Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2020-2021 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

Stefano Perna

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

THEOREMS

Poynting

Time domain – Phasor domain

Uniqueness (Interior problem – Exterior problem)

Time domain – Phasor domain

Equivalence

Phasor domain

Image Theory

Reciprocity

Phasor domain

Maxwell Equations (Spectral Domains)

James Clerk Maxwell 1831-1879

$$
\begin{cases}
\nabla \times \vec{\mathbf{E}} = -j\omega\vec{\mathbf{B}} \\
\nabla \times \vec{\mathbf{H}} = j\omega\vec{\mathbf{D}} + \vec{\mathbf{J}} \\
\nabla \cdot \vec{\mathbf{D}} = \rho \\
\nabla \cdot \vec{\mathbf{B}} = 0
\end{cases}
$$

Maxwell Equations (Spectral Domains)

Magnetic Sources

James Clerk Maxwell 1831-1879

$$\begin{cases} \nabla \times \vec{\mathbf{E}} = -j\omega \vec{\mathbf{B}} - \vec{\mathbf{J}}_{\mathrm{m}} \\ \nabla \times \vec{\mathbf{H}} = j\omega \vec{\mathbf{D}} + \vec{\mathbf{J}} \\ \nabla \cdot \vec{\mathbf{D}} = \rho \\ \nabla \cdot \vec{\mathbf{B}} = \rho_{\mathrm{m}} \end{cases}$$

$$\left[\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t\right)\right]: \frac{Volt}{m} \qquad \left[\vec{\mathbf{b}}\left(\vec{\mathbf{r}},t\right)\right]: \frac{Weber}{m^2}$$

$$\left[\vec{\mathbf{j}}_m\left(\vec{\mathbf{r}},t\right)\right]: \frac{Volt}{m^2} \qquad \left[\rho_m\left(\vec{\mathbf{r}},t\right)\right]: \frac{Weber}{m^3}$$

Equivalence theorem

$\vec{\mathbf{E}}_0, \vec{\mathbf{H}}_0$

Consider a source distribution $\vec{\mathbf{J}}_0$ with its associated electromagnetic field $\left(\vec{\mathbf{E}}_0, \vec{\mathbf{H}}_0\right)$

Equivalence theorem

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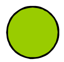
$\vec{\mathbf{E}}_0, \vec{\mathbf{H}}_0$

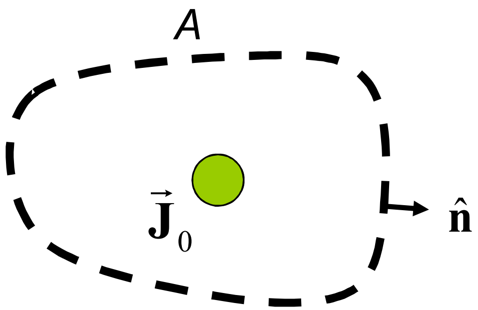


$\vec{\mathbf{J}}_0$

Consider a source distribution $\vec{\mathbf{J}}_0$ with its associated electromagnetic field $\left(\vec{\mathbf{E}}_0, \vec{\mathbf{H}}_0\right)$

$$\vec{\mathbf{J}}_0 \rightarrow \left(\vec{\mathbf{E}}_0, \vec{\mathbf{H}}_0\right)$$

Equivalence theorem



$$\vec{\mathbf{E}}_0, \vec{\mathbf{H}}_0$$

Consider a source distribution $\vec{\mathbf{J}}_0$ with its associated electromagnetic field $\left(\vec{\mathbf{E}}_0, \vec{\mathbf{H}}_0\right)$

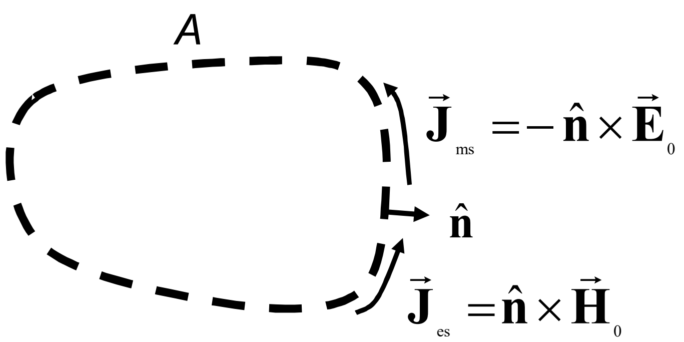
Consider a (smooth) surface *A* with an everywhere defined unit normal $\hat{\mathbf{n}}$

$$\vec{\mathbf{J}}_0 \rightarrow \left(\vec{\mathbf{E}}_0, \vec{\mathbf{H}}_0\right)$$

Equivalence theorem

$$\left[\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right]: \frac{Volt}{m} \qquad \left[\vec{\mathbf{j}}_m(\vec{\mathbf{r}},t)\right]: \frac{Volt}{m^2}$$

$$\left[\vec{\mathbf{j}}_{ms}(\vec{\mathbf{r}},t)\right]: \frac{Volt}{m}$$



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The original sources $\vec{\mathbf{J}}_0$ enclosed in *A* can be removed and substituted by equivalent sources, i.e., electric $\vec{\mathbf{J}}_{es} = \hat{\mathbf{n}} \times \vec{\mathbf{H}}_0$ and magnetic $\vec{\mathbf{J}}_{ms} = -\hat{\mathbf{n}} \times \vec{\mathbf{E}}_0$ current densities distributed over the surface *A*.

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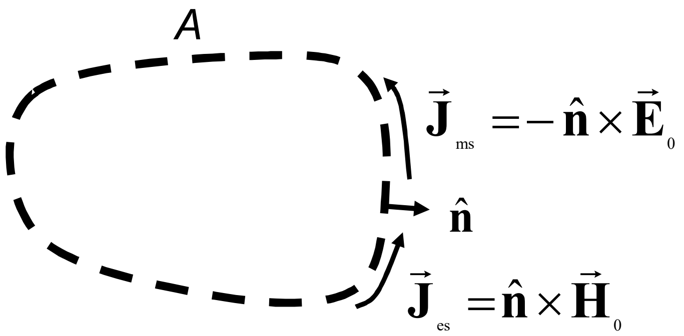

$\left[\vec{h}(\vec{\mathbf{r}},t)\right] : \frac{Ampere}{m}$ $\quad$ $\left[\vec{\mathbf{j}}_e(\vec{\mathbf{r}},t)\right] : \frac{Ampere}{m^2}$

$\left[\vec{\mathbf{j}}_{ms}(\vec{\mathbf{r}},t)\right] : \frac{Volt}{m}$ $\quad$ $\left[\vec{\mathbf{j}}_{es}(\vec{\mathbf{r}},t)\right] : \frac{Ampere}{m}$

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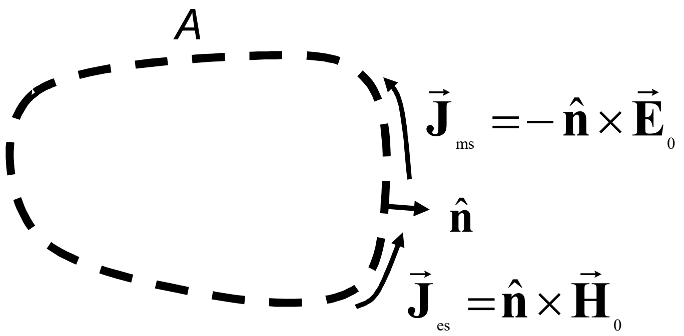

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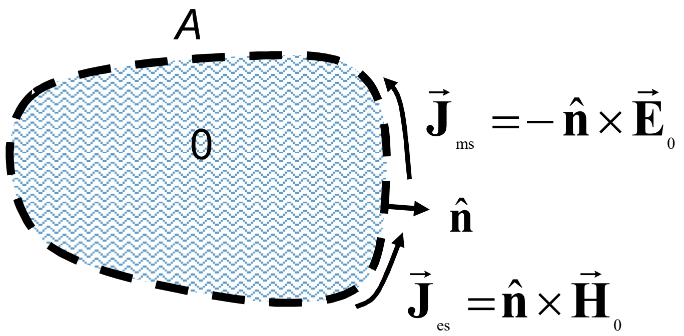
$\left[\vec{\mathbf{j}}_m(\vec{\mathbf{r}},t)\right]:\dfrac{Volt}{m^2}$ $\left[\rho_m(\vec{\mathbf{r}},t)\right]:\dfrac{Weber}{m^3}$

Equivalence theorem

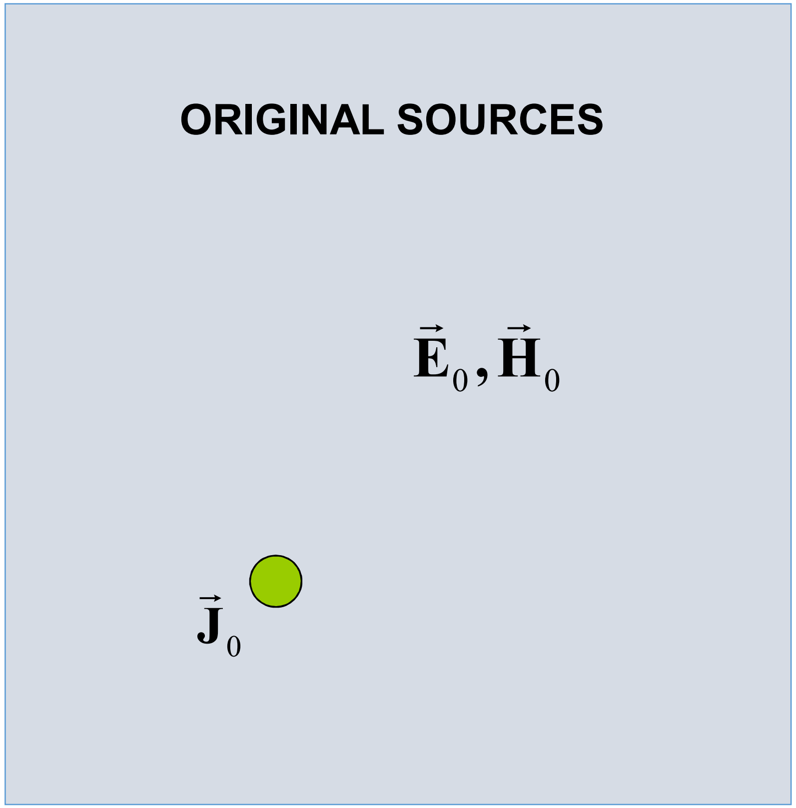
$\vec{\mathbf{E}}_0, \vec{\mathbf{H}}_0$



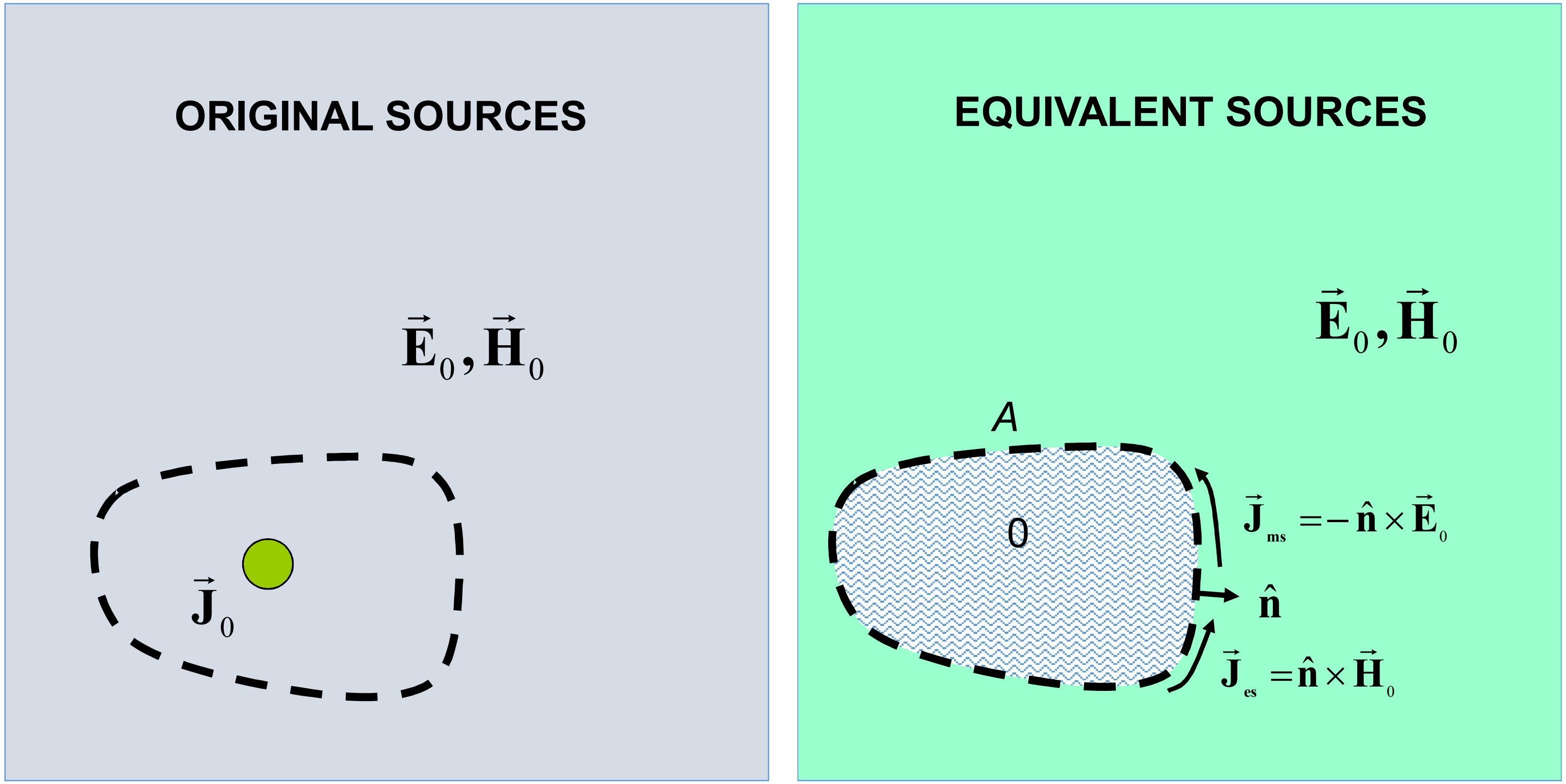
The Equivalence Theorem states that the equivalent sources $\vec{\mathbf{J}}_{es}$ and $\vec{\mathbf{J}}_{ms}$ generate a field $\left(\vec{\mathbf{E}}', \vec{\mathbf{H}}'\right)$ coincident with $\left(\vec{\mathbf{E}}_0, \vec{\mathbf{H}}_0\right)$ outside *A* and identically equal to zero inside

$$\vec{\mathbf{J}}_0 \rightarrow \left(\vec{\mathbf{E}}_0, \vec{\mathbf{H}}_0\right)$$

Equivalence theorem



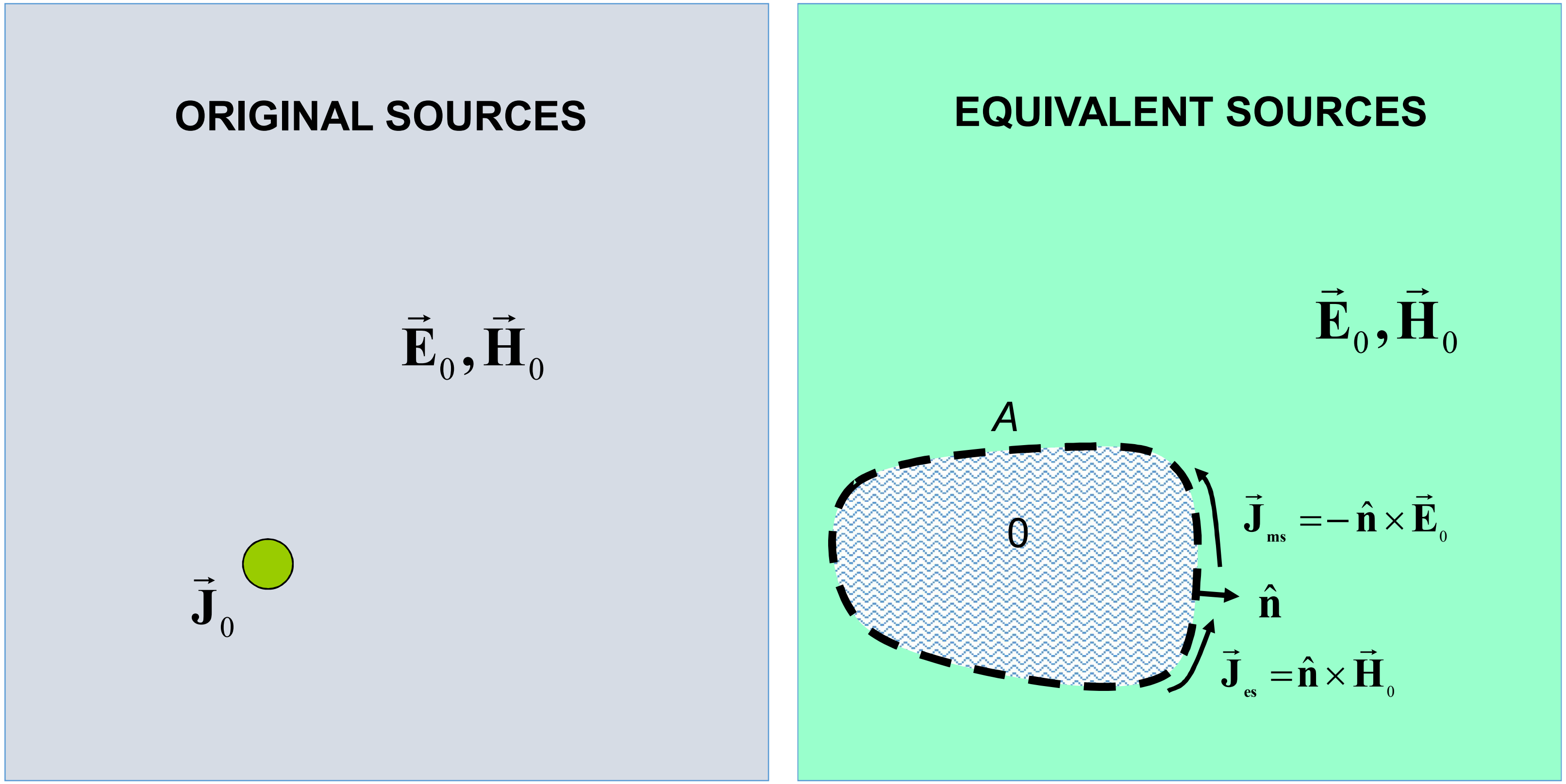
Equivalence theorem



Equivalence theorem

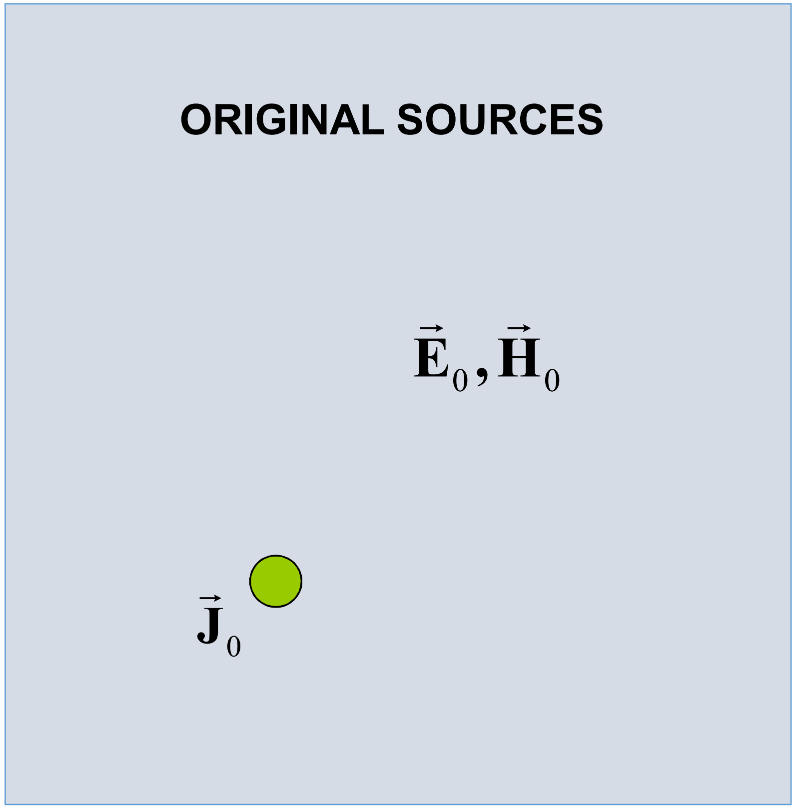
It's a powerful theorem that allows calculating the e.m. field in **all** the space, starting from the knowledge of its value **just on a surface**.

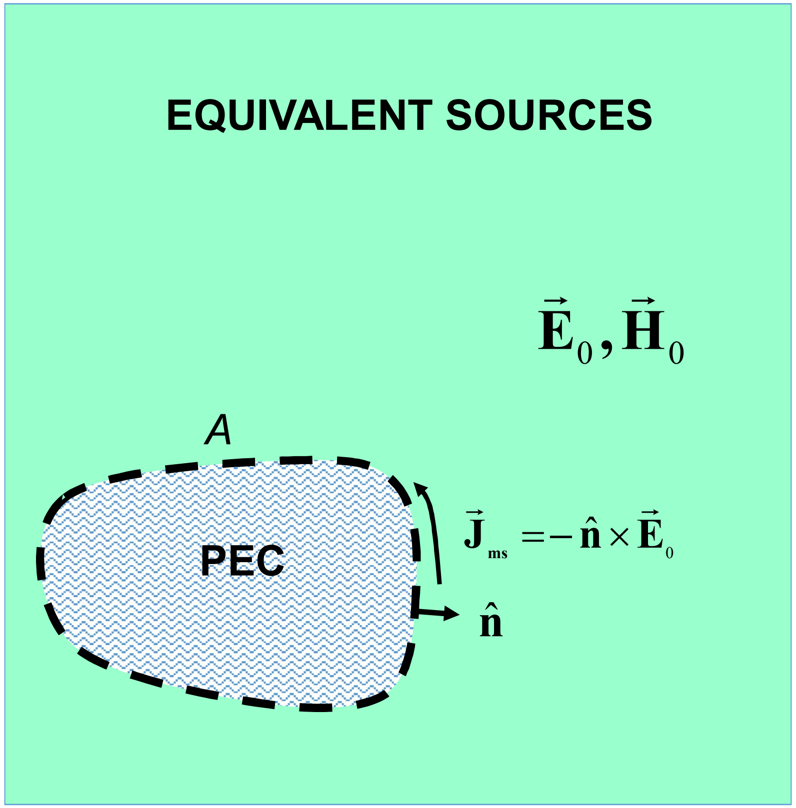
Equivalence theorem



Equivalence theorem

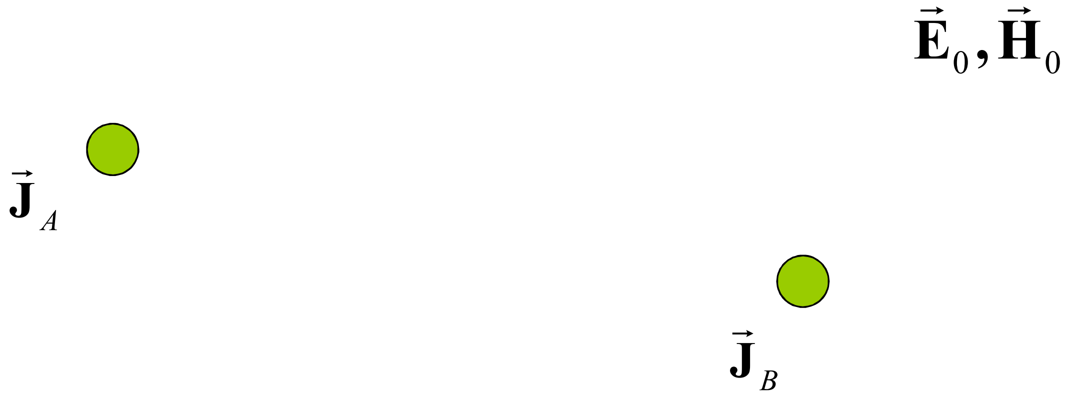
Alternative formulation





Equivalence theorem

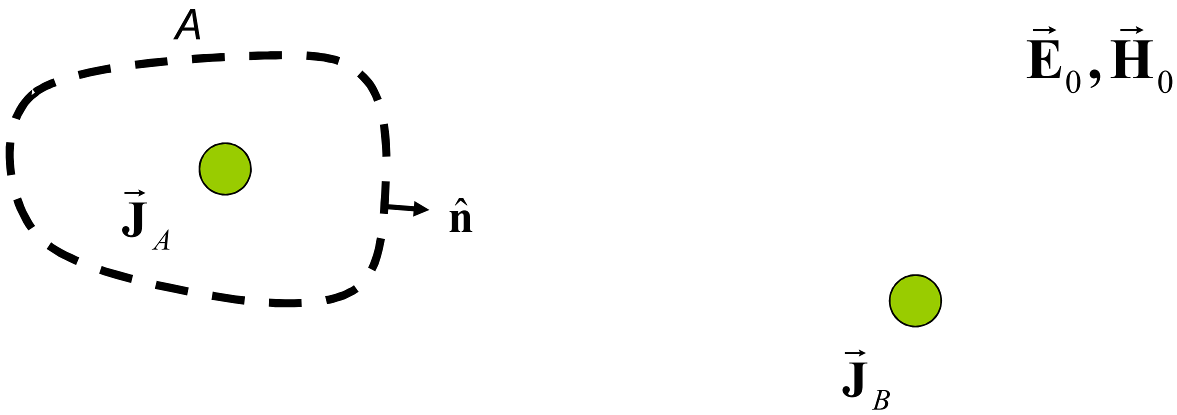
More general formulation



$$\vec{\mathbf{J}}_A + \vec{\mathbf{J}}_B \rightarrow \left(\vec{\mathbf{E}}_0, \vec{\mathbf{H}}_0\right)$$

Equivalence theorem

More general formulation



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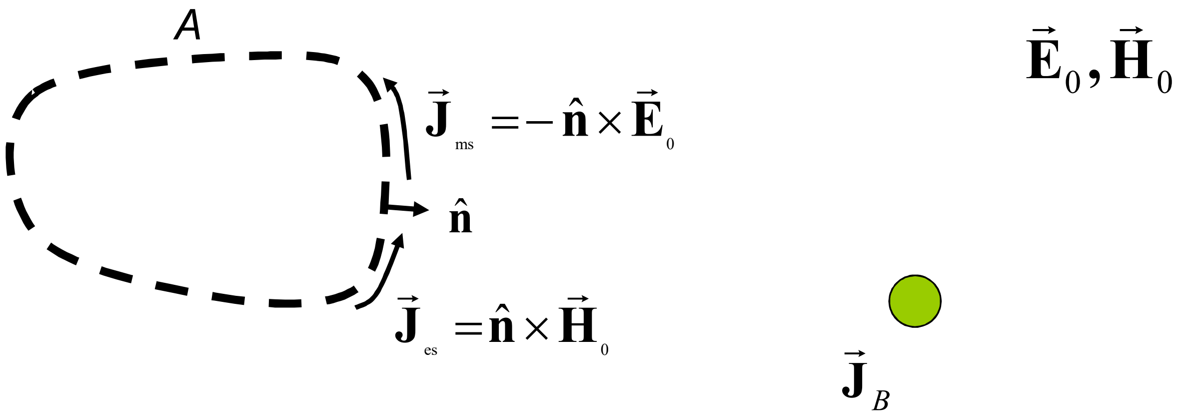

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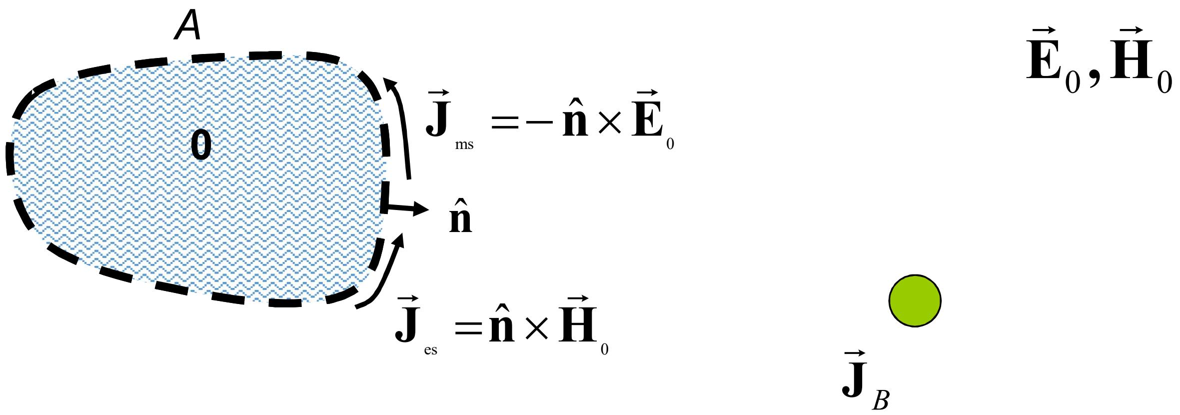
Equivalence theorem

More general formulation



$$\vec{\mathbf{J}}_A + \vec{\mathbf{J}}_B \rightarrow \left(\vec{\mathbf{E}}_0, \vec{\mathbf{H}}_0\right)$$

THEOREMS

Poynting

Time domain – Phasor domain

Uniqueness (Interior problem – Exterior problem)

Time domain – Phasor domain

Equivalence

Phasor domain

Image Theory

Reciprocity

Phasor domain

Image theory

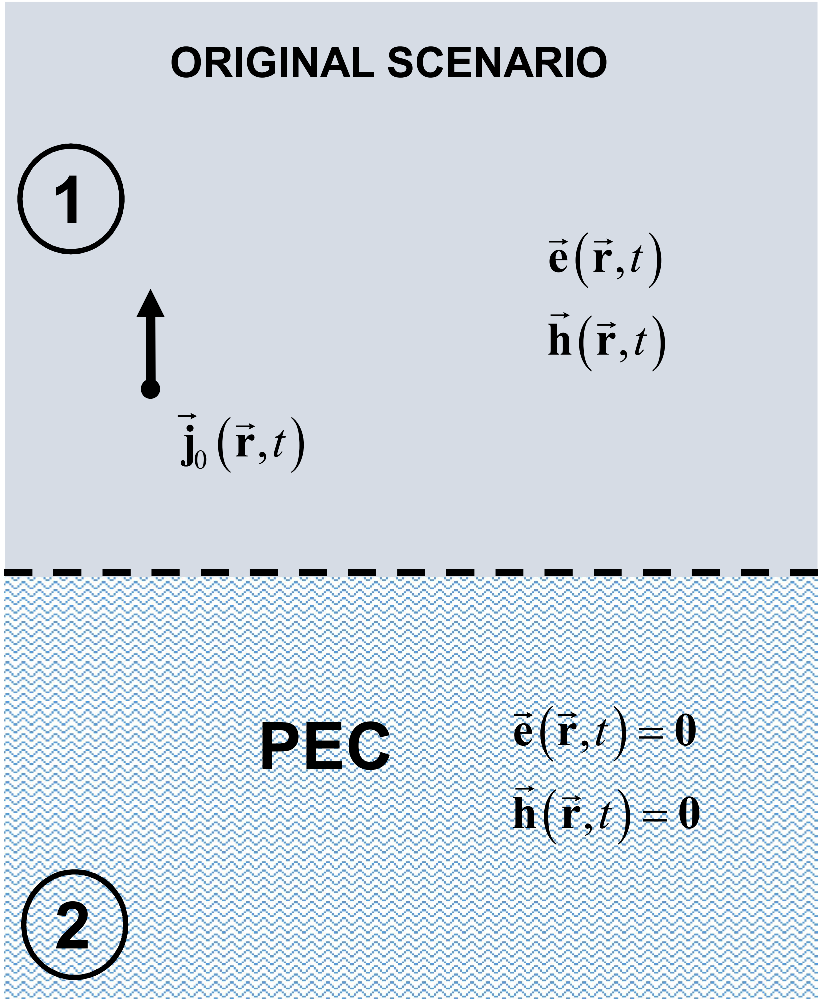
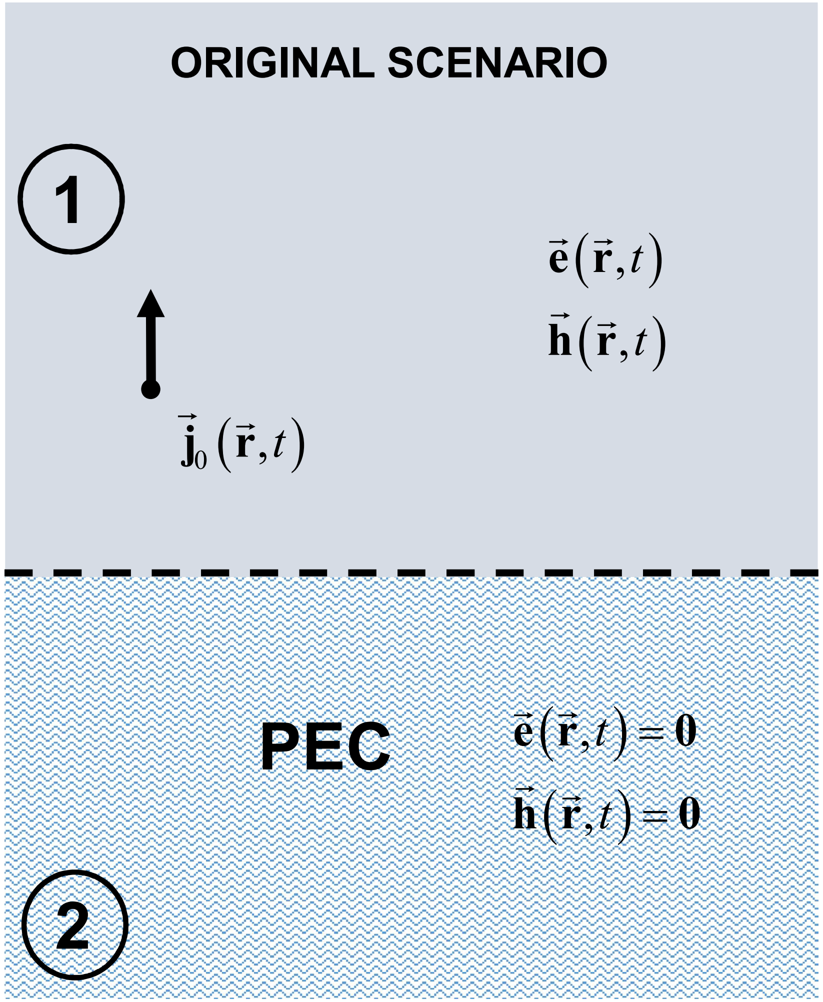
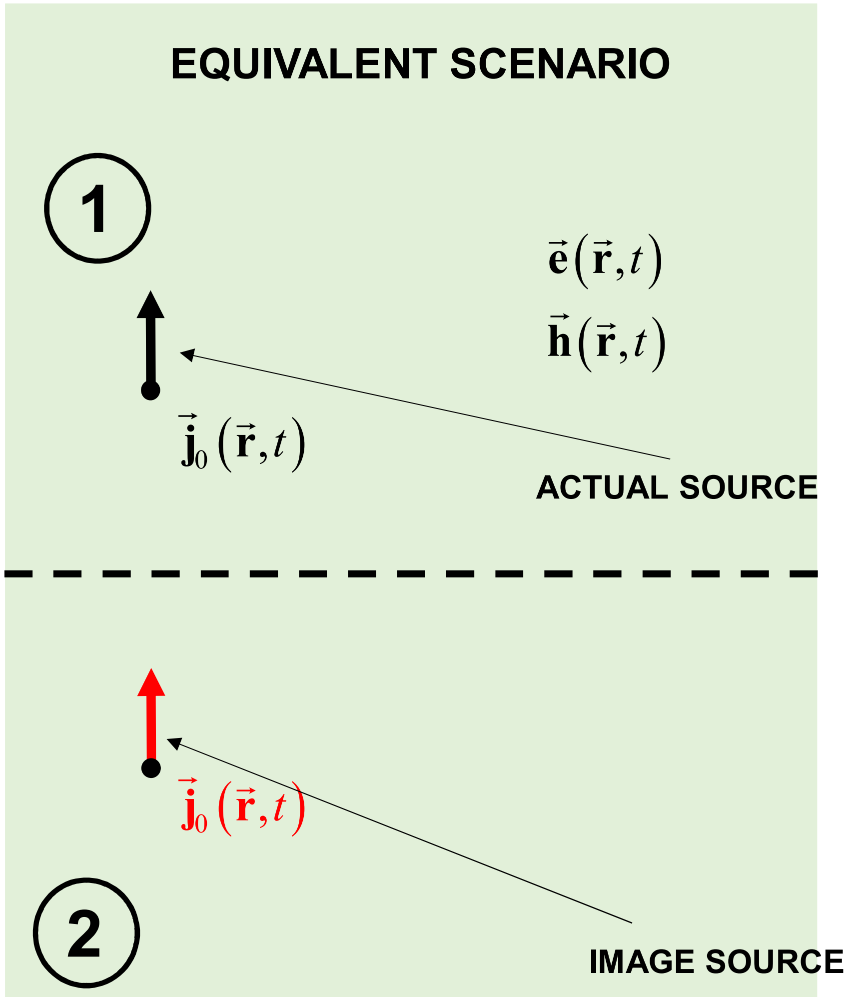


Image theory





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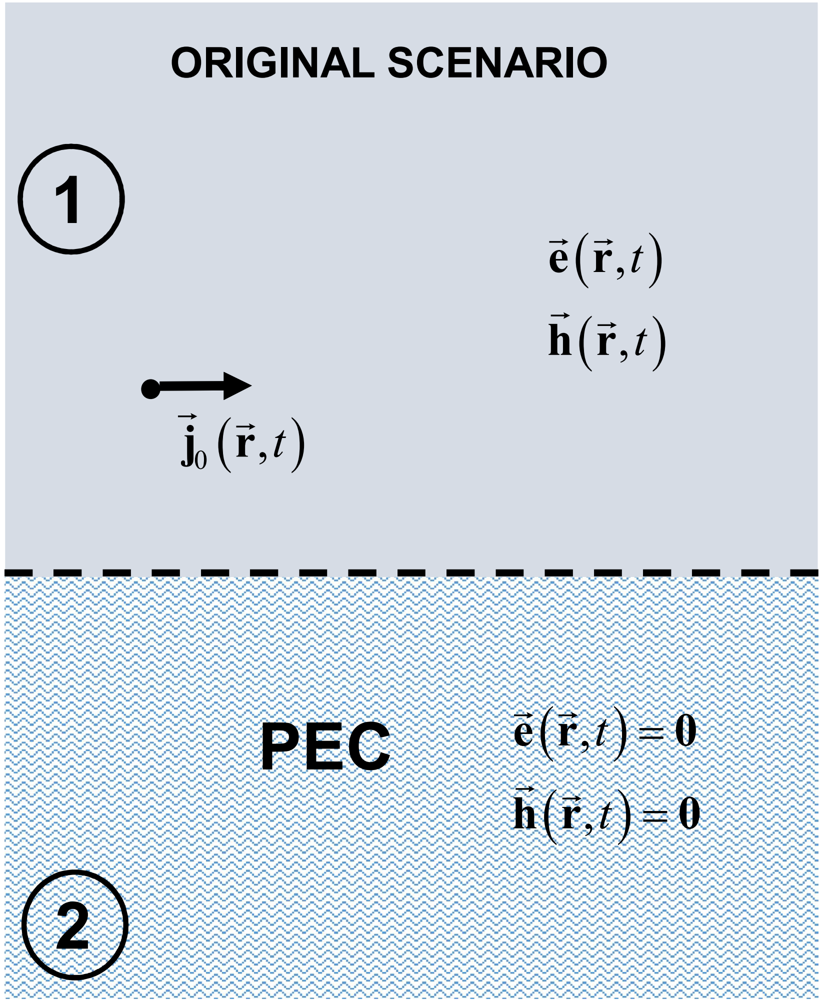


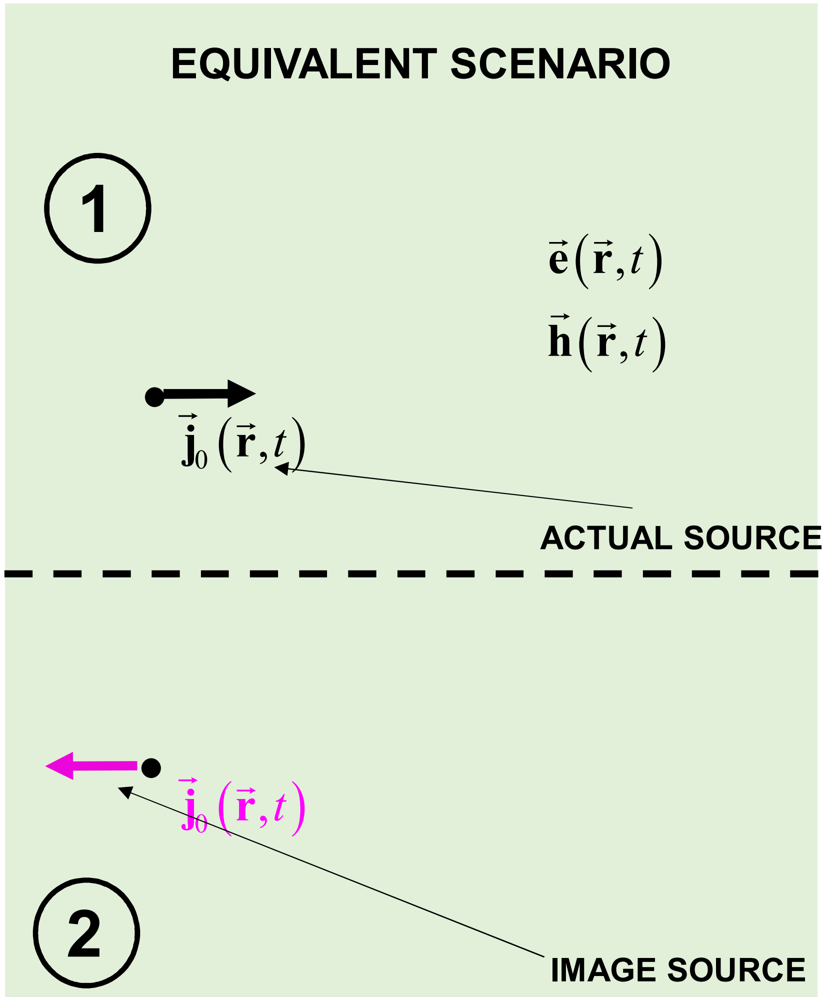
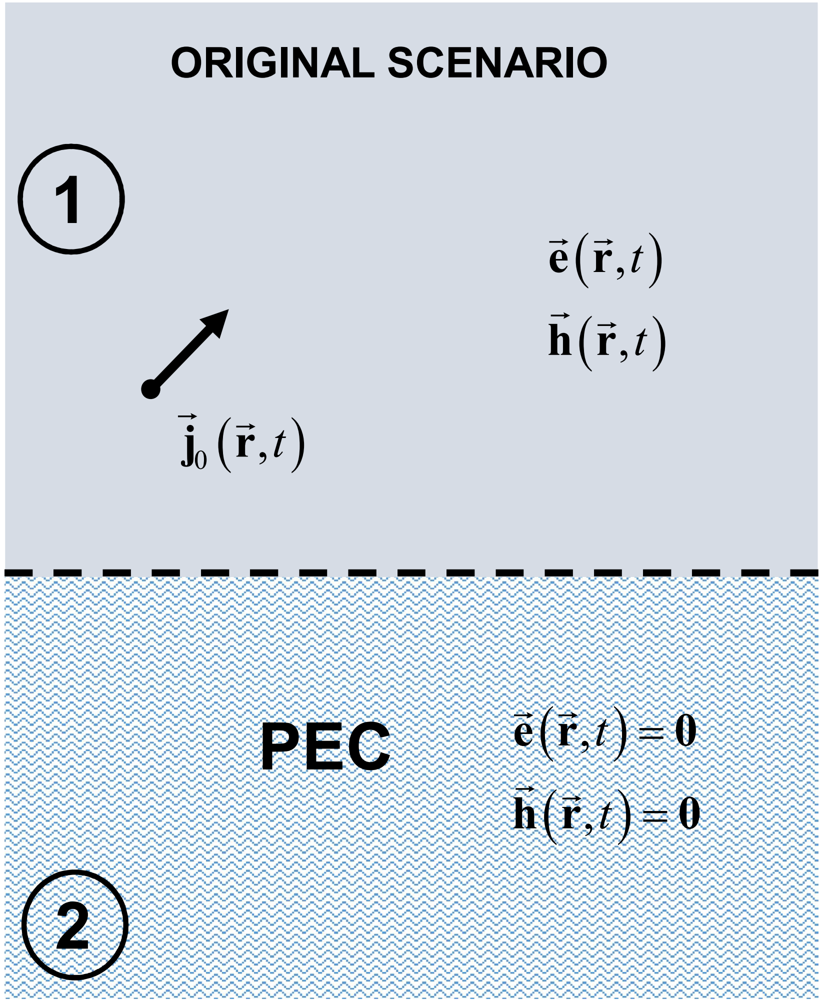
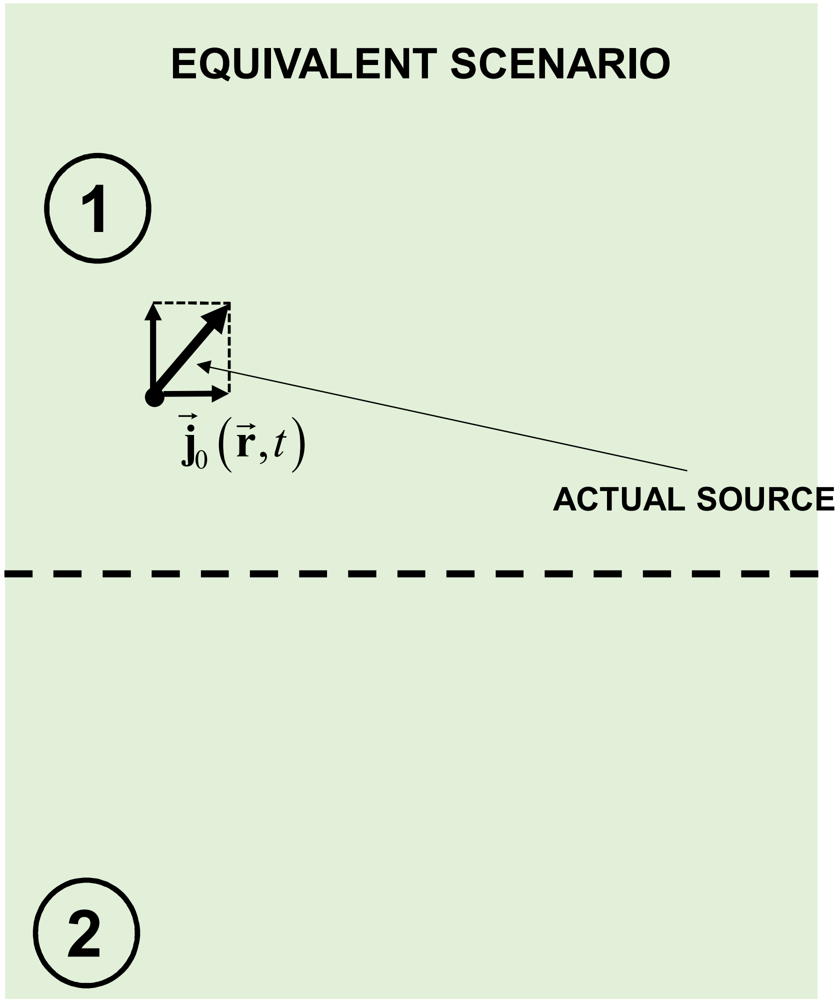


Image theory





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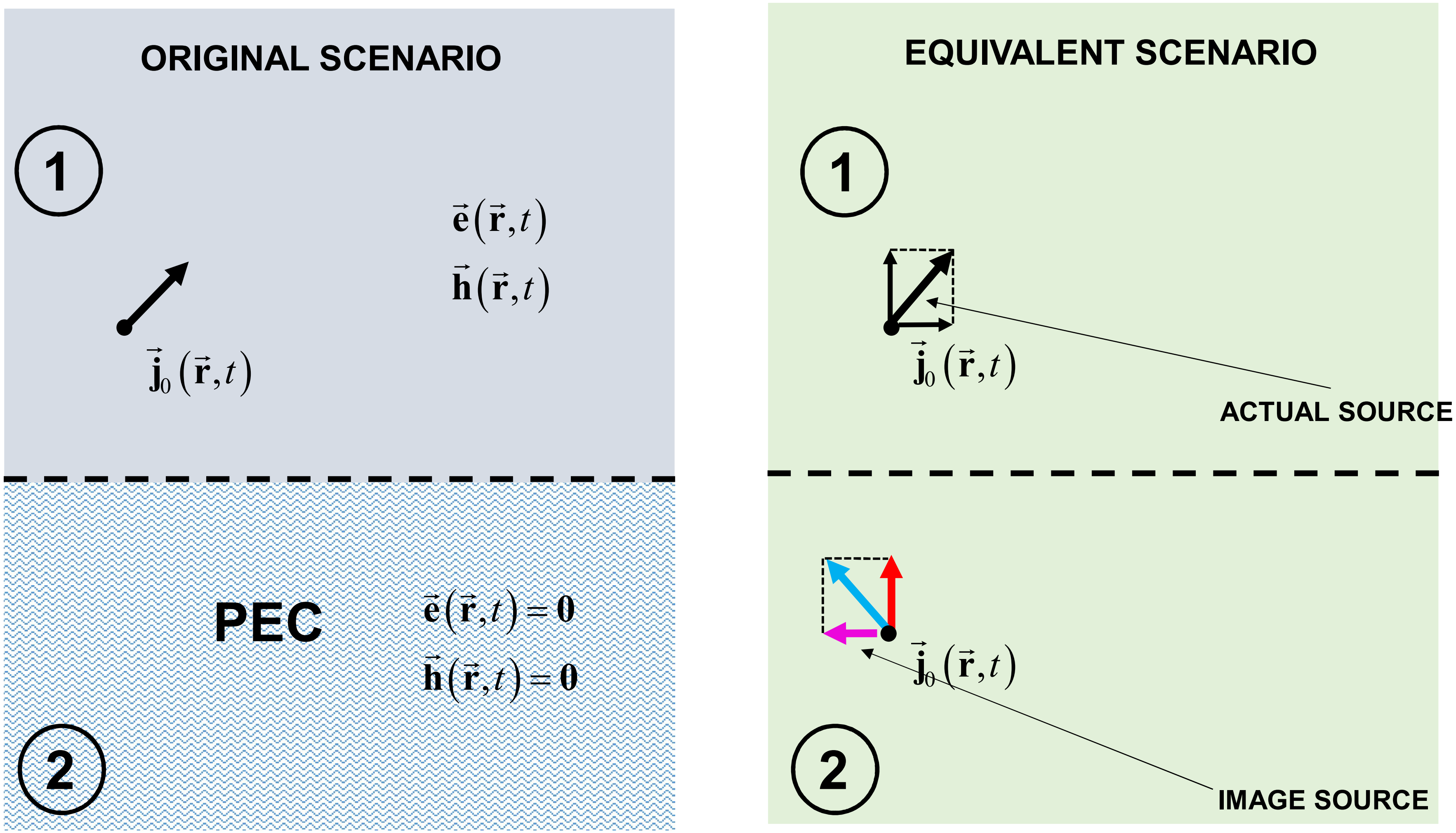

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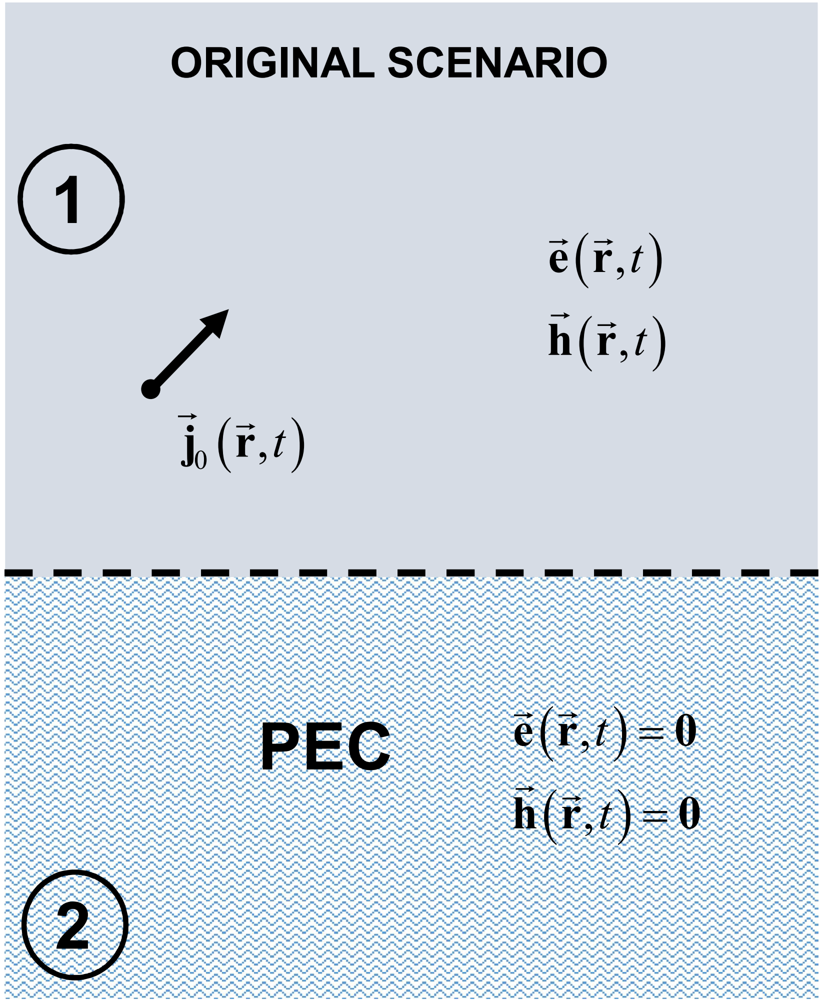


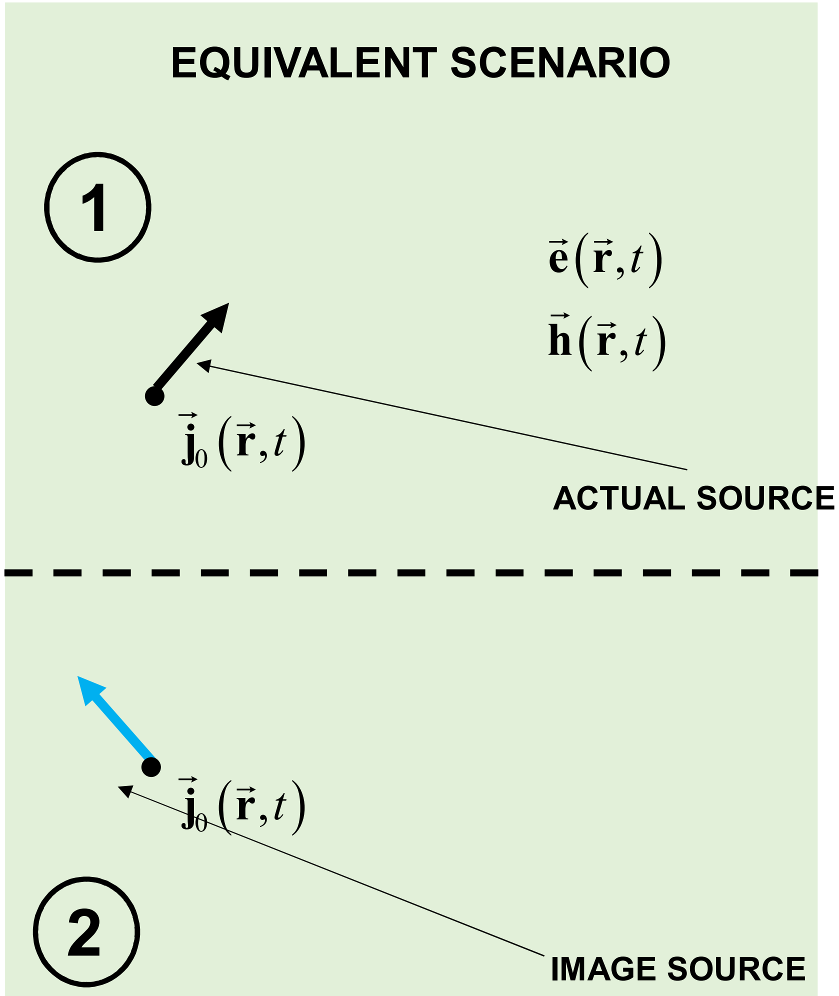
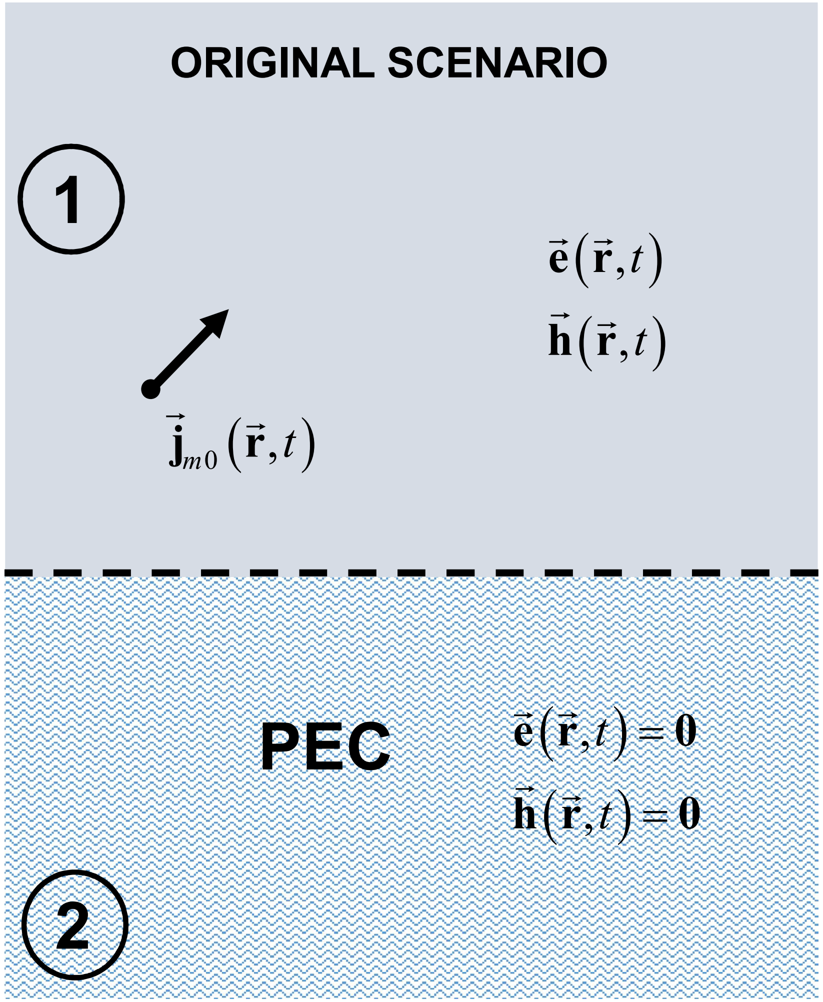
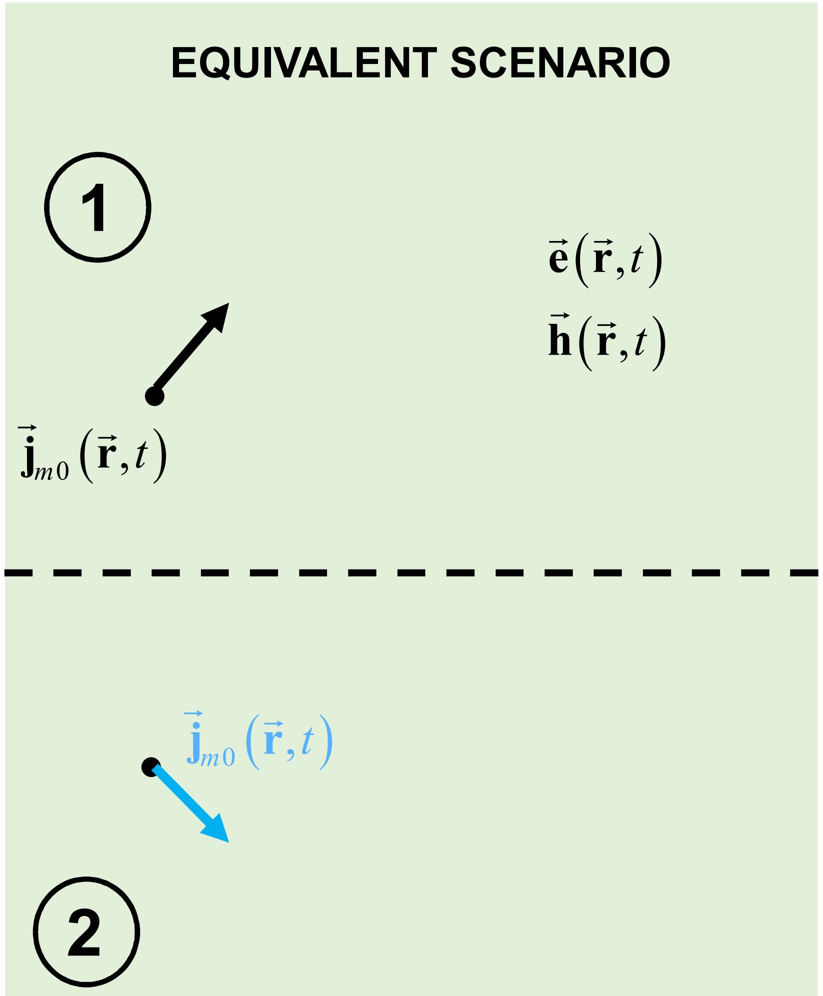


Image theory

Image theory (magnetic sources)





THEOREMS

Poynting

Time domain – Phasor domain

Uniqueness (Interior problem – Exterior problem)

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Equivalence

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Reciprocity

Phasor domain

Reciprocity theorem

$\vec{\mathbf{E}}_1, \vec{\mathbf{H}}_1$

Consider a source distribution $\vec{\mathbf{J}}_{01}$ with its associated electromagnetic field $\left(\vec{\mathbf{E}}_1, \vec{\mathbf{H}}_1\right)$

Reciprocity theorem

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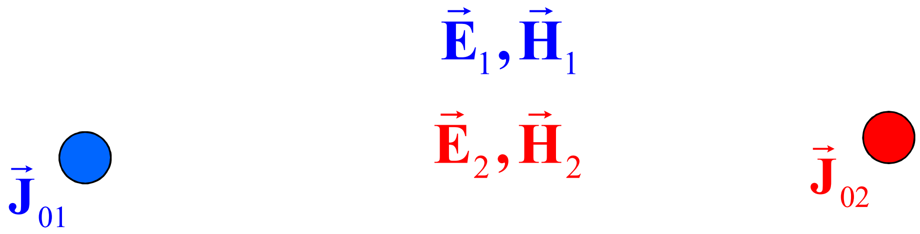
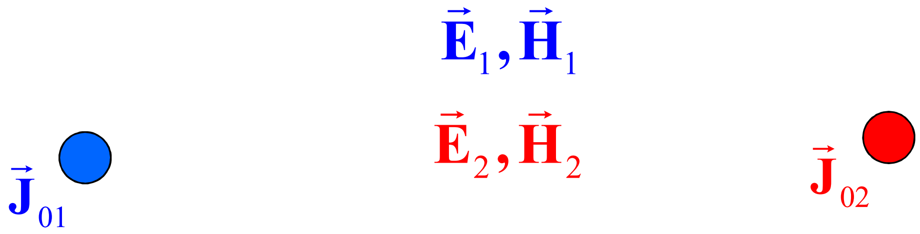


Consider a source distribution $\vec{\mathbf{J}}_{01}$ with its associated electromagnetic field $\left(\vec{\mathbf{E}}_1, \vec{\mathbf{H}}_1\right)$

Consider a source distribution $\vec{\mathbf{J}}_{02}$ with its associated electromagnetic field $\left(\vec{\mathbf{E}}_2, \vec{\mathbf{H}}_2\right)$

Reciprocity theorem



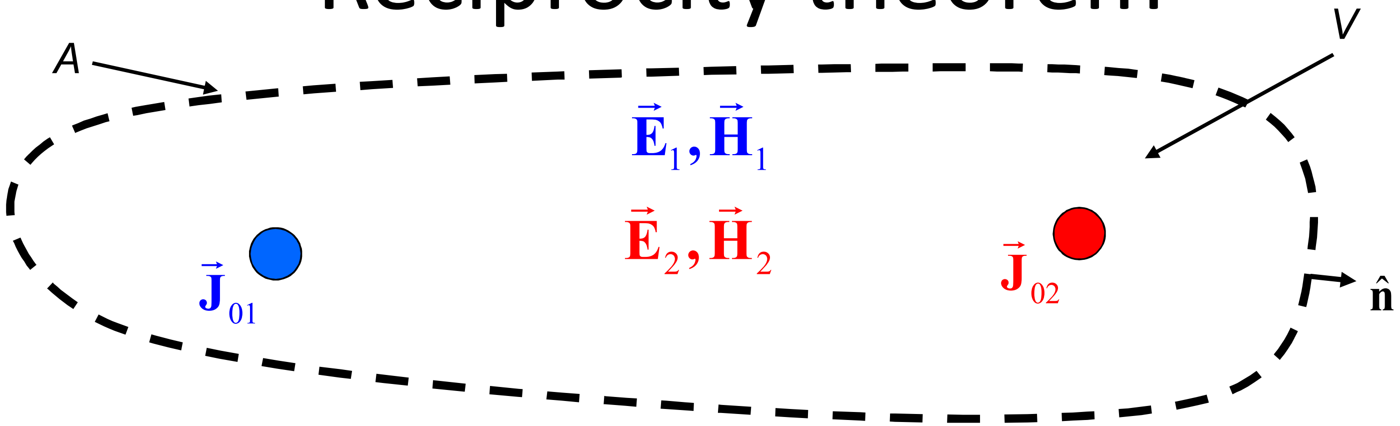
Consider a source distribution $\vec{\mathbf{J}}_{01}$ with its associated electromagnetic field $\left(\vec{\mathbf{E}}_1, \vec{\mathbf{H}}_1\right)$

Consider a source distribution $\vec{\mathbf{J}}_{02}$ with its associated electromagnetic field $\left(\vec{\mathbf{E}}_2, \vec{\mathbf{H}}_2\right)$

We define the mixed Poynting-like vector $\vec{\mathbf{S}}_{12}$

$$\vec{\mathbf{S}}_{12} = \vec{\mathbf{E}}_1 \times \vec{\mathbf{H}}_2 - \vec{\mathbf{E}}_2 \times \vec{\mathbf{H}}_1$$

Reciprocity theorem



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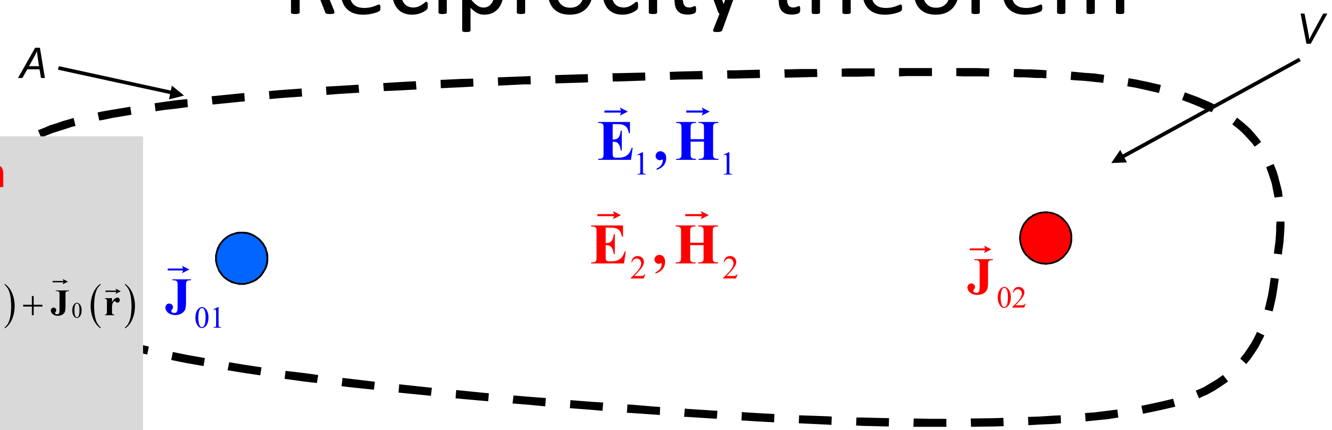
$$\vec{\mathbf{S}}_{12} = \vec{\mathbf{E}}_1 \times \vec{\mathbf{H}}_2 - \vec{\mathbf{E}}_2 \times \vec{\mathbf{H}}_1$$

The reciprocity theorem sates that

$$\oiint_A dA\, \vec{\mathbf{S}}_{12} \cdot \hat{\mathbf{n}} = \iiint_V dV \left[\vec{\mathbf{J}}_{01} \cdot \vec{\mathbf{E}}_2 - \vec{\mathbf{J}}_{02} \cdot \vec{\mathbf{E}}_1 \right]$$

Reciprocity theorem



Phasor domain

$$\nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}})$$

$$\nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}})$$

$$\nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) + \rho_0(\vec{\mathbf{r}})$$

$$\nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) = 0$$

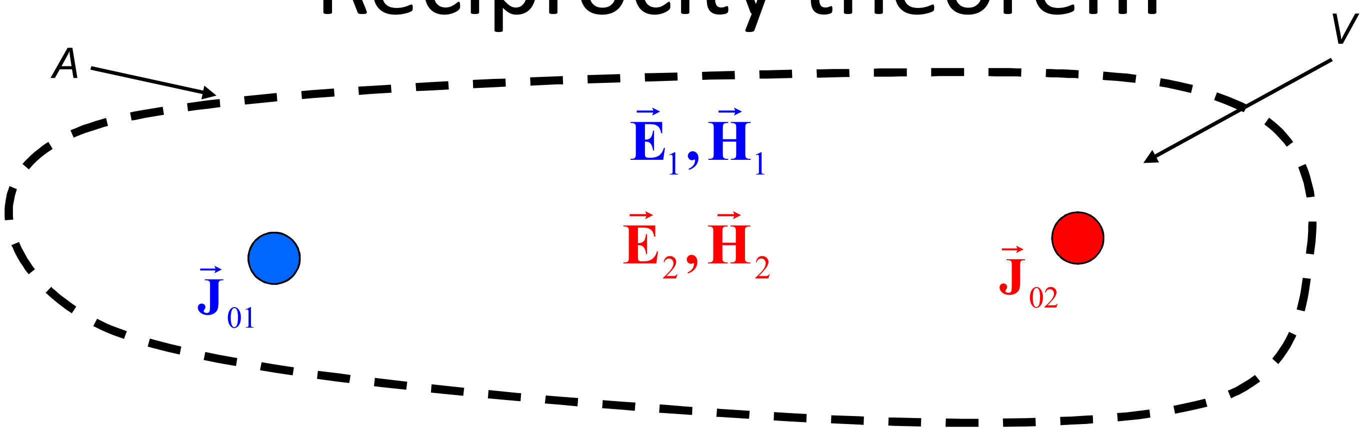
$$\vec{\mathbf{S}}_{12} = \vec{\mathbf{E}}_1 \times \vec{\mathbf{H}}_2 - \vec{\mathbf{E}}_2 \times \vec{\mathbf{H}}_1$$

$$\nabla \cdot \vec{\mathbf{S}}_{12} = \nabla \cdot \left(\vec{\mathbf{E}}_1 \times \vec{\mathbf{H}}_2\right) - \nabla \cdot \left(\vec{\mathbf{E}}_2 \times \vec{\mathbf{H}}_1\right) = \left[\vec{\mathbf{H}}_2 \cdot \left(\nabla \times \vec{\mathbf{E}}_1\right) - \vec{\mathbf{E}}_1 \cdot \left(\nabla \times \vec{\mathbf{H}}_2\right)\right] - \left[\vec{\mathbf{H}}_1 \cdot \left(\nabla \times \vec{\mathbf{E}}_2\right) - \vec{\mathbf{E}}_2 \cdot \left(\nabla \times \vec{\mathbf{H}}_1\right)\right]$$

$$= \left[\vec{\mathbf{H}}_2 \cdot \left(-j\omega_0 \vec{\mathbf{B}}_1\right) - \vec{\mathbf{E}}_1 \cdot \left(j\omega_0 \vec{\mathbf{D}}_2 + \vec{\mathbf{J}}_2 + \vec{\mathbf{J}}_{02}\right)\right] - \left[\vec{\mathbf{H}}_1 \cdot \left(-j\omega_0 \vec{\mathbf{B}}_2(\vec{\mathbf{r}})\right) - \vec{\mathbf{E}}_2 \cdot \left(j\omega_0 \vec{\mathbf{D}}_1 + \vec{\mathbf{J}}_1 + \vec{\mathbf{J}}_{01}\right)\right]$$

$$\nabla \cdot \left[\vec{\mathbf{A}}(\vec{\mathbf{r}}) \times \vec{\mathbf{B}}(\vec{\mathbf{r}})\right] = \vec{\mathbf{B}}(\vec{\mathbf{r}}) \cdot \left[\nabla \times \vec{\mathbf{A}}(\vec{\mathbf{r}})\right] - \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \left[\nabla \times \vec{\mathbf{B}}(\vec{\mathbf{r}})\right]$$

Reciprocity theorem



$$\vec{S}_{12} = \vec{E}_1 \times \vec{H}_2 - \vec{E}_2 \times \vec{H}_1$$

$$\nabla \cdot \vec{S}_{12} = \nabla \cdot \left(\vec{E}_1 \times \vec{H}_2\right) - \nabla \cdot \left(\vec{E}_2 \times \vec{H}_1\right) = \left[\vec{H}_2 \cdot \left(\nabla \times \vec{E}_1\right) - \vec{E}_1 \cdot \left(\nabla \times \vec{H}_2\right)\right] - \left[\vec{H}_1 \cdot \left(\nabla \times \vec{E}_2\right) - \vec{E}_2 \cdot \left(\nabla \times \vec{H}_1\right)\right]$$

$$= \left[\vec{H}_2 \cdot \left(-j\omega_0 \vec{B}_1\right) - \vec{E}_1 \cdot \left(j\omega_0 \vec{D}_2 + \vec{J}_2 + \vec{J}_{02}\right)\right] - \left[\vec{H}_1 \cdot \left(-j\omega_0 \vec{B}_2(\vec{r})\right) - \vec{E}_2 \cdot \left(j\omega_0 \vec{D}_1 + \vec{J}_1 + \vec{J}_{01}\right)\right]$$

$$= \left[\vec{H}_2 \cdot \left(-j\omega_0 \mu \vec{H}_1\right) - \vec{E}_1 \cdot \left(j\omega_0 \varepsilon \vec{E}_2 + \sigma \vec{E}_2 + \vec{J}_{02}\right)\right] - \left[\vec{H}_1 \cdot \left(-j\omega_0 \mu \vec{H}_2\right) - \vec{E}_2 \cdot \left(j\omega_0 \varepsilon \vec{E}_1 + \sigma \vec{E}_1 + \vec{J}_{01}\right)\right]$$

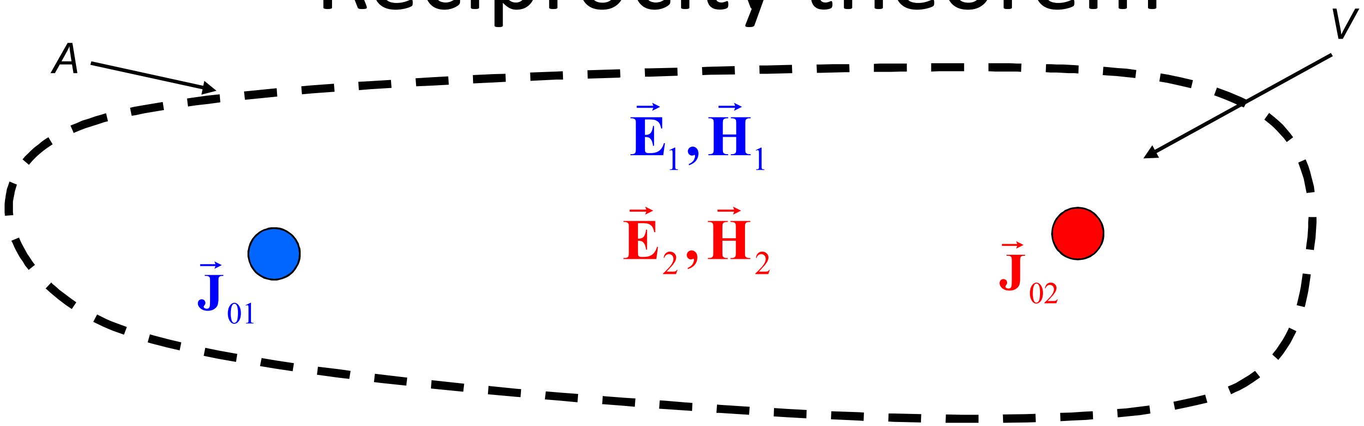
$$= \vec{H}_2 \cdot \left(-j\omega_0 \mu \vec{H}_1\right) - \vec{E}_1 \cdot \left(j\omega_0 \varepsilon \vec{E}_2 + \sigma \vec{E}_2 + \vec{J}_{02}\right) - \vec{H}_1 \cdot \left(-j\omega_0 \mu \vec{H}_2\right) + \vec{E}_2 \cdot \left(j\omega_0 \varepsilon \vec{E}_1 + \sigma \vec{E}_1 + \vec{J}_{01}\right)$$

Hypotheses on the medium (PD)

- **Linear**
- **Isotropic**
- **Space-Nondispersive**
- **Time-invariant**
- **Time-Dispersive**

$$\begin{cases} \vec{D} = \varepsilon \vec{E} \\ \vec{B} = \mu \vec{H} \\ \vec{J} = \sigma \vec{E} \end{cases}$$

Reciprocity theorem



$$\vec{\mathbf{S}}_{12} = \vec{\mathbf{E}}_1 \times \vec{\mathbf{H}}_2 - \vec{\mathbf{E}}_2 \times \vec{\mathbf{H}}_1$$

$$\nabla \cdot \vec{\mathbf{S}}_{12} = \nabla \cdot \left(\vec{\mathbf{E}}_1 \times \vec{\mathbf{H}}_2\right) - \nabla \cdot \left(\vec{\mathbf{E}}_2 \times \vec{\mathbf{H}}_1\right) = \left[\vec{\mathbf{H}}_2 \cdot \left(\nabla \times \vec{\mathbf{E}}_1\right) - \vec{\mathbf{E}}_1 \cdot \left(\nabla \times \vec{\mathbf{H}}_2\right)\right] - \left[\vec{\mathbf{H}}_1 \cdot \left(\nabla \times \vec{\mathbf{E}}_2\right) - \vec{\mathbf{E}}_2 \cdot \left(\nabla \times \vec{\mathbf{H}}_1\right)\right]$$

$$= \left[\vec{\mathbf{H}}_2 \cdot \left(-j\omega_0 \vec{\mathbf{B}}_1\right) - \vec{\mathbf{E}}_1 \cdot \left(j\omega_0 \vec{\mathbf{D}}_2 + \vec{\mathbf{J}}_2 + \vec{\mathbf{J}}_{02}\right)\right] - \left[\vec{\mathbf{H}}_1 \cdot \left(-j\omega_0 \vec{\mathbf{B}}_2(\vec{\mathbf{r}})\right) - \vec{\mathbf{E}}_2 \cdot \left(j\omega_0 \vec{\mathbf{D}}_1 + \vec{\mathbf{J}}_1 + \vec{\mathbf{J}}_{01}\right)\right]$$

$$= \left[\vec{\mathbf{H}}_2 \cdot \left(-j\omega_0 \mu \vec{\mathbf{H}}_1\right) - \vec{\mathbf{E}}_1 \cdot \left(j\omega_0 \varepsilon \vec{\mathbf{E}}_2 + \sigma \vec{\mathbf{E}}_2 + \vec{\mathbf{J}}_{02}\right)\right] - \left[\vec{\mathbf{H}}_1 \cdot \left(-j\omega_0 \mu \vec{\mathbf{H}}_2\right) - \vec{\mathbf{E}}_2 \cdot \left(j\omega_0 \varepsilon \vec{\mathbf{E}}_1 + \sigma \vec{\mathbf{E}}_1 + \vec{\mathbf{J}}_{01}\right)\right]$$

$$= \vec{\mathbf{H}}_2 \cdot \left(-j\omega_0 \mu \vec{\mathbf{H}}_1\right) - \vec{\mathbf{E}}_1 \cdot \left(j\omega_0 \varepsilon \vec{\mathbf{E}}_2 + \sigma \vec{\mathbf{E}}_2 + \vec{\mathbf{J}}_{02}\right) - \vec{\mathbf{H}}_1 \cdot \left(-j\omega_0 \mu \vec{\mathbf{H}}_2\right) + \vec{\mathbf{E}}_2 \cdot \left(j\omega_0 \varepsilon \vec{\mathbf{E}}_1 + \sigma \vec{\mathbf{E}}_1 + \vec{\mathbf{J}}_{01}\right)$$

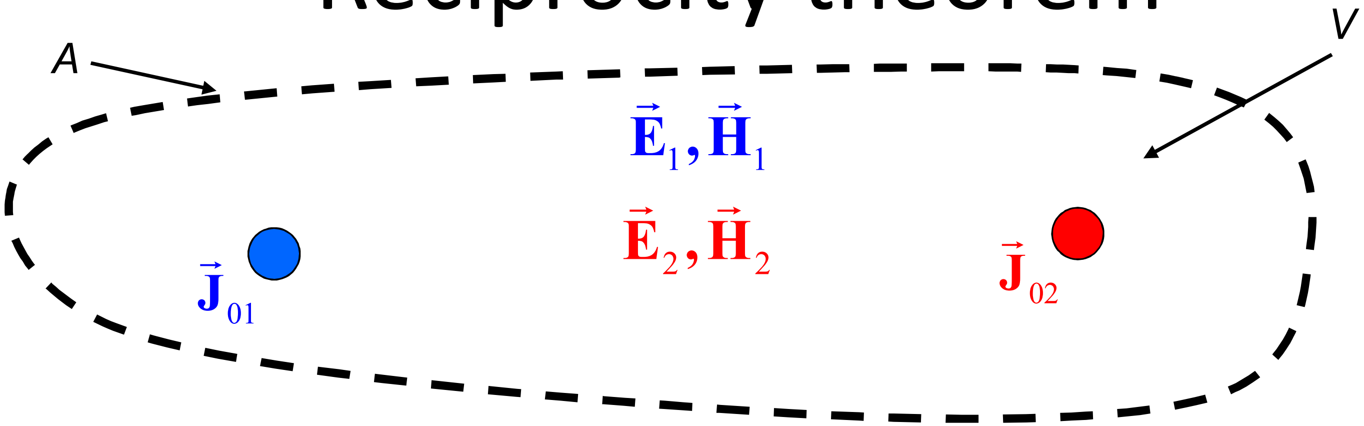
$$= -j\omega_0 \mu \vec{\mathbf{H}}_2 \cdot \vec{\mathbf{H}}_1 - j\omega_0 \varepsilon \vec{\mathbf{E}}_1 \cdot \vec{\mathbf{E}}_2 - \sigma \vec{\mathbf{E}}_1 \cdot \vec{\mathbf{E}}_2 - \vec{\mathbf{E}}_1 \cdot \vec{\mathbf{J}}_{02} + j\omega_0 \mu \vec{\mathbf{H}}_1 \cdot \vec{\mathbf{H}}_2 + j\omega_0 \varepsilon \vec{\mathbf{E}}_2 \cdot \vec{\mathbf{E}}_1 + \sigma \vec{\mathbf{E}}_2 \cdot \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 \cdot \vec{\mathbf{J}}_{01}$$

Hypotheses on the medium (PD)

- Linear
- Isotropic
- Space-Nondispersive
- Time-invariant
- Time-Dispersive

$$\begin{cases} \vec{\mathbf{D}} = \varepsilon \vec{\mathbf{E}} \\ \vec{\mathbf{B}} = \mu \vec{\mathbf{H}} \\ \vec{\mathbf{J}} = \sigma \vec{\mathbf{E}} \end{cases}$$

Reciprocity theorem



$$\vec{\mathbf{S}}_{12} = \vec{\mathbf{E}}_1 \times \vec{\mathbf{H}}_2 - \vec{\mathbf{E}}_2 \times \vec{\mathbf{H}}_1$$

$$\nabla \cdot \vec{\mathbf{S}}_{12} = -j\omega_0 \mu \vec{\mathbf{H}}_2 \cdot \vec{\mathbf{H}}_1 - j\omega_0 \varepsilon \vec{\mathbf{E}}_1 \cdot \vec{\mathbf{E}}_2 - \sigma \vec{\mathbf{E}}_1 \cdot \vec{\mathbf{E}}_2 - \vec{\mathbf{E}}_1 \cdot \vec{\mathbf{J}}_{02} + j\omega_0 \mu \vec{\mathbf{H}}_1 \cdot \vec{\mathbf{H}}_2 + j\omega_0 \varepsilon \vec{\mathbf{E}}_2 \cdot \vec{\mathbf{E}}_1 + \sigma \vec{\mathbf{E}}_2 \cdot \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 \cdot \vec{\mathbf{J}}_{01}$$

$$= -\vec{\mathbf{E}}_1 \cdot \vec{\mathbf{J}}_{02} + \vec{\mathbf{E}}_2 \cdot \vec{\mathbf{J}}_{01}$$

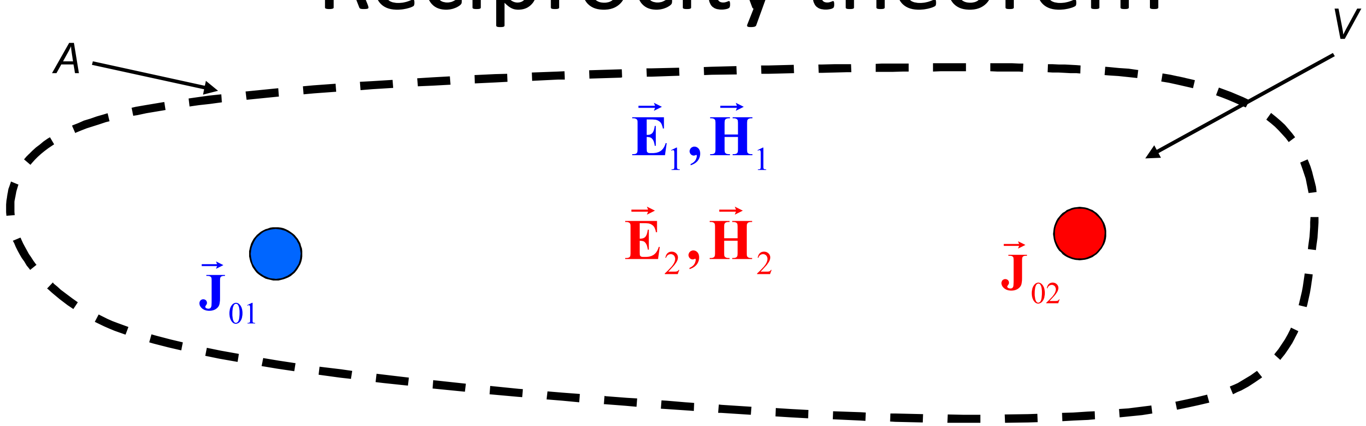
Hypotheses on the medium (PD)

- **Linear**
- **Isotropic**
- **Space-Nondispersive**
- **Time-invariant**
- **Time-Dispersive**

$$\begin{cases} \vec{\mathbf{D}} = \varepsilon \vec{\mathbf{E}} \\ \vec{\mathbf{B}} = \mu \vec{\mathbf{H}} \\ \vec{\mathbf{J}} = \sigma \vec{\mathbf{E}} \end{cases}$$

$$= -j\omega_0 \mu \vec{\mathbf{H}}_2 \cdot \vec{\mathbf{H}}_1 - j\omega_0 \varepsilon \vec{\mathbf{E}}_1 \cdot \vec{\mathbf{E}}_2 - \sigma \vec{\mathbf{E}}_1 \cdot \vec{\mathbf{E}}_2 - \vec{\mathbf{E}}_1 \cdot \vec{\mathbf{J}}_{02} + j\omega_0 \mu \vec{\mathbf{H}}_1 \cdot \vec{\mathbf{H}}_2 + j\omega_0 \varepsilon \vec{\mathbf{E}}_2 \cdot \vec{\mathbf{E}}_1 + \sigma \vec{\mathbf{E}}_2 \cdot \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 \cdot \vec{\mathbf{J}}_{01}$$

Reciprocity theorem



$$\vec{S}_{12} = \vec{E}_1 \times \vec{H}_2 - \vec{E}_2 \times \vec{H}_1$$

$$\nabla \cdot \vec{S}_{12} = -j\omega_0\mu\vec{H}_2 \cdot \vec{H}_1 - j\omega_0\varepsilon\vec{E}_1 \cdot \vec{E}_2 - \sigma\vec{E}_1 \cdot \vec{E}_2 - \vec{E}_1 \cdot \vec{J}_{02} + j\omega_0\mu\vec{H}_1 \cdot \vec{H}_2 + j\omega_0\varepsilon\vec{E}_2 \cdot \vec{E}_1 + \sigma\vec{E}_2 \cdot \vec{E}_1 + \vec{E}_2 \cdot \vec{J}_{01}$$

$$= -\vec{E}_1 \cdot \vec{J}_{02} + \vec{E}_2 \cdot \vec{J}_{01}$$

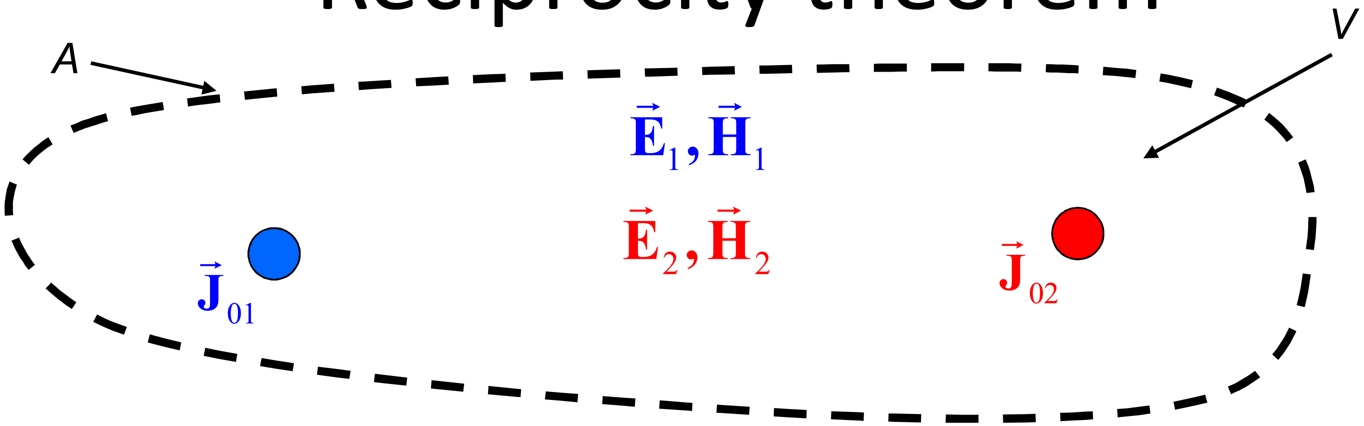
$$\nabla \cdot \vec{S}_{12} = -\vec{E}_1 \cdot \vec{J}_{02} + \vec{E}_2 \cdot \vec{J}_{01}$$

Hypotheses on the medium (PD)

- **Linear**
- **Isotropic**
- **Space-Nondispersive**
- **Time-invariant**
- **Time-Dispersive**

$$\begin{cases} \vec{D} = \varepsilon\vec{E} \\ \vec{B} = \mu\vec{H} \\ \vec{J} = \sigma\vec{E} \end{cases}$$

Reciprocity theorem



$$\vec{\mathbf{S}}_{12} = \vec{\mathbf{E}}_1 \times \vec{\mathbf{H}}_2 - \vec{\mathbf{E}}_2 \times \vec{\mathbf{H}}_1$$

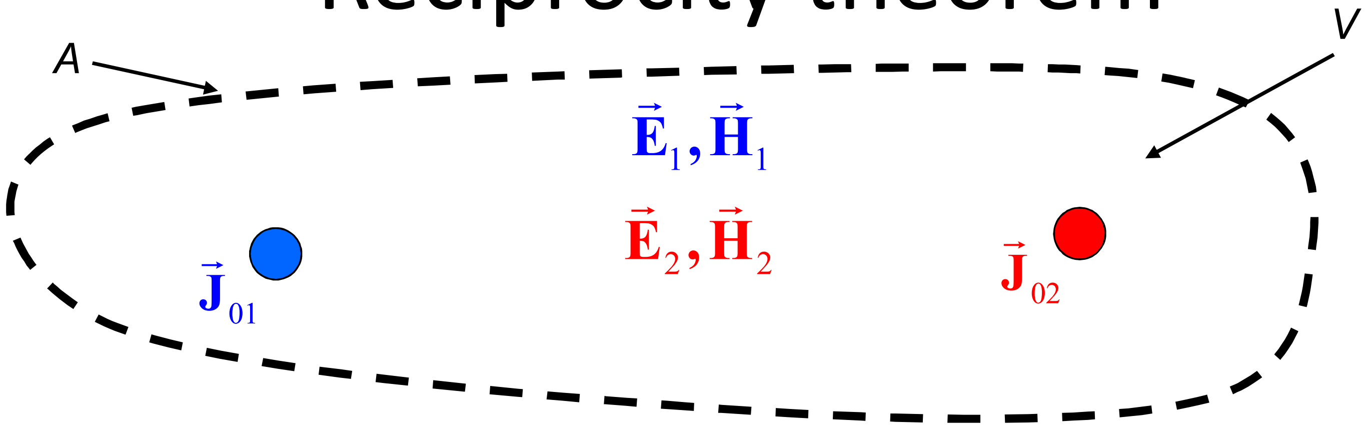
$$\nabla \cdot \vec{\mathbf{S}}_{12} = -\vec{\mathbf{E}}_1 \cdot \vec{\mathbf{J}}_{02} + \vec{\mathbf{E}}_2 \cdot \vec{\mathbf{J}}_{01}$$

Hypotheses on the medium (PD)

- **Linear**
- **Isotropic**
- **Space-Nondispersive**
- **Time-invariant**
- **Time-Dispersive**

$$\begin{cases} \vec{\mathbf{D}} = \varepsilon \vec{\mathbf{E}} \\ \vec{\mathbf{B}} = \mu \vec{\mathbf{H}} \\ \vec{\mathbf{J}} = \sigma \vec{\mathbf{E}} \end{cases}$$

Reciprocity theorem



$$\vec{\mathbf{S}}_{12} = \vec{\mathbf{E}}_1 \times \vec{\mathbf{H}}_2 - \vec{\mathbf{E}}_2 \times \vec{\mathbf{H}}_1$$

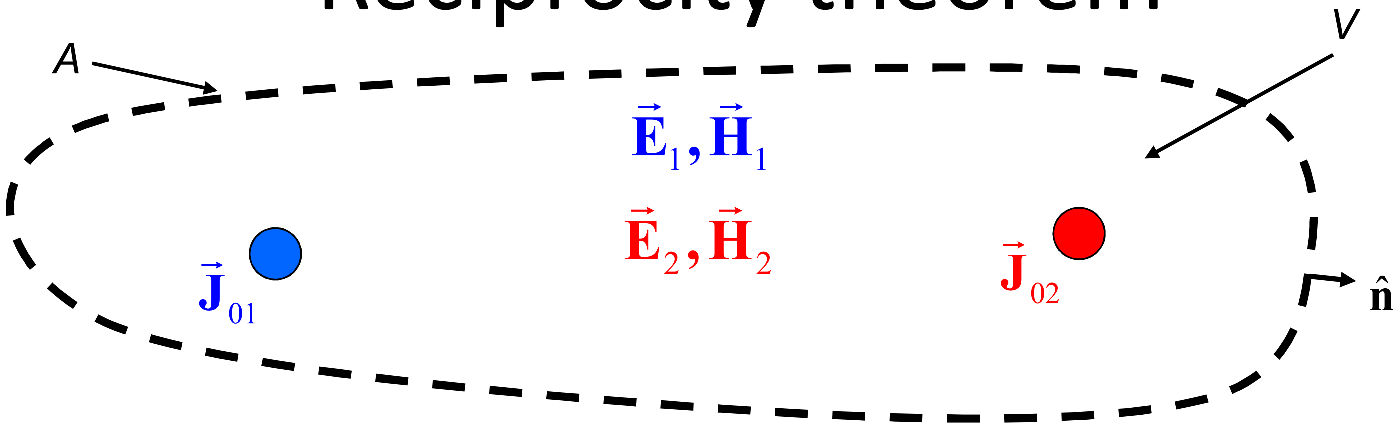
$$\nabla \cdot \vec{\mathbf{S}}_{12} = -\vec{\mathbf{E}}_1 \cdot \vec{\mathbf{J}}_{02} + \vec{\mathbf{E}}_2 \cdot \vec{\mathbf{J}}_{01} \Rightarrow \oiint_A dA\, \vec{\mathbf{S}}_{12} \cdot \hat{\mathbf{n}} = \iiint_V dV \left[\vec{\mathbf{J}}_{01} \cdot \vec{\mathbf{E}}_2 - \vec{\mathbf{J}}_{02} \cdot \vec{\mathbf{E}}_1 \right]$$

Hypotheses on the medium (PD)

- **Linear**
- **Isotropic**
- **Space-Nondispersive**
- **Time-invariant**
- **Time-Dispersive**

$$\begin{cases} \vec{\mathbf{D}} = \varepsilon \vec{\mathbf{E}} \\ \vec{\mathbf{B}} = \mu \vec{\mathbf{H}} \\ \vec{\mathbf{J}} = \sigma \vec{\mathbf{E}} \end{cases}$$

Reciprocity theorem



Consider a source distribution $\vec{\mathbf{J}}_{01}$ with its associated electromagnetic field $\left(\vec{\mathbf{E}}_1, \vec{\mathbf{H}}_1\right)$

Consider a source distribution $\vec{\mathbf{J}}_{02}$ with its associated electromagnetic field $\left(\vec{\mathbf{E}}_2, \vec{\mathbf{H}}_2\right)$

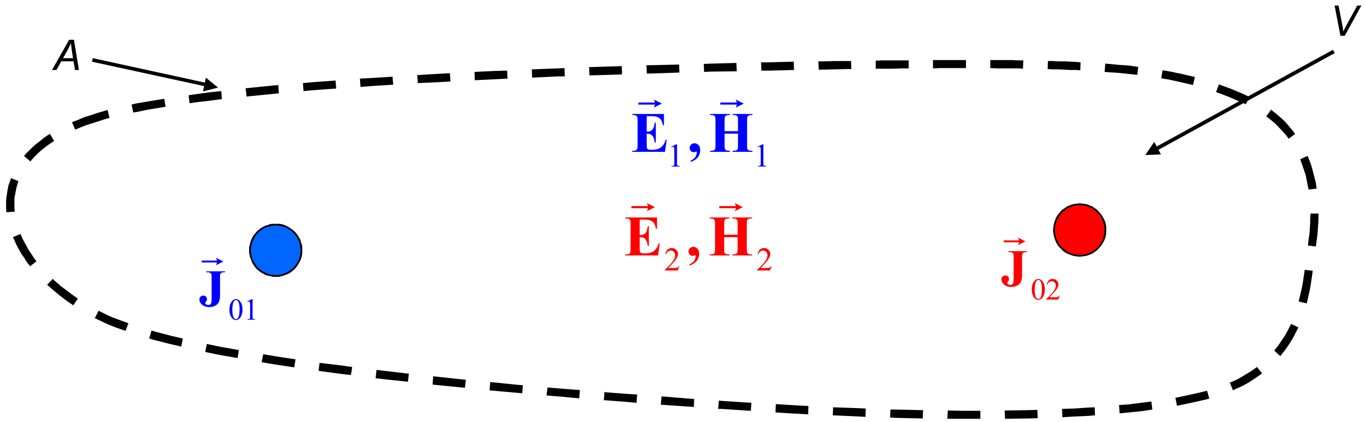
We define the mixed Poynting-like vector $\vec{\mathbf{S}}_{12}$

$$\vec{\mathbf{S}}_{12} = \vec{\mathbf{E}}_1 \times \vec{\mathbf{H}}_2 - \vec{\mathbf{E}}_2 \times \vec{\mathbf{H}}_1$$

The reciprocity theorem sates that

$$\oiint_A dA\, \vec{\mathbf{S}}_{12} \cdot \hat{\mathbf{n}} = \iiint_V dV \left[\vec{\mathbf{J}}_{01} \cdot \vec{\mathbf{E}}_2 - \vec{\mathbf{J}}_{02} \cdot \vec{\mathbf{E}}_1\right]$$

Reciprocity theorem: one consideration

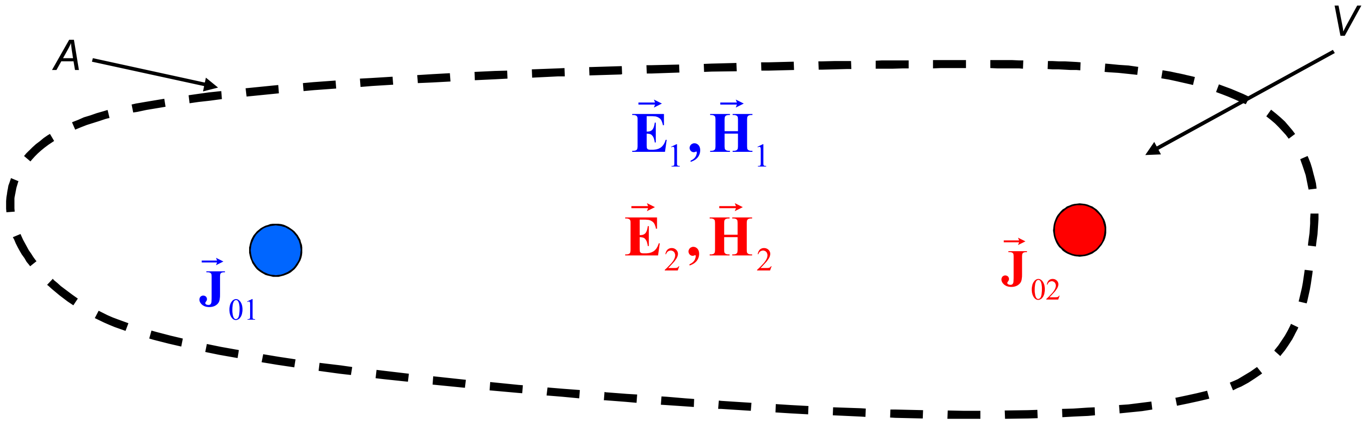


Hypotheses on the medium (PD)

- Linear
- Isotropic
- Space-Nondispersive
- Time-invariant
- Time-Dispersive

$$\begin{cases} \vec{D} = \varepsilon \vec{E} \\ \vec{B} = \mu \vec{H} \\ \vec{J} = \sigma \vec{E} \end{cases}$$

Reciprocity theorem: one consideration

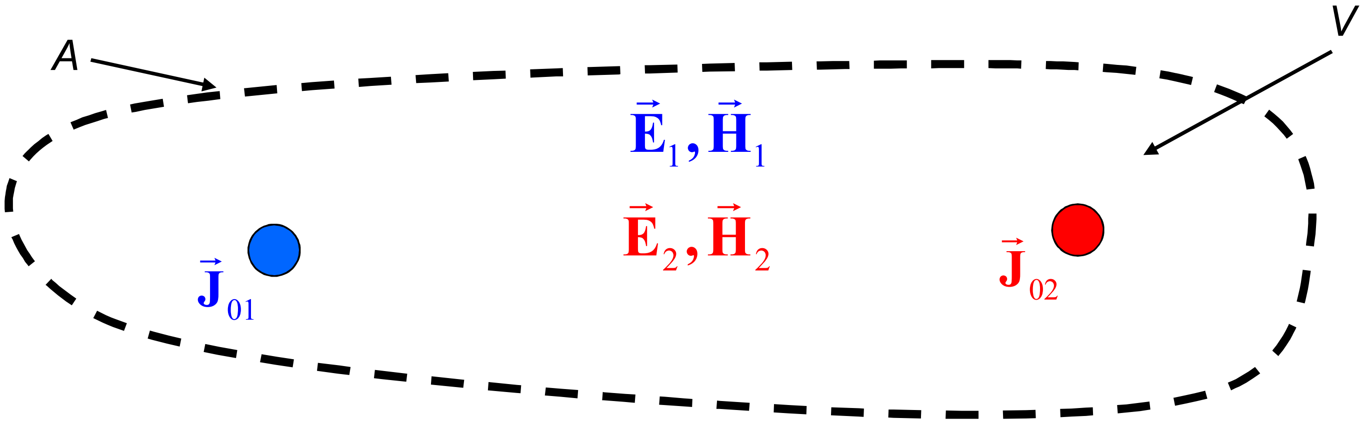


Hypotheses on the medium (PD)

- **Linear**
- **Isotropic**
- **Space-Nondispersive**
- **Time-invariant**
- **Time-Dispersive**

$$\begin{cases} \vec{D} = \varepsilon \vec{E} \\ \vec{B} = \mu \vec{H} \\ \vec{J} = \sigma \vec{E} \end{cases}$$

Reciprocity theorem: one consideration



$$\vec{\mathbf{S}}_{12} = \vec{\mathbf{E}}_1 \times \vec{\mathbf{H}}_2 - \vec{\mathbf{E}}_2 \times \vec{\mathbf{H}}_1$$

$$\nabla \cdot \vec{\mathbf{S}}_{12} = \nabla \cdot \left(\vec{\mathbf{E}}_1 \times \vec{\mathbf{H}}_2 \right) - \nabla \cdot \left(\vec{\mathbf{E}}_2 \times \vec{\mathbf{H}}_1 \right) = \left[\vec{\mathbf{H}}_2 \cdot \left(\nabla \times \vec{\mathbf{E}}_1 \right) - \vec{\mathbf{E}}_1 \cdot \left(\nabla \times \vec{\mathbf{H}}_2 \right) \right] - \left[\vec{\mathbf{H}}_1 \cdot \left(\nabla \times \vec{\mathbf{E}}_2 \right) - \vec{\mathbf{E}}_2 \cdot \left(\nabla \times \vec{\mathbf{H}}_1 \right) \right]$$

$$= \left[\vec{\mathbf{H}}_2 \cdot \left(-j\omega_0 \vec{\mathbf{B}}_1 \right) - \vec{\mathbf{E}}_1 \cdot \left(j\omega_0 \vec{\mathbf{D}}_2 + \vec{\mathbf{J}}_2 + \vec{\mathbf{J}}_{02} \right) \right] - \left[\vec{\mathbf{H}}_1 \cdot \left(-j\omega_0 \vec{\mathbf{B}}_2(\vec{\mathbf{r}}) \right) - \vec{\mathbf{E}}_2 \cdot \left(j\omega_0 \vec{\mathbf{D}}_1 + \vec{\mathbf{J}}_1 + \vec{\mathbf{J}}_{01} \right) \right]$$

$$= \left[\vec{\mathbf{H}}_2 \cdot \left(-j\omega_0 \mu \vec{\mathbf{H}}_1 \right) - \vec{\mathbf{E}}_1 \cdot \left(j\omega_0 \varepsilon \vec{\mathbf{E}}_2 + \sigma \vec{\mathbf{E}}_2 + \vec{\mathbf{J}}_{02} \right) \right] - \left[\vec{\mathbf{H}}_1 \cdot \left(-j\omega_0 \mu \vec{\mathbf{H}}_2 \right) - \vec{\mathbf{E}}_2 \cdot \left(j\omega_0 \varepsilon \vec{\mathbf{E}}_1 + \sigma \vec{\mathbf{E}}_1 + \vec{\mathbf{J}}_{01} \right) \right]$$

$$= \vec{\mathbf{H}}_2 \cdot \left(-j\omega_0 \mu \vec{\mathbf{H}}_1 \right) - \vec{\mathbf{E}}_1 \cdot \left(j\omega_0 \varepsilon \vec{\mathbf{E}}_2 + \sigma \vec{\mathbf{E}}_2 + \vec{\mathbf{J}}_{02} \right) - \vec{\mathbf{H}}_1 \cdot \left(-j\omega_0 \mu \vec{\mathbf{H}}_2 \right) + \vec{\mathbf{E}}_2 \cdot \left(j\omega_0 \varepsilon \vec{\mathbf{E}}_1 + \sigma \vec{\mathbf{E}}_1 + \vec{\mathbf{J}}_{01} \right)$$

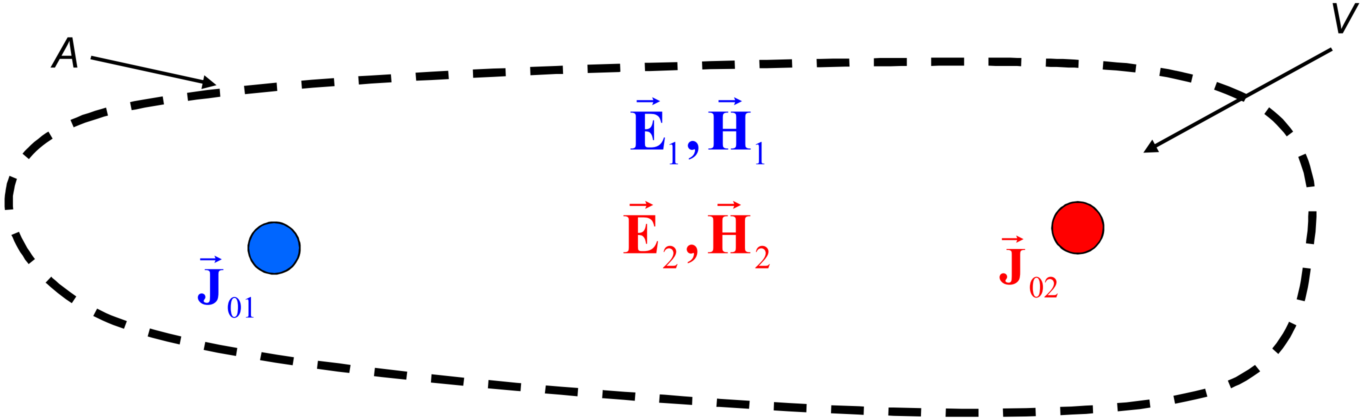
$$= -j\omega_0 \mu \vec{\mathbf{H}}_2 \cdot \vec{\mathbf{H}}_1 - j\omega_0 \varepsilon \vec{\mathbf{E}}_1 \cdot \vec{\mathbf{E}}_2 - \sigma \vec{\mathbf{E}}_1 \cdot \vec{\mathbf{E}}_2 - \vec{\mathbf{E}}_1 \cdot \vec{\mathbf{J}}_{02} + j\omega_0 \mu \vec{\mathbf{H}}_1 \cdot \vec{\mathbf{H}}_2 + j\omega_0 \varepsilon \vec{\mathbf{E}}_2 \cdot \vec{\mathbf{E}}_1 + \sigma \vec{\mathbf{E}}_2 \cdot \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 \cdot \vec{\mathbf{J}}_{01}$$

Hypotheses on the medium (PD)

- **Linear**
- **Isotropic**
- **Space-Nondispersive**
- **Time-invariant**
- **Time-Dispersive**

$$\begin{cases} \vec{\mathbf{D}} = \varepsilon \vec{\mathbf{E}} \\ \vec{\mathbf{B}} = \mu \vec{\mathbf{H}} \\ \vec{\mathbf{J}} = \sigma \vec{\mathbf{E}} \end{cases}$$

Reciprocity theorem: one consideration



$$= \vec{\mathbf{H}}_2 \cdot \left(-j\omega_0\mu\vec{\mathbf{H}}_1\right) - \vec{\mathbf{E}}_1 \cdot \left(j\omega_0\varepsilon\vec{\mathbf{E}}_2 + \sigma\vec{\mathbf{E}}_2 + \vec{\mathbf{J}}_{02}\right) - \vec{\mathbf{H}}_1 \cdot \left(-j\omega_0\mu\vec{\mathbf{H}}_2\right) + \vec{\mathbf{E}}_2 \cdot \left(j\omega_0\varepsilon\vec{\mathbf{E}}_1 + \sigma\vec{\mathbf{E}}_1 + \vec{\mathbf{J}}_{01}\right)$$

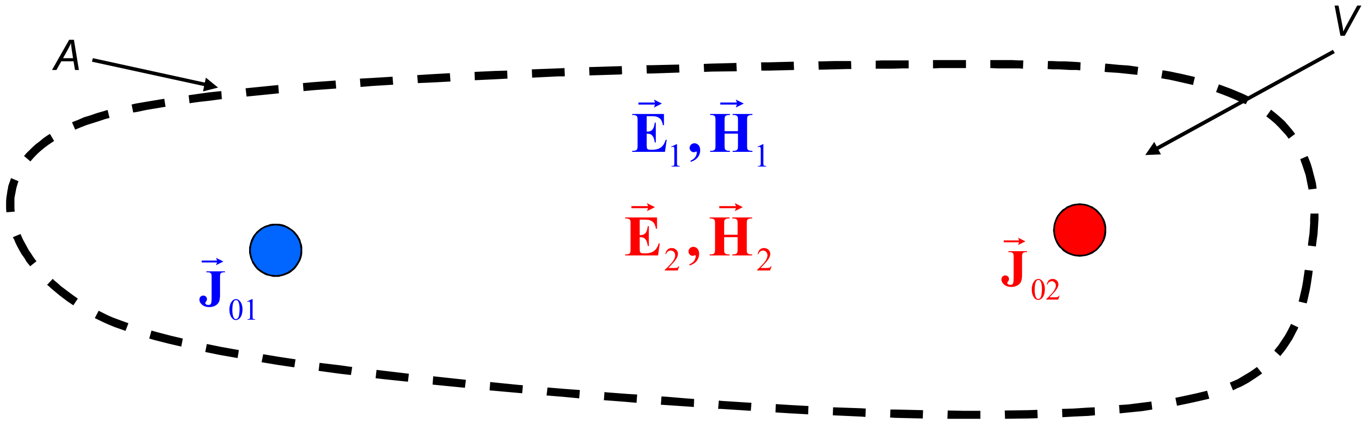
$$= -j\omega_0\mu\vec{\mathbf{H}}_2 \cdot \vec{\mathbf{H}}_1 - j\omega_0\varepsilon\vec{\mathbf{E}}_1 \cdot \vec{\mathbf{E}}_2 - \sigma\vec{\mathbf{E}}_1 \cdot \vec{\mathbf{E}}_2 - \vec{\mathbf{E}}_1 \cdot \vec{\mathbf{J}}_{02} + j\omega_0\mu\vec{\mathbf{H}}_1 \cdot \vec{\mathbf{H}}_2 + j\omega_0\varepsilon\vec{\mathbf{E}}_2 \cdot \vec{\mathbf{E}}_1 + \sigma\vec{\mathbf{E}}_2 \cdot \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 \cdot \vec{\mathbf{J}}_{01}$$

Hypotheses on the medium (PD)

- Linear
- Isotropic
- Space-Nondispersive
- Time-invariant
- Time-Dispersive

$$\begin{cases} \vec{\mathbf{D}} = \varepsilon\vec{\mathbf{E}} \\ \vec{\mathbf{B}} = \mu\vec{\mathbf{H}} \\ \vec{\mathbf{J}} = \sigma\vec{\mathbf{E}} \end{cases}$$

Reciprocity theorem: one consideration



$$\vec{\mathbf{H}}_1 \cdot \mu \vec{\mathbf{H}}_2 = \vec{\mathbf{H}}_2 \cdot \mu \vec{\mathbf{H}}_1$$

$$= \vec{\mathbf{H}}_2 \cdot \left(-j\omega_0 \mu \vec{\mathbf{H}}_1\right) - \vec{\mathbf{E}}_1 \cdot \left(j\omega_0 \varepsilon \vec{\mathbf{E}}_2 + \sigma \vec{\mathbf{E}}_2 + \vec{\mathbf{J}}_{02}\right) - \vec{\mathbf{H}}_1 \cdot \left(-j\omega_0 \mu \vec{\mathbf{H}}_2\right) + \vec{\mathbf{E}}_2 \cdot \left(j\omega_0 \varepsilon \vec{\mathbf{E}}_1 + \sigma \vec{\mathbf{E}}_1 + \vec{\mathbf{J}}_{01}\right)$$

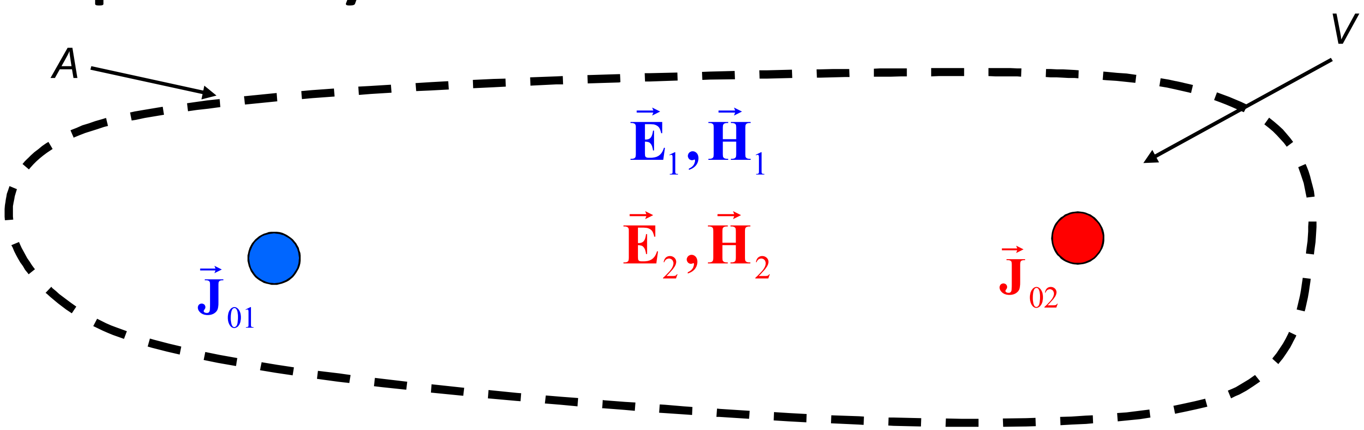
$$= -j\omega_0 \mu \vec{\mathbf{H}}_2 \cdot \vec{\mathbf{H}}_1 - j\omega_0 \varepsilon \vec{\mathbf{E}}_1 \cdot \vec{\mathbf{E}}_2 - \sigma \vec{\mathbf{E}}_1 \cdot \vec{\mathbf{E}}_2 - \vec{\mathbf{E}}_1 \cdot \vec{\mathbf{J}}_{02} + j\omega_0 \mu \vec{\mathbf{H}}_1 \cdot \vec{\mathbf{H}}_2 + j\omega_0 \varepsilon \vec{\mathbf{E}}_2 \cdot \vec{\mathbf{E}}_1 + \sigma \vec{\mathbf{E}}_2 \cdot \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 \cdot \vec{\mathbf{J}}_{01}$$

Hypotheses on the medium (PD)

- Linear
- Isotropic
- Space-Nondispersive
- Time-invariant
- Time-Dispersive

$$\begin{cases} \vec{\mathbf{D}} = \varepsilon \vec{\mathbf{E}} \\ \vec{\mathbf{B}}(\vec{\mathbf{r}}) = \mu \vec{\mathbf{H}} \\ \vec{\mathbf{J}}(\vec{\mathbf{r}}) = \sigma \vec{\mathbf{E}} \end{cases}$$

Reciprocity theorem: one consideration



~~$\vec{\mathbf{H}}_1 \cdot \mu \vec{\mathbf{H}}_2 = \vec{\mathbf{H}}_2 \cdot \mu \vec{\mathbf{H}}_1$~~

$$\vec{\mathbf{H}}_1 \cdot \left[\boldsymbol{\mu} \cdot \vec{\mathbf{H}}_2 \right] \neq \vec{\mathbf{H}}_2 \cdot \left[\boldsymbol{\mu} \cdot \vec{\mathbf{H}}_1 \right]$$

$$\vec{\mathbf{E}}_1 \cdot \left[\boldsymbol{\varepsilon} \cdot \vec{\mathbf{E}}_2 \right] \neq \vec{\mathbf{E}}_2 \cdot \left[\boldsymbol{\varepsilon} \cdot \vec{\mathbf{E}}_1 \right]$$

Note however that when the matrixes μ and ε are symmetrical (reciprocal media):

$$\vec{\mathbf{H}}_1 \cdot \left[\boldsymbol{\mu} \cdot \vec{\mathbf{H}}_2 \right] = \vec{\mathbf{H}}_2 \cdot \left[\boldsymbol{\mu} \cdot \vec{\mathbf{H}}_1 \right]$$

$$\vec{\mathbf{E}}_1 \cdot \left[\boldsymbol{\varepsilon} \cdot \vec{\mathbf{E}}_2 \right] = \vec{\mathbf{E}}_2 \cdot \left[\boldsymbol{\varepsilon} \cdot \vec{\mathbf{E}}_1 \right]$$

Hypotheses on the medium (PD)

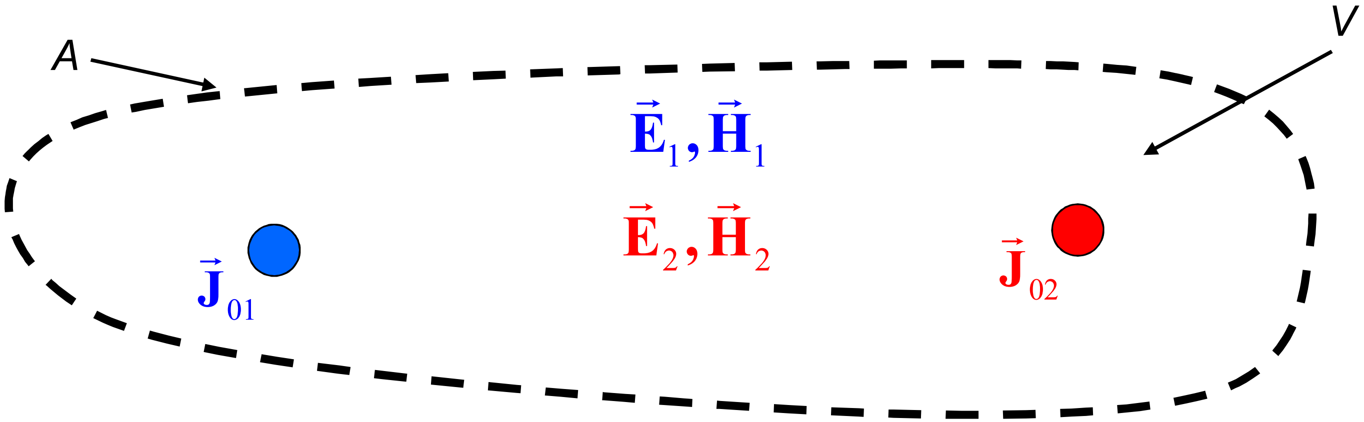
- **Linear**
- **Anisotropic**
- **Space-Nondispersive**
- **Time-invariant**
- **Time-Dispersive**

$$\begin{cases} \vec{\mathbf{D}} = \boldsymbol{\varepsilon} \cdot \vec{\mathbf{E}} \\ \vec{\mathbf{B}}(\vec{\mathbf{r}}) = \boldsymbol{\mu} \cdot \vec{\mathbf{H}} \\ \vec{\mathbf{J}}(\vec{\mathbf{r}}) = \sigma \vec{\mathbf{E}} \end{cases}$$

$$= \vec{\mathbf{H}}_2 \cdot \left(-j\omega_0 \mu \vec{\mathbf{H}}_1 \right) - \vec{\mathbf{E}}_1 \cdot \left(j\omega_0 \varepsilon \vec{\mathbf{E}}_2 + \sigma \vec{\mathbf{E}}_2 + \vec{\mathbf{J}}_{02} \right) - \vec{\mathbf{H}}_1 \cdot \left(-j\omega_0 \mu \vec{\mathbf{H}}_2 \right) + \vec{\mathbf{E}}_2 \cdot \left(j\omega_0 \varepsilon \vec{\mathbf{E}}_1 + \sigma \vec{\mathbf{E}}_1 + \vec{\mathbf{J}}_{01} \right)$$

$$= -j\omega_0 \mu \vec{\mathbf{H}}_2 \cdot \vec{\mathbf{H}}_1 - j\omega_0 \varepsilon \vec{\mathbf{E}}_1 \cdot \vec{\mathbf{E}}_2 - \sigma \vec{\mathbf{E}}_1 \cdot \vec{\mathbf{E}}_2 - \vec{\mathbf{E}}_1 \cdot \vec{\mathbf{J}}_{02} + j\omega_0 \mu \vec{\mathbf{H}}_1 \cdot \vec{\mathbf{H}}_2 + j\omega_0 \varepsilon \vec{\mathbf{E}}_2 \cdot \vec{\mathbf{E}}_1 + \sigma \vec{\mathbf{E}}_2 \cdot \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 \cdot \vec{\mathbf{J}}_{01}$$

Reciprocity theorem: one consideration



~~$\vec{\mathbf{H}}_1 \cdot \mu \vec{\mathbf{H}}_2 = \vec{\mathbf{H}}_2 \cdot \mu \vec{\mathbf{H}}_1$~~

$$\vec{\mathbf{H}}_1 \cdot \left[\boldsymbol{\mu} \cdot \vec{\mathbf{H}}_2 \right] \neq \vec{\mathbf{H}}_2 \cdot \left[\boldsymbol{\mu} \cdot \vec{\mathbf{H}}_1 \right]$$

$$\vec{\mathbf{E}}_1 \cdot \left[\boldsymbol{\varepsilon} \cdot \vec{\mathbf{E}}_2 \right] \neq \vec{\mathbf{E}}_2 \cdot \left[\boldsymbol{\varepsilon} \cdot \vec{\mathbf{E}}_1 \right]$$

Note however that when the matrixes μ and ε are symmetrical (reciprocal media):

$$\vec{\mathbf{H}}_1 \cdot \left[\boldsymbol{\mu} \cdot \vec{\mathbf{H}}_2 \right] = \vec{\mathbf{H}}_2 \cdot \left[\boldsymbol{\mu} \cdot \vec{\mathbf{H}}_1 \right]$$

$$\vec{\mathbf{E}}_1 \cdot \left[\boldsymbol{\varepsilon} \cdot \vec{\mathbf{E}}_2 \right] = \vec{\mathbf{E}}_2 \cdot \left[\boldsymbol{\varepsilon} \cdot \vec{\mathbf{E}}_1 \right]$$

Hypotheses on the medium (PD)

- **Linear**
- **Reciprocal**
- **Space-Nondispersive**
- **Time-invariant**
- **Time-Dispersive**

$$\begin{cases} \vec{\mathbf{D}} = \boldsymbol{\varepsilon} \cdot \vec{\mathbf{E}} \\ \vec{\mathbf{B}}(\vec{\mathbf{r}}) = \boldsymbol{\mu} \cdot \vec{\mathbf{H}} \\ \vec{\mathbf{J}}(\vec{\mathbf{r}}) = \sigma \vec{\mathbf{E}} \end{cases}$$

$$= \vec{\mathbf{H}}_2 \cdot \left(-j\omega_0 \mu \vec{\mathbf{H}}_1 \right) - \vec{\mathbf{E}}_1 \cdot \left(j\omega_0 \varepsilon \vec{\mathbf{E}}_2 + \sigma \vec{\mathbf{E}}_2 + \vec{\mathbf{J}}_{02} \right) - \vec{\mathbf{H}}_1 \cdot \left(-j\omega_0 \mu \vec{\mathbf{H}}_2 \right) + \vec{\mathbf{E}}_2 \cdot \left(j\omega_0 \varepsilon \vec{\mathbf{E}}_1 + \sigma \vec{\mathbf{E}}_1 + \vec{\mathbf{J}}_{01} \right)$$

$$= -j\omega_0 \mu \vec{\mathbf{H}}_2 \cdot \vec{\mathbf{H}}_1 - j\omega_0 \varepsilon \vec{\mathbf{E}}_1 \cdot \vec{\mathbf{E}}_2 - \sigma \vec{\mathbf{E}}_1 \cdot \vec{\mathbf{E}}_2 - \vec{\mathbf{E}}_1 \cdot \vec{\mathbf{J}}_{02} + j\omega_0 \mu \vec{\mathbf{H}}_1 \cdot \vec{\mathbf{H}}_2 + j\omega_0 \varepsilon \vec{\mathbf{E}}_2 \cdot \vec{\mathbf{E}}_1 + \sigma \vec{\mathbf{E}}_2 \cdot \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 \cdot \vec{\mathbf{J}}_{01}$$

Reciprocity theorem

$$\vec{\mathbf{S}}_{12} = \vec{\mathbf{E}}_1 \times \vec{\mathbf{H}}_2 - \vec{\mathbf{E}}_2 \times \vec{\mathbf{H}}_1$$

$$\oiint_A dA\,\vec{\mathbf{S}}_{12} \cdot \hat{\mathbf{n}} = \iiint_V dV \left[\vec{\mathbf{J}}_{01} \cdot \vec{\mathbf{E}}_2 - \vec{\mathbf{J}}_{02} \cdot \vec{\mathbf{E}}_1 \right]$$

An interesting case

If

$$\oiint_A dA\,\vec{\mathbf{S}}_{12} \cdot \hat{\mathbf{n}} = 0$$

(f.i., when the surface material is a PEC or when the volume encompasses all the space), the reciprocity theorem simplifies as:

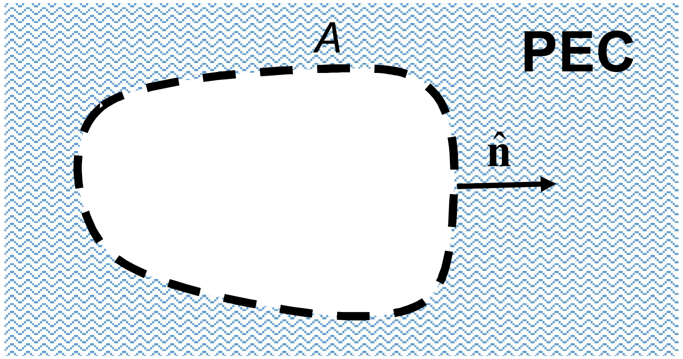
$$\iiint_V dV\,\vec{\mathbf{J}}_{01} \cdot \vec{\mathbf{E}}_2 = \iiint_V dV\,\vec{\mathbf{J}}_{02} \cdot \vec{\mathbf{E}}_1$$

Reciprocity theorem

$$\vec{\mathbf{S}}_{12} = \vec{\mathbf{E}}_1 \times \vec{\mathbf{H}}_2 - \vec{\mathbf{E}}_2 \times \vec{\mathbf{H}}_1$$

$$\oiint_A dA \vec{\mathbf{S}}_{12} \cdot \hat{\mathbf{n}} = \oiint_A dA \left[\vec{\mathbf{E}}_1 \times \vec{\mathbf{H}}_2 \right] \cdot \hat{\mathbf{n}} - \oiint_A dA \left[\vec{\mathbf{E}}_2 \times \vec{\mathbf{H}}_1 \right] \cdot \hat{\mathbf{n}} +$$

$$= \oiint_A dA \left[\hat{\mathbf{n}} \times \vec{\mathbf{E}}_1 \right] \cdot \vec{\mathbf{H}}_2 - \oiint_A dA \left[\hat{\mathbf{n}} \times \vec{\mathbf{E}}_2 \right] \cdot \vec{\mathbf{H}}_1 = 0$$



$$\vec{\mathbf{A}} \cdot \left[\vec{\mathbf{B}} \times \vec{\mathbf{C}} \right] = \vec{\mathbf{C}} \cdot \left[\vec{\mathbf{A}} \times \vec{\mathbf{B}} \right] = \vec{\mathbf{B}} \cdot \left[\vec{\mathbf{C}} \times \vec{\mathbf{A}} \right]$$