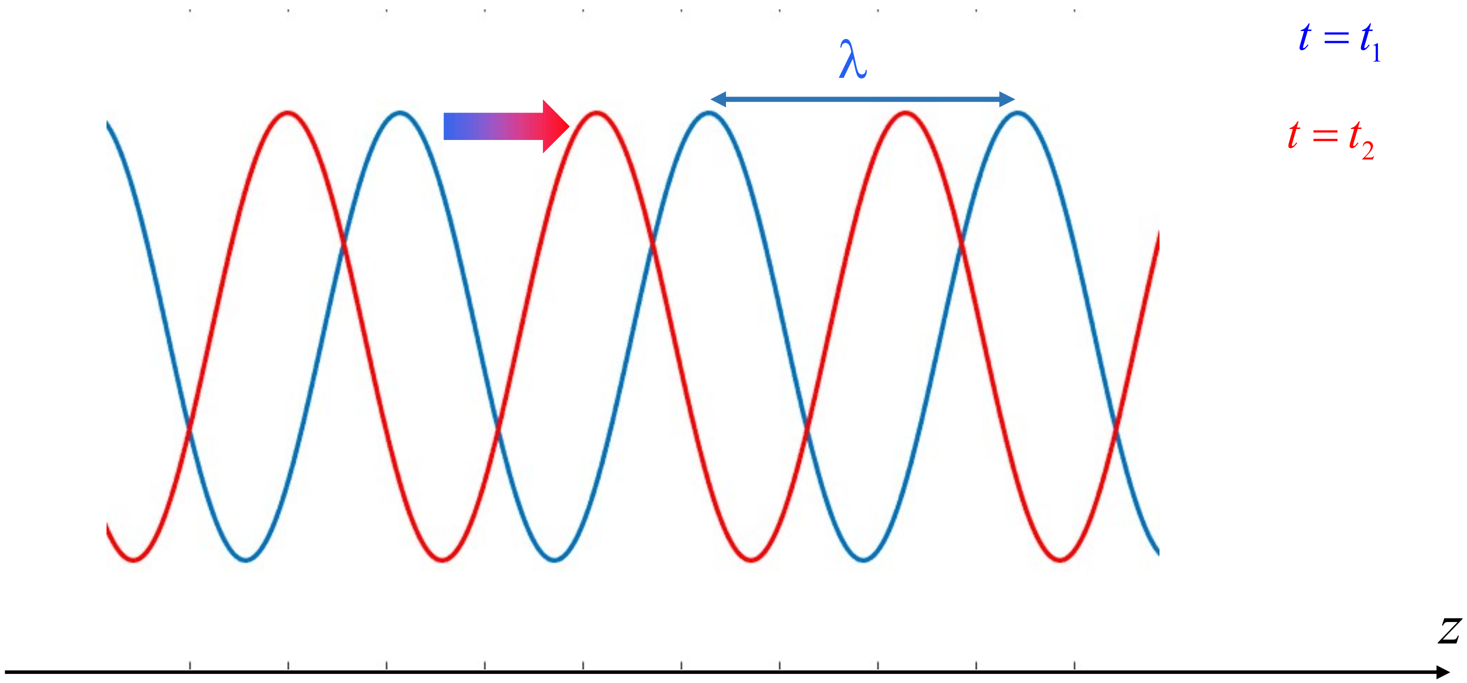

Plane Waves (Phasor Domain)

Time dispersive & lossy medium

$$e_x^+(z,t) = |E^+| e^{-\alpha z} \cos(\omega_o t - \beta z + \varphi^+) = e^{-\alpha z} e_x^+(z - v_p t)$$

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z

Plane Waves (Phasor Domain)

Time dispersive & lossy medium

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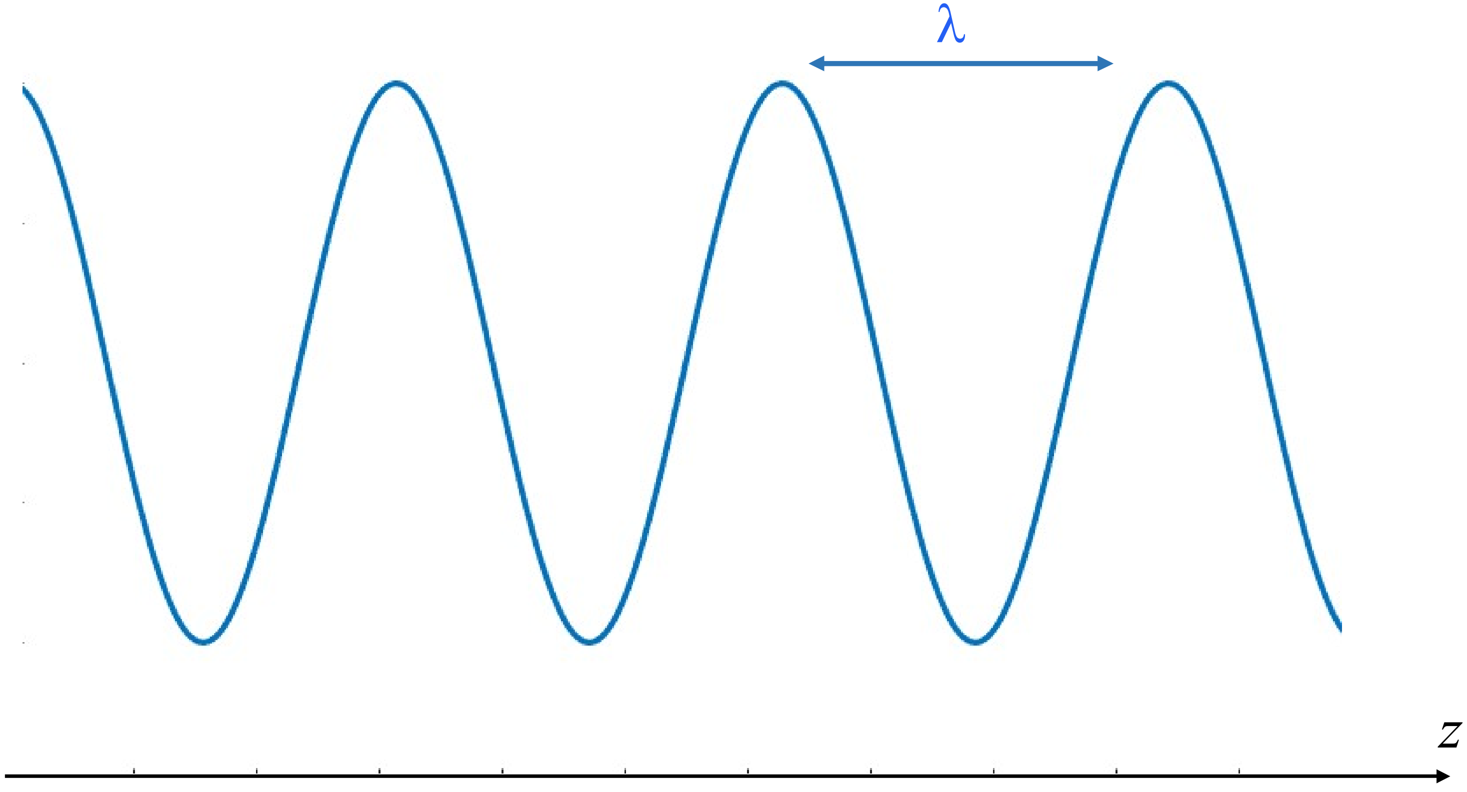

$$\cos(\omega_o t - \beta z + \varphi^+)$$

Plane Waves (Phasor Domain)

Time dispersive & lossy medium

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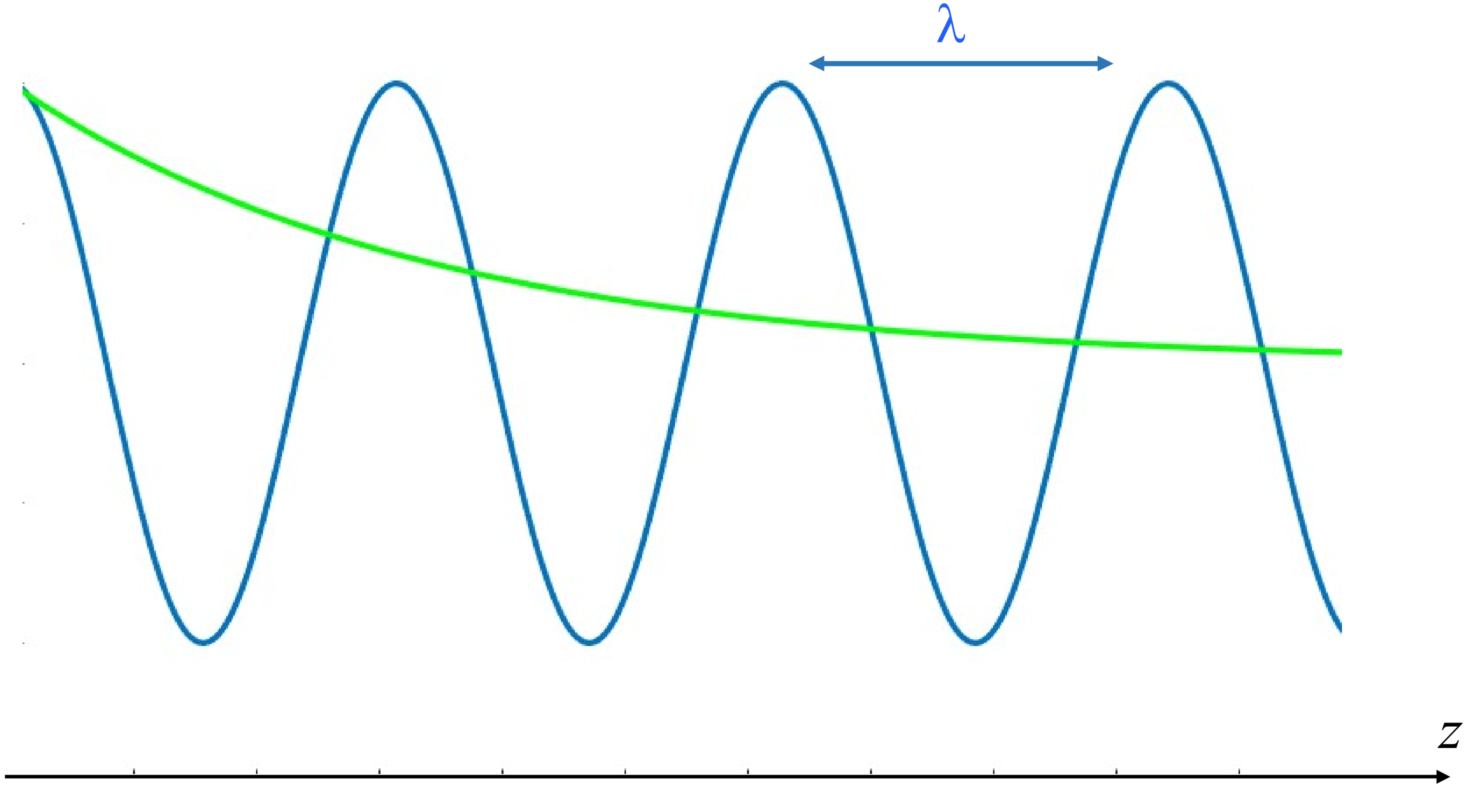

$\cos(\omega_o t - \beta z + \varphi^+)$

$e^{-\alpha z}$

Plane Waves (Phasor Domain)

Time dispersive & lossy medium

$$e_x^+(z,t) = \left|E^+\right| e^{-\alpha z} \cos\left(\omega_o t - \beta z + \varphi^+\right) = e^{-\alpha z} e_x^+\left(z - v_p t\right)$$

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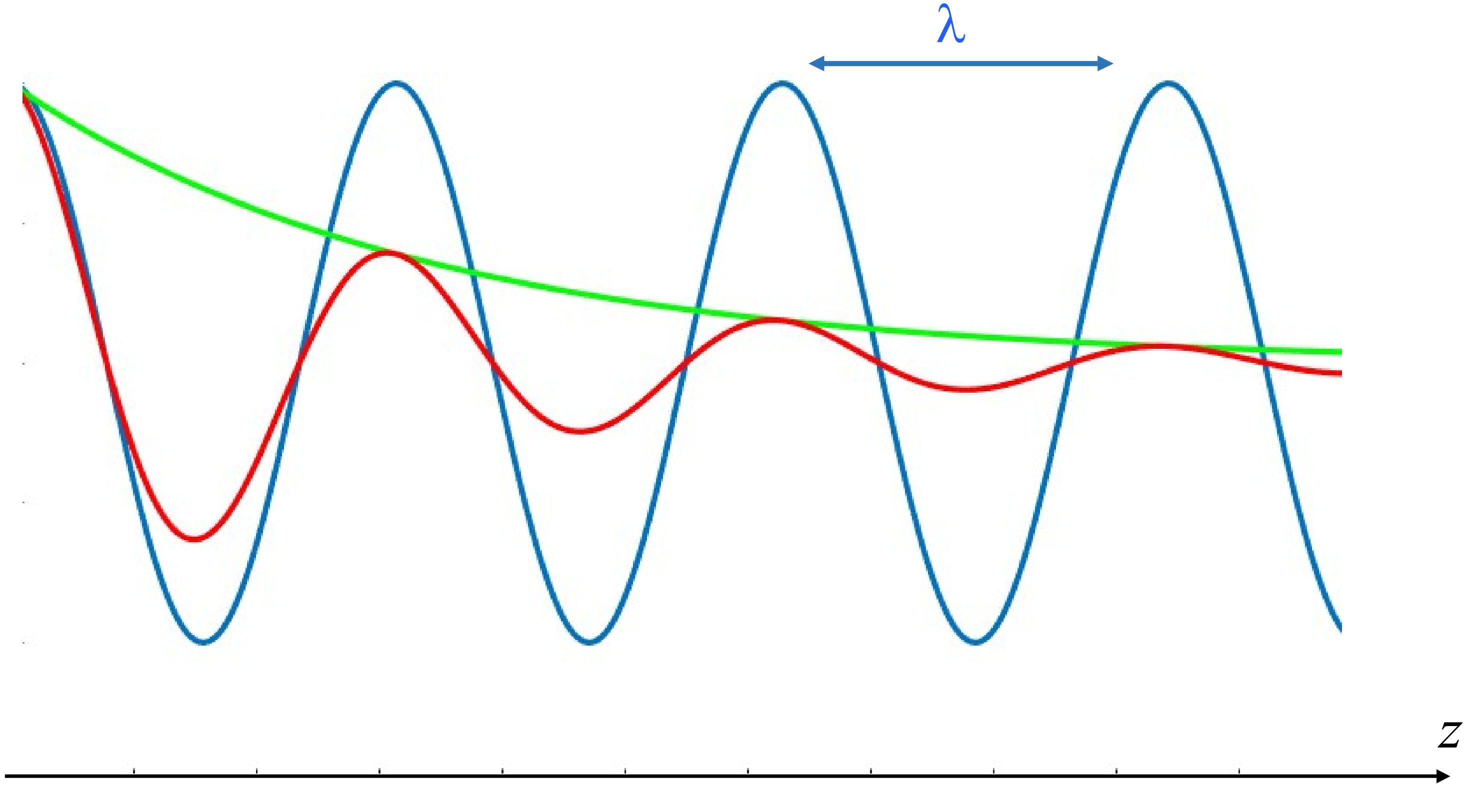

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Plane Waves (Phasor Domain)

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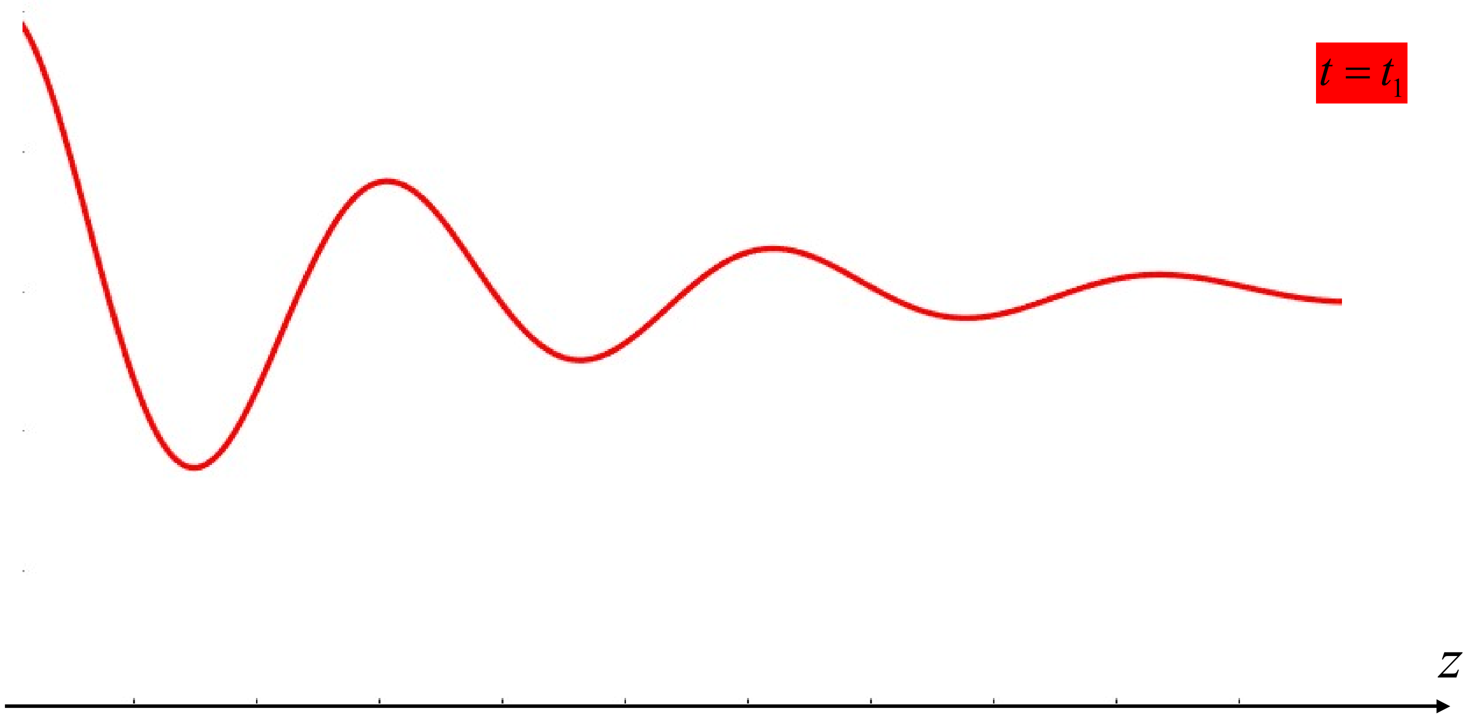

Plane Waves (Phasor Domain)

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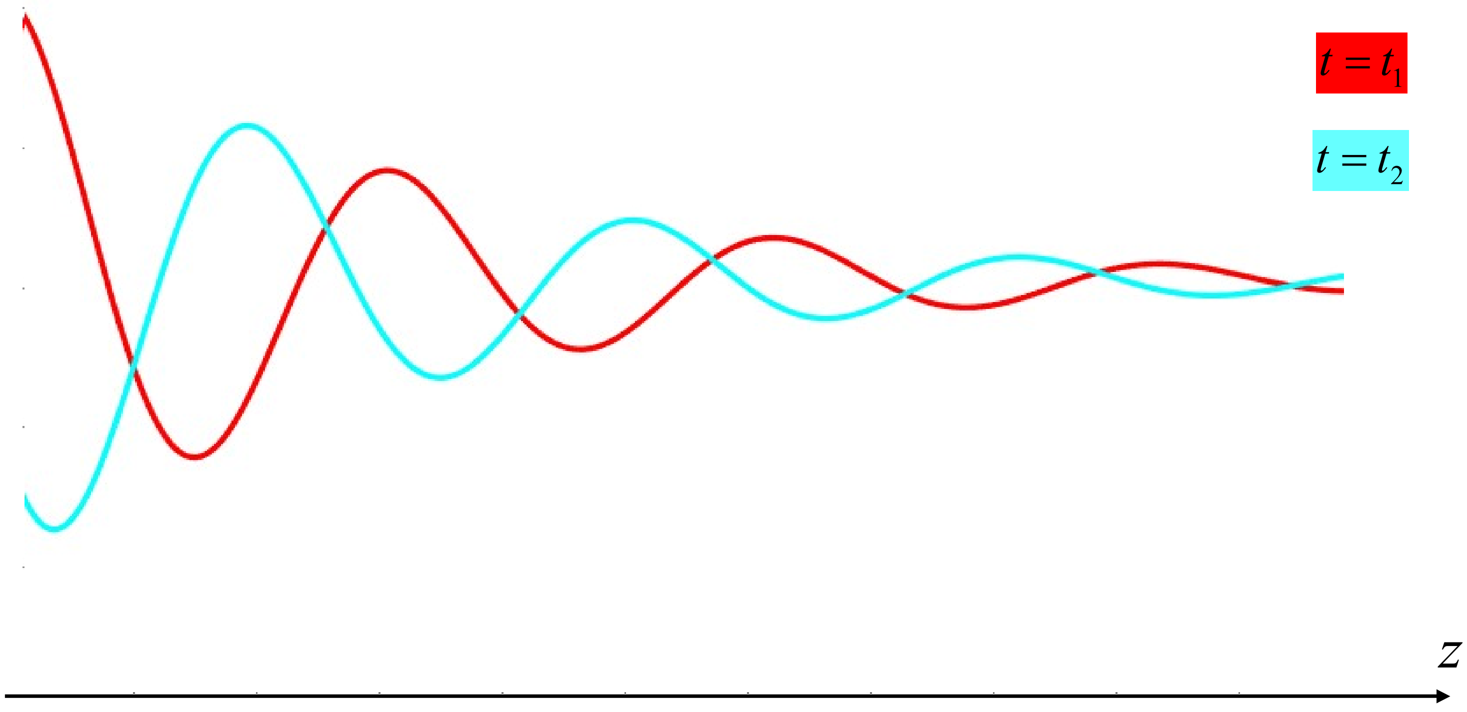

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

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Time dispersive (lossy)

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Time nondispersive & lossless

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Independent each other

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Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

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- **Lossy**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$, $\{E_x, H_y\}$ **Independent each other**

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$k(\omega_0) = \omega_0\sqrt{\mu(\omega_0)\varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

$$E_x^+(z) = E^+ e^{-jkz}$$
$$\zeta H_y^+(z) = E^+ e^{-jkz}$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\mathrm{Re}\left\{\omega_0\sqrt{\mu(\omega_0)\varepsilon(\omega_0)}\right\}} = \frac{1}{\mathrm{Re}\left\{\sqrt{\mu(\omega_0)\varepsilon(\omega_0)}\right\}} = v_p(\omega_0)$$

$$E^+ e^{-jkz} \quad \rightarrow \quad e^+(z,t) = e^{-\alpha z}\left|E^+\right|\cos\left(\omega_o t - \beta z + \varphi^+\right)$$

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma : real \end{cases}$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$
$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = \left|E^+\right|\cos\left(\omega_o t - \beta z + \varphi^+\right) = e_x^+(z - v_p t)$$

Source-free

Medium
- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- Lossy

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

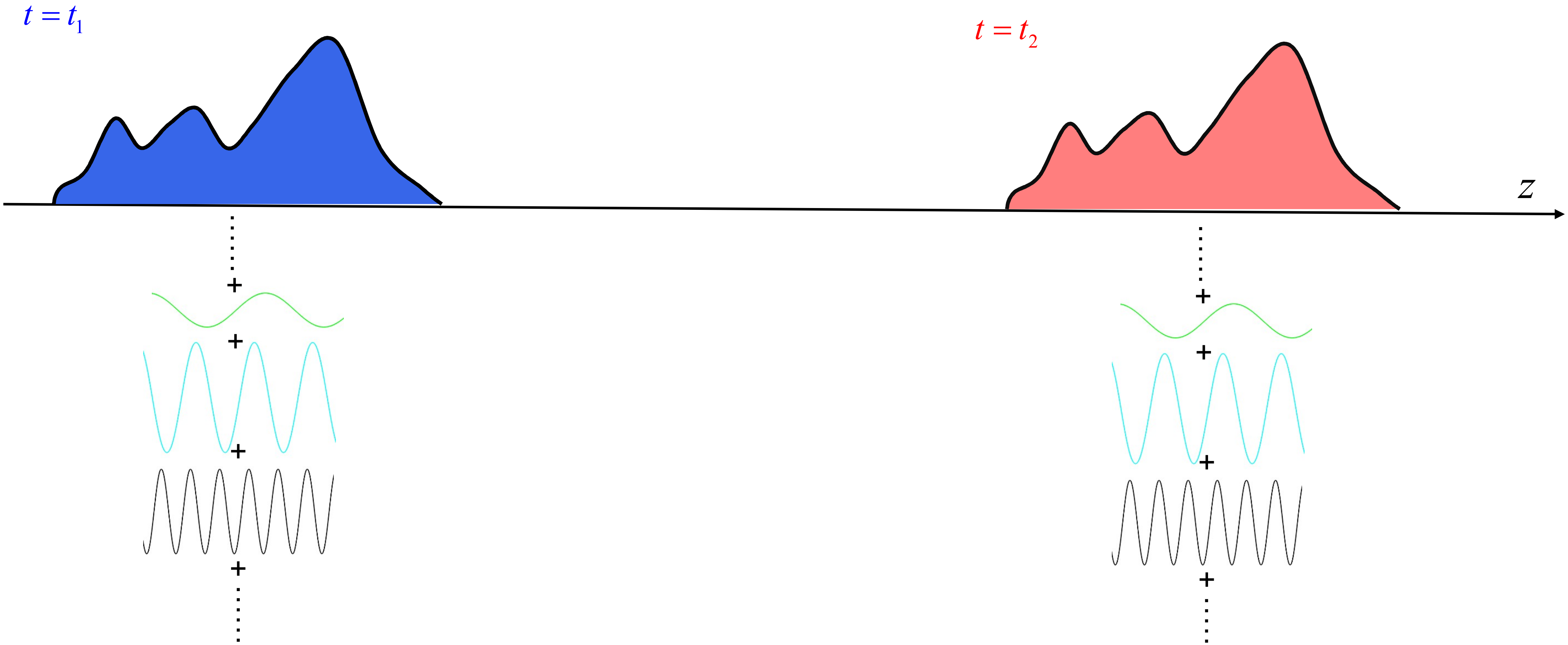
$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

Plane Waves

Time nondispersive & lossless medium

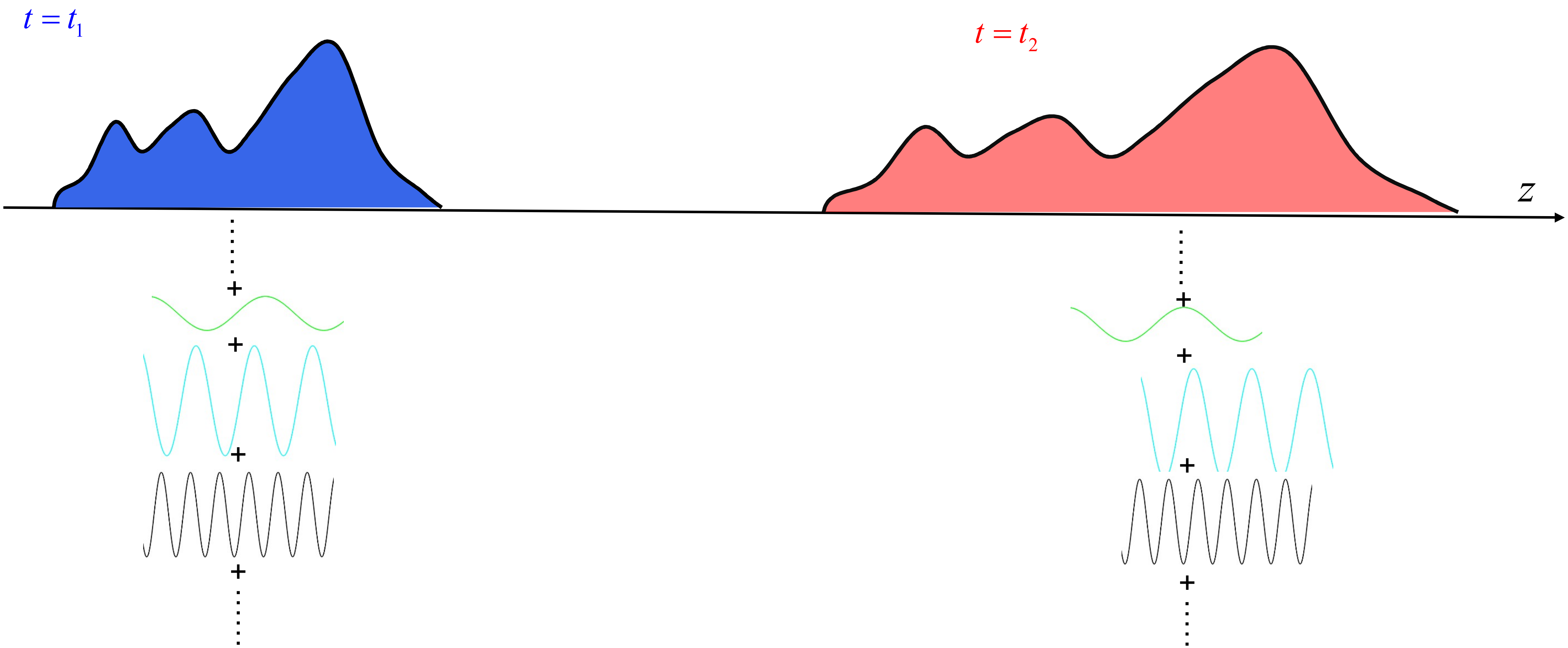
$v_p = c$



Plane Waves

Time dispersive medium

$$v_p = v_p(\omega_0)$$



Plane Waves (Phasor Domain)

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta} = c$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma : \ real \end{cases}$$

Attenuation

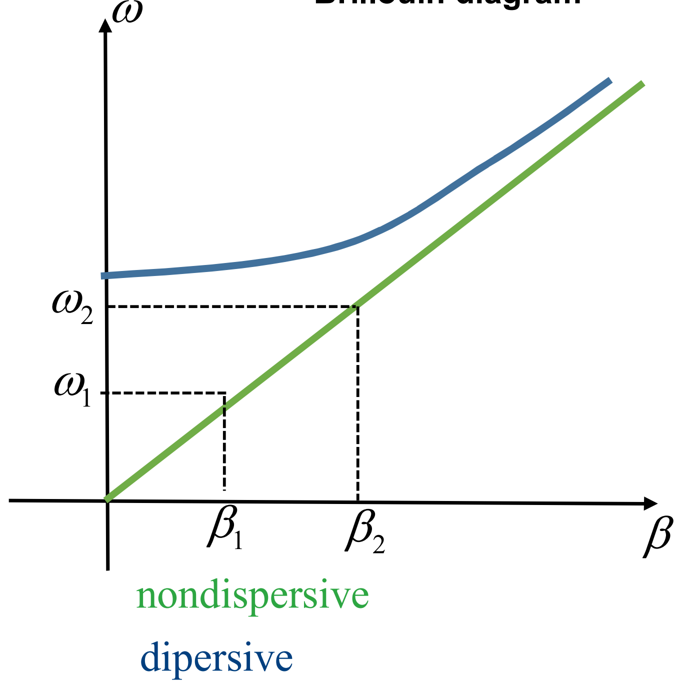
$$\alpha \neq 0$$

Distortion

$$v_p = \frac{\omega_0}{\beta} = v_p(\omega_0)$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0)\varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

Brillouin diagram



$$v_p = \frac{\omega_0}{\beta} = v_p(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta} = c$$

Plane Waves (Phasor Domain)

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta} = c$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma : real \end{cases}$$

Attenuation

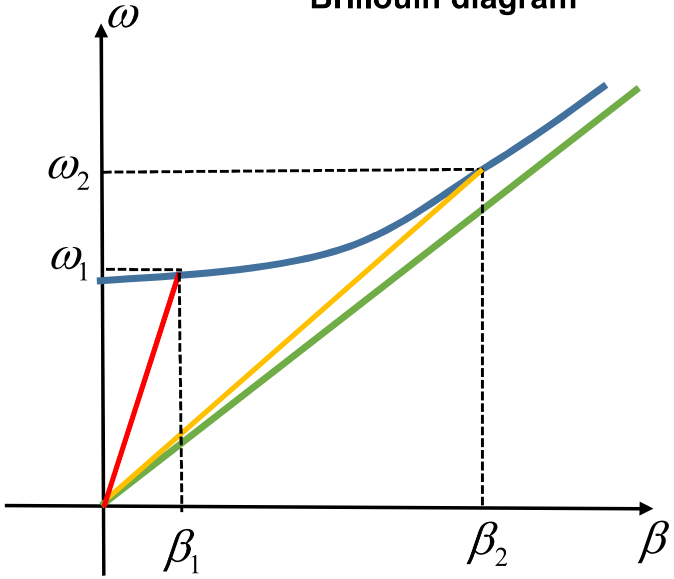
$$\alpha \neq 0$$

Distortion

$$v_p = \frac{\omega_0}{\beta} = v_p(\omega_0)$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0)\varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

Brillouin diagram



nondispersive

dipersive

$$v_p = \frac{\omega_0}{\beta} = v_p(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta} = c$$

Plane Waves (Spectral Domains)

$$\{E_x, H_y\}$$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega_0 \mu H_y \\ \dfrac{dH_y}{dz} = -j\omega_0 \varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x(z) = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y(z) = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \end{cases}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0)\varepsilon(\omega_0)}$$

$$k(\omega_0) = \beta(\omega_0) - j\alpha(\omega_0)$$

Phasor Domain

$$k = \omega\sqrt{\mu\varepsilon}$$

$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\{E_x, H_y\}$$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega \mu H_y \\ \dfrac{dH_y}{dz} = -j\omega \varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x(z,\omega) = E_x^+(\omega) e^{-jkz} + E_x^-(\omega) e^{jkz}$$

$$\zeta H_y(z,\omega) = E_x^+(\omega) e^{-jkz} - E_x^-(\omega) e^{jkz}$$

$$\begin{cases} \varepsilon(\omega) = \varepsilon_1(\omega) - j\varepsilon_2(\omega) \\ \mu(\omega) = \mu_1(\omega) - j\mu_2(\omega) \end{cases}$$

$$k(\omega) = \omega \sqrt{\mu(\omega)\varepsilon(\omega)}$$

$$k(\omega) = \beta(\omega) - j\alpha(\omega)$$

Fourier Domain

Source-free

Medium

- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- ~~Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{e}}(z, t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{h}}(z, t)$$

⬇

$$E_z = H_z = 0$$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

Plane Waves (Fourier Domain)

$$k = \omega\sqrt{\mu\varepsilon}$$
$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases}$$

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x(z,\omega) = E_x^+(\omega)e^{-jkz} + E_x^-(\omega)e^{jkz}$$

$$\zeta H_y(z,\omega) = E_x^+(\omega)e^{-jkz} - E_x^-(\omega)e^{jkz}$$

$$\begin{cases} \varepsilon(\omega) = \varepsilon_1(\omega) - j\varepsilon_2(\omega) \\ \mu(\omega) = \mu_1(\omega) - j\mu_2(\omega) \end{cases}$$

$$k(\omega) = \omega\sqrt{\mu(\omega)\varepsilon(\omega)}$$
$$k(\omega) = \beta(\omega) - j\alpha(\omega)$$

Fourier Domain

Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

Plane Waves (Fourier Domain)

$$k=\omega\sqrt{\mu\varepsilon}$$
$$k=\beta-j\alpha$$

$$\zeta=\sqrt{\frac{\mu}{\varepsilon}}$$

$$\left\{E_x,H_y\right\}$$

$$\begin{cases}\dfrac{dE_x}{dz}=-j\omega\mu H_y\\ \dfrac{dH_y}{dz}=-j\omega\varepsilon E_x\end{cases}$$

$$\frac{d^2E_x}{dz^2}+k^2E_x=0$$

$$E_x^+(z,\omega)=E^+(\omega)e^{-jkz}$$

$$\zeta H_y^+(z,\omega)=E^+(\omega)e^{-jkz}$$

$$\begin{cases}\varepsilon(\omega)=\varepsilon_1(\omega)-j\varepsilon_2(\omega)\\ \mu(\omega)=\mu_1(\omega)-j\mu_2(\omega)\end{cases}$$

$$k(\omega)=\omega\sqrt{\mu(\omega)\varepsilon(\omega)}$$
$$k(\omega)=\beta(\omega)-j\alpha(\omega)$$

Fourier Domain

Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

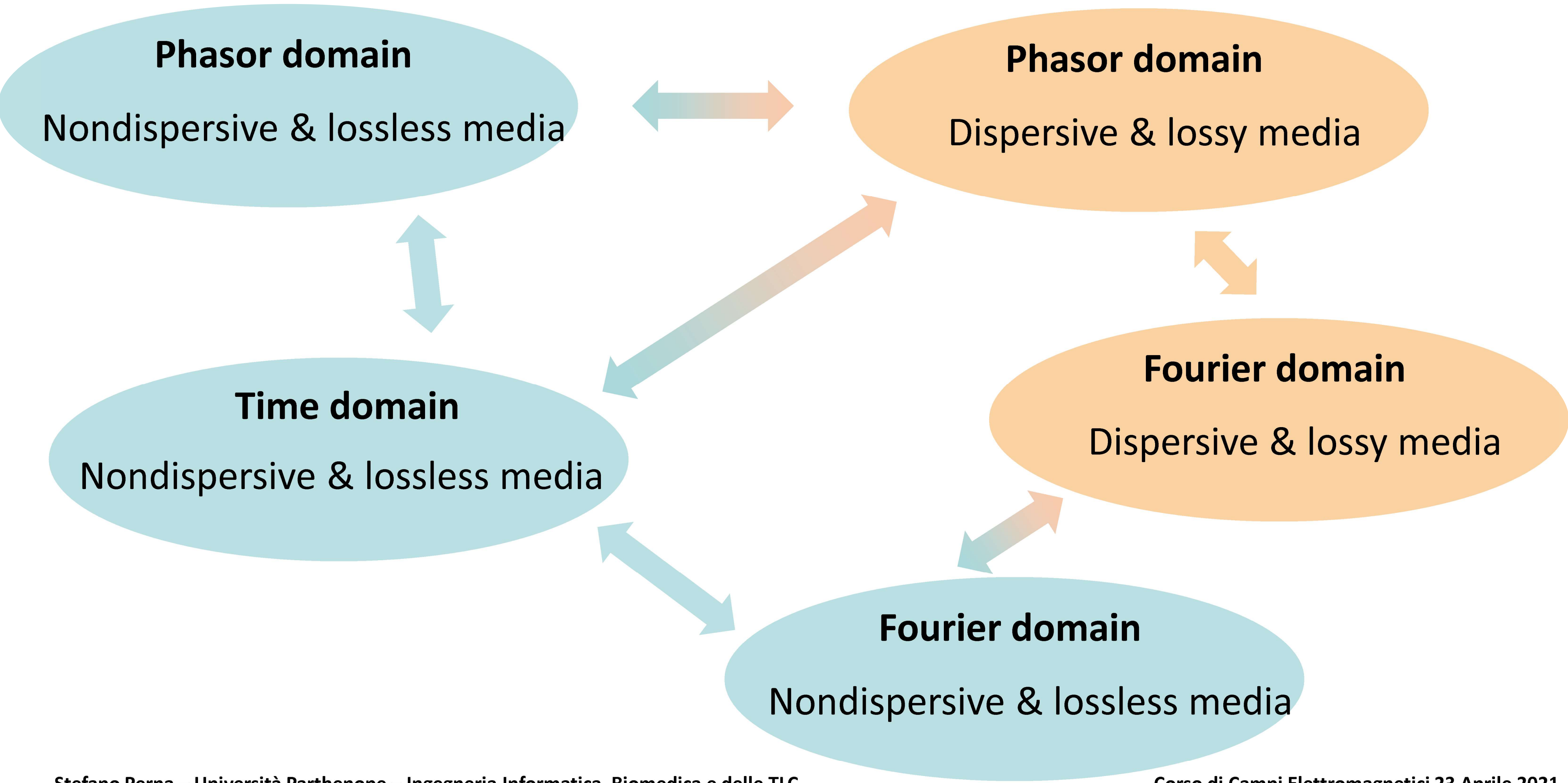
$$\frac{\partial}{\partial x}=\frac{\partial}{\partial y}=0$$
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(z,t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}(z,t)$$

$$E_z=H_z=0$$

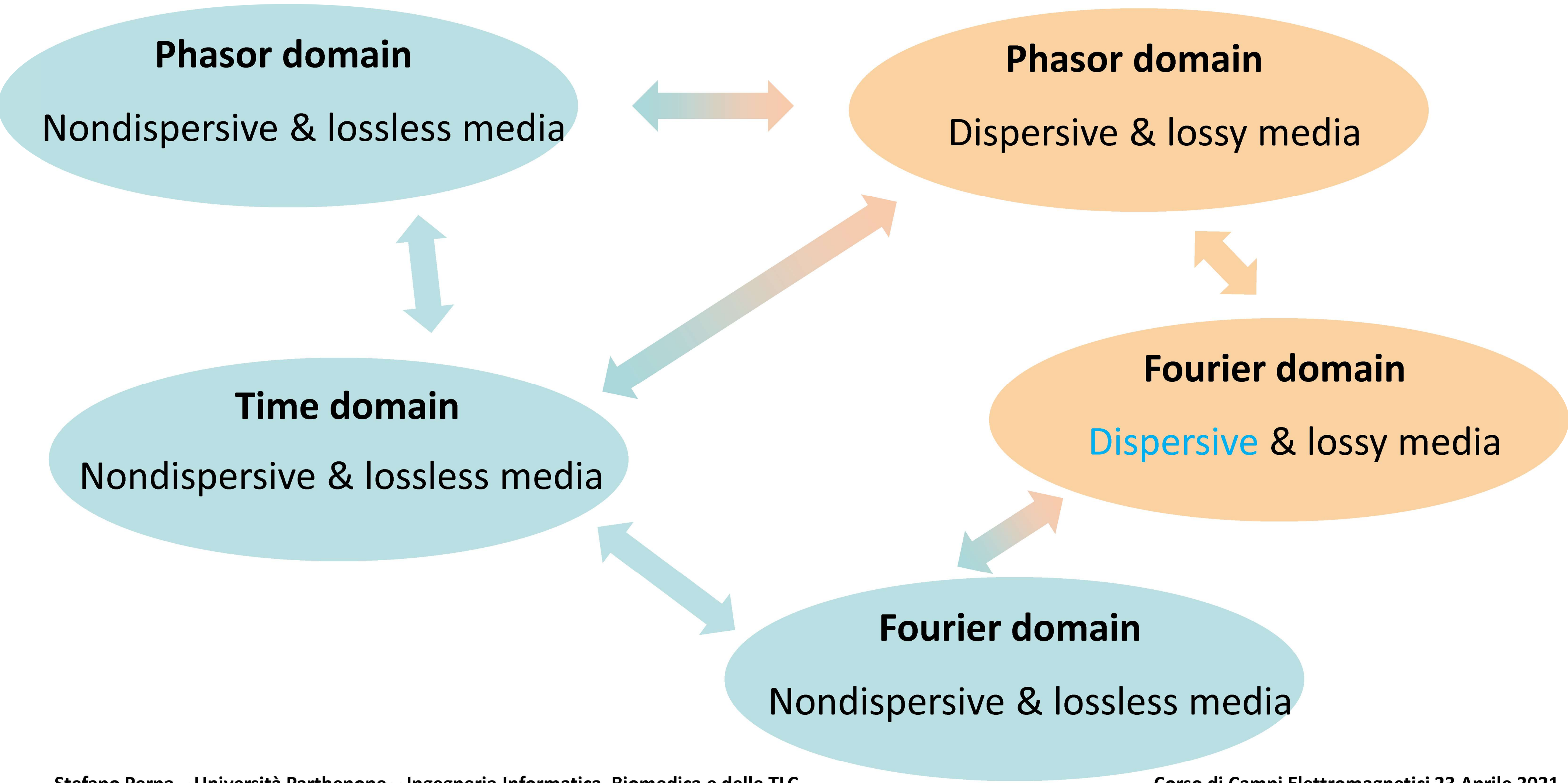
$$\left\{E_y,H_x\right\}$$
$$\left\{E_x,H_y\right\}$$

Independent each other

Rationale


Phasor domain
Nondispersive & lossless media
Phasor domain
Dispersive & lossy media
Time domain
Nondispersive & lossless media
Fourier domain
Dispersive & lossy media
Fourier domain
Nondispersive & lossless media

Rationale


Phasor domain
Nondispersive & lossless media
Phasor domain
Dispersive & lossy media
Time domain
Nondispersive & lossless media
Fourier domain
Dispersive & lossy media
Fourier domain
Nondispersive & lossless media

Plane Waves (Fourier Domain)

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega) = \varepsilon_1(\omega) - j\varepsilon_2(\omega) \\ \mu(\omega) = \mu_1(\omega) - j\mu_2(\omega) \\ \sigma : real \end{cases}$$

$$\varepsilon_{eq}(\omega) = \varepsilon(\omega)\left[1 - \frac{j\sigma}{\omega\varepsilon(\omega)}\right]$$

$$k(\omega) = \omega\sqrt{\mu(\omega)\varepsilon(\omega)}$$

$$k(\omega) = \beta(\omega) - j\alpha(\omega)$$

$$k = \omega\sqrt{\mu\varepsilon}$$

$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases}$$

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x^+(z,\omega) = E^+(\omega)e^{-jkz}$$

$$\zeta H_y^+(z,\omega) = E^+(\omega)e^{-jkz}$$

Fourier Domain

Source-free

Medium

- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

Plane Waves (Fourier Domain)

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega) = \varepsilon_1(\omega) - j\varepsilon_2(\omega) \\ \mu(\omega) = \mu_1(\omega) - j\mu_2(\omega) \\ \sigma : real \end{cases}$$

$$\varepsilon_{eq}(\omega) = \varepsilon(\omega)\left[1 - \frac{j\sigma}{\omega\varepsilon(\omega)}\right]$$

$$k(\omega) = \omega\sqrt{\mu(\omega)\varepsilon(\omega)}$$
$$k(\omega) = \beta(\omega) - j\alpha(\omega)$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad \varepsilon_{eq} = \varepsilon$$

$$k(\omega) = \omega\sqrt{\mu\varepsilon} = \beta(\omega)$$

$$k = \omega\sqrt{\mu\varepsilon}$$
$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\{E_x, H_y\}$$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \qquad \frac{d^2E_x}{dz^2} + k^2E_x = 0$$

$$E_x^+(z,\omega) = E^+(\omega)e^{-jkz}$$
$$\zeta H_y^+(z,\omega) = E^+(\omega)e^{-jkz}$$

Fourier Domain

Source-free

Medium
- **Linear**
- **Time nondispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

Plane Waves (Fourier Domain)

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega) = \varepsilon_1(\omega) - j\varepsilon_2(\omega) \\ \mu(\omega) = \mu_1(\omega) - j\mu_2(\omega) \\ \sigma : real \end{cases}$$

$$\varepsilon_{eq}(\omega) = \varepsilon(\omega)\left[1 - \frac{j\sigma}{\omega\varepsilon(\omega)}\right]$$

$$k(\omega) = \omega\sqrt{\mu(\omega)\varepsilon(\omega)}$$

$$k(\omega) = \beta(\omega) - j\alpha(\omega)$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad \varepsilon_{eq} = \varepsilon$$

$$k(\omega) = \omega\sqrt{\mu\varepsilon} = \beta(\omega)$$

$$k = \omega\sqrt{\mu\varepsilon}$$

$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x^+(z,\omega) = E^+(\omega)e^{-j\beta z}$$

$$\zeta H_y^+(z,\omega) = E^+(\omega)e^{-j\beta z}$$

Fourier Domain

Source-free

Medium

- **Linear**
- **Time nondispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

Plane Waves (Fourier Domain)

$\left\{E_x, H_y\right\}$

$$E_x^+(z,\omega) = E^+(\omega)e^{-j\beta z}$$

$$\zeta H_y^+(z,\omega) = E^+(\omega)e^{-j\beta z}$$

$$k(\omega) = \omega\sqrt{\mu\varepsilon} = \beta(\omega)$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\beta = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{c}$$

$$E_x^+(z,\omega) = E^+(\omega)e^{-j\beta z}$$

$$e_x^+(z,t) = \frac{1}{2\pi}\int E_x^+(z,\omega)e^{j\omega t}d\omega = \frac{1}{2\pi}\int E^+(\omega)e^{-j\beta z}e^{j\omega t}d\omega$$

$$= \frac{1}{2\pi}\int E^+(\omega)e^{-j\frac{\omega}{c}z}e^{j\omega t}d\omega = \frac{1}{2\pi}\int E^+(\omega)e^{j\omega\left(t-\frac{z}{c}\right)}d\omega$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

Source-free

Medium

- **Linear**
- **Time nondispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

↓

$$E_z = H_z = 0$$

$\left\{E_y, H_x\right\}$

$\left\{E_x, H_y\right\}$

Independent each other

Plane Waves (Fourier Domain)

$\{E_x, H_y\}$

$$k(\omega) = \omega\sqrt{\mu\varepsilon} = \beta(\omega)$$

$$E_x^+(z,\omega) = E^+(\omega)e^{-j\beta z}$$

$$\zeta H_y^+(z,\omega) = E^+(\omega)e^{-j\beta z}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\beta = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{c}$$

$$E_x^+(z,\omega) = E^+(\omega)e^{-j\beta z}$$

$$e_x^+(z,t) =$$

$$= \frac{1}{2\pi}\int E^+(\omega)e^{j\omega\left(t-\frac{z}{c}\right)}d\omega$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

Source-free

Medium

- **Linear**
- **Time nondispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

Plane Waves (Fourier Domain)

$\{E_x, H_y\}$

$$k(\omega) = \omega\sqrt{\mu\varepsilon} = \beta(\omega)$$

$$E_x^+(z,\omega) = E^+(\omega)e^{-j\beta z}$$

$$\zeta H_y^+(z,\omega) = E^+(\omega)e^{-j\beta z}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\beta = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{c}$$

$$E_x^+(z,\omega) = E^+(\omega)e^{-j\beta z}$$

$$e_x^+(z,t) = \frac{1}{2\pi}\int E^+(\omega)e^{j\omega\left(t-\frac{z}{c}\right)}d\omega$$

Progressive plane wave

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$e_x^+(z=0,t) = \frac{1}{2\pi}\int E^+(\omega)e^{j\omega t}d\omega = f(t)$$

$$e_x^+(z>0,t) = \frac{1}{2\pi}\int E^+(\omega)e^{j\omega\left(t-\frac{z}{c}\right)}d\omega = f\left(t-\frac{z}{c}\right) = f\left[-\frac{1}{c}(z-ct)\right] = f\left[(z-ct)\right]$$

Source-free

Medium
- Linear
- Time nondispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- Lossless

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

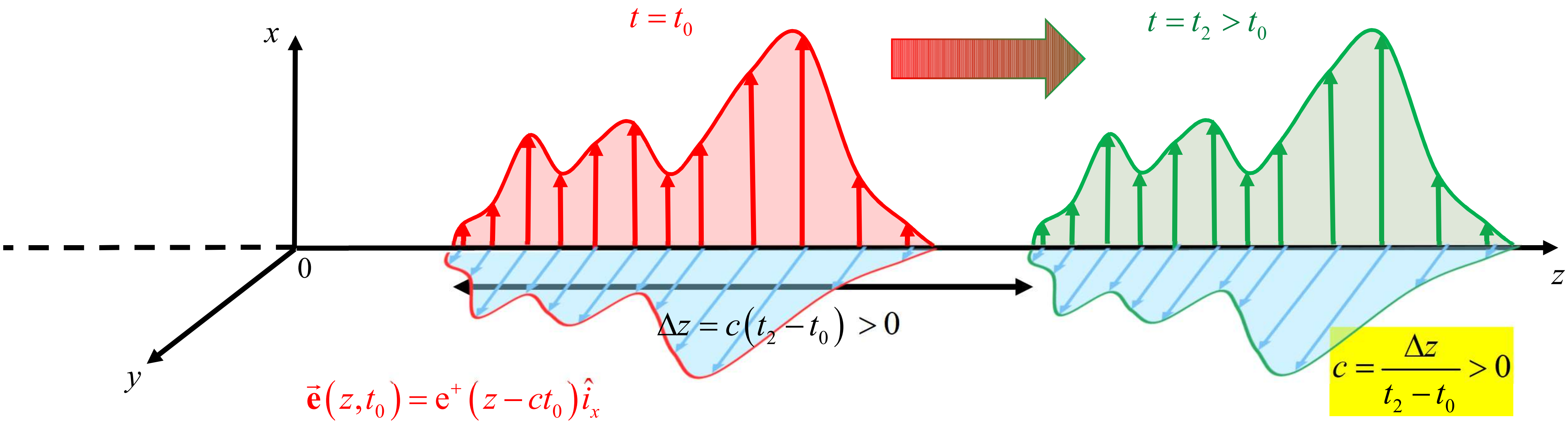
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

Plane Waves



$$\vec{\mathbf{e}}(z,t_0)=\mathrm{e}^+(z-ct_0)\hat{i}_x$$

$$\vec{\mathbf{e}}(z,t_2)=\mathrm{e}^+(z-ct_2)\hat{i}_x=\mathrm{e}^+(z-ct_0+ct_0-ct_2)\hat{i}_x=\mathrm{e}^+(z-ct_0-c[t_2-t_0])\hat{i}_x$$

The electromagnetic perturbation **propagates** without deformation and with constant speed *c* along the positive sense of the *z*-axis

$$\begin{cases} e^+(z-ct) \\ h^+(z-ct) \end{cases}$$ is referred to as electromagnetic **<u>progressive</u> plane wave**

Plane Waves (Fourier Domain)

Source-free

Medium

- **Linear**
- **Time nondispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$ $\{E_x, H_y\}$ **Independent each other**

$\{E_x, H_y\}$

$$k(\omega) = \omega\sqrt{\mu\varepsilon} = \beta(\omega)$$

$$E_x^+(z,\omega) = E^+(\omega)e^{-j\beta z}$$

$$\zeta H_y^+(z,\omega) = E^+(\omega)e^{-j\beta z}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\beta = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{c}$$

$$E_x^+(z,\omega) = E^+(\omega)e^{-j\beta z}$$

$$e_x^+(z,t) = \frac{1}{2\pi}\int E_x^+(z,\omega)e^{j\omega t}d\omega = \frac{1}{2\pi}\int E^+(\omega)e^{-j\beta z}e^{j\omega t}d\omega$$

$$= \frac{1}{2\pi}\int E^+(\omega)e^{-j\frac{\omega}{c}z}e^{j\omega t}d\omega = \frac{1}{2\pi}\int E^+(\omega)e^{j\omega\left(t-\frac{z}{c}\right)}d\omega$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

Plane Waves (Fourier Domain)

$\{E_x, H_y\}$

$$k(\omega)=\omega\sqrt{\mu\varepsilon}=\beta(\omega)$$

$$E_x^+(z,\omega)=E^+(\omega)e^{-j\beta z}$$

$$\zeta H_y^+(z,\omega)=E^+(\omega)e^{-j\beta z}$$

$$\zeta=\sqrt{\frac{\mu}{\varepsilon}}$$

$$\beta=\omega\sqrt{\mu\varepsilon}=\frac{\omega}{c}$$

$$E_x^+(z,\omega)=E^+(\omega)e^{-j\beta z}$$

$$e_x^+(z,t)=\frac{1}{2\pi}\int E_x^+(z,\omega)e^{j\omega t}d\omega=\frac{1}{2\pi}\int E^+(\omega)e^{-j\beta z}e^{j\omega t}d\omega$$

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Source-free

Medium
- Linear
- Time nondispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- Lossless

$$\frac{\partial}{\partial x}=\frac{\partial}{\partial y}=0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}(z,t)$$

$$E_z=H_z=0$$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

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~~Time nondispersive & lossless~~

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$\{E_y, H_x\}$ $\{E_x, H_y\}$ **Independent each other**