Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2020-2021 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

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Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Plane Waves

Time domain (TD)

Spectral domains

- Phasor Domain (PD)
- Fourier Domain (FD)
- Dispersive media: attenuation, distortion, phase velocity and group velocity

General expression of plane waves (PD)

Incidence

Plane Waves

Time domain (TD)

Spectral domains

- Phasor Domain (PD)
- Fourier Domain (FD)
- Dispersive media: attenuation, distortion, phase velocity and group velocity

General expression of plane waves (PD)

Incidence

Plane Waves

Time domain

Plane Waves (TD)

$\{e_x, h_y\}$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

$$e_x(z,t) = e_x^+(z-ct) + e_x^-(z+ct)$$

$$\zeta h_y(z,t) = e_x^+(z-ct) - e_x^-(z+ct)$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

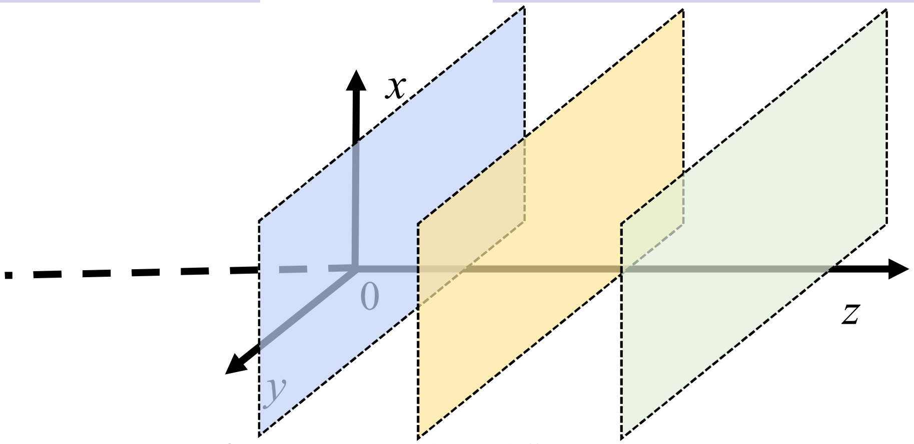
$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{e_y, h_x\}$

$$\begin{cases} \dfrac{\partial e_y}{\partial z} = \mu \dfrac{\partial h_x}{\partial t} \\ \dfrac{\partial h_x}{\partial z} = \varepsilon \dfrac{\partial e_y}{\partial t} \end{cases} \qquad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2} = 0$$

$$e_y(z,t) = e_y^+(z-ct) + e_y^-(z+ct)$$

$$-\zeta h_x(z,t) = e_y^+(z-ct) - e_y^-(z+ct)$$



Source-free

Medium
- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

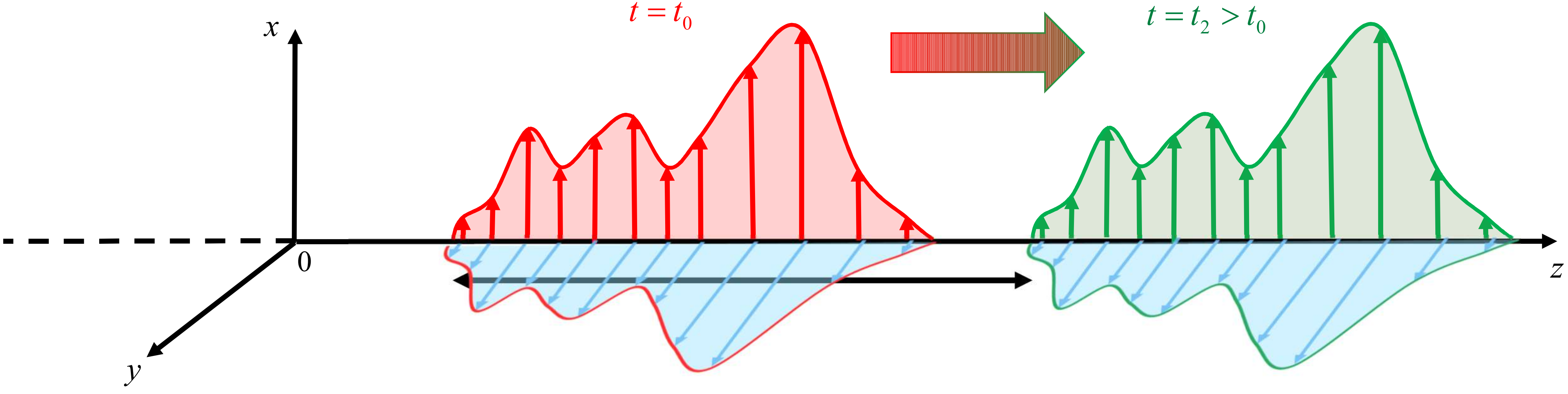
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$, $\{e_x, h_y\}$ **Independent each other**

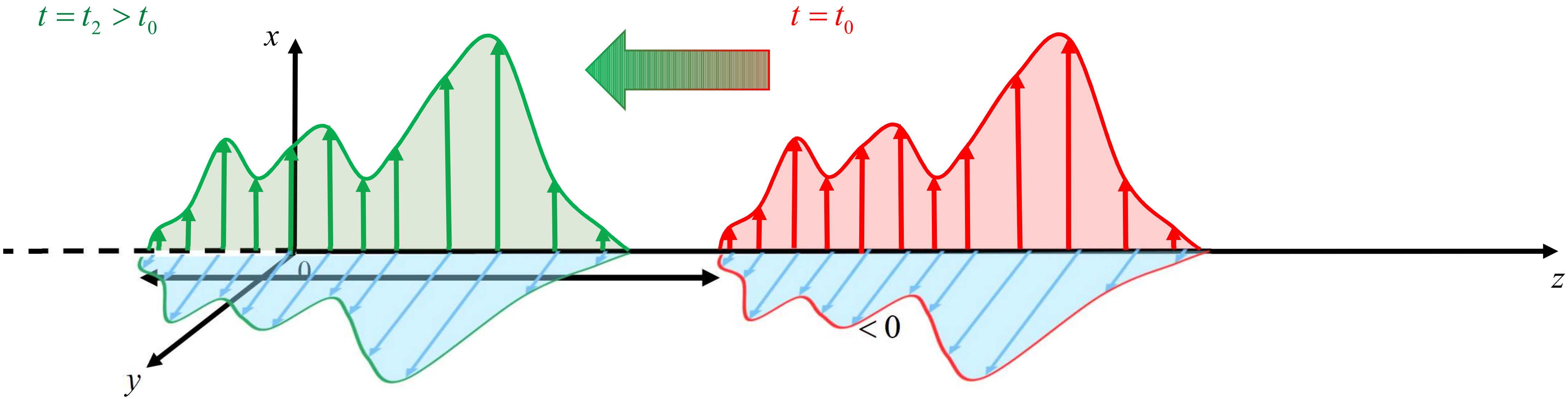
Plane Waves (TD)



The electromagnetic perturbation **propagates** without deformation and with constant speed *c* along the positive sense of the z-axis

$$\begin{cases} e^{+}\left(z-ct\right) \\ h^{+}\left(z-ct\right) \end{cases}$$ is referred to as electromagnetic **<u>progressive</u> plane wave**

Plane Waves (TD)



The electromagnetic perturbation **propagates** without deformation and with constant speed ***c*** along the negative sense of the z-axis

$$\begin{cases} e^{-}(z+ct) \\ h^{-}(z+ct) \end{cases}$$ is referred to as electromagnetic **<u>regressive</u> plane wave**

Plane Waves (TD)

Source-free

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- **Time non-dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

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$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

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$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$
$\{e_x, h_y\}$
Independent each other

$\{e_x^+, h_y^+\}$

$$\begin{cases} e_x^+(z,t) = e_x^+(z-ct) \\ \zeta h_y^+(z,t) = e_x^+(z-ct) \end{cases}$$

$\{e_y^+, h_x^+\}$

$$\begin{cases} e_y^+(z,t) = e_y^+(z-ct) \\ \zeta h_x^+(z,t) = -e_y^+(z-ct) \end{cases}$$

the e.m. field propagates along $\hat{i}_p = \hat{i}_z$

$\{e_x^-, h_y^-\}$

$$\begin{cases} e_x^-(z,t) = e_x^-(z+ct) \\ \zeta h_y^-(z,t) = -e_x^-(z+ct) \end{cases}$$

$\{e_y^-, h_x^-\}$

$$\begin{cases} e_y^-(z,t) = e_y^-(z+ct) \\ \zeta h_x^-(z,t) = e_y^-(z+ct) \end{cases}$$

the e.m. field propagates along $\hat{i}_p = -\hat{i}_z$

- the e.m. field lies on the plane xy orthogonal to the propagation direction
- The Poynting vector is directed along the direction of propagation
- $|\vec{\mathbf{e}}|$ and $|\vec{\mathbf{h}}|$ are proportional through ζ
- $\zeta \vec{\mathbf{h}} = \hat{i}_p \times \vec{\mathbf{e}}$

Plane Waves

Spectral domains

Plane Waves (Spectral Domains)

Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~Lossless~~

Phasor domain (PD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(z) = -j\omega_0 \mu \vec{\mathbf{H}}(z) \\ \nabla \times \vec{\mathbf{H}}(z) = j\omega_0 \varepsilon \vec{\mathbf{E}}(z) \end{cases}$$

Fourier domain (FD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(z,\omega) = -j\omega \mu \vec{\mathbf{H}}(z,\omega) \\ \nabla \times \vec{\mathbf{H}}(z,\omega) = j\omega \varepsilon \vec{\mathbf{E}}(z,\omega) \end{cases}$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = E_x(z)\hat{i}_x + E_y(z)\hat{i}_y + E_z(z)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = H_x(z)\hat{i}_x + H_y(z)\hat{i}_y + H_z(z)\hat{i}_z$$

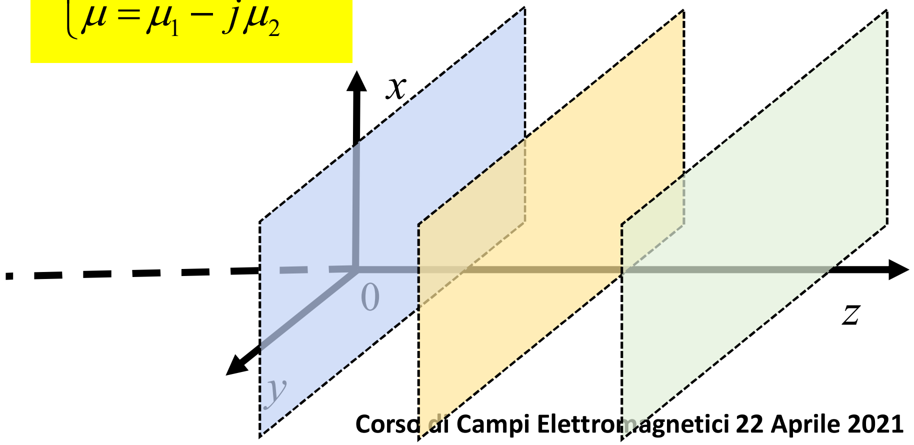
PD

$$\begin{cases} \varepsilon = \varepsilon_1 - j\varepsilon_2 \\ \mu = \mu_1 - j\mu_2 \end{cases}$$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = E_x(z,\omega)\hat{i}_x + E_y(z,\omega)\hat{i}_y + E_z(z,\omega)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = H_x(z,\omega)\hat{i}_x + H_y(z,\omega)\hat{i}_y + H_z(z,\omega)\hat{i}_z$$

FD



Plane Waves (Spectral Domains)

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$$E_z = H_z = 0$$

$\{E_y, H_x\}$, $\{E_x, H_y\}$ **Independent each other**

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$k = \omega\sqrt{\mu\varepsilon}$$

$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{E_y, H_x\}$

$$\begin{cases} \dfrac{dE_y}{dz} = j\omega\mu H_x \\ \dfrac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases} \qquad \frac{d^2 E_y}{dz^2} + k^2 E_y = 0$$

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Plane Waves (Spectral Domains)

$\{E_x, H_y\}$

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↓

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Phasor Domain

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Fourier Domain

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Plane Waves (Spectral Domains)

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Plane Waves (Spectral Domains)

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↓

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Plane Waves (Spectral Domains)

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Plane Waves (Spectral Domains)

$\{E_x, H_y\}$

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$$E_x(z,\omega) = E_x^+(\omega) e^{-jkz} + E_x^-(\omega) e^{jkz}$$

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$$E_x(z) = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

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$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \end{cases}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0)\varepsilon(\omega_0)}$$

$$k(\omega_0) = \beta(\omega_0) - j\alpha(\omega_0)$$

Phasor Domain

$$k = \omega\sqrt{\mu\varepsilon}$$

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$\{E_x, H_y\}$

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$$\begin{cases} \varepsilon(\omega) = \varepsilon_1(\omega) - j\varepsilon_2(\omega) \\ \mu(\omega) = \mu_1(\omega) - j\mu_2(\omega) \end{cases}$$

$$k(\omega) = \omega \sqrt{\mu(\omega)\varepsilon(\omega)}$$

$$k(\omega) = \beta(\omega) - j\alpha(\omega)$$

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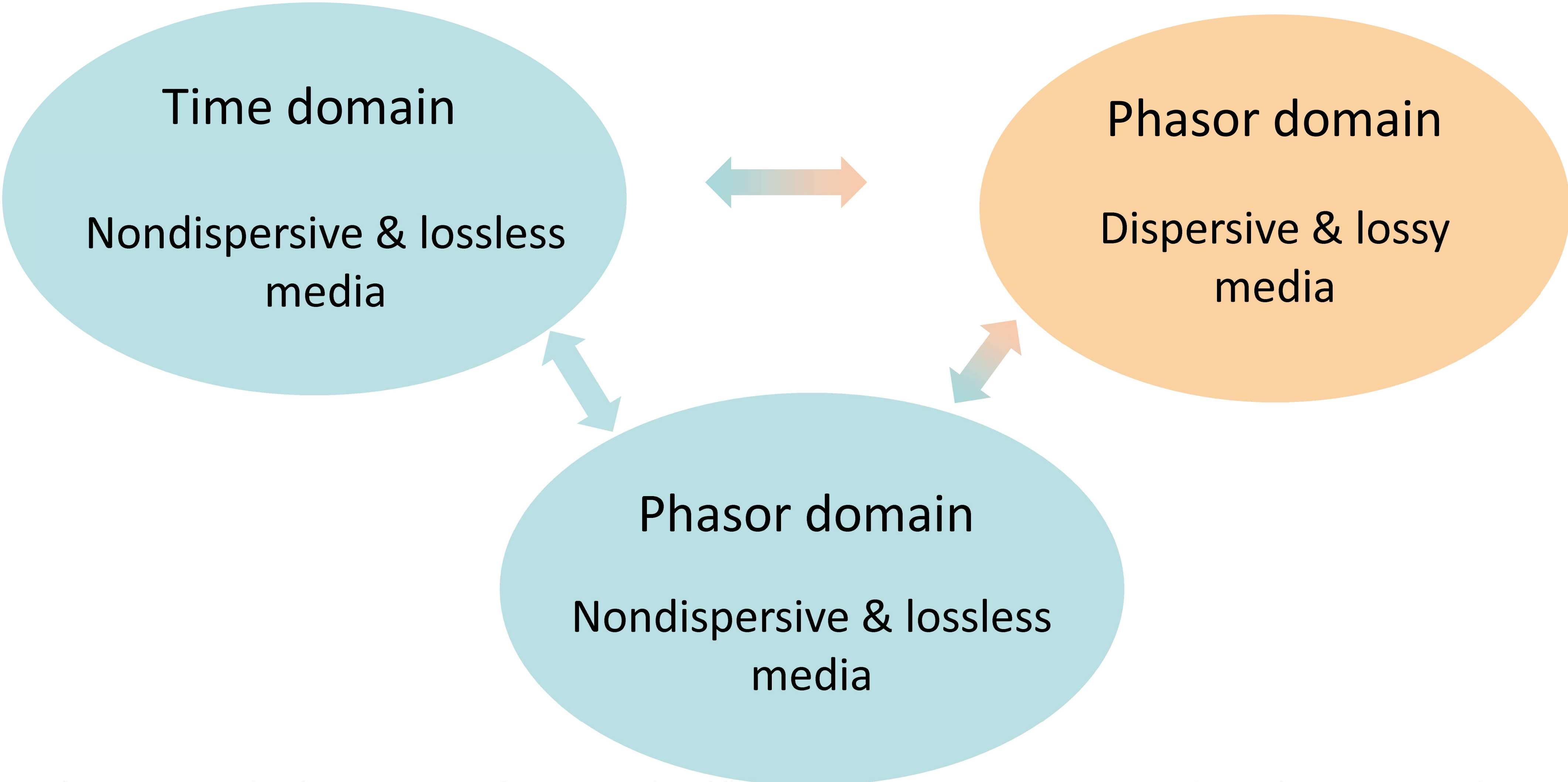
Phasor domain

Rationale


Time domain
Nondispersive & lossless media
Phasor domain
Dispersive & lossy media
Phasor domain
Nondispersive & lossless media

Plane Waves (Spectral Domains)

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega_0 \mu H_y \\ \dfrac{dH_y}{dz} = -j\omega_0 \varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x(z) = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y(z) = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \end{cases}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0)\varepsilon(\omega_0)}$$

$$k(\omega_0) = \beta(\omega_0) - j\alpha(\omega_0)$$

Phasor Domain

$$k = \omega\sqrt{\mu\varepsilon}$$

$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega \mu H_y \\ \dfrac{dH_y}{dz} = -j\omega \varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x(z,\omega) = E_x^+(\omega) e^{-jkz} + E_x^-(\omega) e^{jkz}$$

$$\zeta H_y(z,\omega) = E_x^+(\omega) e^{-jkz} - E_x^-(\omega) e^{jkz}$$

$$\begin{cases} \varepsilon(\omega) = \varepsilon_1(\omega) - j\varepsilon_2(\omega) \\ \mu(\omega) = \mu_1(\omega) - j\mu_2(\omega) \end{cases}$$

$$k(\omega) = \omega \sqrt{\mu(\omega)\varepsilon(\omega)}$$

$$k(\omega) = \beta(\omega) - j\alpha(\omega)$$

Fourier Domain

Source-free

Medium

- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{e}}(z, t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{h}}(z, t)$$

↓

$$E_z = H_z = 0$$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

Plane Waves (Phasor Domain)

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega_0 \mu H_y \\ \dfrac{dH_y}{dz} = -j\omega_0 \varepsilon E_x \end{cases}$$

$\{E_x, H_y\}$

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Phasor Domain

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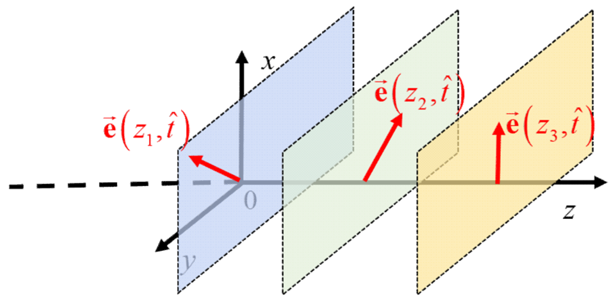

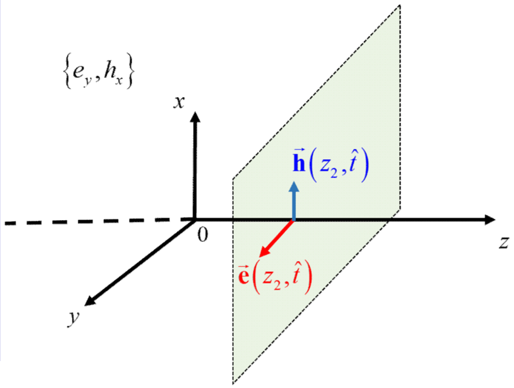

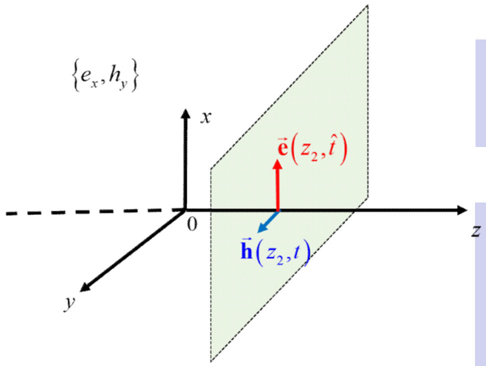

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$\{E_y, H_x\}$
$\{E_x, H_y\}$ **Independent each other**

Plane Waves (Phasor Domain)

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega_0 \mu H_y \\ \dfrac{dH_y}{dz} = -j\omega_0 \varepsilon E_x \end{cases}$$

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Phasor Domain

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Independent each other

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0)\varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

$$E_x(z) = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y(z) = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$\zeta(\omega_0) = \sqrt{\frac{\mu(\omega_0)}{\varepsilon(\omega_0)}}$$

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma : \ real \end{cases}$$

$$\varepsilon_{eq}(\omega_0) = \varepsilon(\omega_0)\left[1 - \frac{j\sigma}{\omega_0 \varepsilon(\omega_0)}\right]$$

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$\{E_y, H_x\}$

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Independent each other

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$\{E_x, H_y\}$

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Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$\varepsilon_{eq} = \varepsilon$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

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Independent each other

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$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta} \qquad \zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E_x^+ = |E_x^+| e^{j\varphi^+}$$

$$E_x^+ e^{-j\beta z} = |E_x^+| e^{j\varphi^+} e^{-j\beta z}$$

$$e_x^+(z,t) = \mathrm{Re}\left\{|E_x^+| e^{j\varphi^+} e^{-j\beta z} e^{j\omega_0 t}\right\} = |E_x^+|\cos(\omega_o t - \beta z + \varphi^+)$$

$$= |E_x^+|\cos\left(-\beta\left[z - \frac{\omega_0}{\beta}t\right] + \varphi^+\right) = |E_x^+|\cos\left(-\beta\left[z - v_p t\right] + \varphi^+\right)$$

$$= e_x^+(z - v_p t) = e_x^+(z - ct)$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

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Plane Waves (Phasor Domain)

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$$E_x(z) = E_x^+ e^{-j\beta z} + E_x^- e^{j\beta z}$$

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$$v_p = \frac{\omega_0}{\beta} \qquad \zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E_x^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = |E_x^+| \cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

$$E_x^+ e^{-j\beta z} = |E_x^+| e^{j\varphi^+} e^{-j\beta z}$$

$$e_x^+(z,t) = \mathrm{Re}\left\{|E_x^+| e^{j\varphi^+} e^{-j\beta z} e^{j\omega_0 t}\right\} = |E_x^+| \cos(\omega_o t - \beta z + \varphi^+)$$

$$= |E_x^+| \cos\left(-\beta\left[z - \frac{\omega_0}{\beta} t\right] + \varphi^+\right) = |E_x^+| \cos\left(-\beta\left[z - v_p t\right] + \varphi^+\right)$$

$$= e_x^+(z - v_p t) = e_x^+(z - ct)$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

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$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

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Plane Waves (Phasor Domain)

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$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E_x^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = |E_x^+| \cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\varepsilon_{eq} = \varepsilon$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

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$$E_x^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = |E_x^+| \cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

$$E_x^- = |E_x^-| e^{j\varphi^-}$$

$$E_x^- e^{j\beta z} = |E_x^-| e^{j\varphi^-} e^{j\beta z}$$

$$e_x^-(z,t) = \mathrm{Re}\left\{|E_x^-| e^{j\varphi^-} e^{j\beta z} e^{j\omega_0 t}\right\} = |E_x^-| \cos(\omega_o t + \beta z + \varphi^-)$$

$$= e_x^-(z + v_p t) = e_x^-(z + ct)$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

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$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

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$$E_x(z) = E_x^+ e^{-j\beta z} + E_x^- e^{j\beta z}$$

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$$v_p = \frac{\omega_0}{\beta} \qquad \zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

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$$E_x^+ e^{-j\beta z} \;\rightarrow e_x^+(z,t) = \left|E_x^+\right|\cos\left(\omega_o t - \beta z + \varphi^+\right) = e_x^+\left(z - v_p t\right)$$

Progressive plane wave

$$E_x^- e^{j\beta z} \;\rightarrow e_x^-(z,t) = \left|E_x^-\right|\cos\left(\omega_o t + \beta z + \varphi^-\right) = e_x^-\left(z + v_p t\right)$$

$$E_x^- e^{j\beta z} = \left|E_x^-\right| e^{j\varphi^-} e^{j\beta z}$$

$$e_x^-(z,t) = \mathrm{Re}\left\{\left|E_x^-\right| e^{j\varphi^-} e^{j\beta z} e^{j\omega_0 t}\right\} = \left|E_x^-\right|\cos\left(\omega_o t + \beta z + \varphi^-\right)$$

$$= e_x^-\left(z + v_p t\right) = e_x^-\left(z + ct\right)$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

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Independent each other

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<u>Progressive</u> plane wave

$$E_x^- e^{j\beta z} \rightarrow e_x^-(z,t) = |E_x^-| \cos(\omega_o t + \beta z + \varphi^-) = e_x^-(z + v_p t)$$

<u>Regressive</u> plane wave

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\varepsilon_{eq} = \varepsilon$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

Source-free

Medium
- **Linear**
- **Time nondispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$, $\{E_x, H_y\}$ **Independent each other**

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$E_x^+(z) = E_x^+ e^{-j\beta z}$$
$$\zeta H_y^+(z) = E_x^+ e^{-j\beta z}$$

$$E_x^-(z) = E_x^- e^{j\beta z}$$
$$\zeta H_y^-(z) = -E_x^- e^{j\beta z}$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta} \qquad \zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E_x^+ e^{-j\beta z} \;\to e_x^+(z,t) = |E_x^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

<u>Progressive</u> plane wave

$$E_x^- e^{j\beta z} \;\to e_x^-(z,t) = |E_x^-|\cos(\omega_o t + \beta z + \varphi^-) = e_x^-(z + v_p t)$$

<u>Regressive</u> plane wave

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\varepsilon_{eq} = \varepsilon$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

Source-free

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- Time nondispersive
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$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$E_x^+(z) = E^+ e^{-j\beta z}$

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$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$

$v_p = \dfrac{\omega_0}{\beta}$ $\zeta = \sqrt{\dfrac{\mu}{\varepsilon}}$

$E^+ e^{-j\beta z} \quad \rightarrow e_x^+(z,t) = |E^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$

Progressive plane wave

$E^- e^{j\beta z} \quad \rightarrow e_x^-(z,t) = |E^-|\cos(\omega_o t + \beta z + \varphi^-) = e_x^-(z + v_p t)$

Regressive plane wave

$v_p = \dfrac{\omega_0}{\beta} = \dfrac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \dfrac{1}{\sqrt{\mu\varepsilon}} = c$

Time nondispersive & lossless

$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$ $c = \dfrac{1}{\sqrt{\mu\varepsilon}}$

$\varepsilon_{eq} = \varepsilon$

$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$

Source-free

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$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$

$E_z = H_z = 0$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

Plane Waves (Phasor Domain)

Source-free

Medium
- **Linear**
- **Time nondispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

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$$E_z = H_z = 0$$

$\{E_y, H_x\}$ $\{E_x, H_y\}$ **Independent each other**

$\{E_x, H_y\}$

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$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = |E^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

<u>Progressive</u> plane wave

$$E^- e^{j\beta z} \rightarrow e_x^-(z,t) = |E^-|\cos(\omega_o t + \beta z + \varphi^-) = e_x^-(z + v_p t)$$

<u>Regressive</u> plane wave

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\varepsilon_{eq} = \varepsilon$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

Plane Waves (Phasor Domain)

Source-free

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- **Linear**
- **Time nondispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$ **Independent each other**

$\{E_x, H_y\}$

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$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E^+ e^{-j\beta z} \quad \rightarrow e_x^+(z,t) = \left|E^+\right| \cos\left(\omega_o t - \beta z + \varphi^+\right) = e_x^+(z - v_p t)$$

<u>Progressive</u> plane wave

$$E^- e^{j\beta z} \quad \rightarrow e_x^-(z,t) = \left|E^-\right| \cos\left(\omega_o t + \beta z + \varphi^-\right) = e_x^-(z + v_p t)$$

<u>Regressive</u> plane wave

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$\varepsilon_{\ } = \varepsilon$

$\beta(\omega_0)$

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \frac{1}{2}\vec{\mathbf{E}}(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^*(\vec{\mathbf{r}})$$

$$\vec{\mathbf{S}}^+(\vec{\mathbf{r}}) = \frac{1}{2}\vec{\mathbf{E}}^+(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^{+*}(\vec{\mathbf{r}}) = \frac{1}{2}E_x^+(z)\hat{i}_x \times H_y^{+*}(z)\hat{i}_y = \frac{1}{2}\left(E^+ e^{-j\beta z}\right) \cdot \frac{\left(E^+ e^{-j\beta z}\right)^*}{\zeta}\hat{i}_z$$

$$= \frac{\left|E^+\right|^2}{2\zeta}\hat{i}_z = \frac{\left|E_x^+(z)\right|^2}{2\zeta}\hat{i}_z = \zeta\frac{\left|H_y^+(z)\right|^2}{2}\hat{i}_z$$

$$\hat{i}_z = \hat{i}_x \times \hat{i}_y$$

$$\hat{i}_y = \hat{i}_z \times \hat{i}_x$$

$$\hat{i}_x = \hat{i}_y \times \hat{i}_z$$

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

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$$E^+ e^{-j\beta z} \quad \to e_x^+(z,t) = |E^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

$$E^- e^{j\beta z} \quad \to e_x^-(z,t) = |E^-|\cos(\omega_o t + \beta z + \varphi^-) = e_x^-(z + v_p t)$$

Regressive plane wave

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \frac{1}{2}\vec{\mathbf{E}}(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^*(\vec{\mathbf{r}})$$

$$\vec{\mathbf{S}}^-(\vec{\mathbf{r}}) = \frac{1}{2}\vec{\mathbf{E}}^-(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^{-*}(\vec{\mathbf{r}}) = \frac{1}{2}E_x^-(z)\hat{i}_x \times H_y^{-*}(z)\hat{i}_y = -\frac{|E_x^-(z)|^2}{2\zeta}\hat{i}_z$$
$$= -\zeta\frac{|H_y^-(z)|^2}{2}\hat{i}_z$$

$$\hat{i}_z = \hat{i}_x \times \hat{i}_y$$
$$\hat{i}_y = \hat{i}_z \times \hat{i}_x$$
$$\hat{i}_x = \hat{i}_y \times \hat{i}_z$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$\varepsilon_{\ } = \varepsilon$

$\beta(\omega_0)$

Source-free

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$\{E_y, H_x\}$ $\{E_x, H_y\}$ **Independent each other**

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

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$v_p = \dfrac{\omega_0}{\beta} = \dfrac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \dfrac{1}{\sqrt{\mu\varepsilon}} = c$

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$E^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = |E^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$

Progressive plane wave

$E^- e^{j\beta z} \rightarrow e_x^-(z,t) = |E^-|\cos(\omega_o t + \beta z + \varphi^-) = e_x^-(z + v_p t)$

Regressive plane wave

Time nondispersive & lossless

$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$

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$\varepsilon_{eq} = \varepsilon$

$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$

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$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

Plane Waves (Phasor Domain)

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<u>Progressive</u> plane wave

Time nondispersive & lossless

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$\{E_y, H_x\}$
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Independent each other

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

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$$\omega_o = 2\pi f_0$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

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Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

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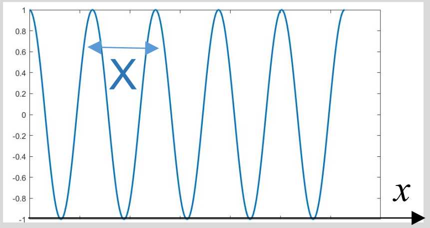
$$E^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = |E^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

<u>Progressive</u> plane wave

Memo



$$\cos(2\pi\nu x) = \cos\left(\frac{2\pi}{X}x\right)$$

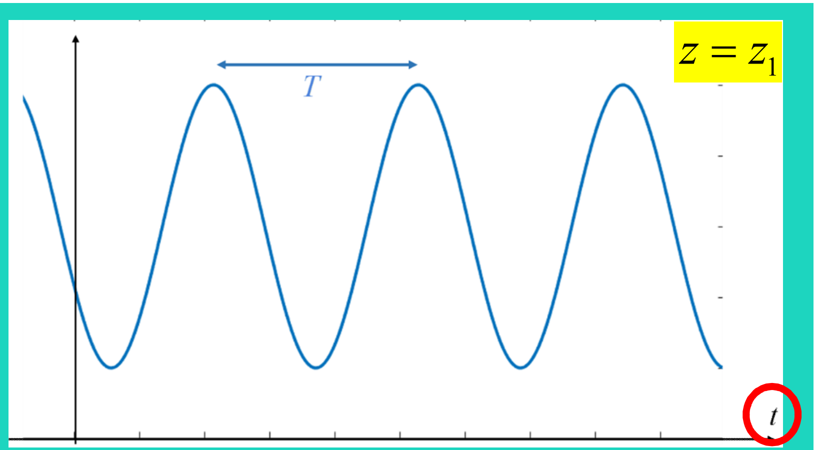
ν: frequency

$X = \frac{1}{\nu}$: period

$$e_x^+(z = z_1, t) = |E^+|\cos(\omega_o t - \beta z_1 + \varphi^+) = |E^+|\cos\left(2\pi\left[\frac{\omega_o}{2\pi}\right]t - \beta z_1 + \varphi^+\right)$$

frequency: $f_0 = \frac{\omega_o}{2\pi}$

period: $T = \frac{1}{f_0} = \frac{2\pi}{\omega_o}$



Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$\omega_o = 2\pi f_0$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

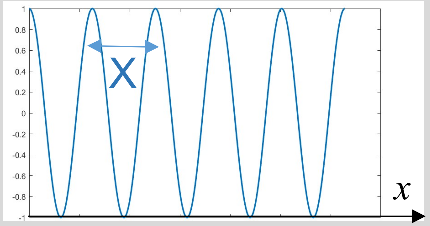
$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f_0}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = |E^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

Memo



$$\cos(2\pi\nu x) = \cos\left(\frac{2\pi}{X}x\right)$$

ν : frequency

$X = \frac{1}{\nu}$: period

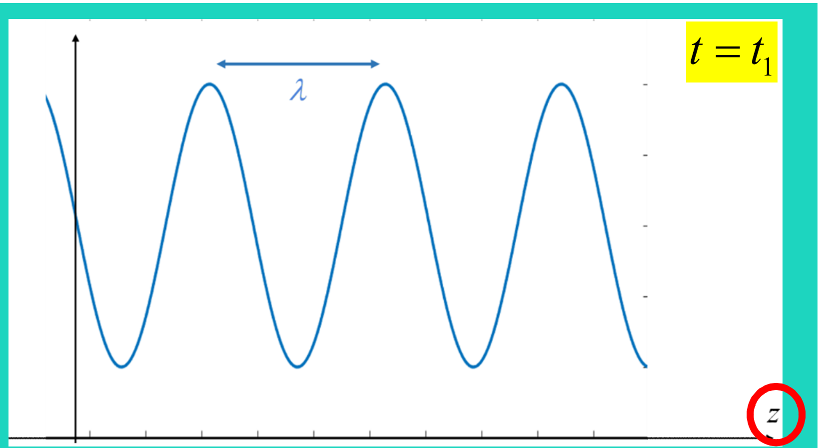
$$e_x^+(z, t = t_1) = |E^+|\cos(\omega_o t_1 - \beta z + \varphi^+) = |E^+|\cos(-\omega_o t_1 + \beta z - \varphi^+)$$

$$= |E^+|\cos\left(2\pi\left[\frac{\beta}{2\pi}\right]z - \omega_o t_1 - \varphi^+\right)$$

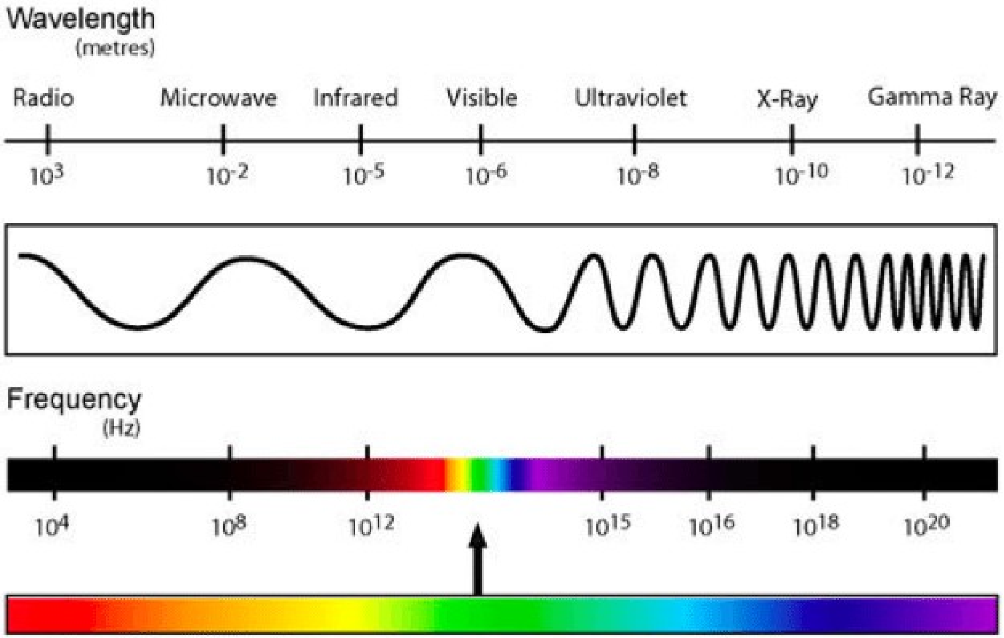
frequency : $\nu = \frac{\beta}{2\pi}$

period : $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega_0\sqrt{\mu\varepsilon}} = \frac{c}{f_0}$

λ : wavelength



Electromagnetic spectrum



Free space (Linear, isotropic, local, homogeneous)

$$\lambda = \frac{c}{f}$$

f: frequency
λ: wavelength
c: light speed

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$\omega_o = 2\pi f_0$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f_0}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = |E^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

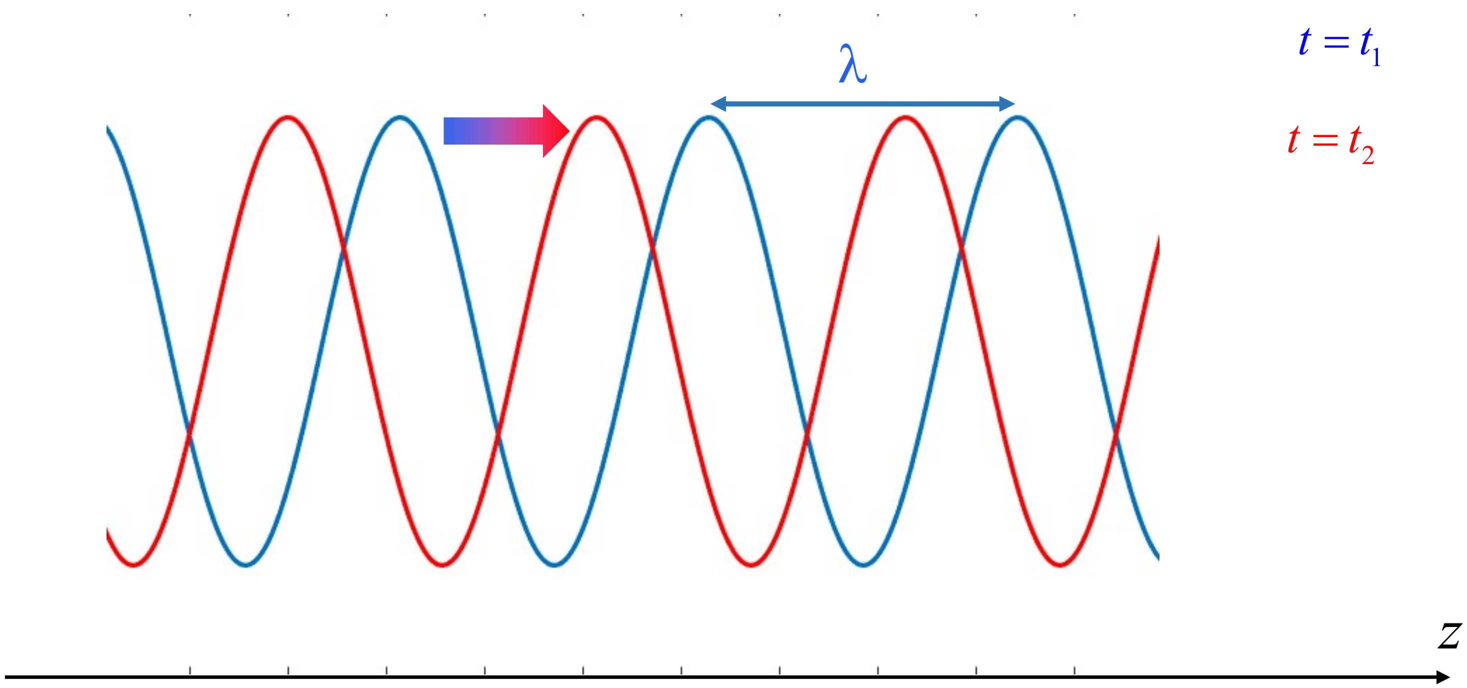
<u>Progressive</u> plane wave

- The term $e^{-j\beta z}$ is related to the propagation along the (positive sense of the) z-axis

Plane Waves (Phasor Domain)

$$e_x^+(z,t) = \left|E^+\right| \cos\left(\omega_o t - \beta z + \varphi^+\right) = e_x^+\left(z - v_p t\right)$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$



Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$E_x^+(z) = E^+ e^{-j\beta z}$$
$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

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Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

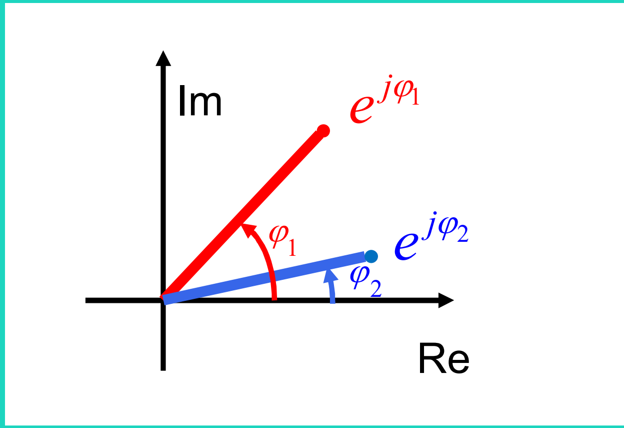
$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f_0}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = |E^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

<u>Progressive</u> plane wave

$$e^{-j\beta z} = e^{-j\frac{2\pi}{\lambda}z}$$

$$z << \lambda \Rightarrow \frac{2\pi}{\lambda} z << 2\pi \Rightarrow e^{-j\frac{2\pi}{\lambda}z} \approx 1$$



Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

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Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f_0}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = |E^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

<u>Progressive</u> plane wave

- The term $e^{-j\beta z}$ is related to the propagation along the (positive sense of the) z-axis
- When *z* is small with respect to λ, the propagation effects become negligible