Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2020-2021 - Laurea "Triennale" – Secondo semestre - Secondo anno

Università degli Studi di Napoli "Parthenope"

Stefano Perna

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Plane Waves

Time domain (TD)

Spectral domains

Phasor Domain (PD)

Fourier Domain (FD)

Dispersive media: attenuation, distortion, phase velocity and group velocity

General expression of plane waves (PD)

Incidence

Plane Waves

Time domain

Plane Waves (TD)

Medium

- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\begin{cases} \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \varepsilon \vec{\mathbf{e}}(\vec{\mathbf{r}},t) \\ \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = \mu \vec{\mathbf{h}}(\vec{\mathbf{r}},t) \\ \sigma = 0 \end{cases}$$

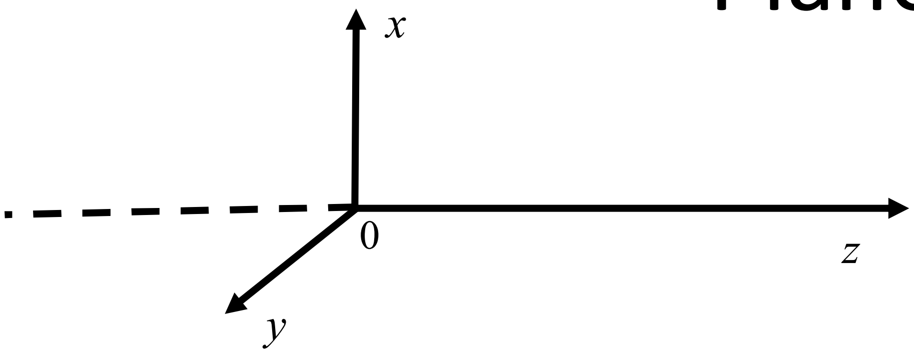
Time domain - Differential form

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) + \vec{\mathbf{j}}_0(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) + \rho_0(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \end{cases}$$

Time domain - Differential form

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\mu \dfrac{\partial \vec{\mathbf{h}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \varepsilon \dfrac{\partial \vec{\mathbf{e}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}_0(\vec{\mathbf{r}},t) \\ \varepsilon \nabla \cdot \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \rho_0(\vec{\mathbf{r}},t) \\ \mu \nabla \cdot \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = 0 \end{cases}$$

Plane Waves (TD)



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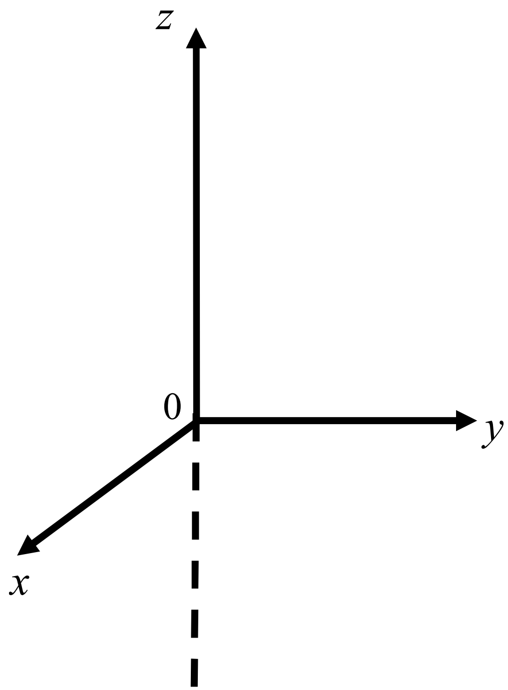
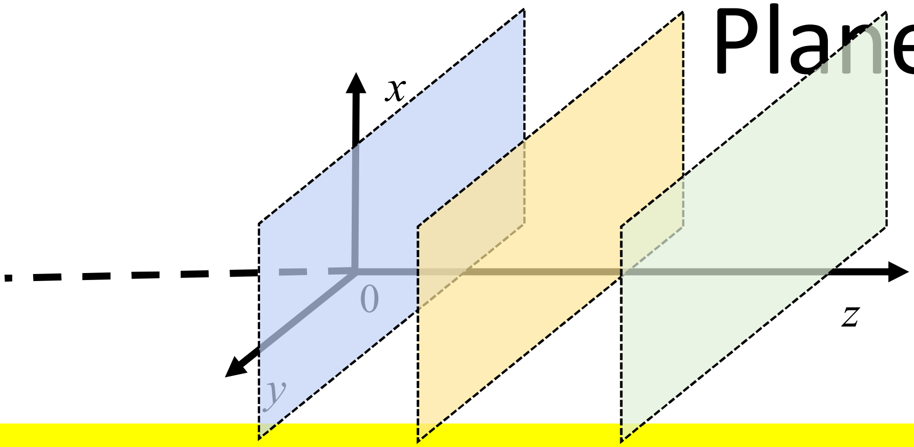
Time domain - Differential form

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\mu \dfrac{\partial \vec{\mathbf{h}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \varepsilon \dfrac{\partial \vec{\mathbf{e}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}_0(\vec{\mathbf{r}},t) \\ \varepsilon \nabla \cdot \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \rho_0(\vec{\mathbf{r}},t) \\ \mu \nabla \cdot \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = 0 \end{cases}$$



Plane Waves (TD)



Medium

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$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=e_x(\vec{\mathbf{r}},t)\hat{i}_x+e_y(\vec{\mathbf{r}},t)\hat{i}_y+e_z(\vec{\mathbf{r}},t)\hat{i}_z=e_x(\cancel{x,y},z,t)\hat{i}_x+e_y(\cancel{x,y},z,t)\hat{i}_y+e_z(\cancel{x,y},z,t)\hat{i}_z$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=h_x(\vec{\mathbf{r}},t)\hat{i}_x+h_y(\vec{\mathbf{r}},t)\hat{i}_y+h_z(\vec{\mathbf{r}},t)\hat{i}_z=h_x(\cancel{x,y},z,t)\hat{i}_x+h_y(\cancel{x,y},z,t)\hat{i}_y+h_z(\cancel{x,y},z,t)\hat{i}_z$$

$$\frac{\partial}{\partial x}=\frac{\partial}{\partial y}=0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}(z,t)$$

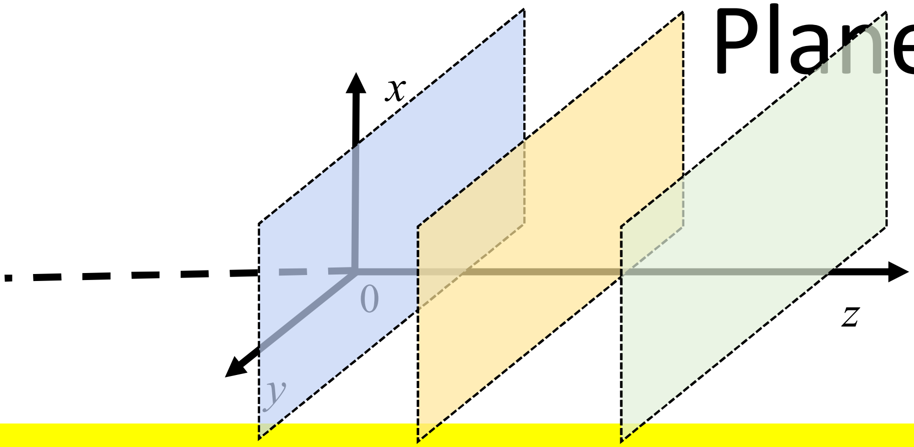
Time domain - Differential form

$$\begin{cases}\nabla\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=-\mu\dfrac{\partial\vec{\mathbf{h}}(\vec{\mathbf{r}},t)}{\partial t}\\ \nabla\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\varepsilon\dfrac{\partial\vec{\mathbf{e}}(\vec{\mathbf{r}},t)}{\partial t}+\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)\\ \varepsilon\nabla\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\rho_0(\vec{\mathbf{r}},t)\\ \mu\nabla\cdot\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=0\end{cases}$$

$$\nabla\times\vec{\mathbf{e}}=\left(\cancel{\frac{\partial e_z}{\partial y}}-\frac{\partial e_y}{\partial z}\right)\hat{i}_x+\left(\frac{\partial e_x}{\partial z}-\cancel{\frac{\partial e_z}{\partial x}}\right)\hat{i}_y+\left(\cancel{\frac{\partial e_y}{\partial x}}-\cancel{\frac{\partial e_x}{\partial y}}\right)\hat{i}_z$$

$$\nabla\times\vec{\mathbf{h}}=\left(\cancel{\frac{\partial h_z}{\partial y}}-\frac{\partial h_y}{\partial z}\right)\hat{i}_x+\left(\frac{\partial h_x}{\partial z}-\cancel{\frac{\partial h_z}{\partial x}}\right)\hat{i}_y+\left(\cancel{\frac{\partial h_y}{\partial x}}-\cancel{\frac{\partial h_x}{\partial y}}\right)\hat{i}_z$$

Plane Waves (TD)



Source-free

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$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=e_x(\vec{\mathbf{r}},t)\hat{i}_x+e_y(\vec{\mathbf{r}},t)\hat{i}_y+e_z(\vec{\mathbf{r}},t)\hat{i}_z=e_x(z,t)\hat{i}_x+e_y(z,t)\hat{i}_y+e_z(z,t)\hat{i}_z$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=h_x(\vec{\mathbf{r}},t)\hat{i}_x+h_y(\vec{\mathbf{r}},t)\hat{i}_y+h_z(\vec{\mathbf{r}},t)\hat{i}_z=h_x(z,t)\hat{i}_x+h_y(z,t)\hat{i}_y+h_z(z,t)\hat{i}_z$$

$$\frac{\partial}{\partial x}=\frac{\partial}{\partial y}=0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}(z,t)$$

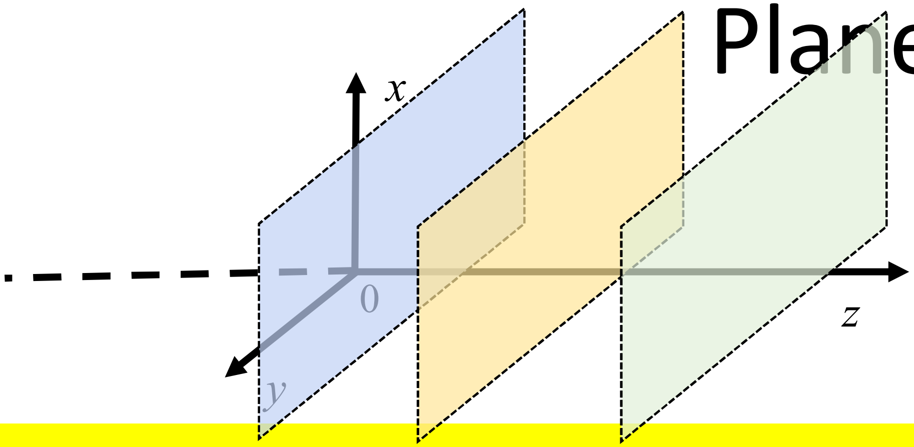
Time domain - Differential form

$$\begin{cases}\nabla\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=-\mu\dfrac{\partial\vec{\mathbf{h}}(\vec{\mathbf{r}},t)}{\partial t}\\ \nabla\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\varepsilon\dfrac{\partial\vec{\mathbf{e}}(\vec{\mathbf{r}},t)}{\partial t}+\cancel{\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)}\\ \varepsilon\nabla\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\cancel{\rho_0(\vec{\mathbf{r}},t)}\\ \mu\nabla\cdot\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=0\end{cases}$$

$$\nabla\times\vec{\mathbf{e}}=\left(-\frac{\partial e_y}{\partial z}\right)\hat{i}_x+\left(\frac{\partial e_x}{\partial z}\right)\hat{i}_y$$

$$\nabla\times\vec{\mathbf{h}}=\left(-\frac{\partial h_y}{\partial z}\right)\hat{i}_x+\left(\frac{\partial h_x}{\partial z}\right)\hat{i}_y$$

Plane Waves (TD)



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$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = e_x(\vec{\mathbf{r}},t)\hat{i}_x + e_y(\vec{\mathbf{r}},t)\hat{i}_y + e_z(\vec{\mathbf{r}},t)\hat{i}_z = e_x(z,t)\hat{i}_x + e_y(z,t)\hat{i}_y + e_z(z,t)\hat{i}_z$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = h_x(\vec{\mathbf{r}},t)\hat{i}_x + h_y(\vec{\mathbf{r}},t)\hat{i}_y + h_z(\vec{\mathbf{r}},t)\hat{i}_z = h_x(z,t)\hat{i}_x + h_y(z,t)\hat{i}_y + h_z(z,t)\hat{i}_z$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

Time domain - Differential form

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(z,t) = -\mu \dfrac{\partial \vec{\mathbf{h}}(z,t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(z,t) = \varepsilon \dfrac{\partial \vec{\mathbf{e}}(z,t)}{\partial t} \\ \varepsilon \nabla \cdot \vec{\mathbf{e}}(z,t) = 0 \\ \mu \nabla \cdot \vec{\mathbf{h}}(z,t) = 0 \end{cases}$$

$$\nabla \times \vec{\mathbf{e}} = \left(-\frac{\partial e_y}{\partial z}\right)\hat{i}_x + \left(\frac{\partial e_x}{\partial z}\right)\hat{i}_y$$

$$\nabla \times \vec{\mathbf{h}} = \left(-\frac{\partial h_y}{\partial z}\right)\hat{i}_x + \left(\frac{\partial h_x}{\partial z}\right)\hat{i}_y$$

Plane Waves (TD)

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$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$\Downarrow$$

$$e_z(z,t) = h_z(z,t) = 0$$

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(z,t) = -\mu \dfrac{\partial \vec{\mathbf{h}}(z,t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(z,t) = \varepsilon \dfrac{\partial \vec{\mathbf{e}}(z,t)}{\partial t} \end{cases}$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = e_x(z,t)\hat{i}_x + e_y(z,t)\hat{i}_y + \cancel{e_z(z,t)\hat{i}_z}$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = h_x(z,t)\hat{i}_x + h_y(z,t)\hat{i}_y + \cancel{h_z(z,t)\hat{i}_z}$$

$$\nabla \times \vec{\mathbf{e}} = \left(-\frac{\partial e_y}{\partial z}\right)\hat{i}_x + \left(\frac{\partial e_x}{\partial z}\right)\hat{i}_y$$

$$-\mu \frac{\partial \vec{\mathbf{h}}}{\partial t} = -\mu \frac{\partial h_x}{\partial t}\hat{i}_x - \mu \frac{\partial h_y}{\partial t}\hat{i}_y - \mu \frac{\partial h_z}{\partial t}\hat{i}_z$$

$$\nabla \times \vec{\mathbf{h}} = \left(-\frac{\partial h_y}{\partial z}\right)\hat{i}_x + \left(\frac{\partial h_x}{\partial z}\right)\hat{i}_y$$

$$\varepsilon \frac{\partial \vec{\mathbf{e}}(\vec{\mathbf{r}},t)}{\partial t} = \varepsilon \frac{\partial e_x}{\partial t}\hat{i}_x + \varepsilon \frac{\partial e_y}{\partial t}\hat{i}_y + \varepsilon \frac{\partial e_z}{\partial t}\hat{i}_z$$

$$\frac{\partial e_z}{\partial t} = 0 \qquad \frac{\partial h_z}{\partial t} = 0 \;\Rightarrow\; e_z(z,t) = \mathrm{const} \qquad h_z(z,t) = \mathrm{const} \;\Rightarrow\; e_z(z,t) = 0 \qquad h_z(z,t) = 0 \;\Rightarrow$$

TEM fields

Plane Waves (TD)

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(z,t) = -\mu \dfrac{\partial \vec{\mathbf{h}}(z,t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(z,t) = \varepsilon \dfrac{\partial \vec{\mathbf{e}}(z,t)}{\partial t} \end{cases}$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = e_x(z,t)\hat{i}_x + e_y(z,t)\hat{i}_y$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = h_x(z,t)\hat{i}_x + h_y(z,t)\hat{i}_y$$

$$\nabla \times \vec{\mathbf{e}} = \left(-\frac{\partial e_y}{\partial z}\right)\hat{i}_x + \left(\frac{\partial e_x}{\partial z}\right)\hat{i}_y$$

$$-\mu \frac{\partial \vec{\mathbf{h}}}{\partial t} = -\mu \frac{\partial h_x}{\partial t}\hat{i}_x - \mu \frac{\partial h_y}{\partial t}\hat{i}_y - \mu \frac{\partial h_z}{\partial t}\hat{i}_z$$

$$\nabla \times \vec{\mathbf{h}} = \left(-\frac{\partial h_y}{\partial z}\right)\hat{i}_x + \left(\frac{\partial h_x}{\partial z}\right)\hat{i}_y$$

$$\varepsilon \frac{\partial \vec{\mathbf{e}}(\vec{\mathbf{r}},t)}{\partial t} = \varepsilon \frac{\partial e_x}{\partial t}\hat{i}_x + \varepsilon \frac{\partial e_y}{\partial t}\hat{i}_y + \varepsilon \frac{\partial e_z}{\partial t}\hat{i}_z$$

Source-free

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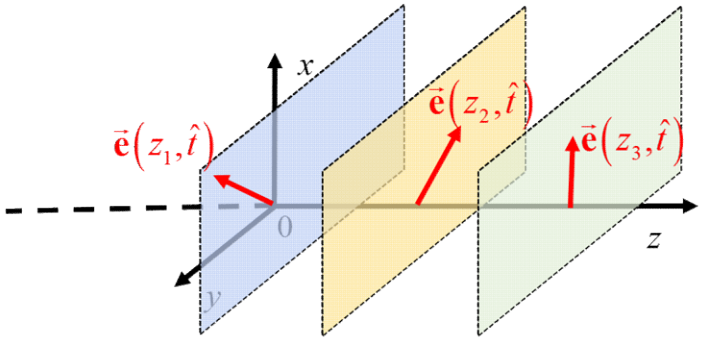
$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$e_z(z,t) = h_z(z,t) = 0$$



Plane Waves (TD)

Source-free

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$$\begin{cases}\nabla\times\vec{\mathbf{e}}(z,t) = -\mu\dfrac{\partial\vec{\mathbf{h}}(z,t)}{\partial t}\\ \nabla\times\vec{\mathbf{h}}(z,t) = \varepsilon\dfrac{\partial\vec{\mathbf{e}}(z,t)}{\partial t}\end{cases}$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = e_x(z,t)\hat{i}_x + e_y(z,t)\hat{i}_y$$

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$$-\mu\frac{\partial\vec{\mathbf{h}}}{\partial t} = -\mu\frac{\partial h_x}{\partial t}\hat{i}_x - \mu\frac{\partial h_y}{\partial t}\hat{i}_y - \mu\frac{\partial h_z}{\partial t}\hat{i}_z$$

$$\nabla\times\vec{\mathbf{h}} = \left(-\frac{\partial h_y}{\partial z}\right)\hat{i}_x + \left(\frac{\partial h_x}{\partial z}\right)\hat{i}_y$$

$$\varepsilon\frac{\partial\vec{\mathbf{e}}(\vec{\mathbf{r}},t)}{\partial t} = \varepsilon\frac{\partial e_x}{\partial t}\hat{i}_x + \varepsilon\frac{\partial e_y}{\partial t}\hat{i}_y + \varepsilon\frac{\partial e_z}{\partial t}\hat{i}_z$$

$$\frac{\partial e_y}{\partial z} = \mu\frac{\partial h_x}{\partial t}$$

$$\frac{\partial e_x}{\partial z} = -\mu\frac{\partial h_y}{\partial t}$$

$$\frac{\partial h_y}{\partial z} = -\varepsilon\frac{\partial e_x}{\partial t}$$

$$\frac{\partial h_x}{\partial z} = \varepsilon\frac{\partial e_y}{\partial t}$$

$\{e_y, h_x\}$

$\{e_x, h_y\}$

Independent each other

Plane Waves (TD)

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(z,t) = -\mu \dfrac{\partial \vec{\mathbf{h}}(z,t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(z,t) = \varepsilon \dfrac{\partial \vec{\mathbf{e}}(z,t)}{\partial t} \end{cases}$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = e_x(z,t)\hat{i}_x + e_y(z,t)\hat{i}_y$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = h_x(z,t)\hat{i}_x + h_y(z,t)\hat{i}_y$$

$$\frac{\partial e_x}{\partial z} = -\mu \frac{\partial h_y}{\partial t}$$

$$\frac{\partial h_y}{\partial z} = -\varepsilon \frac{\partial e_x}{\partial t}$$

$$\frac{\partial e_y}{\partial z} = \mu \frac{\partial h_x}{\partial t}$$

$$\frac{\partial h_x}{\partial z} = \varepsilon \frac{\partial e_y}{\partial t}$$

$$\frac{\partial e_y}{\partial z} = \mu \frac{\partial h_x}{\partial t}$$

$$\frac{\partial e_x}{\partial z} = -\mu \frac{\partial h_y}{\partial t}$$

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$\{e_y, h_x\}$

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Independent each other

Plane Waves (TD)

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=e_x(z,t)\hat{i}_x+e_y(z,t)\hat{i}_y$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=h_x(z,t)\hat{i}_x+h_y(z,t)\hat{i}_y$$

$$\begin{cases}\dfrac{\partial e_x}{\partial z}=-\mu\dfrac{\partial h_y}{\partial t}\\[2mm]\dfrac{\partial h_y}{\partial z}=-\varepsilon\dfrac{\partial e_x}{\partial t}\end{cases}$$

$$\begin{cases}\dfrac{\partial e_y}{\partial z}=\mu\dfrac{\partial h_x}{\partial t}\\[2mm]\dfrac{\partial h_x}{\partial z}=\varepsilon\dfrac{\partial e_y}{\partial t}\end{cases}$$

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$\{e_y,h_x\}$

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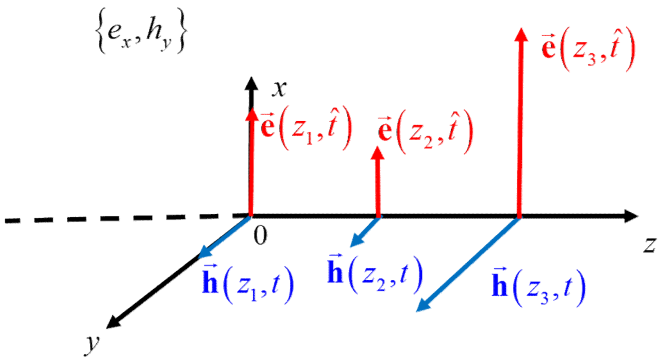
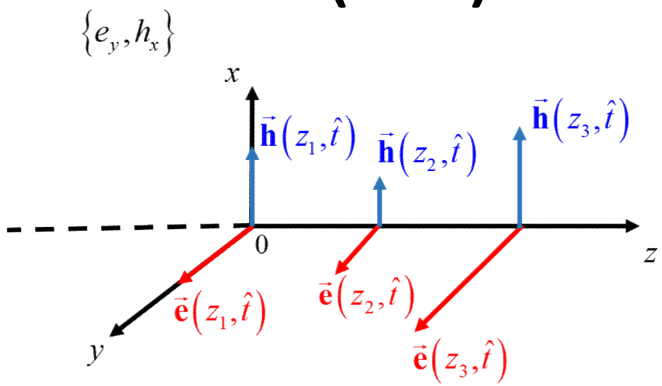
Independent each other

Plane Waves (TD)





$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases}$$

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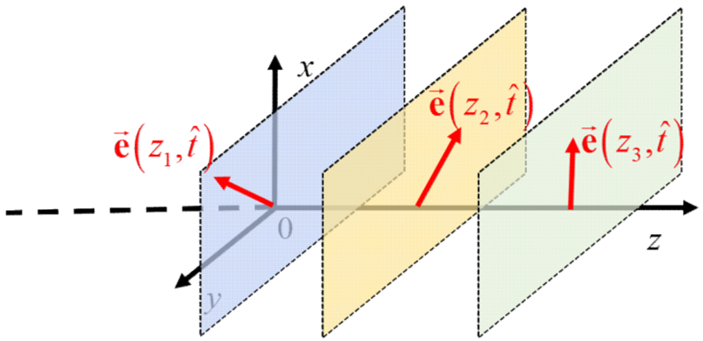

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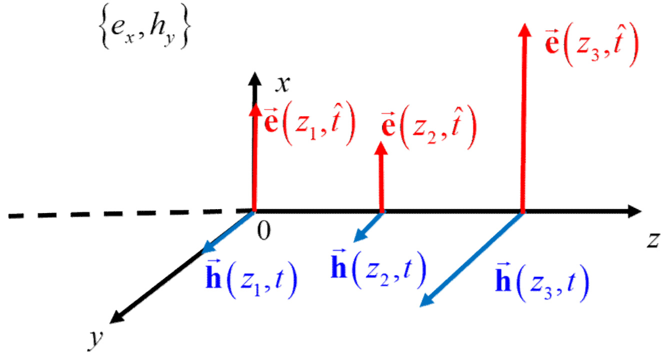
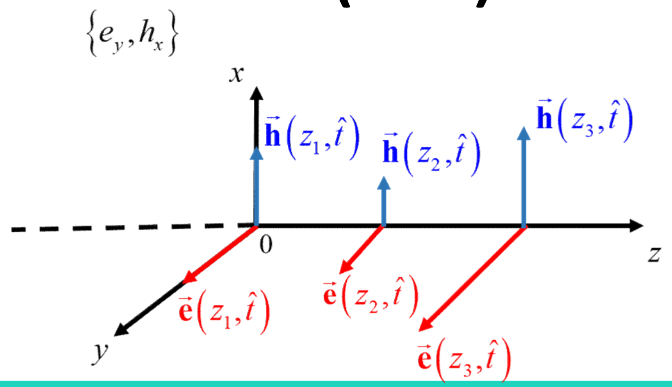
$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$
$\{e_x, h_y\}$ **Independent each other**

Plane Waves (TD)





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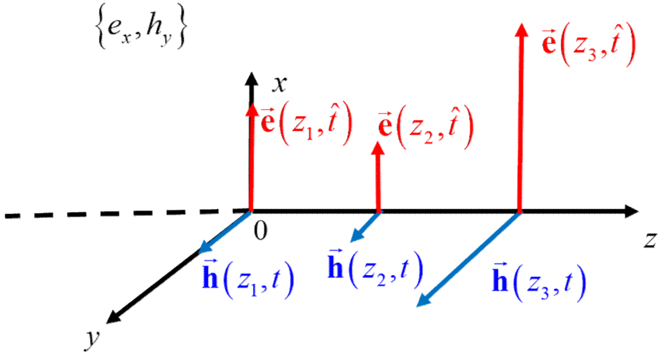
$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$, $\{e_x, h_y\}$ **Independent each other**

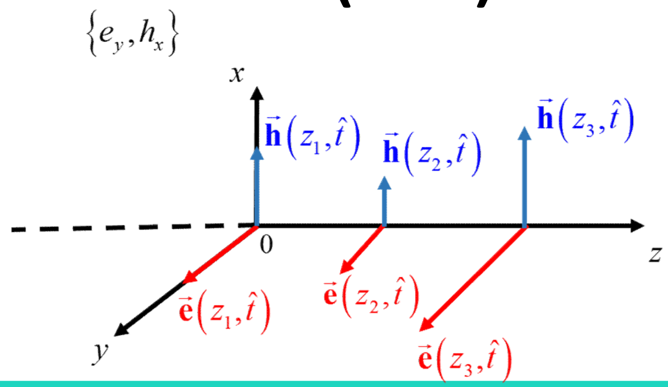
$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \Rightarrow \begin{array}{l} \dfrac{\partial^2 e_x}{\partial z^2} = -\mu \dfrac{\partial}{\partial z}\dfrac{\partial h_y}{\partial t} = -\mu \dfrac{\partial}{\partial t}\dfrac{\partial h_y}{\partial z} = \mu\varepsilon \dfrac{\partial^2 e_x}{\partial t^2} \\ -\dfrac{\partial}{\partial t}\dfrac{\partial h_y}{\partial z} = \varepsilon \dfrac{\partial^2 e_x}{\partial t^2} \end{array} \Rightarrow \frac{\partial^2 e_x}{\partial z^2} = \mu\varepsilon \frac{\partial^2 e_x}{\partial t^2}$$

$$\begin{cases} \dfrac{\partial e_y}{\partial z} = \mu \dfrac{\partial h_x}{\partial t} \\ \dfrac{\partial h_x}{\partial z} = \varepsilon \dfrac{\partial e_y}{\partial t} \end{cases} \Rightarrow \begin{array}{l} \dfrac{\partial^2 e_y}{\partial z^2} = \mu \dfrac{\partial}{\partial z}\dfrac{\partial h_x}{\partial t} = \mu \dfrac{\partial}{\partial t}\dfrac{\partial h_x}{\partial z} = \mu\varepsilon \dfrac{\partial^2 e_y}{\partial t^2} \\ \dfrac{\partial}{\partial t}\dfrac{\partial h_x}{\partial z} = \varepsilon \dfrac{\partial^2 e_y}{\partial t^2} \end{array} \Rightarrow \frac{\partial^2 e_y}{\partial z^2} = \mu\varepsilon \frac{\partial^2 e_y}{\partial t^2}$$

Plane Waves (TD)



$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$



$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} = \mu\varepsilon \frac{\partial^2 e_x}{\partial t^2} \qquad \frac{\partial^2 e_x(z,t)}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x(z,t)}{\partial t^2} = 0$$

$$\begin{cases} \dfrac{\partial e_y}{\partial z} = \mu \dfrac{\partial h_x}{\partial t} \\ \dfrac{\partial h_x}{\partial z} = \varepsilon \dfrac{\partial e_y}{\partial t} \end{cases} \qquad \frac{\partial^2 e_y}{\partial z^2} = \mu\varepsilon \frac{\partial^2 e_y}{\partial t^2} \qquad \frac{\partial^2 e_y(z,t)}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_y(z,t)}{\partial t^2} = 0$$

Source-free

Medium
- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$
$\{e_x, h_y\}$ **Independent each other**

Plane Waves (TD)

Source-free

Medium
- **Linear**
- **Local (TND & SND)**
- **Isotropic**
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$$\frac{\partial}{\partial x}=\frac{\partial}{\partial y}=0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}(z,t)$$

$$e_z(z,t)=h_z(z,t)=0$$

$\{e_y,h_x\}$, $\{e_x,h_y\}$ **Independent each other**

$\{e_x,h_y\}$

$$\begin{cases}\dfrac{\partial e_x}{\partial z}=-\mu\dfrac{\partial h_y}{\partial t}\\[2mm]\dfrac{\partial h_y}{\partial z}=-\varepsilon\dfrac{\partial e_x}{\partial t}\end{cases}\qquad \frac{\partial^2 e_x}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2}=0$$

$$c=\frac{1}{\sqrt{\mu\varepsilon}}$$

$\{e_y,h_x\}$

$$\begin{cases}\dfrac{\partial e_y}{\partial z}=\mu\dfrac{\partial h_x}{\partial t}\\[2mm]\dfrac{\partial h_x}{\partial z}=\varepsilon\dfrac{\partial e_y}{\partial t}\end{cases}\qquad \frac{\partial^2 e_y}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2}=0$$

$$\begin{cases}\dfrac{\partial e_x}{\partial z}=-\mu\dfrac{\partial h_y}{\partial t}\\[2mm]\dfrac{\partial h_y}{\partial z}=-\varepsilon\dfrac{\partial e_x}{\partial t}\end{cases}\qquad \frac{\partial^2 e_x}{\partial z^2}=\mu\varepsilon\frac{\partial^2 e_x}{\partial t^2}\qquad \frac{\partial^2 e_x(z,t)}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2 e_x(z,t)}{\partial t^2}=0$$

$$\begin{cases}\dfrac{\partial e_y}{\partial z}=\mu\dfrac{\partial h_x}{\partial t}\\[2mm]\dfrac{\partial h_x}{\partial z}=\varepsilon\dfrac{\partial e_y}{\partial t}\end{cases}\qquad \frac{\partial^2 e_y}{\partial z^2}=\mu\varepsilon\frac{\partial^2 e_y}{\partial t^2}\qquad \frac{\partial^2 e_y(z,t)}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2 e_y(z,t)}{\partial t^2}=0$$

Plane Waves (TD)

$\{e_x, h_y\}$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$\{e_y, h_x\}$

$$\begin{cases} \dfrac{\partial e_y}{\partial z} = \mu \dfrac{\partial h_x}{\partial t} \\ \dfrac{\partial h_x}{\partial z} = \varepsilon \dfrac{\partial e_y}{\partial t} \end{cases} \qquad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2} = 0$$

Source-free

Medium

- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{e}}(z, t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{h}}(z, t)$$

$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$
$\{e_x, h_y\}$ **Independent each other**

Plane Waves (TD)

$\{e_x, h_y\}$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$\{e_y, h_x\}$

$$\begin{cases} \dfrac{\partial e_y}{\partial z} = \mu \dfrac{\partial h_x}{\partial t} \\ \dfrac{\partial h_x}{\partial z} = \varepsilon \dfrac{\partial e_y}{\partial t} \end{cases} \qquad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2} = 0$$

$$f(z,t) = e^+(z - ct) \qquad \alpha = z - ct \qquad \frac{\partial e^+}{\partial z} = -\frac{1}{c}\frac{\partial e^+}{\partial t}$$

$$\frac{\partial e^+}{\partial z} = \frac{\partial e^+}{\partial \alpha}\frac{\partial \alpha}{\partial z} = \frac{\partial e^+}{\partial \alpha} \Rightarrow \frac{\partial e^+}{\partial \alpha} = \frac{\partial e^+}{\partial z}$$

$$\frac{\partial e^+}{\partial t} = \frac{\partial e^+}{\partial \alpha}\frac{\partial \alpha}{\partial t} = -c\frac{\partial e^+}{\partial \alpha} \Rightarrow \frac{\partial e^+}{\partial \alpha} = -\frac{1}{c}\frac{\partial e^+}{\partial t}$$

$$\Rightarrow \frac{\partial e^+}{\partial z} = -\frac{1}{c}\frac{\partial e^+}{\partial t}$$

Source-free

Medium
- **Linear**
- **Local (TND & SND)**
- **Isotropic**
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$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$
$\{e_x, h_y\}$ **Independent each other**

Plane Waves (TD)

$\{e_x, h_y\}$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$\{e_y, h_x\}$

$$\begin{cases} \dfrac{\partial e_y}{\partial z} = \mu \dfrac{\partial h_x}{\partial t} \\ \dfrac{\partial h_x}{\partial z} = \varepsilon \dfrac{\partial e_y}{\partial t} \end{cases} \qquad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2} = 0$$

$$f(z,t) = e^+(z - ct) \qquad \alpha = z - ct \qquad \frac{\partial e^+}{\partial z} = -\frac{1}{c}\frac{\partial e^+}{\partial t}$$

$$\Rightarrow \frac{\partial^2 e^+}{\partial z^2} = \frac{\partial}{\partial z}\left[-\frac{1}{c}\frac{\partial e^+}{\partial t}\right] = -\frac{1}{c}\frac{\partial}{\partial z}\frac{\partial e^+}{\partial t} = -\frac{1}{c}\frac{\partial}{\partial t}\frac{\partial e^+}{\partial z} = -\frac{1}{c}\frac{\partial}{\partial t}\left[-\frac{1}{c}\frac{\partial e^+}{\partial t}\right] = \frac{1}{c^2}\frac{\partial^2 e^+}{\partial t^2}$$

$$\Rightarrow \frac{\partial^2 e^+}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 e^+}{\partial t^2} \Rightarrow \frac{\partial^2 e^+}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e^+}{\partial t^2} = 0$$

Source-free

Medium
- **Linear**
- **Local (TND & SND)**
- **Isotropic**
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$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

⬇

$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$ / $\{e_x, h_y\}$ **Independent each other**

Plane Waves (TD)

$$\left\{e_x, h_y\right\}$$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\left\{e_y, h_x\right\}$$

$$\begin{cases} \dfrac{\partial e_y}{\partial z} = \mu \dfrac{\partial h_x}{\partial t} \\ \dfrac{\partial h_x}{\partial z} = \varepsilon \dfrac{\partial e_y}{\partial t} \end{cases} \qquad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2} = 0$$

$$f(z,t) = e^+(z - ct) \qquad \frac{\partial^2 e^+}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e^+}{\partial t^2} = 0 \qquad \frac{\partial e^+}{\partial z} = -\frac{1}{c}\frac{\partial e^+}{\partial t}$$

$$\Rightarrow \frac{\partial^2 e^+}{\partial z^2} = \frac{\partial}{\partial z}\left[-\frac{1}{c}\frac{\partial e^+}{\partial t}\right] = -\frac{1}{c}\frac{\partial}{\partial z}\frac{\partial e^+}{\partial t} = -\frac{1}{c}\frac{\partial}{\partial t}\frac{\partial e^+}{\partial z} = -\frac{1}{c}\frac{\partial}{\partial t}\left[-\frac{1}{c}\frac{\partial e^+}{\partial t}\right] = \frac{1}{c^2}\frac{\partial^2 e^+}{\partial t^2}$$

$$\Rightarrow \frac{\partial^2 e^+}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 e^+}{\partial t^2} \Rightarrow \frac{\partial^2 e^+}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e^+}{\partial t^2} = 0$$

Source-free

Medium
- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Homogeneous (TI – SI)**
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$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$e_z(z,t) = h_z(z,t) = 0$$

$\left\{e_y, h_x\right\}$, $\left\{e_x, h_y\right\}$ **Independent each other**

Plane Waves (TD)

Source-free

Medium
- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\{e_x, h_y\}$$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\{e_y, h_x\}$$

$$\begin{cases} \dfrac{\partial e_y}{\partial z} = \mu \dfrac{\partial h_x}{\partial t} \\ \dfrac{\partial h_x}{\partial z} = \varepsilon \dfrac{\partial e_y}{\partial t} \end{cases} \qquad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2} = 0$$

$$f(z,t) = e^+(z - ct) \qquad \frac{\partial^2 e^+}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e^+}{\partial t^2} = 0 \qquad \frac{\partial e^+}{\partial z} = -\frac{1}{c}\frac{\partial e^+}{\partial t}$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$

$\{e_x, h_y\}$

Independent each other

Plane Waves (TD)

$\{e_x, h_y\}$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$\{e_y, h_x\}$

$$\begin{cases} \dfrac{\partial e_y}{\partial z} = \mu \dfrac{\partial h_x}{\partial t} \\ \dfrac{\partial h_x}{\partial z} = \varepsilon \dfrac{\partial e_y}{\partial t} \end{cases} \qquad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2} = 0$$

$$f(z,t) = e^+(z - ct) \qquad \frac{\partial^2 e^+}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e^+}{\partial t^2} = 0 \qquad \frac{\partial e^+}{\partial z} = -\frac{1}{c}\frac{\partial e^+}{\partial t}$$

Source-free

Medium
- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$
$\{e_x, h_y\}$ **Independent each other**

Plane Waves (TD)

$\{e_x, h_y\}$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$\{e_y, h_x\}$

$$\begin{cases} \dfrac{\partial e_y}{\partial z} = \mu \dfrac{\partial h_x}{\partial t} \\ \dfrac{\partial h_x}{\partial z} = \varepsilon \dfrac{\partial e_y}{\partial t} \end{cases} \qquad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2} = 0$$

$$f(z,t) = e^+(z - ct) \qquad \frac{\partial^2 e^+}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e^+}{\partial t^2} = 0 \qquad \frac{\partial e^+}{\partial z} = -\frac{1}{c}\frac{\partial e^+}{\partial t}$$

$$f(z,t) = e^-(z + ct) \qquad \beta = z + ct \qquad \frac{\partial e^-}{\partial z} = \frac{1}{c}\frac{\partial e^-}{\partial t}$$

$$\frac{\partial e^-}{\partial z} = \frac{\partial e^-}{\partial \beta}\frac{\partial \beta}{\partial z} = \frac{\partial e^-}{\partial \beta} \Rightarrow \frac{\partial e^-}{\partial \beta} = \frac{\partial e^-}{\partial z}$$

$$\frac{\partial e^-}{\partial t} = \frac{\partial e^-}{\partial \beta}\frac{\partial \beta}{\partial t} = c\frac{\partial e^-}{\partial \beta} \Rightarrow \frac{\partial e^-}{\partial \beta} = \frac{1}{c}\frac{\partial e^-}{\partial t}$$

$$\Rightarrow \frac{\partial e^-}{\partial z} = \frac{1}{c}\frac{\partial e^-}{\partial t}$$

Source-free

Medium
- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$\Downarrow$$

$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$, $\{e_x, h_y\}$ **Independent each other**

Plane Waves (TD)

Source-free

Medium

- **Linear**
- **Local (TND & SND)**
- **Isotropic**
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- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$ $\{e_x, h_y\}$ **Independent each other**

$\{e_x, h_y\}$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$\{e_y, h_x\}$

$$\begin{cases} \dfrac{\partial e_y}{\partial z} = \mu \dfrac{\partial h_x}{\partial t} \\ \dfrac{\partial h_x}{\partial z} = \varepsilon \dfrac{\partial e_y}{\partial t} \end{cases} \qquad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2} = 0$$

$$f(z,t) = e^+(z-ct) \qquad \frac{\partial^2 e^+}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e^+}{\partial t^2} = 0 \qquad \frac{\partial e^+}{\partial z} = -\frac{1}{c}\frac{\partial e^+}{\partial t}$$

$$f(z,t) = e^-(z+ct) \qquad \beta = z + ct \qquad \frac{\partial e^-}{\partial z} = \frac{1}{c}\frac{\partial e^-}{\partial t}$$

$$\Rightarrow \frac{\partial^2 e^-}{\partial z^2} = \frac{\partial}{\partial z}\left[\frac{1}{c}\frac{\partial e^-}{\partial t}\right] = \frac{1}{c}\frac{\partial}{\partial t}\frac{\partial e^-}{\partial z} = \frac{1}{c}\frac{\partial}{\partial t}\left[\frac{1}{c}\frac{\partial e^-}{\partial t}\right] = \frac{1}{c^2}\frac{\partial^2 e^-}{\partial t^2}$$

$$\Rightarrow \frac{\partial^2 e^-}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 e^-}{\partial t^2} \Rightarrow \frac{\partial^2 e^-}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e^-}{\partial t^2} = 0$$

Plane Waves (TD)

Source-free

Medium
- Linear
- Local (TND & SND)
- Isotropic
- Homogeneous (TI – SI)
- Lossless

$\{e_x, h_y\}$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$\{e_y, h_x\}$

$$\begin{cases} \dfrac{\partial e_y}{\partial z} = \mu \dfrac{\partial h_x}{\partial t} \\ \dfrac{\partial h_x}{\partial z} = \varepsilon \dfrac{\partial e_y}{\partial t} \end{cases} \qquad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2} = 0$$

$$f(z,t) = e^+(z-ct) \qquad \frac{\partial^2 e^+}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e^+}{\partial t^2} = 0 \qquad \frac{\partial e^+}{\partial z} = -\frac{1}{c}\frac{\partial e^+}{\partial t}$$

$$f(z,t) = e^-(z+ct) \qquad \frac{\partial^2 e^-}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e^-}{\partial t^2} = 0 \qquad \frac{\partial e^-}{\partial z} = \frac{1}{c}\frac{\partial e^-}{\partial t}$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$e_z(z,t) = h_z(z,t) = 0$$

$$\Rightarrow \frac{\partial^2 e^-}{\partial z^2} = \frac{\partial}{\partial z}\left[\frac{1}{c}\frac{\partial e^-}{\partial t}\right] = \frac{1}{c}\frac{\partial}{\partial t}\frac{\partial e^-}{\partial z} = \frac{1}{c}\frac{\partial}{\partial t}\left[\frac{1}{c}\frac{\partial e^-}{\partial t}\right] = \frac{1}{c^2}\frac{\partial^2 e^-}{\partial t^2}$$

$$\Rightarrow \frac{\partial^2 e^-}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 e^-}{\partial t^2} \Rightarrow \frac{\partial^2 e^-}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e^-}{\partial t^2} = 0$$

$\{e_y, h_x\}$
$\{e_x, h_y\}$
Independent each other

Plane Waves (TD)

$\{e_x, h_y\}$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$\{e_y, h_x\}$

$$\begin{cases} \dfrac{\partial e_y}{\partial z} = \mu \dfrac{\partial h_x}{\partial t} \\ \dfrac{\partial h_x}{\partial z} = \varepsilon \dfrac{\partial e_y}{\partial t} \end{cases} \qquad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2} = 0$$

$$f(z,t) = e^+(z - ct) \qquad \frac{\partial^2 e^+}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e^+}{\partial t^2} = 0 \qquad \frac{\partial e^+}{\partial z} = -\frac{1}{c}\frac{\partial e^+}{\partial t}$$

$$f(z,t) = e^-(z + ct) \qquad \frac{\partial^2 e^-}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e^-}{\partial t^2} = 0 \qquad \frac{\partial e^-}{\partial z} = \frac{1}{c}\frac{\partial e^-}{\partial t}$$

Source-free

Medium
- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

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$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$
$\{e_x, h_y\}$
Independent each other

Plane Waves (TD)

Source-free

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$\{e_y, h_x\}$, $\{e_x, h_y\}$ **Independent each other**

$\{e_x, h_y\}$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

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Plane Waves (TD)

$\{e_x, h_y\}$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases}$$

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$\{e_y, h_x\}$

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$$\frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2} = 0$$

$$f(z,t) = e^+(z - ct)$$

$$\frac{\partial^2 e^+}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e^+}{\partial t^2} = 0$$

$$\frac{\partial e^+}{\partial z} = -\frac{1}{c}\frac{\partial e^+}{\partial t}$$

$$f(z,t) = e^-(z + ct)$$

$$\frac{\partial^2 e^-}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e^-}{\partial t^2} = 0$$

$$\frac{\partial e^-}{\partial z} = \frac{1}{c}\frac{\partial e^-}{\partial t}$$

$$e_x(z,t) = e_x^+(z - ct) + e_x^-(z + ct)$$

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Plane Waves (TD)

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$$f(z,t) = e^+(z - ct) \qquad \frac{\partial^2 e^+}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e^+}{\partial t^2} = 0 \qquad \frac{\partial e^+}{\partial z} = -\frac{1}{c}\frac{\partial e^+}{\partial t}$$

$$f(z,t) = e^-(z + ct) \qquad \frac{\partial^2 e^-}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e^-}{\partial t^2} = 0 \qquad \frac{\partial e^-}{\partial z} = \frac{1}{c}\frac{\partial e^-}{\partial t}$$

$$e_x(z,t) = e_x^+(z - ct) + e_x^-(z + ct)$$

$$e_y(z,t) = e_y^+(z - ct) + e_y^-(z + ct)$$

Plane Waves (TD)

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$$e_z(z,t) = h_z(z,t) = 0$$

$$\frac{\partial e^+}{\partial z} = -\frac{1}{c}\frac{\partial e^+}{\partial t} \qquad \frac{\partial e^-}{\partial z} = \frac{1}{c}\frac{\partial e^-}{\partial t}$$

$$e_x(z,t) = e_x^+(z - ct) + e_x^-(z + ct)$$

$$e_y(z,t) = e_y^+(z - ct) + e_y^-(z + ct)$$

$\{e_y, h_x\}$
$\{e_x, h_y\}$ **Independent each other**

Plane Waves (TD)

$\{e_x, h_y\}$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

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$\{e_y, h_x\}$
$\{e_x, h_y\}$ **Independent each other**

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$$e_x(z,t) = e_x^+(z-ct) + e_x^-(z+ct)$$

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Plane Waves (TD)

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$$-\mu\frac{\partial h_y}{\partial t} = \frac{\partial e_x}{\partial z} = \frac{\partial e_x^+}{\partial z} + \frac{\partial e_x^-}{\partial z} = -\frac{1}{c}\frac{\partial e_x^+}{\partial t} + \frac{1}{c}\frac{\partial e_x^-}{\partial t}$$

$$\Rightarrow \quad \mu c\frac{\partial h_y}{\partial t} = \frac{\partial}{\partial t}\left(e_x^+ - e_x^-\right)$$

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Plane Waves (TD)

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$$e_x(z,t) = e_x^+(z-ct) + e_x^-(z+ct)$$

$$\zeta h_y(z,t) = e_x^+(z-ct) - e_x^-(z+ct)$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}} \qquad \zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

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$$\begin{cases} \dfrac{\partial e_y}{\partial z} = \mu \dfrac{\partial h_x}{\partial t} \\ \dfrac{\partial h_x}{\partial z} = \varepsilon \dfrac{\partial e_y}{\partial t} \end{cases} \qquad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2} = 0$$

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$$\Rightarrow \mu c\frac{\partial h_y}{\partial t} = \frac{\partial}{\partial t}\left(e_x^+ - e_x^-\right) \Rightarrow \frac{\partial}{\partial t}\left[\zeta h_y - \left(e_x^+ - e_x^-\right)\right] = 0$$

$$\zeta h_y = \left(e_x^+ - e_x^-\right)$$

Plane Waves (TD)

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$$e_z(z,t)=h_z(z,t)=0$$

$\{e_y,h_x\}$
$\{e_x,h_y\}$
Independent each other

$\{e_x,h_y\}$

$$\begin{cases}\dfrac{\partial e_x}{\partial z}=-\mu\dfrac{\partial h_y}{\partial t}\\ \dfrac{\partial h_y}{\partial z}=-\varepsilon\dfrac{\partial e_x}{\partial t}\end{cases}\qquad \frac{\partial^2 e_x}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2}=0$$

$$e_x(z,t)=e_x^+(z-ct)+e_x^-(z+ct)$$

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$$\frac{\partial e^+}{\partial z}=-\frac{1}{c}\frac{\partial e^+}{\partial t}$$

$$\frac{\partial e^-}{\partial z}=\frac{1}{c}\frac{\partial e^-}{\partial t}$$

$$\mu\frac{\partial h_x}{\partial t}=\frac{\partial e_y}{\partial z}\quad=\frac{\partial e_y^+}{\partial z}+\frac{\partial e_y^-}{\partial z}=-\frac{1}{c}\frac{\partial e_y^+}{\partial t}\ +\frac{1}{c}\frac{\partial e_y^-}{\partial t}$$

$$\Rightarrow\ \mu c\frac{\partial h_x}{\partial t}=-\frac{\partial}{\partial t}\left(e_y^+-e_y^-\right)\ \Rightarrow\ \frac{\partial}{\partial t}\left[\zeta h_x+\left(e_y^+-e_y^-\right)\right]=0$$

$$-\zeta h_x=\left(e_y^+-e_y^-\right)$$

Plane Waves (TD)

$\{e_x, h_y\}$

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$$e_y(z,t)=e_y^+(z-ct)+e_y^-(z+ct)$$

$$-\zeta h_x(z,t)=e_y^+(z-ct)-e_y^-(z+ct)$$

$$[\vec{\mathbf{e}}]:\frac{Volt}{m}\qquad [\vec{\mathbf{h}}]:\frac{Ampere}{m}$$

$$[\zeta]\frac{Ampere}{m}=\frac{Volt}{m}\;\Rightarrow\;[\zeta]=\frac{Volt}{Ampere}=\Omega$$

ζ : intrinsic resistance of the medium

in free space

$$\zeta_0=\sqrt{\frac{\mu_0}{\varepsilon_0}}=377\Omega$$

$\mu_0=4\pi\times10^{-7}\quad Henry/m$

$\varepsilon_0=8.8\times10^{-12}\quad Farad/m$

Plane Waves (TD)

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$$\mu_0 = 4\pi \times 10^{-7} \quad Henry/m$$

$$\varepsilon_0 = 8.8 \times 10^{-12} \quad Farad/m$$

$$\frac{Volt}{m}\frac{1}{m^2} = \frac{1}{[c]^2}\frac{Volt}{m}\frac{1}{s^2} \Rightarrow [c]^2 = \left(\frac{m}{s}\right)^2 \Rightarrow [c] = \frac{m}{s}$$

c is a speed

in free space

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 3 \times 10^8 \ m/s$$

$\{e_y, h_x\}$
$\{e_x, h_y\}$
Independent each other

Plane Waves (TD)

$\{e_x, h_y\}$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

$$e_x(z,t) = e_x^+(z-ct) + e_x^-(z+ct)$$

$$\zeta h_y(z,t) = e_x^+(z-ct) - e_x^-(z+ct)$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{e_y, h_x\}$

$$\begin{cases} \dfrac{\partial e_y}{\partial z} = \mu \dfrac{\partial h_x}{\partial t} \\ \dfrac{\partial h_x}{\partial z} = \varepsilon \dfrac{\partial e_y}{\partial t} \end{cases} \qquad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2} = 0$$

$$e_y(z,t) = e_y^+(z-ct) + e_y^-(z+ct)$$

$$-\zeta h_x(z,t) = e_y^+(z-ct) - e_y^-(z+ct)$$

Source-free

Medium
- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

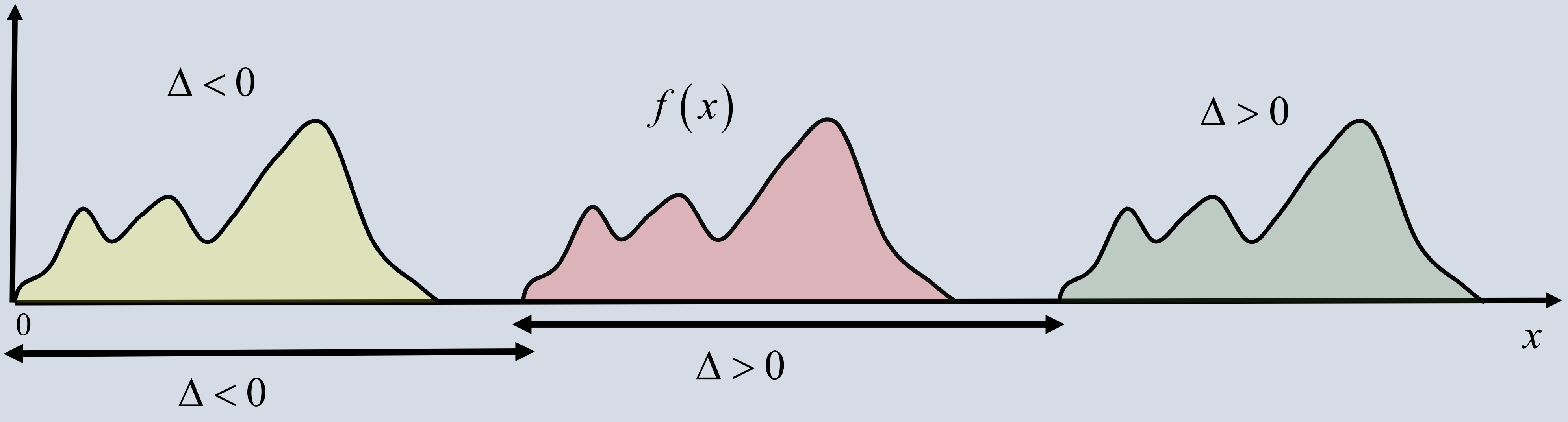
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$e_z(z,t) = h_z(z,t) = 0$$

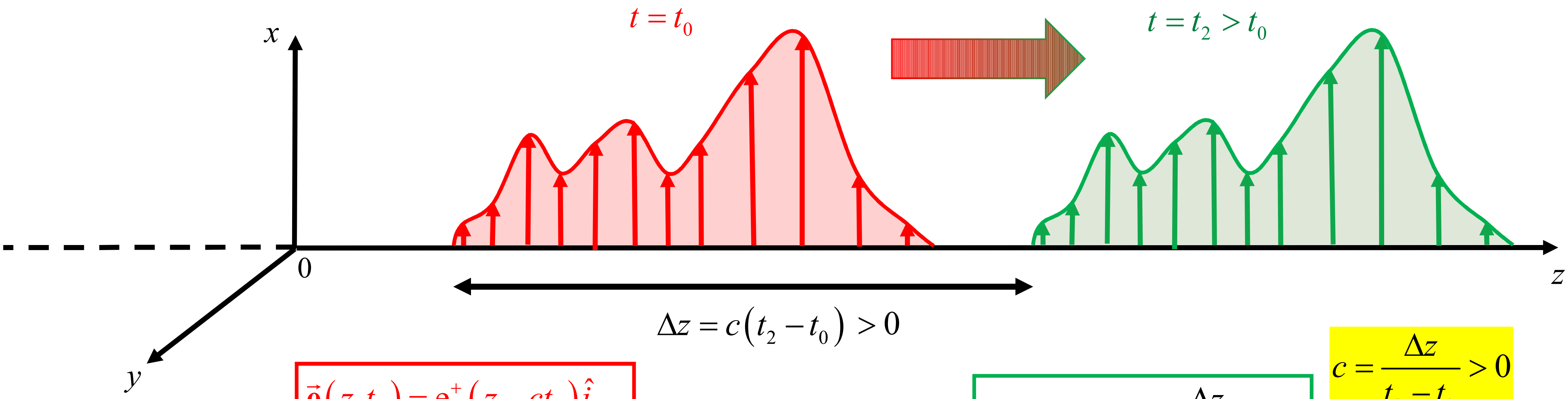
$\{e_y, h_x\}$, $\{e_x, h_y\}$ **Independent each other**

MEMO

$f(x-\Delta)$



Plane Waves (TD)



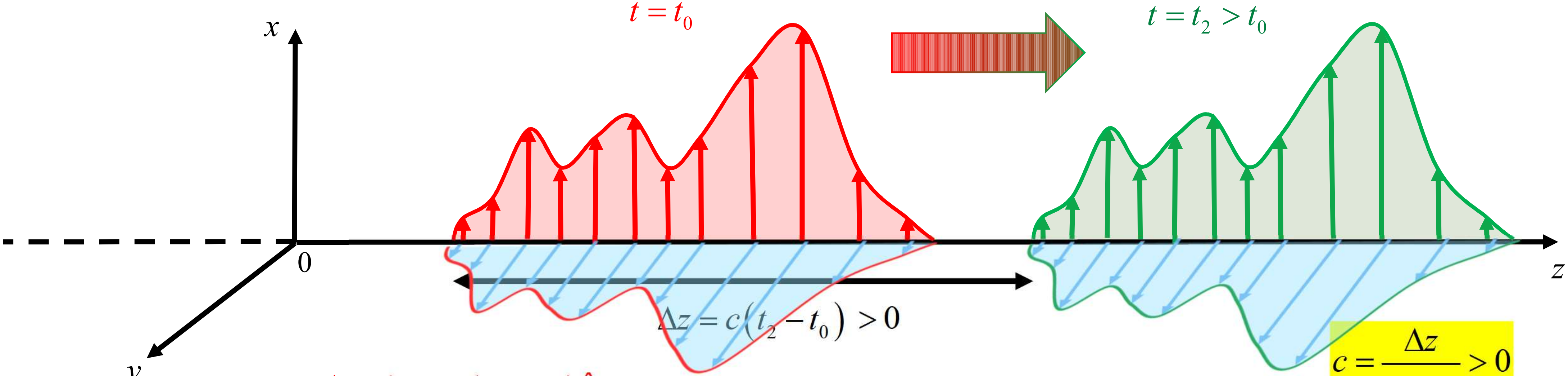
$$\vec{\mathbf{e}}(z,t_0) = \mathrm{e}^+(z - ct_0)\hat{i}_x$$

$$c = \frac{\Delta z}{t_2 - t_0} > 0$$

$$\vec{\mathbf{e}}(z,t_2) = \mathrm{e}^+(z - ct_2)\hat{i}_x = \mathrm{e}^+(z - ct_0 + ct_0 - ct_2)\hat{i}_x = \mathrm{e}^+\big(z - ct_0 - \overbrace{c[t_2 - t_0]}^{\Delta z}\big)\hat{i}_x$$

The electromagnetic perturbation **propagates** without deformation and with constant speed ***c*** along the positive sense of the z-axis

Plane Waves (TD)



$$c = \frac{\Delta z}{t_2 - t_0} > 0$$

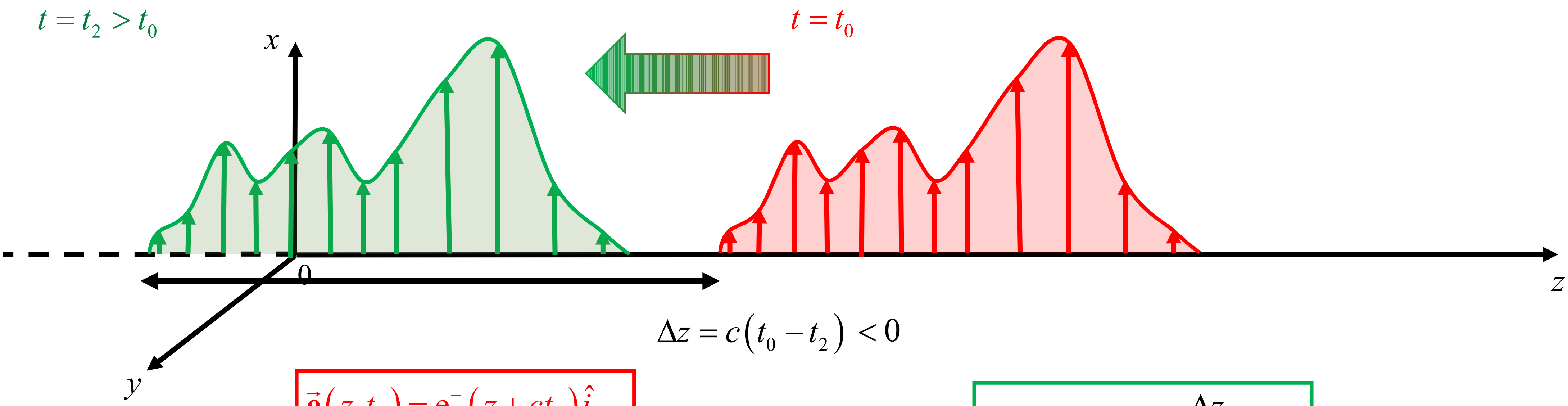
$$\vec{\mathbf{e}}(z, t_0) = \mathrm{e}^{+}(z - ct_0)\,\hat{i}_x$$

$$\vec{\mathbf{e}}(z, t_2) = \mathrm{e}^{+}(z - ct_2)\,\hat{i}_x = \mathrm{e}^{+}(z - ct_0 + ct_0 - ct_2)\,\hat{i}_x = \mathrm{e}^{+}(z - ct_0 - c[t_2 - t_0])\,\hat{i}_x$$

The electromagnetic perturbation **propagates** without deformation and with constant speed *c* along the positive sense of the *z*-axis

$$\begin{cases} e^{+}(z - ct) \\ h^{+}(z - ct) \end{cases}$$ is referred to as electromagnetic **<u>progressive</u> plane wave**

Plane Waves (TD)

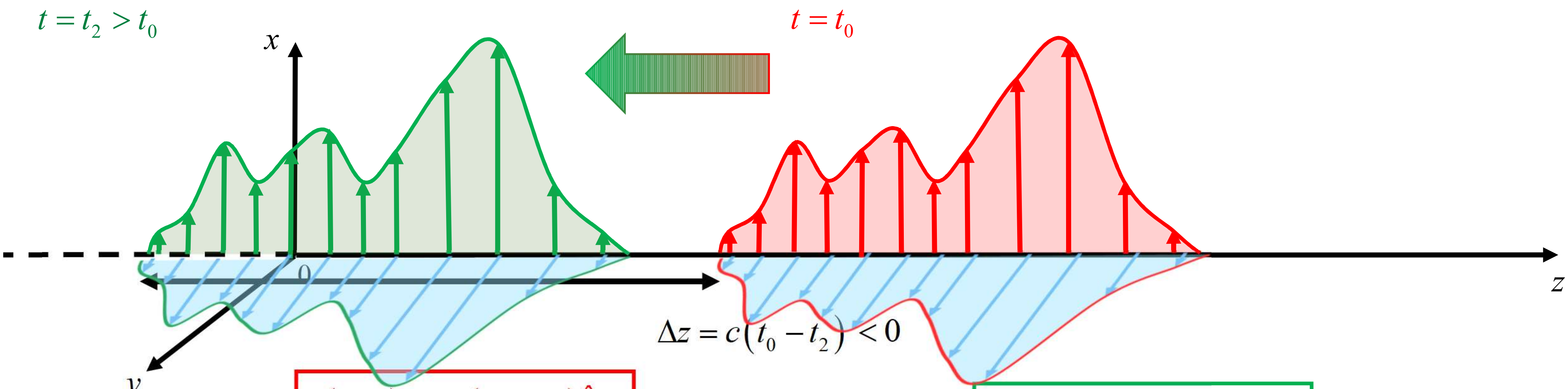


$$\vec{\mathbf{e}}\left(z, t_0\right) = \mathrm{e}^{-}\left(z + ct_0\right)\hat{i}_x$$

$$\vec{\mathbf{e}}\left(z, t_2\right) = \mathrm{e}^{-}\left(z + ct_2\right)\hat{i}_x = \mathrm{e}^{-}\left(z + ct_0 - ct_0 + ct_2\right)\hat{i}_x = \mathrm{e}^{-}\left(z + ct_0 - \overbrace{c\left[t_0 - t_2\right]}^{\Delta z}\right)\hat{i}_x$$

The electromagnetic perturbation **propagates** without deformation and with constant speed ***c*** along the negative sense of the z-axis

Plane Waves (TD)



$$\vec{\mathbf{e}}(z,t_0) = \mathrm{e}^-(z+ct_0)\hat{i}_x$$

$$\vec{\mathbf{e}}(z,t_2) = \mathrm{e}^-(z+ct_2)\hat{i}_x = \mathrm{e}^-(z+ct_0-ct_0+ct_2)\hat{i}_x = \mathrm{e}^-\Big(z+ct_0-\overbrace{c[t_0-t_2]}^{\Delta z}\Big)\hat{i}_x$$

The electromagnetic perturbation **propagates** without deformation and with constant speed ***c*** along the negative sense of the z-axis

$$\begin{cases} e^-(z+ct) \\ h^-(z+ct) \end{cases}$$ is referred to as electromagnetic **<u>regressive</u> plane wave**