Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2020-2021 - Laurea "Triennale" – Secondo semestre - Secondo anno

Università degli Studi di Napoli "Parthenope"

Stefano Perna

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Plane Waves

Time domain (TD)

Spectral domains

- Phasor Domain (PD)
- Fourier Domain (FD)
- Dispersive media: attenuation, distortion, phase velocity and group velocity

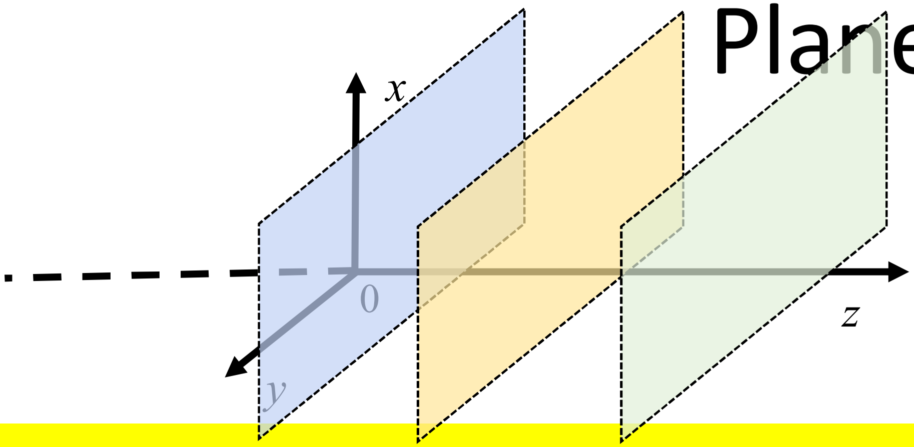
General expression of plane waves (PD)

Incidence

Plane Waves

Time domain

Plane Waves (TD)



Source-free

Medium
- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = e_x(\vec{\mathbf{r}},t)\hat{i}_x + e_y(\vec{\mathbf{r}},t)\hat{i}_y + e_z(\vec{\mathbf{r}},t)\hat{i}_z = e_x(z,t)\hat{i}_x + e_y(z,t)\hat{i}_y + e_z(z,t)\hat{i}_z$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = h_x(\vec{\mathbf{r}},t)\hat{i}_x + h_y(\vec{\mathbf{r}},t)\hat{i}_y + h_z(\vec{\mathbf{r}},t)\hat{i}_z = h_x(z,t)\hat{i}_x + h_y(z,t)\hat{i}_y + h_z(z,t)\hat{i}_z$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

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Time domain - Differential form

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(z,t) = -\mu \dfrac{\partial \vec{\mathbf{h}}(z,t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(z,t) = \varepsilon \dfrac{\partial \vec{\mathbf{e}}(z,t)}{\partial t} \\ \varepsilon \nabla \cdot \vec{\mathbf{e}}(z,t) = 0 \\ \mu \nabla \cdot \vec{\mathbf{h}}(z,t) = 0 \end{cases}$$

$$\nabla \times \vec{\mathbf{e}} = \left(-\frac{\partial e_y}{\partial z}\right)\hat{i}_x + \left(\frac{\partial e_x}{\partial z}\right)\hat{i}_y$$

$$\nabla \times \vec{\mathbf{h}} = \left(-\frac{\partial h_y}{\partial z}\right)\hat{i}_x + \left(\frac{\partial h_x}{\partial z}\right)\hat{i}_y$$

Plane Waves (TD)

$\{e_x, h_y\}$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

$$e_x(z,t) = e_x^+(z-ct) + e_x^-(z+ct)$$

$$\zeta h_y(z,t) = e_x^+(z-ct) - e_x^-(z+ct)$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

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$\{e_y, h_x\}$
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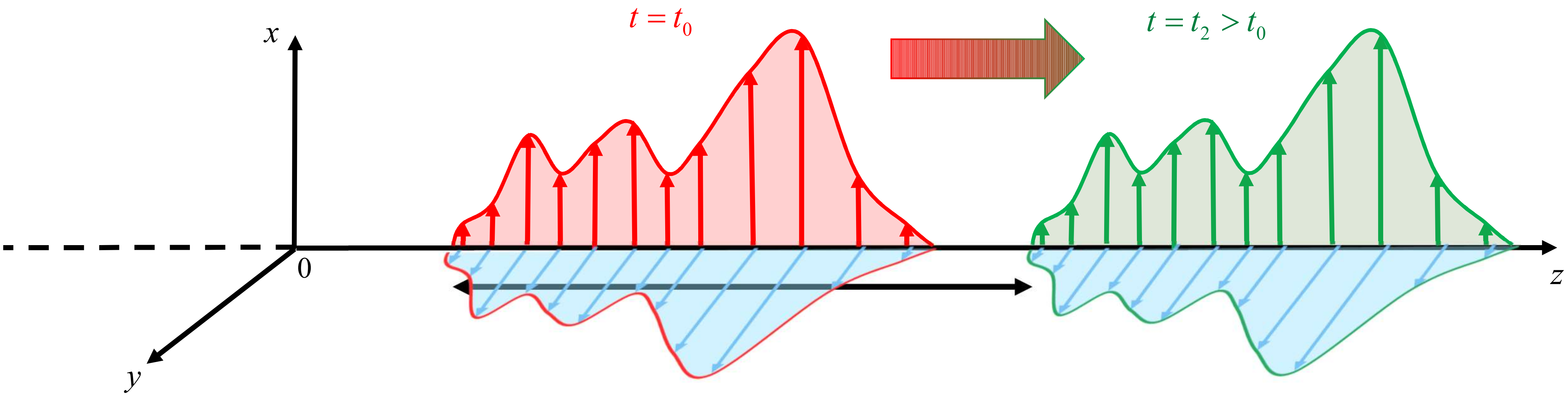
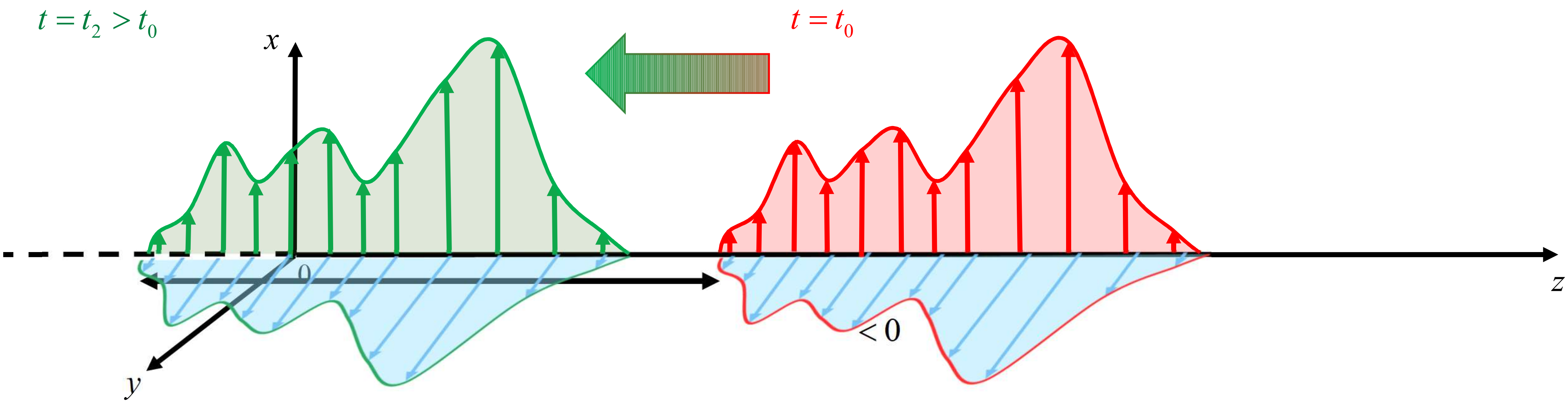


The electromagnetic perturbation **propagates** without deformation and with constant speed *c* along the positive sense of the *z*-axis

$$\begin{cases} e^{+}(z - ct) \\ h^{+}(z - ct) \end{cases}$$ is referred to as electromagnetic **progressive plane wave**

Plane Waves (TD)



The electromagnetic perturbation **propagates** without deformation and with constant speed ***c*** along the negative sense of the z-axis

$$\begin{cases} e^{-}(z+ct) \\ h^{-}(z+ct) \end{cases}$$ is referred to as electromagnetic **regressive** **plane wave**

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⬇

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$$\vec{\mathbf{s}}^+ = \frac{\left|e_x^+(z-ct)\right|^2}{\zeta}\hat{i}_z$$

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$$\vec{\mathbf{s}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(\vec{\mathbf{r}},t) \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{s}}^+ = \vec{\mathbf{e}}^+ \times \vec{\mathbf{h}}^+ = e_x^+\hat{i}_x \times h_y^+\hat{i}_y = e_x^+\hat{i}_x \times \frac{e_x^+}{\zeta}\hat{i}_y = \frac{\left|e_x^+\right|^2}{\zeta}\left(\hat{i}_x \times \hat{i}_y\right) = \frac{\left|e_x^+\right|^2}{\zeta}\hat{i}_z$$

$$\hat{i}_z = \hat{i}_x \times \hat{i}_y$$
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$$\begin{cases}\dfrac{\partial e_x}{\partial z}=-\mu\dfrac{\partial h_y}{\partial t}\\[2mm]\dfrac{\partial h_y}{\partial z}=-\varepsilon\dfrac{\partial e_x}{\partial t}\end{cases}\qquad \frac{\partial^2 e_x}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2}=0$$

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$$\begin{cases}e_x^+(z,t)=e_x^+(z-ct)\\ \zeta h_y^+(z,t)=e_x^+(z-ct)\end{cases}$$

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$$\vec{\mathbf{s}}^-=\vec{\mathbf{e}}^-\times\vec{\mathbf{h}}^-=e_x^-\hat{i}_x\times h_y^-\hat{i}_y=e_x^-\hat{i}_x\times\left(-\frac{e_x^-}{\zeta}\right)\hat{i}_y=-\frac{\left|e_x^-\right|^2}{\zeta}\left(\hat{i}_x\times\hat{i}_y\right)=-\frac{\left|e_x^-\right|^2}{\zeta}\hat{i}_z$$

$$\hat{i}_z=\hat{i}_x\times\hat{i}_y\qquad \hat{i}_y=\hat{i}_z\times\hat{i}_x\qquad \hat{i}_x=\hat{i}_y\times\hat{i}_z$$

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$$\begin{cases} \dfrac{\partial e_y}{\partial z} = \mu \dfrac{\partial h_x}{\partial t} \\ \dfrac{\partial h_x}{\partial z} = \varepsilon \dfrac{\partial e_y}{\partial t} \end{cases} \qquad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2} = 0$$

$$\begin{cases} e_y^+(z,t) = e_y^+(z-ct) \\ \zeta h_x^+(z,t) = -e_y^+(z-ct) \end{cases} \qquad \{e_y^+, h_x^+\}$$

$$\begin{cases} e_y^-(z,t) = e_y^-(z+ct) \\ \zeta h_x^-(z,t) = e_y^-(z+ct) \end{cases} \qquad \{e_y^-, h_x^-\}$$

$$\vec{\mathbf{s}}^+ = \frac{\left|e_y^+(z-ct)\right|^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{\left|e_y^-(z+ct)\right|^2}{\zeta}\hat{i}_z$$

$\{e_x^+, h_y^+\}$ $\{e_x^-, h_y^-\}$ $\{e_y^+, h_x^+\}$ $\{e_y^-, h_x^-\}$

In all these 4 cases the Poynting vector is directed along the direction of propagation

Source-free

Medium

- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$
$\{e_x, h_y\}$ **Independent each other**

Plane Waves (TD)

$\{e_x, h_y\}$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

$$\begin{cases} e_x^+(z,t) = e_x^+(z-ct) \\ \zeta h_y^+(z,t) = e_x^+(z-ct) \end{cases} \qquad \{e_x^+, h_y^+\}$$

$$\begin{cases} e_x^-(z,t) = e_x^-(z+ct) \\ \zeta h_y^-(z,t) = -e_x^-(z+ct) \end{cases} \qquad \{e_x^-, h_y^-\}$$

$$\vec{\mathbf{s}}^+ = \frac{\left|e_x^+(z-ct)\right|^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{\left|e_x^-(z+ct)\right|^2}{\zeta}\hat{i}_z$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{e_y, h_x\}$

$$\begin{cases} \dfrac{\partial e_y}{\partial z} = \mu \dfrac{\partial h_x}{\partial t} \\ \dfrac{\partial h_x}{\partial z} = \varepsilon \dfrac{\partial e_y}{\partial t} \end{cases} \qquad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2} = 0$$

$$\begin{cases} e_y^+(z,t) = e_y^+(z-ct) \\ \zeta h_x^+(z,t) = -e_y^+(z-ct) \end{cases} \qquad \{e_y^+, h_x^+\}$$

$$\begin{cases} e_y^-(z,t) = e_y^-(z+ct) \\ \zeta h_x^-(z,t) = e_y^-(z+ct) \end{cases} \qquad \{e_y^-, h_x^-\}$$

$$\vec{\mathbf{s}}^+ = \frac{\left|e_y^+(z-ct)\right|^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{\left|e_y^-(z+ct)\right|^2}{\zeta}\hat{i}_z$$

$$\zeta\vec{\mathbf{h}} = \hat{i}_p \times \vec{\mathbf{e}}$$

Source-free

Medium

- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

⬇

$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$

$\{e_x, h_y\}$

Independent each other

Plane Waves (TD)

$\{e_x, h_y\}$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

$$\begin{cases} e_x^+(z,t) = e_x^+(z-ct) \\ \zeta h_y^+(z,t) = e_x^+(z-ct) \end{cases} \qquad \{e_x^+, h_y^+\}$$

$$\begin{cases} e_x^-(z,t) = e_x^-(z+ct) \\ \zeta h_y^-(z,t) = -e_x^-(z+ct) \end{cases} \qquad \{e_x^-, h_y^-\}$$

$$\vec{\mathbf{s}}^+ = \frac{\left|e_x^+(z-ct)\right|^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{\left|e_x^-(z+ct)\right|^2}{\zeta}\hat{i}_z$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}} \qquad \zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{e_y, h_x\}$

$$\begin{cases} \dfrac{\partial e_y}{\partial z} = \mu \dfrac{\partial h_x}{\partial t} \\ \dfrac{\partial h_x}{\partial z} = \varepsilon \dfrac{\partial e_y}{\partial t} \end{cases} \qquad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2} = 0$$

$$\begin{cases} e_y^+(z,t) = e_y^+(z-ct) \\ \zeta h_x^+(z,t) = -e_y^+(z-ct) \end{cases} \qquad \{e_y^+, h_x^+\}$$

$$\begin{cases} e_y^-(z,t) = e_y^-(z+ct) \\ \zeta h_x^-(z,t) = e_y^-(z+ct) \end{cases} \qquad \{e_y^-, h_x^-\}$$

$$\vec{\mathbf{s}}^+ = \frac{\left|e_y^+(z-ct)\right|^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{\left|e_y^-(z+ct)\right|^2}{\zeta}\hat{i}_z$$

$\{e_x^+, h_y^+\}$

$$\zeta\vec{\mathbf{h}} = \hat{i}_p \times \vec{\mathbf{e}}$$

$$\zeta\vec{\mathbf{h}} = \zeta h_y^+ \hat{i}_y = e_x^+ \hat{i}_y$$

$$\hat{i}_p = \hat{i}_z\,;\ \vec{\mathbf{e}} = e_x^+\hat{i}_x \Rightarrow \hat{i}_p \times \vec{\mathbf{e}} = \hat{i}_z \times e_x^+\hat{i}_x = e_x^+\left(\hat{i}_z \times \hat{i}_x\right) = e_x^+\hat{i}_y$$

$$\hat{i}_z = \hat{i}_x \times \hat{i}_y \qquad \hat{i}_y = \hat{i}_z \times \hat{i}_x \qquad \hat{i}_x = \hat{i}_y \times \hat{i}_z$$

Source-free

Medium
- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Homogeneous (TI – SI)**
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$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

⬇

$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$, $\{e_x, h_y\}$ **Independent each other**

Plane Waves (TD)

$\{e_x, h_y\}$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

$\{e_x^+, h_y^+\}$

$$\begin{cases} e_x^+(z,t) = e_x^+(z-ct) \\ \zeta h_y^+(z,t) = e_x^+(z-ct) \end{cases}$$

$\{e_x^-, h_y^-\}$

$$\begin{cases} e_x^-(z,t) = e_x^-(z+ct) \\ \zeta h_y^-(z,t) = -e_x^-(z+ct) \end{cases}$$

$$\vec{\mathbf{s}}^+ = \frac{|e_x^+(z-ct)|^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{|e_x^-(z+ct)|^2}{\zeta}\hat{i}_z$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}} \qquad \zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{e_y, h_x\}$

$$\begin{cases} \dfrac{\partial e_y}{\partial z} = \mu \dfrac{\partial h_x}{\partial t} \\ \dfrac{\partial h_x}{\partial z} = \varepsilon \dfrac{\partial e_y}{\partial t} \end{cases} \qquad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2} = 0$$

$\{e_y^+, h_x^+\}$

$$\begin{cases} e_y^+(z,t) = e_y^+(z-ct) \\ \zeta h_x^+(z,t) = -e_y^+(z-ct) \end{cases}$$

$\{e_y^-, h_x^-\}$

$$\begin{cases} e_y^-(z,t) = e_y^-(z+ct) \\ \zeta h_x^-(z,t) = e_y^-(z+ct) \end{cases}$$

$$\vec{\mathbf{s}}^+ = \frac{|e_y^+(z-ct)|^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{|e_y^-(z+ct)|^2}{\zeta}\hat{i}_z$$

Source-free

Medium
- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$, $\{e_x, h_y\}$ **Independent each other**

$$\zeta\vec{\mathbf{h}} = \hat{i}_p \times \vec{\mathbf{e}}$$

$\{e_x^-, h_y^-\}$

$$\zeta\vec{\mathbf{h}} = \zeta h_y^- \hat{i}_y = -e_x^- \hat{i}_y$$

$$\hat{i}_p = -\hat{i}_z\,;\ \vec{\mathbf{e}} = e_x^- \hat{i}_x \Rightarrow \hat{i}_p \times \vec{\mathbf{e}} = -\hat{i}_z \times e_x^- \hat{i}_x = -e_x^-(\hat{i}_z \times \hat{i}_x) = -e_x^- \hat{i}_y$$

$$\hat{i}_z = \hat{i}_x \times \hat{i}_y \qquad \hat{i}_y = \hat{i}_z \times \hat{i}_x \qquad \hat{i}_x = \hat{i}_y \times \hat{i}_z$$

Plane Waves (TD)

$\{e_x, h_y\}$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

$$\begin{cases} e_x^+(z,t) = e_x^+(z-ct) \\ \zeta h_y^+(z,t) = e_x^+(z-ct) \end{cases} \qquad \{e_x^+, h_y^+\}$$

$$\begin{cases} e_x^-(z,t) = e_x^-(z+ct) \\ \zeta h_y^-(z,t) = -e_x^-(z+ct) \end{cases} \qquad \{e_x^-, h_y^-\}$$

$$\vec{\mathbf{s}}^+ = \frac{|e_x^+(z-ct)|^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{|e_x^-(z+ct)|^2}{\zeta}\hat{i}_z$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{e_y, h_x\}$

$$\begin{cases} \dfrac{\partial e_y}{\partial z} = \mu \dfrac{\partial h_x}{\partial t} \\ \dfrac{\partial h_x}{\partial z} = \varepsilon \dfrac{\partial e_y}{\partial t} \end{cases} \qquad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2} = 0$$

$$\begin{cases} e_y^+(z,t) = e_y^+(z-ct) \\ \zeta h_x^+(z,t) = -e_y^+(z-ct) \end{cases} \qquad \{e_y^+, h_x^+\}$$

$$\begin{cases} e_y^-(z,t) = e_y^-(z+ct) \\ \zeta h_x^-(z,t) = e_y^-(z+ct) \end{cases} \qquad \{e_y^-, h_x^-\}$$

$$\vec{\mathbf{s}}^+ = \frac{|e_y^+(z-ct)|^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{|e_y^-(z+ct)|^2}{\zeta}\hat{i}_z$$

$\{e_x^+, h_y^+\}$ $\{e_x^-, h_y^-\}$ $\{e_y^+, h_x^+\}$ $\{e_y^-, h_x^-\}$

In all these 4 cases the electric and magnetic fields are related each other through the following relation:

$$\zeta\vec{\mathbf{h}} = \hat{i}_p \times \vec{\mathbf{e}}$$

where $\hat{i}_p$ points to the propagation direction

Source-free

Medium
- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

⬇

$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$
$\{e_x, h_y\}$ **Independent each other**

Plane Waves (TD)

Source-free

Medium
- Linear
- Local (TND & SND)
- Isotropic
- Homogeneous (TI – SI)
- Lossless

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$, $\{e_x, h_y\}$ **Independent each other**

$\{e_x, h_y\}$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}} \qquad \zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{e_x^+, h_y^+\}$

$$\begin{cases} e_x^+(z,t) = e_x^+(z-ct) \\ \zeta h_y^+(z,t) = e_x^+(z-ct) \end{cases}$$

$\{e_x^-, h_y^-\}$

$$\begin{cases} e_x^-(z,t) = e_x^-(z+ct) \\ \zeta h_y^-(z,t) = -e_x^-(z+ct) \end{cases}$$

$$\vec{\mathbf{s}}^+ = \frac{\left|e_x^+(z-ct)\right|^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{\left|e_x^-(z+ct)\right|^2}{\zeta}\hat{i}_z$$

$$\vec{\mathbf{s}}^+ = \zeta\left|h_y^+(z-ct)\right|^2\hat{i}_z$$

$\{e_y, h_x\}$

$$\begin{cases} \dfrac{\partial e_y}{\partial z} = \mu \dfrac{\partial h_x}{\partial t} \\ \dfrac{\partial h_x}{\partial z} = \varepsilon \dfrac{\partial e_y}{\partial t} \end{cases} \qquad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2} = 0$$

$\{e_y^+, h_x^+\}$

$$\begin{cases} e_y^+(z,t) = e_y^+(z-ct) \\ \zeta h_x^+(z,t) = -e_y^+(z-ct) \end{cases}$$

$\{e_y^-, h_x^-\}$

$$\begin{cases} e_y^-(z,t) = e_y^-(z+ct) \\ \zeta h_x^-(z,t) = e_y^-(z+ct) \end{cases}$$

$$\vec{\mathbf{s}}^+ = \frac{\left|e_y^+(z-ct)\right|^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{\left|e_y^-(z+ct)\right|^2}{\zeta}\hat{i}_z$$

$$\left|e_x^+(z-ct)\right| = \zeta\left|h_y^+(z-ct)\right|$$

$$\vec{\mathbf{s}}^+ = \frac{\left|e_x^+(z-ct)\right|^2}{\zeta}\hat{i}_z = \zeta\left|h_y^+(z-ct)\right|^2\hat{i}_z$$

Plane Waves (TD)

$\{e_x, h_y\}$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

$$\begin{cases} e_x^+(z,t) = e_x^+(z-ct) \\ \zeta h_y^+(z,t) = e_x^+(z-ct) \end{cases} \qquad \{e_x^+, h_y^+\}$$

$$\begin{cases} e_x^-(z,t) = e_x^-(z+ct) \\ \zeta h_y^-(z,t) = -e_x^-(z+ct) \end{cases} \qquad \{e_x^-, h_y^-\}$$

$$\vec{\mathbf{s}}^+ = \frac{\left|e_x^+(z-ct)\right|^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{\left|e_x^-(z+ct)\right|^2}{\zeta}\hat{i}_z$$

$$\vec{\mathbf{s}}^+ = \zeta\left|h_y^+(z-ct)\right|^2\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\zeta\left|h_y^-(z+ct)\right|^2\hat{i}_z$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{e_y, h_x\}$

$$\begin{cases} \dfrac{\partial e_y}{\partial z} = \mu \dfrac{\partial h_x}{\partial t} \\ \dfrac{\partial h_x}{\partial z} = \varepsilon \dfrac{\partial e_y}{\partial t} \end{cases} \qquad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2} = 0$$

$$\begin{cases} e_y^+(z,t) = e_y^+(z-ct) \\ \zeta h_x^+(z,t) = -e_y^+(z-ct) \end{cases} \qquad \{e_y^+, h_x^+\}$$

$$\begin{cases} e_y^-(z,t) = e_y^-(z+ct) \\ \zeta h_x^-(z,t) = e_y^-(z+ct) \end{cases} \qquad \{e_y^-, h_x^-\}$$

$$\vec{\mathbf{s}}^+ = \frac{\left|e_y^+(z-ct)\right|^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{\left|e_y^-(z+ct)\right|^2}{\zeta}\hat{i}_z$$

$$\vec{\mathbf{s}}^+ = \zeta\left|h_x^+(z-ct)\right|^2\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\zeta\left|h_x^-(z+ct)\right|^2\hat{i}_z$$

Source-free

Medium
- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$ $\{e_x, h_y\}$ **Independent each other**

Plane Waves (TD)

Source-free

Medium
- **Linear**
- **Time non-dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

⬇

$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$ and $\{e_x, h_y\}$: **Independent each other**

$\{e_x^+, h_y^+\}$

$$\begin{cases} e_x^+(z,t) = e_x^+(z-ct) \\ \zeta h_y^+(z,t) = e_x^+(z-ct) \end{cases}$$

$\{e_y^+, h_x^+\}$

$$\begin{cases} e_y^+(z,t) = e_y^+(z-ct) \\ \zeta h_x^+(z,t) = -e_y^+(z-ct) \end{cases}$$

the e.m. field propagates along $\hat{i}_p = \hat{i}_z$

$\{e_x^-, h_y^-\}$

$$\begin{cases} e_x^-(z,t) = e_x^-(z+ct) \\ \zeta h_y^-(z,t) = -e_x^-(z+ct) \end{cases}$$

$\{e_y^-, h_x^-\}$

$$\begin{cases} e_y^-(z,t) = e_y^-(z+ct) \\ \zeta h_x^-(z,t) = e_y^-(z+ct) \end{cases}$$

the e.m. field propagates along $\hat{i}_p = -\hat{i}_z$

- the e.m. field lies on the plane xy orthogonal to the propagation direction
- $|\vec{\mathbf{e}}|$ and $|\vec{\mathbf{h}}|$ are proportional through ζ
- $\zeta \vec{\mathbf{h}} = \hat{i}_p \times \vec{\mathbf{e}}$

Plane Waves

Time domain (TD)

Spectral domains

- Phasor Domain (PD)
- Fourier Domain (FD)
- Dispersive media: attenuation, distortion, phase velocity and group velocity

General expression of plane waves (PD)

Incidence

Plane Waves

Spectral domains

Plane Waves (Spectral Domains)

Source-free

Medium

- **Linear**
- **Time non-dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

Phasor domain (PD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) + \rho_0(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) = 0 \end{cases}$$

Fourier domain (FD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega \vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = j\omega \vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) + \vec{\mathbf{J}}(\vec{\mathbf{r}},\omega) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}},\omega) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) = \rho(\vec{\mathbf{r}},\omega) + \rho_0(\vec{\mathbf{r}},\omega) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) = 0 \end{cases}$$

Plane Waves (Spectral Domains)

Source-free

Medium
- **Linear**
- **Time non-dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

Phasor domain (PD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) + \rho_0(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) = 0 \end{cases}$$

Fourier domain (FD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega \vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = j\omega \vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) + \vec{\mathbf{J}}(\vec{\mathbf{r}},\omega) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}},\omega) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) = \rho(\vec{\mathbf{r}},\omega) + \rho_0(\vec{\mathbf{r}},\omega) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) = 0 \end{cases}$$

$$\vec{\mathbf{D}} = \varepsilon \vec{\mathbf{E}} \qquad \vec{\mathbf{B}} = \mu \vec{\mathbf{H}} \qquad \vec{\mathbf{J}} = \sigma \vec{\mathbf{E}}$$

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

Plane Waves (Spectral Domains)

Source-free

Medium
- **Linear**
- **Time non-dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

Phasor domain (PD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) + \rho_0(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) = 0 \end{cases}$$

Fourier domain (FD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega \vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = j\omega \vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) + \vec{\mathbf{J}}(\vec{\mathbf{r}},\omega) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}},\omega) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) = \rho(\vec{\mathbf{r}},\omega) + \rho_0(\vec{\mathbf{r}},\omega) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) = 0 \end{cases}$$

$$\begin{cases} \vec{\mathbf{J}}_0 = \mathbf{0} \\ \rho_0 = 0 \end{cases}$$

$$\vec{\mathbf{D}} = \varepsilon \vec{\mathbf{E}} \qquad \vec{\mathbf{B}} = \mu \vec{\mathbf{H}} \qquad \vec{\mathbf{J}} = \sigma \vec{\mathbf{E}}$$

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PD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = E_x(z)\hat{i}_x + E_y(z)\hat{i}_y + E_z(z)\hat{i}_z$$

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FD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = E_x(z,\omega)\hat{i}_x + E_y(z,\omega)\hat{i}_y + E_z(z,\omega)\hat{i}_z$$

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PD

$$\begin{cases} \vec{\mathbf{J}}_0 = \mathbf{0} \\ \rho_0 = 0 \end{cases}$$

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FD

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Fourier domain (FD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega \vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = j\omega \vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) + \vec{\mathbf{J}}(\vec{\mathbf{r}},\omega) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}},\omega) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) = \rho(\vec{\mathbf{r}},\omega) + \rho_0(\vec{\mathbf{r}},\omega) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) = 0 \end{cases}$$

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FD

$$\vec{\mathbf{D}} = \varepsilon \vec{\mathbf{E}} \qquad \vec{\mathbf{B}} = \mu \vec{\mathbf{H}} \qquad \vec{\mathbf{J}} = \sigma \vec{\mathbf{E}}$$

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

Plane Waves (Spectral Domains)

Source-free

Medium

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Phasor domain (PD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) + \rho_0(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) = 0 \end{cases}$$

Fourier domain (FD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega \vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = j\omega \vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) + \vec{\mathbf{J}}(\vec{\mathbf{r}},\omega) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}},\omega) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) = \rho(\vec{\mathbf{r}},\omega) + \rho_0(\vec{\mathbf{r}},\omega) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) = 0 \end{cases}$$

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$$\begin{cases} \vec{\mathbf{J}}_0 = \mathbf{0} \\ \rho_0 = 0 \end{cases}$$

$$\vec{\mathbf{D}} = \varepsilon \vec{\mathbf{E}}$$

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$$\vec{\mathbf{J}} = \sigma \vec{\mathbf{E}}$$

$$\begin{cases} \varepsilon = \varepsilon_1 - j\varepsilon_2 \quad (= real) \\ \mu = \mu_1 - j\mu_2 \quad (= real) \\ \sigma = 0 \quad (= real) \end{cases}$$

Plane Waves (Spectral Domains)

Source-free

Medium
- **Linear**
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- **Space non-dispersive**
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- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

Phasor domain (PD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \mu \vec{\mathbf{H}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \varepsilon \vec{\mathbf{E}}(\vec{\mathbf{r}}) + \sigma \vec{\mathbf{E}}(\vec{\mathbf{r}}) \\ \nabla \cdot \varepsilon \vec{\mathbf{E}}(\vec{\mathbf{r}}) = 0 \\ \nabla \cdot \mu \vec{\mathbf{H}}(\vec{\mathbf{r}}) = 0 \end{cases}$$

Fourier domain (FD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega \mu \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = j\omega \varepsilon \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) + \sigma \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) \\ \nabla \cdot \varepsilon \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = 0 \\ \nabla \cdot \mu \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = 0 \end{cases}$$

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PD

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Fourier domain (FD)

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Phasor domain (PD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \mu \vec{\mathbf{H}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \varepsilon_{eq} \vec{\mathbf{E}}(\vec{\mathbf{r}}) \\ \nabla \cdot \varepsilon \vec{\mathbf{E}}(\vec{\mathbf{r}}) = 0 \\ \nabla \cdot \mu \vec{\mathbf{H}}(\vec{\mathbf{r}}) = 0 \end{cases}$$

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- ~~Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

Phasor domain (PD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \mu \vec{\mathbf{H}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \varepsilon \vec{\mathbf{E}}(\vec{\mathbf{r}}) \end{cases}$$

Fourier domain (FD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega \mu \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = j\omega \varepsilon \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) \end{cases}$$

PD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = E_x(z)\hat{i}_x + E_y(z)\hat{i}_y + E_z(z)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = H_x(z)\hat{i}_x + H_y(z)\hat{i}_y + H_z(z)\hat{i}_z$$

FD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = E_x(z,\omega)\hat{i}_x + E_y(z,\omega)\hat{i}_y + E_z(z,\omega)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = H_x(z,\omega)\hat{i}_x + H_y(z,\omega)\hat{i}_y + H_z(z,\omega)\hat{i}_z$$

$$\begin{cases} \varepsilon = \varepsilon_1 - j\varepsilon_2 \\ \mu = \mu_1 - j\mu_2 \end{cases}$$

Plane Waves (Spectral Domains)

Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

Phasor domain (PD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(z) = -j\omega_0 \mu \vec{\mathbf{H}}(z) \\ \nabla \times \vec{\mathbf{H}}(z) = j\omega_0 \varepsilon \vec{\mathbf{E}}(z) \end{cases}$$

Fourier domain (FD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(z,\omega) = -j\omega \mu \vec{\mathbf{H}}(z,\omega) \\ \nabla \times \vec{\mathbf{H}}(z,\omega) = j\omega \varepsilon \vec{\mathbf{E}}(z,\omega) \end{cases}$$

PD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = E_x(z)\hat{i}_x + E_y(z)\hat{i}_y + E_z(z)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = H_x(z)\hat{i}_x + H_y(z)\hat{i}_y + H_z(z)\hat{i}_z$$

FD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = E_x(z,\omega)\hat{i}_x + E_y(z,\omega)\hat{i}_y + E_z(z,\omega)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = H_x(z,\omega)\hat{i}_x + H_y(z,\omega)\hat{i}_y + H_z(z,\omega)\hat{i}_z$$

$$\begin{cases} \varepsilon = \varepsilon_1 - j\varepsilon_2 \\ \mu = \mu_1 - j\mu_2 \end{cases}$$

Plane Waves (Spectral Domains)

Source-free

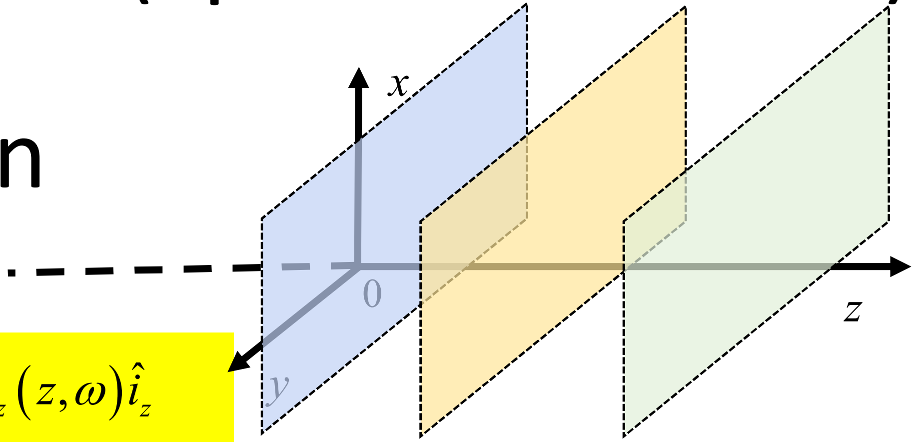
Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

Fourier Domain



$$\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = E_x(z,\omega)\hat{i}_x + E_y(z,\omega)\hat{i}_y + E_z(z,\omega)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = H_x(z,\omega)\hat{i}_x + H_y(z,\omega)\hat{i}_y + H_z(z,\omega)\hat{i}_z$$

Fourier domain (FD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(z,\omega) = -j\omega\mu\vec{\mathbf{H}}(z,\omega) \\ \nabla \times \vec{\mathbf{H}}(z,\omega) = j\omega\varepsilon\vec{\mathbf{E}}(z,\omega) \end{cases}$$

$$\nabla \times \vec{\mathbf{E}} = \left(-\frac{dE_y}{dz}\right)\hat{i}_x + \left(\frac{dE_x}{dz}\right)\hat{i}_y$$

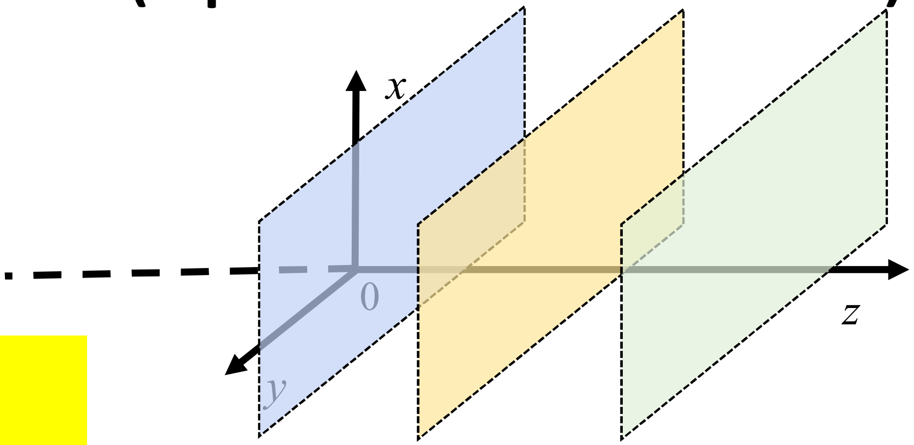
$$\nabla \times \vec{\mathbf{H}} = \left(-\frac{dH_y}{dz}\right)\hat{i}_x + \left(\frac{dH_x}{dz}\right)\hat{i}_y$$

Plane Waves (Spectral Domains)

Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

Phasor Domain



$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = E_x(z)\hat{i}_x + E_y(z)\hat{i}_y + E_z(z)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = H_x(z)\hat{i}_x + H_y(z)\hat{i}_y + H_z(z)\hat{i}_z$$

Phasor domain (PD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(z) = -j\omega_0 \mu \vec{\mathbf{H}}(z) \\ \nabla \times \vec{\mathbf{H}}(z) = j\omega_0 \varepsilon \vec{\mathbf{E}}(z) \end{cases}$$

$$\nabla \times \vec{\mathbf{E}} = \left(-\frac{dE_y}{dz}\right)\hat{i}_x + \left(\frac{dE_x}{dz}\right)\hat{i}_y$$

$$\nabla \times \vec{\mathbf{H}} = \left(-\frac{dH_y}{dz}\right)\hat{i}_x + \left(\frac{dH_x}{dz}\right)\hat{i}_y$$

Plane Waves (Spectral Domains)

$$\begin{cases}\nabla\times\vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}} \\ \nabla\times\vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}}\end{cases}$$

$$\vec{\mathbf{E}} = E_x\hat{i}_x + E_y\hat{i}_y + \cancel{E_z\hat{i}_z}$$

$$\vec{\mathbf{H}} = H_x\hat{i}_x + H_y\hat{i}_y + \cancel{H_z\hat{i}_z}$$

$$\nabla\times\vec{\mathbf{E}} = \left(-\frac{dE_y}{dz}\right)\hat{i}_x + \left(\frac{dE_x}{dz}\right)\hat{i}_y$$

$$-j\omega\mu\vec{\mathbf{H}} = -j\omega\mu H_x\hat{i}_x - j\omega\mu H_y\hat{i}_y - j\omega\mu H_z\hat{i}_z$$

$$\nabla\times\vec{\mathbf{H}} = \left(-\frac{dH_y}{dz}\right)\hat{i}_x + \left(\frac{dH_x}{dz}\right)\hat{i}_y$$

$$j\omega\varepsilon\vec{\mathbf{E}} = j\omega\varepsilon E_x\hat{i}_x + j\omega\varepsilon E_y\hat{i}_y + j\omega\varepsilon E_z\hat{i}_z$$

$$E_z = 0 \quad H_z = 0 \;\Rightarrow\; e_z(z,t) = 0 \quad h_z(z,t) = 0 \;\Rightarrow\; \textbf{TEM fields}$$

Source-free

Medium

- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- ~~Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

Plane Waves (Spectral Domains)

$$\begin{cases} \nabla \times \vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}} \\ \nabla \times \vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}} \end{cases}$$

$$\vec{\mathbf{E}} = E_x \hat{i}_x + E_y \hat{i}_y$$

$$\vec{\mathbf{H}} = H_x \hat{i}_x + H_y \hat{i}_y$$

$$\nabla \times \vec{\mathbf{E}} = \left(-\frac{dE_y}{dz}\right)\hat{i}_x + \left(\frac{dE_x}{dz}\right)\hat{i}_y$$

$$-j\omega\mu\vec{\mathbf{H}} = -j\omega\mu H_x \hat{i}_x - j\omega\mu H_y \hat{i}_y - j\omega\mu H_z \hat{i}_z$$

$$\nabla \times \vec{\mathbf{H}} = \left(-\frac{dH_y}{dz}\right)\hat{i}_x + \left(\frac{dH_x}{dz}\right)\hat{i}_y$$

$$j\omega\varepsilon\vec{\mathbf{E}} = j\omega\varepsilon E_x \hat{i}_x + j\omega\varepsilon E_y \hat{i}_y + j\omega\varepsilon E_z \hat{i}_z$$

Source-free

Medium

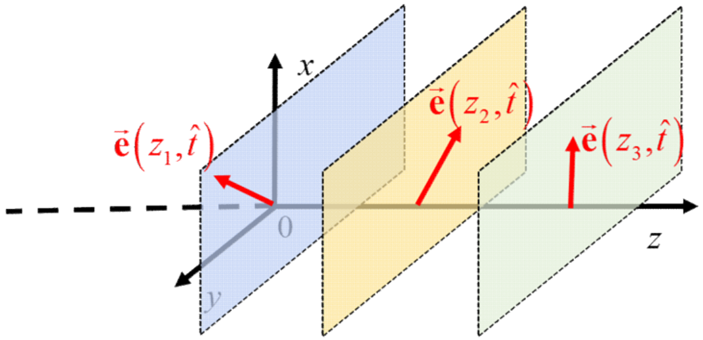
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$



Plane Waves (Spectral Domains)

$$\begin{cases} \nabla\times\vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}} \\ \nabla\times\vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}} \end{cases}$$

$$\vec{\mathbf{E}} = E_x\hat{i}_x + E_y\hat{i}_y$$

$$\vec{\mathbf{H}} = H_x\hat{i}_x + H_y\hat{i}_y$$

$$\nabla\times\vec{\mathbf{E}} = \left(-\frac{dE_y}{dz}\right)\hat{i}_x + \left(\frac{dE_x}{dz}\right)\hat{i}_y$$

$$-j\omega\mu\vec{\mathbf{H}} = -j\omega\mu H_x\hat{i}_x - j\omega\mu H_y\hat{i}_y - j\omega\mu H_z\hat{i}_z$$

$$\nabla\times\vec{\mathbf{H}} = \left(-\frac{dH_y}{dz}\right)\hat{i}_x + \left(\frac{dH_x}{dz}\right)\hat{i}_y$$

$$j\omega\varepsilon\vec{\mathbf{E}} = j\omega\varepsilon E_x\hat{i}_x + j\omega\varepsilon E_y\hat{i}_y + j\omega\varepsilon E_z\hat{i}_z$$

$$\frac{dE_y}{dz} = j\omega\mu H_x$$

$$\frac{dE_x}{dz} = -j\omega\mu H_y$$

$$\frac{dH_y}{dz} = -j\omega\varepsilon E_x$$

$$\frac{dH_x}{dz} = j\omega\varepsilon E_y$$

Source-free

Medium
- **Linear**
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- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$ **Independent each other**

Plane Waves (Spectral Domains)

$$\begin{cases} \nabla \times \vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}} \\ \nabla \times \vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}} \end{cases}$$

$$\vec{\mathbf{E}} = E_x\hat{i}_x + E_y\hat{i}_y$$

$$\vec{\mathbf{H}} = H_x\hat{i}_x + H_y\hat{i}_y$$

$$\frac{dE_x}{dz} = -j\omega\mu H_y$$

$$\frac{dH_y}{dz} = -j\omega\varepsilon E_x$$

$$\frac{dE_y}{dz} = j\omega\mu H_x$$

$$\frac{dH_x}{dz} = j\omega\varepsilon E_y$$

$$\frac{dE_y}{dz} = j\omega\mu H_x$$

$$\frac{dE_x}{dz} = -j\omega\mu H_y$$

$$\frac{dH_y}{dz} = -j\omega\varepsilon E_x$$

$$\frac{dH_x}{dz} = j\omega\varepsilon E_y$$

Source-free

Medium

- Linear
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$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

Plane Waves (Spectral Domains)

Source-free

Medium

- **Linear**
- **Time dispersive**
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$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$, $\{E_x, H_y\}$ **Independent each other**

$\{E_x, H_y\}$

$$\frac{dE_x}{dz} = -j\omega\mu H_y$$

$$\frac{dH_y}{dz} = -j\omega\varepsilon E_x$$

$\{E_y, H_x\}$

$$\frac{dE_y}{dz} = j\omega\mu H_x$$

$$\frac{dH_x}{dz} = j\omega\varepsilon E_y$$

Plane Waves (Spectral Domains)

$\{E_x, H_y\}$

$\{E_y, H_x\}$

Source-free

Medium
- **Linear**
- **Time dispersive**
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$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{h}}(z, t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases}$$

$$\begin{cases} \dfrac{dE_y}{dz} = j\omega\mu H_x \\ \dfrac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases}$$

Plane Waves (Spectral Domains)

Source-free

Medium
- **Linear**
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$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \Rightarrow \frac{d^2 E_x}{dz^2} = -j\omega\mu \frac{dH_y}{dz} = -\omega^2 \mu\varepsilon E_x \Rightarrow \frac{d^2 E_x}{dz^2} + \omega^2 \mu\varepsilon E_x = 0$$

$\{E_y, H_x\}$

$$\begin{cases} \dfrac{dE_y}{dz} = j\omega\mu H_x \\ \dfrac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases} \Rightarrow \frac{d^2 E_y}{dz^2} = j\omega\mu \frac{dH_x}{dz} = -\omega^2 \mu\varepsilon E_y \Rightarrow \frac{d^2 E_y}{dz^2} + \omega^2 \mu\varepsilon E_y = 0$$

$\{E_y, H_x\}$, $\{E_x, H_y\}$ **Independent each other**

Plane Waves (Spectral Domains)

$$k = \omega\sqrt{\mu\varepsilon}$$
$$k = \beta - j\alpha$$

k : (complex) propagation constant

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \qquad \frac{d^2E_x}{dz^2} + \omega^2\mu\varepsilon E_x = 0 \qquad \frac{d^2E_x}{dz^2} + k^2 E_x = 0$$

$\{E_y, H_x\}$

$$\begin{cases} \dfrac{dE_y}{dz} = j\omega\mu H_x \\ \dfrac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases} \qquad \frac{d^2E_y}{dz^2} + \omega^2\mu\varepsilon E_y = 0 \qquad \frac{d^2E_y}{dz^2} + k^2 E_y = 0$$

Source-free

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- **Linear**
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$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

↓

$$E_z = H_z = 0$$

$\{E_y, H_x\}$, $\{E_x, H_y\}$ **Independent each other**

Plane Waves (Spectral Domains)

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$k = \omega\sqrt{\mu\varepsilon}$$
$$k = \beta - j\alpha$$

$\{E_y, H_x\}$

$$\begin{cases} \dfrac{dE_y}{dz} = j\omega\mu H_x \\ \dfrac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases} \qquad \frac{d^2 E_y}{dz^2} + k^2 E_y = 0$$

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + \omega^2\mu\varepsilon E_x = 0 \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$\{E_y, H_x\}$

$$\begin{cases} \dfrac{dE_y}{dz} = j\omega\mu H_x \\ \dfrac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases} \qquad \frac{d^2 E_y}{dz^2} + \omega^2\mu\varepsilon E_y = 0 \qquad \frac{d^2 E_y}{dz^2} + k^2 E_y = 0$$

Source-free

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- **Linear**
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$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{h}}(z, t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$, $\{E_x, H_y\}$ **Independent each other**

Plane Waves

Time domain (TD)

Spectral domains

- Phasor Domain (PD)
- Fourier Domain (FD)
- Dispersive media: attenuation, distortion, phase velocity and group velocity

General expression of plane waves (PD)

Incidence

Plane Waves (Spectral Domains)

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$k = \omega\sqrt{\mu\varepsilon}$$
$$k = \beta - j\alpha$$

$\{E_y, H_x\}$

$$\begin{cases} \dfrac{dE_y}{dz} = j\omega\mu H_x \\ \dfrac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases} \qquad \frac{d^2 E_y}{dz^2} + k^2 E_y = 0$$

Source-free

Medium

- **Linear**
- **Time dispersive**
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$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{e}}(z, t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{h}}(z, t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$, $\{E_x, H_y\}$ **Independent each other**

Plane Waves (Spectral Domains)

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases}$$

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$k = \omega\sqrt{\mu\varepsilon}$$

$$k = \beta - j\alpha$$

$\{E_y, H_x\}$

$$\begin{cases} \dfrac{dE_y}{dz} = j\omega\mu H_x \\ \dfrac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases}$$

$$\frac{d^2 E_y}{dz^2} + k^2 E_y = 0$$

$$E_y = E_y^+ e^{-jkz} + E_y^- e^{jkz}$$

$$\frac{d^2 f(z)}{dz^2} + k^2 f(z) = 0$$

$$\xi^2 + k^2 = 0 \qquad \xi = \pm jk$$

$$f(z) = C_1 e^{-jkz} + C_2 e^{+jkz}$$

Source-free

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$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{h}}(z, t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$, $\{E_x, H_y\}$ **Independent each other**

Plane Waves (Spectral Domains)

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases}$$

$$\frac{d^2E_x}{dz^2} + k^2E_x = 0$$

$$E_x = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$k = \omega\sqrt{\mu\varepsilon}$$

$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{E_y, H_x\}$

$$\begin{cases} \dfrac{dE_y}{dz} = j\omega\mu H_x \\ \dfrac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases}$$

$$\frac{d^2E_y}{dz^2} + k^2E_y = 0$$

$$E_y = E_y^+ e^{-jkz} + E_y^- e^{jkz}$$

Source-free

Medium

- **Linear**
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- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$$-j\omega\mu H_y = \frac{dE_x}{dz} = -jkE_x^+ e^{-jkz} + jkE_x^- e^{jkz}$$

$$\omega\mu H_y = kE_x^+ e^{-jkz} - kE_x^- e^{jkz} \qquad \frac{\omega\mu}{k} H_y = E_x^+ e^{-jkz} - E_x^- e^{jkz} \qquad \zeta H_y = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

ζ : intrinsic impedance of the medium

$$\frac{\omega\mu}{k} = \frac{\omega\mu}{\omega\sqrt{\mu\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}} = \zeta$$

$\{E_y, H_x\}$ $\{E_x, H_y\}$ **Independent each other**

Plane Waves (Spectral Domains)

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$k = \omega\sqrt{\mu\varepsilon}$$
$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{E_y, H_x\}$

$$\begin{cases} \dfrac{dE_y}{dz} = j\omega\mu H_x \\ \dfrac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases} \qquad \frac{d^2 E_y}{dz^2} + k^2 E_y = 0$$

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$$-\omega\mu H_x = kE_y^+ e^{-jkz} - kE_y^- e^{jkz} \qquad -\frac{\omega\mu}{k} H_x = E_y^+ e^{-jkz} - E_y^- e^{jkz} \qquad -\zeta H_y = E_y^+ e^{-jkz} - E_y^- e^{jkz}$$

$$\frac{\omega\mu}{k} = \frac{\omega\mu}{\omega\sqrt{\mu\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}} = \zeta$$

Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

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$$E_z = H_z = 0$$

$\{E_y, H_x\}$, $\{E_x, H_y\}$ **Independent each other**

Plane Waves (Spectral Domains)

$\left\{E_x, H_y\right\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

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$\left\{E_y, H_x\right\}$

$$\begin{cases} \dfrac{dE_y}{dz} = j\omega\mu H_x \\ \dfrac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases} \qquad \frac{d^2 E_y}{dz^2} + k^2 E_y = 0$$

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$\left\{E_y, H_x\right\}$
$\left\{E_x, H_y\right\}$ **Independent each other**