Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2020-2021 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

Stefano Perna

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

THEOREMS

Poynting

Time domain – Phasor domain

Uniqueness (Interior problem – Exterior problem)

Time domain – Phasor domain

Equivalence

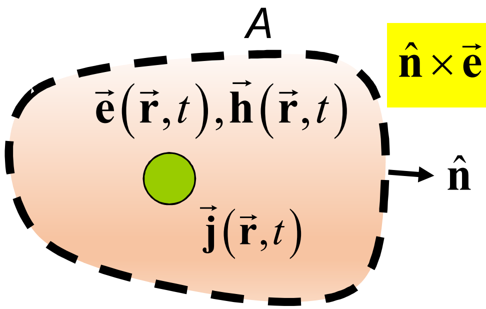
Phasor domain

Image Theory

Reciprocity

Phasor domain

Uniqueness (TD)



$\hat{\mathbf{n}}\times\vec{\mathbf{e}}\ \left(\text{or}\ \ \hat{\mathbf{n}}\times\vec{\mathbf{h}}\right)$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$

Interior Problem

I Consider a source distribution $\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$ with its associated electromagnetic field $\left(\vec{\mathbf{e}},\vec{\mathbf{h}}\right)$

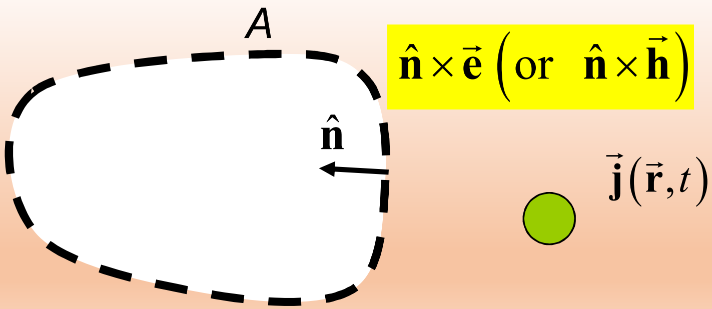
II Consider a (smooth) surface *A* with an everywhere defined unit normal $\hat{\mathbf{n}}$

III Consider the values of the electromagnetic field everywhere in **the finite volume V** bounded by the surface A **at the initial time**; that is, consider $\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$

IV Consider the values of the tangential component of the electric (**or** magnetic) field upon the surface A at any time after the initial one; that is, consider $\hat{\mathbf{n}}\times\vec{\mathbf{e}}$ (**or** $\hat{\mathbf{n}}\times\vec{\mathbf{h}}$) **on the boundary at any time**

The Uniqueness Theorem states that the electromagnetic field produced by the source in (I) within the **finite volume V bounded** by the surface A in (II), enforcing **the initial condition** in (III) and **the boundary condition** in (IV) is unique.

Uniqueness (TD)



$\vec{\mathbf{e}}(\vec{\mathbf{r}},t),\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$ $\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$

Exterior Problem

I Consider a source distribution $\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$ with its associated electromagnetic field $\left(\vec{\mathbf{e}},\vec{\mathbf{h}}\right)$

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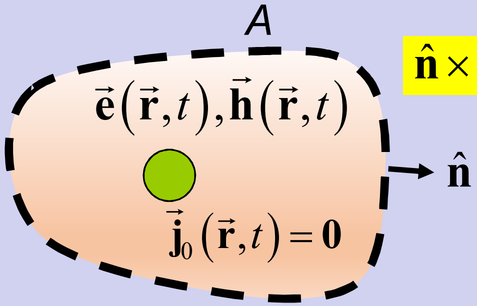
Uniqueness (TD-Interior Problem)



$\hat{\mathbf{n}}\times\vec{\mathbf{e}}=\mathbf{0}$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$$

Let's apply the Poynting theorem (TD)

Medium
- **Linear**
- **Isotropic**
- **Space-Nondispersive**
- **Time-Nondispersive**
- **Time-invariant**

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

Source distribution $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)$ **= 0**

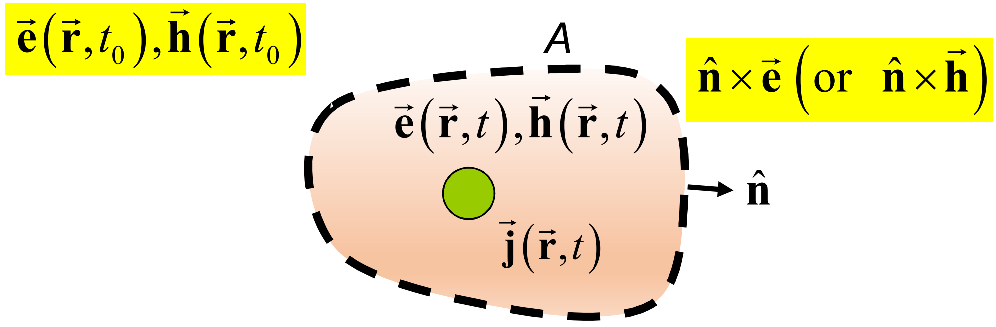
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$$

$\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\mathbf{0}$ on the boundary

$$\frac{d}{dt}W(t)+P_j(t)=0 \;\Rightarrow\; \frac{d}{dt}W(t)=-P_j(t) \;\Rightarrow\; \begin{array}{l} W(t_0)=0 \\ \dfrac{d}{\partial t}W(t)\le 0 \\ W(t)\ge 0 \end{array} \;\Rightarrow\; W(t)=0 \;\Rightarrow\; \begin{array}{l} \vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\mathbf{0} \\ \vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\mathbf{0} \end{array}$$

cvd

Uniqueness (TD-Interior Problem)



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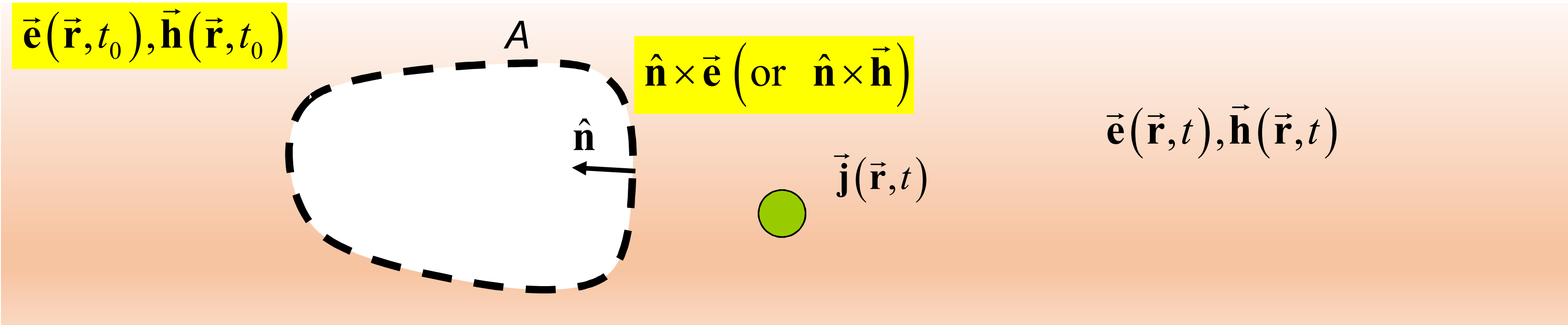
Uniqueness (TD-Exterior Problem)



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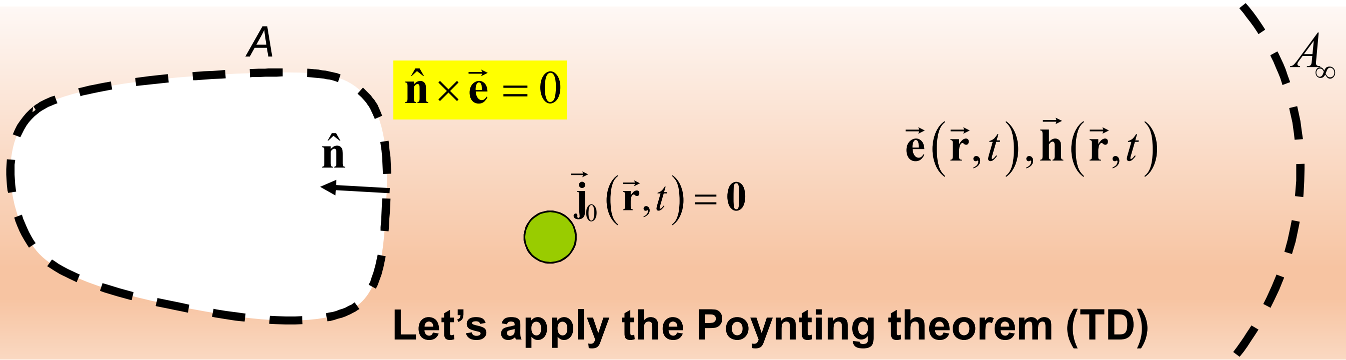

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$\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\mathbf{0}$ on the boundary

$$\cancel{\oint\!\!\oint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)}\cdot\hat{\mathbf{n}}+\oint\!\!\oint_{A_\infty}\cancel{dA_\infty\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}}+\frac{d}{dt}\iiint_V dV\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right]+\iiint_V dV\,\sigma\left|\vec{\mathbf{e}}\right|^2=-\cancel{\iiint_V dV\,\vec{\mathbf{j}}_0}\cdot\vec{\mathbf{e}}$$

$$\Rightarrow\quad \begin{aligned} &W(t_0)=0\\ &\frac{d}{dt}W(t)\le 0\\ &W(t)\ge 0\end{aligned}\quad\Rightarrow\quad \begin{aligned}&\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\mathbf{0}\\ &\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\mathbf{0}\end{aligned}$$

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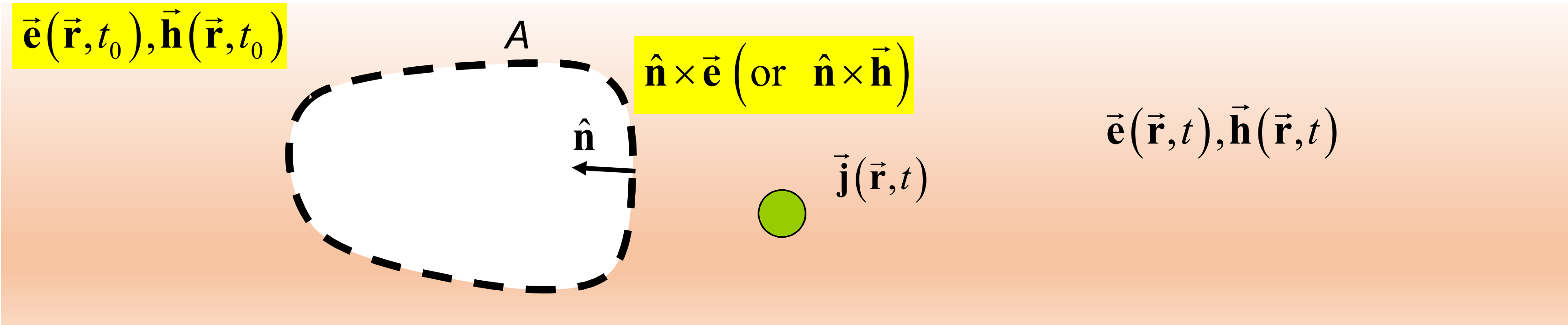
Uniqueness (TD-Exterior Problem)



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Mathematical tools that we will exploit today

$$\vec{\mathbf{A}} \times \left(\vec{\mathbf{B}} \times \vec{\mathbf{C}}\right) = \left(\vec{\mathbf{A}} \cdot \vec{\mathbf{C}}\right)\vec{\mathbf{B}} - \left(\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}\right)\vec{\mathbf{C}}$$

Let A be the surface of a sphere of radius r

$$\oint\!\!\!\oint_A dA\,\Phi\left(\vec{\mathbf{r}}\right) = \oint\!\!\!\oint_A dA\,\Phi\left(r,\vartheta,\varphi\right) = \int_0^{2\pi} d\varphi \int_0^{\pi} d\vartheta r^2 \sin\vartheta\,\Phi\left(r,\vartheta,\varphi\right)$$

$$dA = r^2 \sin\vartheta d\vartheta d\varphi$$

The radiation condition

$$\vec{\mathbf{e}} \cdot \hat{\mathbf{n}} = 0$$

$$\vec{\mathbf{h}} \cdot \hat{\mathbf{n}} = 0 \qquad as \quad r \to \infty$$

$$\vec{\mathbf{e}} - \zeta \vec{\mathbf{h}} \times \hat{\mathbf{n}} \sim o\left(\frac{1}{r}\right)$$

$\zeta = \sqrt{\dfrac{\mu}{\varepsilon}}$ is the intrinsic resistance of the medium, which is assumed homogeneous, isotropic, nondispersive and lossless at infinity

The radiation condition

$$\vec{\mathbf{e}}\cdot\hat{\mathbf{n}}=0$$

$$\boxed{\vec{\mathbf{h}}\cdot\hat{\mathbf{n}}=0}$$

$$\boxed{\vec{\mathbf{e}}-\zeta\vec{\mathbf{h}}\times\hat{\mathbf{n}}\sim o\left(\frac{1}{r}\right)} \qquad \left(\textit{and thus } \zeta\vec{\mathbf{h}}-\hat{\mathbf{n}}\times\vec{\mathbf{e}}\sim o\left(\frac{1}{r}\right) \right) \qquad \textit{as} \quad r\to\infty$$

$\zeta=\sqrt{\dfrac{\mu}{\varepsilon}}$ is the intrinsic resistance of the medium, which is assumed homogeneous, isotropic, nondispersive and lossless at infinity

At infinity

$$\vec{\mathbf{e}}=\zeta\vec{\mathbf{h}}\times\hat{\mathbf{n}} \;\Rightarrow\; \boxed{\hat{\mathbf{n}}\times\vec{\mathbf{e}}}=\hat{\mathbf{n}}\times\left(\zeta\vec{\mathbf{h}}\times\hat{\mathbf{n}}\right)=\left(\hat{\mathbf{n}}\cdot\hat{\mathbf{n}}\right)\zeta\vec{\mathbf{h}}-\cancel{\left(\hat{\mathbf{n}}\cdot\zeta\vec{\mathbf{h}}\right)\hat{\mathbf{n}}}\;\boxed{=\zeta\vec{\mathbf{h}}}$$

$$\Downarrow$$

$$\hat{\mathbf{n}}\times\vec{\mathbf{e}}=\zeta\vec{\mathbf{h}}$$

$$\vec{\mathbf{A}}\times\left(\vec{\mathbf{B}}\times\vec{\mathbf{C}}\right)=\left(\vec{\mathbf{A}}\cdot\vec{\mathbf{C}}\right)\vec{\mathbf{B}}-\left(\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}\right)\vec{\mathbf{C}}$$

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$$\vec{\mathbf{s}}=\frac{|\vec{\mathbf{e}}|^2}{\zeta}\hat{\mathbf{n}}$$

At infinity

$$\vec{\mathbf{s}}=\vec{\mathbf{e}}\times\vec{\mathbf{h}}=\frac{1}{\zeta}\vec{\mathbf{e}}\times\left(\hat{\mathbf{n}}\times\vec{\mathbf{e}}\right)=\frac{1}{\zeta}\left[\left(\vec{\mathbf{e}}\cdot\vec{\mathbf{e}}\right)\hat{\mathbf{n}}-\cancel{\left(\vec{\mathbf{e}}\cdot\hat{\mathbf{n}}\right)\vec{\mathbf{e}}}\right]=\frac{|\vec{\mathbf{e}}|^2}{\zeta}\hat{\mathbf{n}}$$

$$\vec{\mathbf{A}}\times\left(\vec{\mathbf{B}}\times\vec{\mathbf{C}}\right)=\left(\vec{\mathbf{A}}\cdot\vec{\mathbf{C}}\right)\vec{\mathbf{B}}-\left(\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}\right)\vec{\mathbf{C}}$$

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$\zeta=\sqrt{\dfrac{\mu}{\varepsilon}}$ is the intrinsic resistance of the medium, which is assumed homogeneous, isotropic, nondispersive and lossless at infinity

$$\vec{\mathbf{s}}=\frac{\left|\vec{\mathbf{e}}\right|^2}{\zeta}\hat{\mathbf{n}}=\zeta\left|\vec{\mathbf{h}}\right|^2\hat{\mathbf{n}}$$

At infinity

$$\vec{\mathbf{s}}=\vec{\mathbf{e}}\times\vec{\mathbf{h}}=\left(\zeta\vec{\mathbf{h}}\times\hat{\mathbf{n}}\right)\times\vec{\mathbf{h}}=-\vec{\mathbf{h}}\times\left(\zeta\vec{\mathbf{h}}\times\hat{\mathbf{n}}\right)=-\left[\cancel{\left(\vec{\mathbf{h}}\cdot\hat{\mathbf{n}}\right)}\zeta\vec{\mathbf{h}}-\left(\vec{\mathbf{h}}\cdot\zeta\vec{\mathbf{h}}\right)\hat{\mathbf{n}}\right]=\zeta\left|\vec{\mathbf{h}}\right|^2\hat{\mathbf{n}}$$

$$\vec{\mathbf{A}}\times\left(\vec{\mathbf{B}}\times\vec{\mathbf{C}}\right)=\left(\vec{\mathbf{A}}\cdot\vec{\mathbf{C}}\right)\vec{\mathbf{B}}-\left(\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}\right)\vec{\mathbf{C}}$$

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$$\vec{\mathbf{h}} \cdot \hat{\mathbf{n}} = 0 \qquad\qquad as \quad r \to \infty$$

$$\vec{\mathbf{e}} - \zeta \vec{\mathbf{h}} \times \hat{\mathbf{n}} \sim o\left(\frac{1}{r}\right) \qquad \left(and\ thus\ \zeta \vec{\mathbf{h}} - \hat{\mathbf{n}} \times \vec{\mathbf{e}} \sim o\left(\frac{1}{r}\right) \right)$$

$\zeta = \sqrt{\dfrac{\mu}{\varepsilon}}$ is the intrinsic resistance of the medium, which is assumed homogeneous, isotropic, nondispersive and lossless at infinity

$$\vec{\mathbf{s}} = \frac{|\vec{\mathbf{e}}|^2}{\zeta} \hat{\mathbf{n}} = \zeta |\vec{\mathbf{h}}|^2 \hat{\mathbf{n}}$$

$$\Downarrow$$

$$\vec{\mathbf{e}} \sim O\left(\frac{1}{r}\right) \qquad \vec{\mathbf{h}} \sim O\left(\frac{1}{r}\right) \qquad as \quad r \to \infty$$

The radiation condition

$$\hat{\mathbf{n}}\cdot\vec{\mathbf{e}}=\hat{\mathbf{n}}\cdot\vec{\mathbf{h}}=0 \qquad \vec{\mathbf{e}}-\zeta\vec{\mathbf{h}}\times\hat{\mathbf{n}}\sim o\left(\frac{1}{r}\right) \qquad \left(and\ \zeta\vec{\mathbf{h}}-\hat{\mathbf{n}}\times\vec{\mathbf{e}}\sim o\left(\frac{1}{r}\right) \right)$$

$$\boxed{\vec{\mathbf{e}}\sim O\left(\frac{1}{r}\right)} \quad \boxed{\vec{\mathbf{h}}\sim O\left(\frac{1}{r}\right)} \qquad \vec{\mathbf{s}}=\frac{\left|\vec{\mathbf{e}}\right|^2}{\zeta}\hat{\mathbf{n}}=\zeta\left|\vec{\mathbf{h}}\right|^2\hat{\mathbf{n}} \qquad as\ \ r\to\infty$$

TD

$$\oiint_{A_\infty} dA_\infty\, \vec{\mathbf{s}}\cdot\hat{\mathbf{n}} = \oiint_{A_\infty} dA_\infty \frac{\left|\vec{\mathbf{e}}\right|^2}{\zeta} = \lim_{r\to\infty} \int_0^{2\pi} d\varphi \int_0^{\pi} d\vartheta\, r^2 \sin\vartheta \frac{\left|\vec{\mathbf{e}}\right|^2}{\zeta}$$ is a finite nonnegative quantity

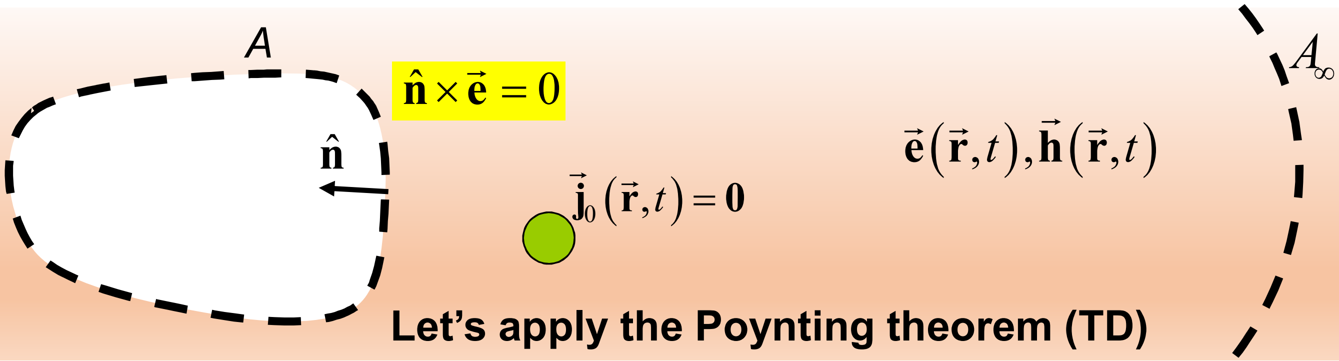
$$\oiint_{A_\infty} dA_\infty\, \vec{\mathbf{s}}\cdot\hat{\mathbf{n}} = \oiint_{A_\infty} dA_\infty\, \zeta\left|\vec{\mathbf{h}}\right|^2 = \lim_{r\to\infty} \int_0^{2\pi} d\varphi \int_0^{\pi} d\vartheta\, r^2 \sin\vartheta\, \zeta\left|\vec{\mathbf{h}}\right|^2$$ is a finite nonnegative quantity

$$\oiint_{A} dA\, \Phi(r,\vartheta,\varphi) = \int_0^{2\pi} d\varphi \int_0^{\pi} d\vartheta\, r^2 \sin\vartheta\, \Phi(r,\vartheta,\varphi)$$

Uniqueness (TD-Exterior Problem)

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$



$\hat{\mathbf{n}}\times\vec{\mathbf{e}}=0$

$\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)=\mathbf{0}$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t),\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$

Let's apply the Poynting theorem (TD)

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$$\Rightarrow\quad \begin{cases} W(t_0)=0\\ \dfrac{d}{dt}W(t)\le 0\\ W(t)\ge 0\end{cases}\quad\Rightarrow\quad \begin{cases}\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\mathbf{0}\\ \vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\mathbf{0}\end{cases}$$

cvd

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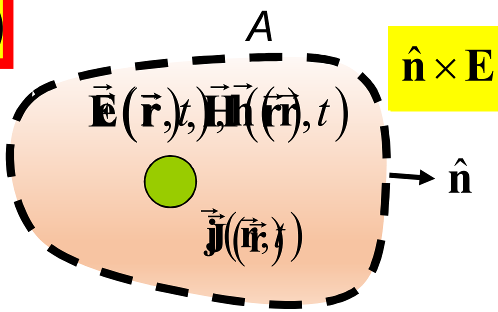

$\hat{\mathbf{n}}\times\mathbf{E}\left(\text{or }\ \hat{\mathbf{n}}\times\vec{\mathbf{H}}\right)$

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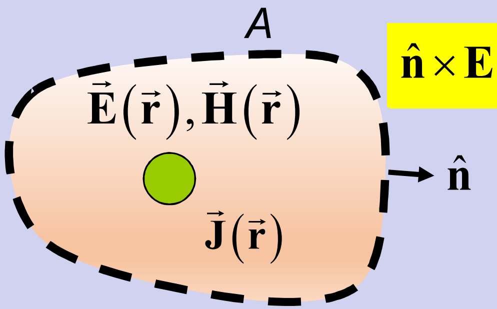
Uniqueness (PD-Interior Problem)



$\hat{\mathbf{n}} \times \mathbf{E} \left(\text{or } \ \hat{\mathbf{n}} \times \vec{\mathbf{H}}\right)$

Source distribution: $\vec{\mathbf{J}}(\vec{\mathbf{r}})$

$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}), \vec{\mathbf{H}}_1(\vec{\mathbf{r}})$ $\qquad$ $\vec{\mathbf{E}}_2(\vec{\mathbf{r}}), \vec{\mathbf{H}}_2(\vec{\mathbf{r}})$

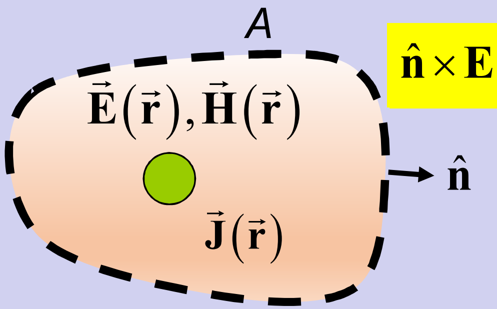
$$\hat{\mathbf{n}} \times \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) = \hat{\mathbf{n}} \times \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) \text{ on the boundary}$$

Field difference: source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}})$ **= 0**

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) \qquad \vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{H}}_2(\vec{\mathbf{r}})$$

$$\hat{\mathbf{n}} \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \hat{\mathbf{n}} \times \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) - \hat{\mathbf{n}} \times \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = 0 \quad \text{on the boundary}$$

Uniqueness (PD-Interior Problem)



$\hat{\mathbf{n}} \times \mathbf{E} \left(\text{or } \; \hat{\mathbf{n}} \times \vec{\mathbf{H}}\right)$

Field difference: source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}})$ = 0

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) \qquad \vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{H}}_2(\vec{\mathbf{r}})$$

$$\hat{\mathbf{n}} \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \hat{\mathbf{n}} \times \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) - \hat{\mathbf{n}} \times \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = 0 \quad \text{on the boundary}$$

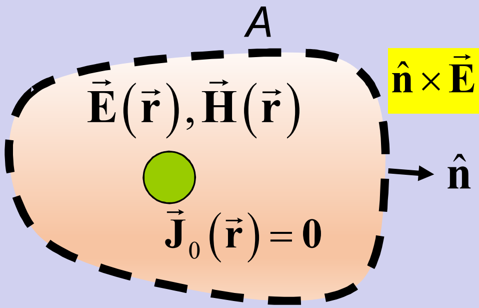
Uniqueness (PD-Interior Problem)

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{E}}_2(\vec{\mathbf{r}})$$
$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{H}}_2(\vec{\mathbf{r}})$$

Source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}})$ **= 0**

$\hat{\mathbf{n}} \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathbf{0}$ on the boundary

Uniqueness (PD-Interior Problem)



$\hat{\mathbf{n}}\times\vec{\mathbf{E}}(\vec{\mathbf{r}})=\mathbf{0}$

Let's apply the Poynting theorem (PD)

Medium

- **Linear**
- **Isotropic**
- **Space-Nondispersive**
- **Time-Dispersive**
- **Time-invariant**

$$\vec{\mathbf{E}}(\vec{\mathbf{r}})=\vec{\mathbf{E}}_1(\vec{\mathbf{r}})-\vec{\mathbf{E}}_2(\vec{\mathbf{r}})$$
$$\vec{\mathbf{H}}(\vec{\mathbf{r}})=\vec{\mathbf{H}}_1(\vec{\mathbf{r}})-\vec{\mathbf{H}}_2(\vec{\mathbf{r}})$$

Source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}})$ **= 0**

$\hat{\mathbf{n}}\times\vec{\mathbf{E}}(\vec{\mathbf{r}})=\mathbf{0}$ on the boundary

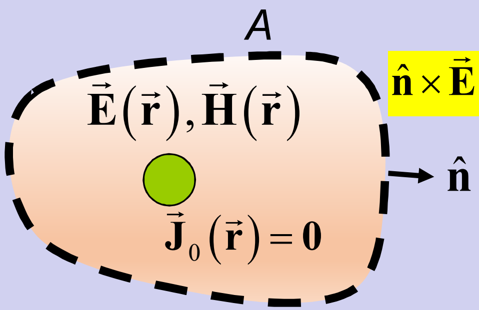
$$\cancel{\oiint_A dA\,\vec{\mathbf{S}}_1(\vec{\mathbf{r}})\cdot\hat{\mathbf{n}}}+\iiint_V dV\left[\frac{1}{2}\omega_0\mu_2\left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2+\frac{1}{2}\omega_0\varepsilon_2\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2+\frac{1}{2}\sigma\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2\right]=\iiint_V dV\left[-\frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{E}}(\vec{\mathbf{r}})\cdot\vec{\mathbf{J}}_0^*(\vec{\mathbf{r}})\right\}\right]$$

$$\oiint_A dA\,\vec{\mathbf{S}}_2(\vec{\mathbf{r}})\cdot\hat{\mathbf{n}}+2\omega_0\iiint_V dV\left[\frac{1}{4}\mu_1\left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2-\frac{1}{4}\varepsilon_1\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2\right]=\iiint_V dV\left[-\frac{1}{2}\mathrm{Im}\left\{\vec{\mathbf{E}}(\vec{\mathbf{r}})\cdot\vec{\mathbf{J}}_0^*(\vec{\mathbf{r}})\right\}\right]$$

$$\oiint_A dA\,\vec{\mathbf{S}}_1(\vec{\mathbf{r}})\cdot\hat{\mathbf{n}}=\mathrm{Re}\left\{\oiint_A dA\left[\frac{1}{2}\vec{\mathbf{E}}(\vec{\mathbf{r}})\times\vec{\mathbf{H}}^*(\vec{\mathbf{r}})\right]\cdot\hat{\mathbf{n}}\right\}=\mathrm{Re}\left\{\oiint_A dA\left[\frac{1}{2}\boxed{\hat{\mathbf{n}}\times\vec{\mathbf{E}}(\vec{\mathbf{r}})}\right]\cdot\vec{\mathbf{H}}^*(\vec{\mathbf{r}})\right\}=\mathrm{Re}\left\{\oiint_A dA\left[\frac{1}{2}\vec{\mathbf{H}}^*(\vec{\mathbf{r}})\times\hat{\mathbf{n}}\right]\cdot\vec{\mathbf{E}}(\vec{\mathbf{r}})\right\}=0$$

$$\vec{\mathbf{A}}\cdot\left[\vec{\mathbf{B}}\times\vec{\mathbf{C}}\right]=\vec{\mathbf{C}}\cdot\left[\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right]=\vec{\mathbf{B}}\cdot\left[\vec{\mathbf{C}}\times\vec{\mathbf{A}}\right]$$

Uniqueness (PD-Interior Problem)



$\hat{\mathbf{n}}\times\vec{\mathbf{E}}(\vec{\mathbf{r}})=\mathbf{0}$

Let's apply the Poynting theorem (PD)

Medium

- **Linear**
- **Isotropic**
- **Space-Nondispersive**
- **Time-Dispersive**
- **Time-invariant**

$\vec{\mathbf{E}}(\vec{\mathbf{r}})=\vec{\mathbf{E}}_1(\vec{\mathbf{r}})-\vec{\mathbf{E}}_2(\vec{\mathbf{r}})$

$\vec{\mathbf{H}}(\vec{\mathbf{r}})=\vec{\mathbf{H}}_1(\vec{\mathbf{r}})-\vec{\mathbf{H}}_2(\vec{\mathbf{r}})$

Source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}})$ **= 0**

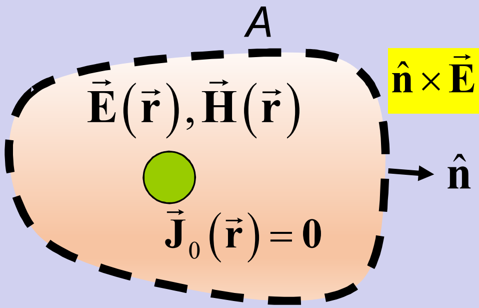
$\hat{\mathbf{n}}\times\vec{\mathbf{E}}(\vec{\mathbf{r}})=\mathbf{0}$ on the boundary

$$\cancel{\oint\!\!\!\oint_A dA\,\vec{\mathbf{S}}_1(\vec{\mathbf{r}})\cdot\hat{\mathbf{n}}}+\iiint_V dV\left[\frac{1}{2}\omega_0\mu_2\left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2+\frac{1}{2}\omega_0\varepsilon_2\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2+\frac{1}{2}\sigma\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2\right]=\iiint_V dV\left[-\frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{E}}(\vec{\mathbf{r}})\cdot\vec{\mathbf{J}}_0^*(\vec{\mathbf{r}})\right\}\right]$$

$$\cancel{\oint\!\!\!\oint_A dA\,\vec{\mathbf{S}}_2(\vec{\mathbf{r}})\cdot\hat{\mathbf{n}}}+2\omega_0\iiint_V dV\left[\frac{1}{4}\mu_1\left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2-\frac{1}{4}\varepsilon_1\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2\right]=\iiint_V dV\left[-\frac{1}{2}\mathrm{Im}\left\{\vec{\mathbf{E}}(\vec{\mathbf{r}})\cdot\vec{\mathbf{J}}_0^*(\vec{\mathbf{r}})\right\}\right]$$

$$\oint\!\!\!\oint_A dA\,\vec{\mathbf{S}}_2(\vec{\mathbf{r}})\cdot\hat{\mathbf{n}}=\mathrm{Im}\left\{\oint\!\!\!\oint_A dA\left[\frac{1}{2}\vec{\mathbf{E}}(\vec{\mathbf{r}})\times\vec{\mathbf{H}}^*(\vec{\mathbf{r}})\right]\cdot\hat{\mathbf{n}}\right\}=\mathrm{Im}\left\{\oint\!\!\!\oint_A dA\left[\frac{1}{2}\boxed{\hat{\mathbf{n}}\times\vec{\mathbf{E}}(\vec{\mathbf{r}})}\right]\cdot\vec{\mathbf{H}}^*(\vec{\mathbf{r}})\right\}=0$$

Uniqueness (PD-Interior Problem)



$\hat{\mathbf{n}}\times\vec{\mathbf{E}}(\vec{\mathbf{r}})=\mathbf{0}$

Let's apply the Poynting theorem (PD)

Medium
- **Linear**
- **Isotropic**
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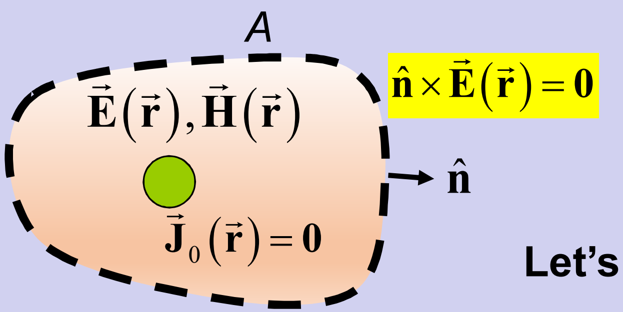
Source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}})=\mathbf{0}$

$\hat{\mathbf{n}}\times\vec{\mathbf{E}}(\vec{\mathbf{r}})=\mathbf{0}$ on the boundary

$$\cancel{\oint\!\!\!\oint_A dA\,\vec{\mathbf{S}}_1(\vec{\mathbf{r}})\cdot\hat{\mathbf{n}}}+\iiint_V dV\left[\frac{1}{2}\omega_0\mu_2\left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2+\frac{1}{2}\omega_0\varepsilon_2\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2+\frac{1}{2}\sigma\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2\right]=\cancel{\iiint_V dV\left[-\frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{E}}(\vec{\mathbf{r}})\cdot\boxed{\vec{\mathbf{J}}_0^*(\vec{\mathbf{r}})}\right\}\right]}$$

$$\cancel{\oint\!\!\!\oint_A dA\,\vec{\mathbf{S}}_2(\vec{\mathbf{r}})\cdot\hat{\mathbf{n}}}+2\omega_0\iiint_V dV\left[\frac{1}{4}\mu_1\left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2-\frac{1}{4}\varepsilon_1\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2\right]=\cancel{\iiint_V dV\left[-\frac{1}{2}\mathrm{Im}\left\{\vec{\mathbf{E}}(\vec{\mathbf{r}})\cdot\boxed{\vec{\mathbf{J}}_0^*(\vec{\mathbf{r}})}\right\}\right]}$$

Uniqueness (PD-Interior Problem)



Let's apply the Poynting theorem (PD)

Medium
- **Linear**
- **Isotropic**
- **Space-Nondispersive**
- **Time-Dispersive**
- **Time-invariant**

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{E}}_2(\vec{\mathbf{r}})$$
$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{H}}_2(\vec{\mathbf{r}})$$

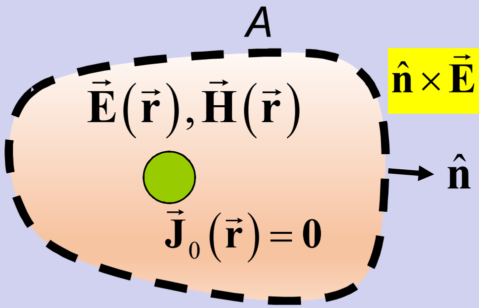
Source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}}) = \mathbf{0}$

$\hat{\mathbf{n}} \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathbf{0}$ on the boundary

$$\iiint_V dV \left[\frac{1}{2}\omega_0 \mu_2 \left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2 + \frac{1}{2}\omega_0 \varepsilon_2 \left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2 + \frac{1}{2}\sigma \left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2 \right] = 0$$

$$2\omega_0 \iiint_V dV \left[\frac{1}{4}\mu_1 \left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2 - \frac{1}{4}\varepsilon_1 \left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2 \right] = 0$$

Uniqueness (PD-Interior Problem)



$\hat{\mathbf{n}}\times\vec{\mathbf{E}}(\vec{\mathbf{r}})=\mathbf{0}$

Let's apply the Poynting theorem (PD)

Medium
- **Linear**
- **Isotropic**
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Source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}})=\mathbf{0}$

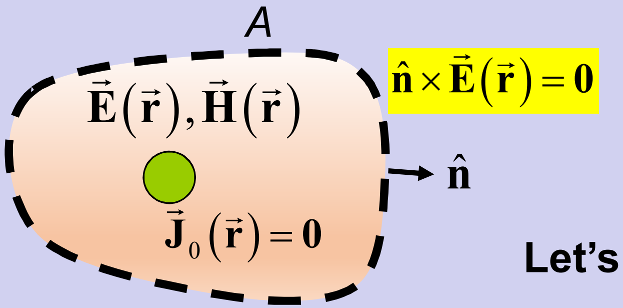
$\hat{\mathbf{n}}\times\vec{\mathbf{E}}(\vec{\mathbf{r}})=\mathbf{0}$ on the boundary

$$\iiint_V dV\left[\frac{1}{2}\omega_0\mu_2\left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2+\frac{1}{2}\omega_0\varepsilon_2\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2+\frac{1}{2}\sigma\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2\right]=0 \Rightarrow \begin{array}{l}\vec{\mathbf{E}}(\vec{\mathbf{r}})=\mathbf{0}\\ \vec{\mathbf{H}}(\vec{\mathbf{r}})=\mathbf{0}\end{array}$$

cvd

$$\iiint_V dV\frac{1}{4}\mu_1\left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2=\iiint_V dV\frac{1}{4}\varepsilon_1\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2$$

Uniqueness (PD-Interior Problem)



Let's apply the Poynting theorem (PD)

Medium
- **Linear**
- **Isotropic**
- **Space-Nondispersive**
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$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{E}}_2(\vec{\mathbf{r}})$$
$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{H}}_2(\vec{\mathbf{r}})$$

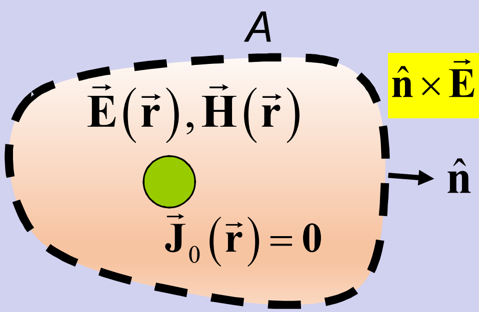
Source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}})$ **= 0**

$\hat{\mathbf{n}} \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathbf{0}$ on the boundary

$$\iiint_V dV \left[\frac{1}{2}\omega_0 \mu_2 \left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2 + \frac{1}{2}\omega_0 \varepsilon_2 \left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2 + \frac{1}{2}\sigma \left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2 \right] = 0$$

$$\iiint_V dV \frac{1}{4}\mu_1 \left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2 = \iiint_V dV \frac{1}{4}\varepsilon_1 \left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2$$

Uniqueness (PD-Interior Problem)



Let's apply the Poynting theorem (PD)

Medium
- **Linear**
- **Isotropic**
- **Space-Nondispersive**
- **-Time-Nondispersive**
- **Time-invariant**

$$\vec{\mathbf{E}}(\vec{\mathbf{r}})=\vec{\mathbf{E}}_1(\vec{\mathbf{r}})-\vec{\mathbf{E}}_2(\vec{\mathbf{r}})$$
$$\vec{\mathbf{H}}(\vec{\mathbf{r}})=\vec{\mathbf{H}}_1(\vec{\mathbf{r}})-\vec{\mathbf{H}}_2(\vec{\mathbf{r}})$$

Source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}})=\mathbf{0}$

$\hat{\mathbf{n}}\times\vec{\mathbf{E}}(\vec{\mathbf{r}})=\mathbf{0}$ on the boundary

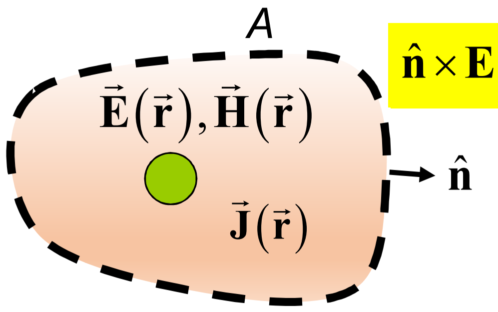
$$\iiint_V dV\left[\cancel{\frac{1}{2}\omega_0\mu_2\left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2}+\cancel{\frac{1}{2}\omega_0\varepsilon_2\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2}+\cancel{\frac{1}{2}\sigma\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2}\right]=0$$

$$\boxed{\iiint_V dV\,\frac{1}{4}\mu_1\left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2=\iiint_V dV\,\frac{1}{4}\varepsilon_1\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2}$$

$$\boxed{\begin{cases}\varepsilon_2=0\\ \mu_2=0\end{cases}} + \boxed{\text{No Homic losses}\quad \sigma=0}$$

Uniqueness is not ensured anymore!

Uniqueness (PD-Interior Problem)



$\hat{\mathbf{n}}\times\mathbf{E}\left(\text{or }\ \hat{\mathbf{n}}\times\vec{\mathbf{H}}\right)$

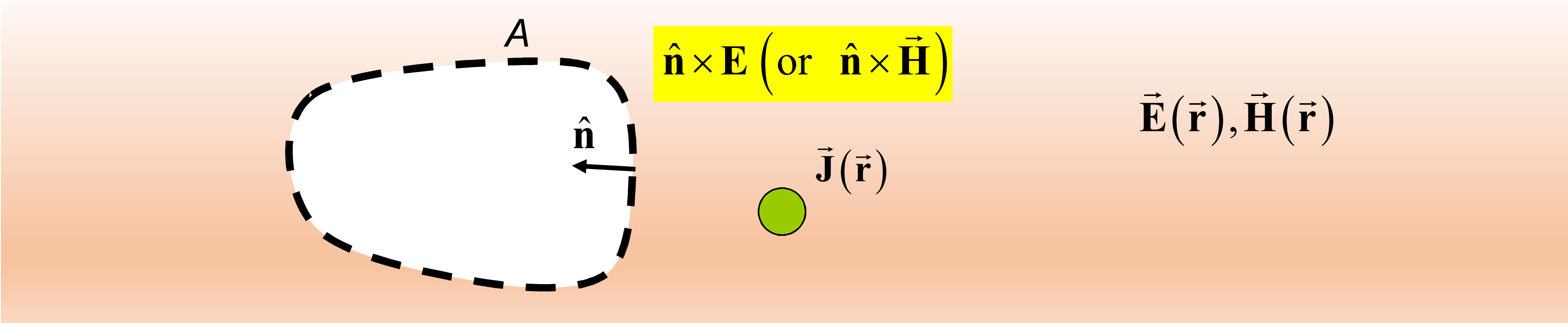
I Consider a source distribution $\vec{\mathbf{J}}(\vec{\mathbf{r}})$ with its associated electromagnetic field $\vec{\mathbf{E}}(\vec{\mathbf{r}}),\vec{\mathbf{H}}(\vec{\mathbf{r}})$

II Consider a (smooth) surface *A* with an everywhere defined unit normal $\hat{\mathbf{n}}$

IV Consider the values of the tangential component of the electric (**or** magnetic) field upon the surface A; that is, consider $\hat{\mathbf{n}}\times\mathbf{E}\left(\text{or }\ \hat{\mathbf{n}}\times\vec{\mathbf{H}}\right)$ **on the boundary**

The Uniqueness Theorem states that the electromagnetic field produced by the source in (I) within the **finite volume V bounded** by the surface A in (II), enforcing **the boundary condition** in (IV) is unique **provided that the considered medium is lossy**.
In a lossless medium, instead, the solution is unique **but for a set of resonant solutions**.

Uniqueness (PD-Exterior Problem)



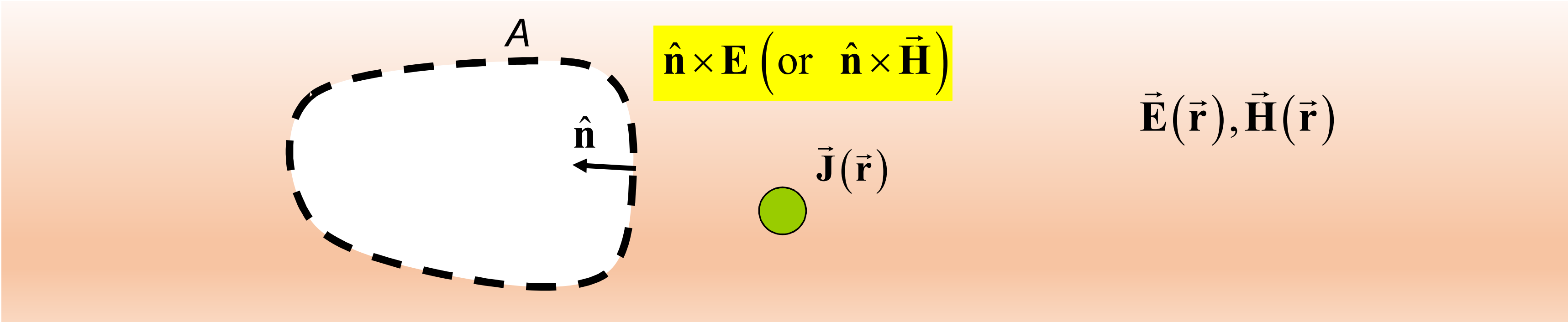
I Consider a source distribution $\vec{\mathbf{J}}(\vec{\mathbf{r}})$ with its associated electromagnetic field $\vec{\mathbf{E}}(\vec{\mathbf{r}}), \vec{\mathbf{H}}(\vec{\mathbf{r}})$

II Consider a (smooth) surface *A* with an everywhere defined unit normal $\hat{\mathbf{n}}$

IV Consider the values of the tangential component of the electric (**or** magnetic) field upon the surface A; that is, consider $\hat{\mathbf{n}} \times \mathbf{E} \left(\text{or} \;\; \hat{\mathbf{n}} \times \vec{\mathbf{H}} \right)$ **on the boundary**

The Uniqueness Theorem states that …….

Uniqueness (PD-Exterior Problem)



Source distribution: $\vec{\mathbf{J}}(\vec{\mathbf{r}})$

$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}), \vec{\mathbf{H}}_1(\vec{\mathbf{r}}) \qquad \vec{\mathbf{E}}_2(\vec{\mathbf{r}}), \vec{\mathbf{H}}_2(\vec{\mathbf{r}})$$

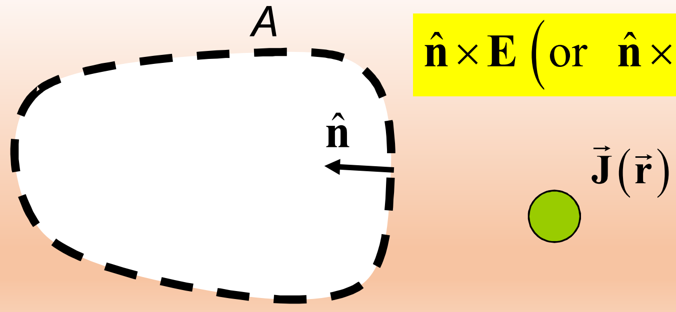
$$\hat{\mathbf{n}} \times \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) = \hat{\mathbf{n}} \times \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) \text{ on the boundary}$$

Field difference: source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}})$ **= 0**

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) \qquad \vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{H}}_2(\vec{\mathbf{r}})$$

$$\hat{\mathbf{n}} \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \hat{\mathbf{n}} \times \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) - \hat{\mathbf{n}} \times \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = 0 \quad \text{on the boundary}$$

Uniqueness (PD-Exterior Problem)



$\hat{\mathbf{n}} \times \mathbf{E}$ (or $\hat{\mathbf{n}} \times \vec{\mathbf{H}}$)

$\vec{\mathbf{E}}(\vec{\mathbf{r}}), \vec{\mathbf{H}}(\vec{\mathbf{r}})$

Field difference: source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}})$ = 0

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) \qquad \vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{H}}_2(\vec{\mathbf{r}})$$

$$\hat{\mathbf{n}} \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \hat{\mathbf{n}} \times \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) - \hat{\mathbf{n}} \times \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = 0 \quad \text{on the boundary}$$

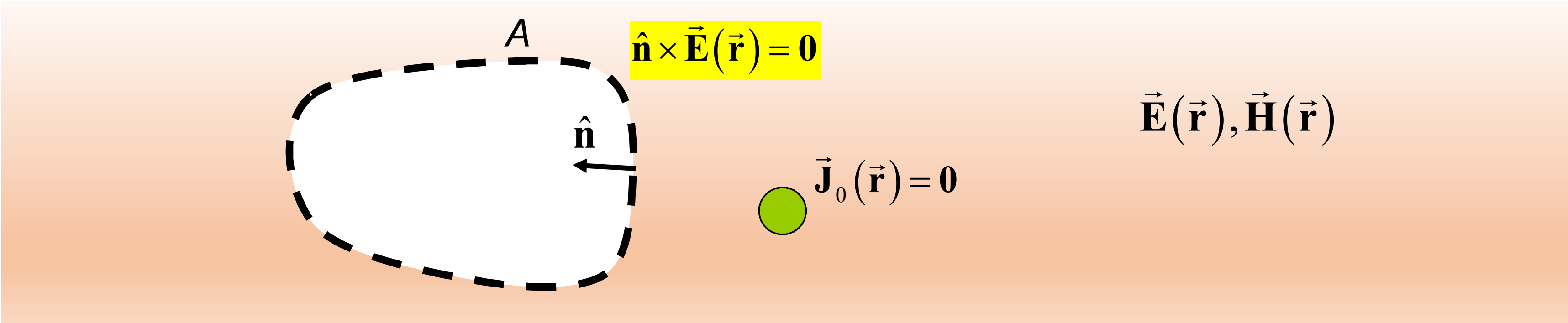
Uniqueness (PD-Exterior Problem)

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{E}}_2(\vec{\mathbf{r}})$$
$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{H}}_2(\vec{\mathbf{r}})$$

Source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}})$ **= 0**

$\hat{\mathbf{n}} \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathbf{0}$ on the boundary

Uniqueness (PD-Exterior Problem)



Medium

- **Linear**
- **Isotropic**
- **Space-Nondispersive**
- **-Time-Dispersive**
- **Time-invariant**

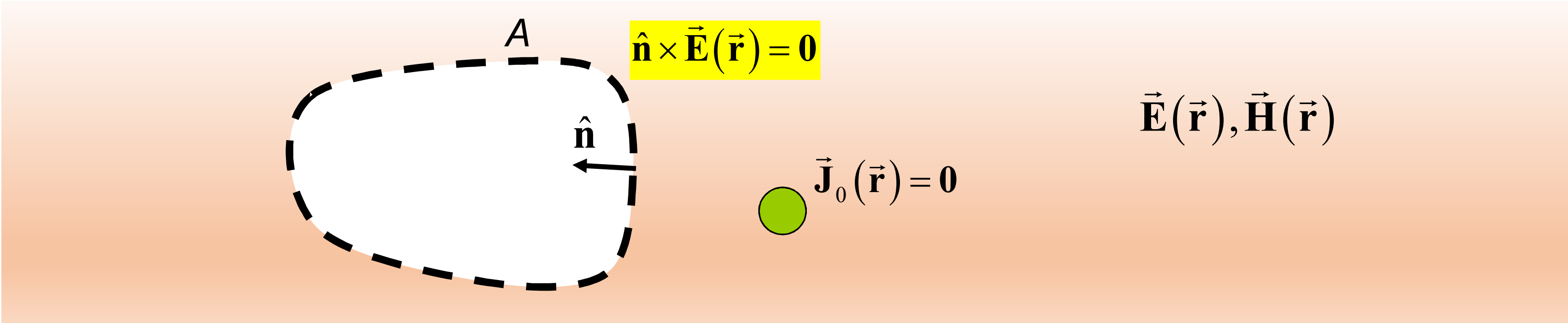
$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{E}}_2(\vec{\mathbf{r}})$$
$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{H}}_2(\vec{\mathbf{r}})$$

Source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}})$ **= 0**

$\hat{\mathbf{n}} \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathbf{0}$ on the boundary

$$\oint\!\!\oint_A dA\, \vec{\mathbf{S}}_1(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}} + \oint\!\!\oint_{A_\infty} dA_\infty\, \vec{\mathbf{S}}_1(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}} + \iiint_V dV \left[\frac{1}{2}\omega_0 \mu_2 \left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2 + \frac{1}{2}\omega_0 \varepsilon_2 \left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2 + \frac{1}{2}\sigma \left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2 \right] = \cancel{\iiint_V dV \left[-\frac{1}{2} \mathrm{Re}\left\{ \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \boxed{\vec{\mathbf{J}}_0^*(\vec{\mathbf{r}})} \right\} \right]}$$

Uniqueness (PD-Exterior Problem)



Medium

- **Linear**
- **Isotropic**
- **Space-Nondispersive**
- **Time-Dispersive**
- **Time-invariant**

$$\vec{\mathbf{E}}(\vec{\mathbf{r}})=\vec{\mathbf{E}}_1(\vec{\mathbf{r}})-\vec{\mathbf{E}}_2(\vec{\mathbf{r}})$$
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Source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}})=\mathbf{0}$

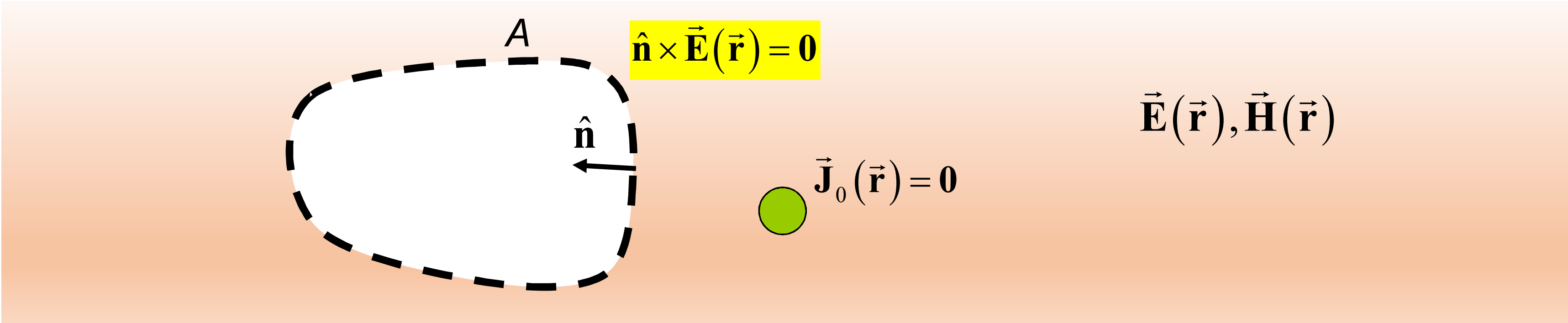
$\hat{\mathbf{n}}\times\vec{\mathbf{E}}(\vec{\mathbf{r}})=\mathbf{0}$ on the boundary

$$\boxed{\cancel{\oint\!\!\oint_A dA\,\vec{\mathbf{S}}_1(\vec{\mathbf{r}})\cdot\hat{\mathbf{n}}}}+\oint\!\!\oint_{A_\infty} dA_\infty\,\vec{\mathbf{S}}_1(\vec{\mathbf{r}})\cdot\hat{\mathbf{n}}+\iiint_V dV\left[\frac{1}{2}\omega_0\mu_2\left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2+\frac{1}{2}\omega_0\varepsilon_2\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2+\frac{1}{2}\sigma\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2\right]=\cancel{\iiint_V dV\left[-\frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{E}}(\vec{\mathbf{r}})\cdot\vec{\mathbf{J}}_0^*(\vec{\mathbf{r}})\right\}\right]}$$

$$\oint\!\!\oint_A dA\,\vec{\mathbf{S}}_1(\vec{\mathbf{r}})\cdot\hat{\mathbf{n}}=\mathrm{Re}\left\{\oint\!\!\oint_A dA\left[\frac{1}{2}\vec{\mathbf{E}}(\vec{\mathbf{r}})\times\vec{\mathbf{H}}^*(\vec{\mathbf{r}})\right]\cdot\hat{\mathbf{n}}\right\}=\mathrm{Re}\left\{\oint\!\!\oint_A dA\left[\frac{1}{2}\boxed{\hat{\mathbf{n}}\times\vec{\mathbf{E}}(\vec{\mathbf{r}})}\right]\cdot\vec{\mathbf{H}}^*(\vec{\mathbf{r}})\right\}=0\,\mathrm{Re}\left\{\oint\!\!\oint_A dA\left[\frac{1}{2}\vec{\mathbf{H}}^*(\vec{\mathbf{r}})\times\hat{\mathbf{n}}\right]\cdot\vec{\mathbf{E}}(\vec{\mathbf{r}})\right\}$$

$$\vec{\mathbf{A}}\cdot\left[\vec{\mathbf{B}}\times\vec{\mathbf{C}}\right]=\vec{\mathbf{C}}\cdot\left[\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right]=\vec{\mathbf{B}}\cdot\left[\vec{\mathbf{C}}\times\vec{\mathbf{A}}\right]$$

Uniqueness (PD-Exterior Problem)



Medium
- Linear
- Isotropic
- Space-Nondispersive
- Time-Dispersive
- Time-invariant

$$\vec{\mathbf{E}}(\vec{\mathbf{r}})=\vec{\mathbf{E}}_1(\vec{\mathbf{r}})-\vec{\mathbf{E}}_2(\vec{\mathbf{r}})$$
$$\vec{\mathbf{H}}(\vec{\mathbf{r}})=\vec{\mathbf{H}}_1(\vec{\mathbf{r}})-\vec{\mathbf{H}}_2(\vec{\mathbf{r}})$$

Source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}})=\mathbf{0}$

$\hat{\mathbf{n}}\times\vec{\mathbf{E}}(\vec{\mathbf{r}})=\mathbf{0}$ on the boundary

$$\cancel{\oint\!\!\!\oint_A dA\,\vec{\mathbf{S}}_1(\vec{\mathbf{r}})\cdot\hat{\mathbf{n}}}+\boxed{\oint\!\!\!\oint_{A_\infty} dA_\infty\,\vec{\mathbf{S}}_1(\vec{\mathbf{r}})\cdot\hat{\mathbf{n}}}+\iiint_V dV\left[\frac{1}{2}\omega_0\mu_2\left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2+\frac{1}{2}\omega_0\varepsilon_2\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2+\frac{1}{2}\sigma\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2\right]=\cancel{\iiint_V dV\left[-\frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{E}}(\vec{\mathbf{r}})\cdot\vec{\mathbf{J}}_0^*(\vec{\mathbf{r}})\right\}\right]}$$

The radiation condition

$$\vec{\mathbf{e}} \sim O\left(\frac{1}{r}\right) \quad \vec{\mathbf{h}} \sim O\left(\frac{1}{r}\right) \quad \vec{\mathbf{e}} - \zeta\vec{\mathbf{h}}\times\hat{\mathbf{n}} \sim o\left(\frac{1}{r}\right) \quad \left(and\ \zeta\vec{\mathbf{h}} - \hat{\mathbf{n}}\times\vec{\mathbf{e}} \sim o\left(\frac{1}{r}\right) \right) \quad as \quad r \to \infty$$

TD

$$\hat{\mathbf{n}}\cdot\vec{\mathbf{e}} = \hat{\mathbf{n}}\cdot\vec{\mathbf{h}} = 0$$

$$\vec{\mathbf{E}} \sim O\left(\frac{1}{r}\right) \quad \vec{\mathbf{H}} \sim O\left(\frac{1}{r}\right) \quad \vec{\mathbf{E}} - \zeta\vec{\mathbf{H}}\times\hat{\mathbf{n}} \sim o\left(\frac{1}{r}\right) \quad \left(and\ \boxed{\zeta\vec{\mathbf{H}} - \hat{\mathbf{n}}\times\vec{\mathbf{E}} \sim o\left(\frac{1}{r}\right)} \right) \quad as \quad r \to \infty$$

PD

$$\boxed{\hat{\mathbf{n}}\cdot\vec{\mathbf{E}} = \hat{\mathbf{n}}\cdot\vec{\mathbf{H}} = 0}$$

$$\vec{\mathbf{S}} = \frac{1}{2}\vec{\mathbf{E}}\times\vec{\mathbf{H}}^* = \frac{1}{2\zeta}\vec{\mathbf{E}}\times\left(\hat{\mathbf{n}}\times\vec{\mathbf{E}}\right)^* = \frac{1}{2\zeta}\vec{\mathbf{E}}\times\left(\hat{\mathbf{n}}\times\vec{\mathbf{E}}^*\right) = \frac{1}{2\zeta}\left[\left(\vec{\mathbf{E}}\cdot\vec{\mathbf{E}}^*\right)\hat{\mathbf{n}} - \cancel{\left(\hat{\mathbf{n}}\cdot\vec{\mathbf{E}}\right)\vec{\mathbf{E}}^*}\right] = \frac{\left|\vec{\mathbf{E}}\right|^2}{2\zeta}\hat{\mathbf{n}}$$

$$\vec{\mathbf{S}} = \frac{1}{2}\vec{\mathbf{E}}\times\vec{\mathbf{H}}^* = \left(\zeta\vec{\mathbf{H}}\times\hat{\mathbf{n}}\right)\times\vec{\mathbf{H}}^* = -\vec{\mathbf{H}}^*\times\left(\zeta\vec{\mathbf{H}}\times\hat{\mathbf{n}}\right) = -\left[\cancel{\left(\hat{\mathbf{n}}\cdot\vec{\mathbf{H}}^*\right)}\zeta\vec{\mathbf{H}} - \left(\zeta\vec{\mathbf{H}}\cdot\vec{\mathbf{H}}^*\right)\hat{\mathbf{n}}\right] = \frac{\zeta}{2}\left|\vec{\mathbf{H}}\right|^2\hat{\mathbf{n}}$$

$$\Rightarrow \begin{cases} \vec{\mathbf{S}}_1 = \dfrac{1}{2\zeta}\left|\vec{\mathbf{E}}\right|^2\hat{\mathbf{n}} = \dfrac{\zeta}{2}\left|\vec{\mathbf{H}}\right|^2\hat{\mathbf{n}} \\ \vec{\mathbf{S}}_2 = \mathbf{0} \end{cases}$$

$$\vec{\mathbf{A}}\times\left(\vec{\mathbf{B}}\times\vec{\mathbf{C}}\right) = \left(\vec{\mathbf{A}}\cdot\vec{\mathbf{C}}\right)\vec{\mathbf{B}} - \left(\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}\right)\vec{\mathbf{C}}$$

The radiation condition

$$\vec{\mathbf{e}} \sim O\left(\frac{1}{r}\right) \quad \vec{\mathbf{h}} \sim O\left(\frac{1}{r}\right) \quad \vec{\mathbf{e}} - \zeta \vec{\mathbf{h}} \times \hat{\mathbf{n}} \sim o\left(\frac{1}{r}\right) \quad \left(and \ \zeta \vec{\mathbf{h}} - \hat{\mathbf{n}} \times \vec{\mathbf{e}} \sim o\left(\frac{1}{r}\right) \right) \quad as \quad r \to \infty$$

TD

$$\hat{\mathbf{n}} \cdot \vec{\mathbf{e}} = \hat{\mathbf{n}} \cdot \vec{\mathbf{h}} = 0$$

$$\boxed{\vec{\mathbf{E}} \sim O\left(\frac{1}{r}\right)} \quad \vec{\mathbf{H}} \sim O\left(\frac{1}{r}\right) \quad \vec{\mathbf{E}} - \zeta \vec{\mathbf{H}} \times \hat{\mathbf{n}} \sim o\left(\frac{1}{r}\right) \quad \left(and \ \zeta \vec{\mathbf{H}} - \hat{\mathbf{n}} \times \vec{\mathbf{E}} \sim o\left(\frac{1}{r}\right) \right) \quad as \quad r \to \infty$$

PD

$$\hat{\mathbf{n}} \cdot \vec{\mathbf{E}} = \hat{\mathbf{n}} \cdot \vec{\mathbf{H}} = 0$$

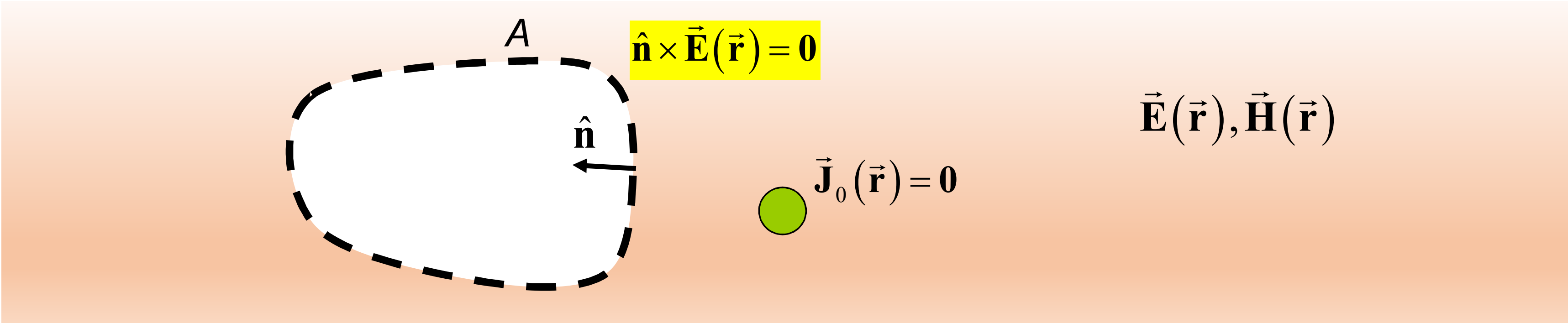
$$\oiint_{A_\infty} dA_\infty \, \vec{\mathbf{S}}_1(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}} = \oiint_{A_\infty} dA_\infty \frac{1}{2\zeta} \left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2 = \lim_{r \to \infty} \int_0^{2\pi} d\varphi \int_0^{\pi} d\vartheta r^2 \sin\vartheta \, \frac{\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2}{2\zeta}$$

is a finite nonnegative quantity

$$\begin{cases} \vec{\mathbf{S}}_1 = \dfrac{1}{2\zeta} \left|\vec{\mathbf{E}}\right|^2 \hat{\mathbf{n}} = \dfrac{\zeta}{2} \left|\vec{\mathbf{H}}\right|^2 \hat{\mathbf{n}} \\ \vec{\mathbf{S}}_2 = \mathbf{0} \end{cases}$$

$$\oiint_A dA \, \Phi(r,\vartheta,\varphi) = \int_0^{2\pi} d\varphi \int_0^{\pi} d\vartheta r^2 \sin\vartheta \, \Phi(r,\vartheta,\varphi)$$

Uniqueness (PD-Exterior Problem)



Medium
- **Linear**
- **Isotropic**
- **Space-Nondispersive**
- **-Time-Dispersive**
- **Time-invariant**

$$\vec{\mathbf{E}}(\vec{\mathbf{r}})=\vec{\mathbf{E}}_1(\vec{\mathbf{r}})-\vec{\mathbf{E}}_2(\vec{\mathbf{r}})$$
$$\vec{\mathbf{H}}(\vec{\mathbf{r}})=\vec{\mathbf{H}}_1(\vec{\mathbf{r}})-\vec{\mathbf{H}}_2(\vec{\mathbf{r}})$$

Source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}})=\mathbf{0}$

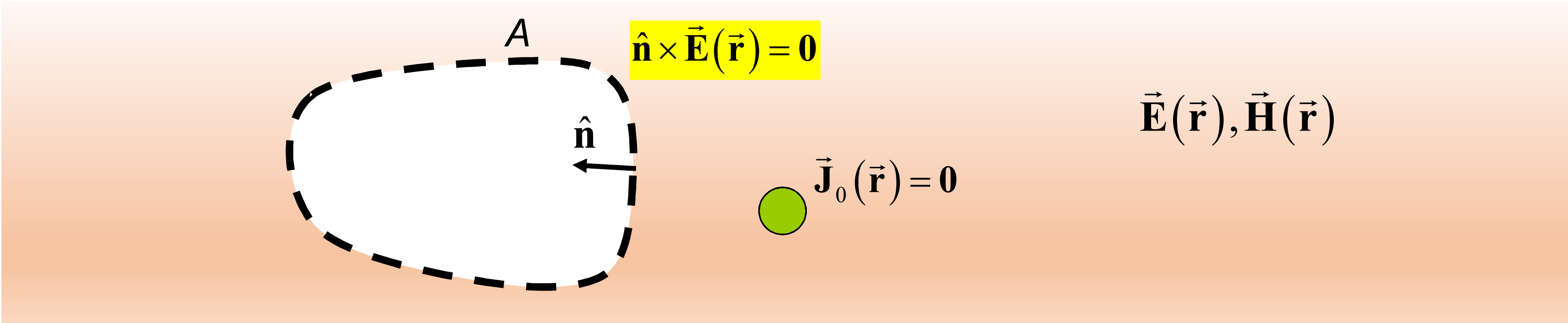
$\hat{\mathbf{n}}\times\vec{\mathbf{E}}(\vec{\mathbf{r}})=\mathbf{0}$ on the boundary

Radiation condition at infinity

$$\cancel{\oint\!\!\!\oint_A dA\,\vec{\mathbf{S}}_1(\vec{\mathbf{r}})\cdot\hat{\mathbf{n}}}+\boxed{\oint\!\!\!\oint_{A_\infty} dA_\infty\,\vec{\mathbf{S}}_1(\vec{\mathbf{r}})\cdot\hat{\mathbf{n}}}+\iiint_V dV\left[\frac{1}{2}\omega_0\mu_2\left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2+\frac{1}{2}\omega_0\varepsilon_2\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2+\frac{1}{2}\sigma\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2\right]=\cancel{\iiint_V dV\left[-\frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{E}}(\vec{\mathbf{r}})\cdot\vec{\mathbf{J}}_0^*(\vec{\mathbf{r}})\right\}\right]}$$

$$\oint\!\!\!\oint_{A_\infty} dA_\infty\,\vec{\mathbf{S}}_1(\vec{\mathbf{r}})\cdot\hat{\mathbf{n}}\geq 0$$

Uniqueness (PD-Exterior Problem)



Medium

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$$\vec{\mathbf{H}}(\vec{\mathbf{r}})=\vec{\mathbf{H}}_1(\vec{\mathbf{r}})-\vec{\mathbf{H}}_2(\vec{\mathbf{r}})$$

Source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}})$= 0

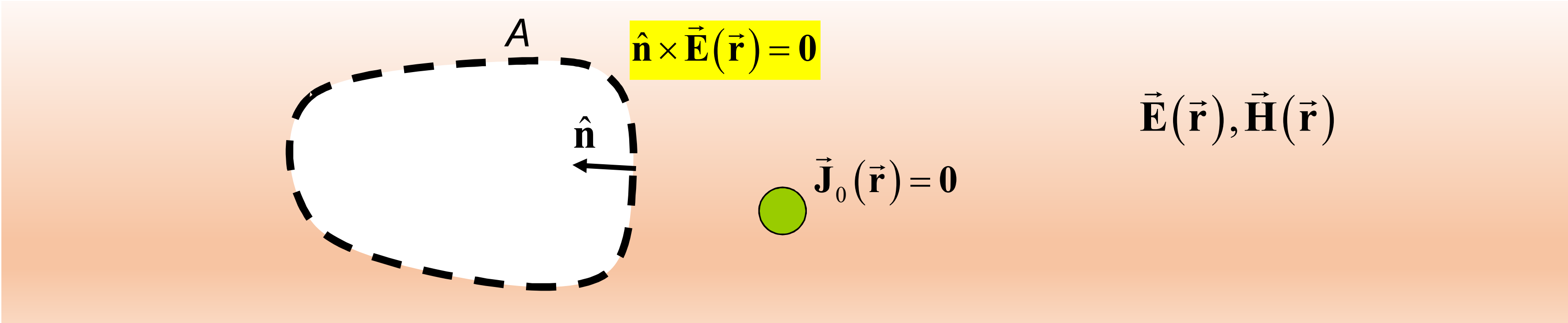
$\hat{\mathbf{n}}\times\vec{\mathbf{E}}(\vec{\mathbf{r}})=\mathbf{0}$ on the boundary

Radiation condition at infinity

$$\cancel{\oint\!\!\!\oint_A dA\,\vec{\mathbf{S}}_1(\vec{\mathbf{r}})\cdot\hat{\mathbf{n}}}+\boxed{\oint\!\!\!\oint_{A_\infty} dA_\infty\,\vec{\mathbf{S}}_1(\vec{\mathbf{r}})\cdot\hat{\mathbf{n}}}+\iiint_V dV\left[\frac{1}{2}\omega_0\mu_2\left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2+\frac{1}{2}\omega_0\varepsilon_2\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2+\frac{1}{2}\sigma\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2\right]=\cancel{\iiint_V dV\left[-\frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{E}}(\vec{\mathbf{r}})\cdot\vec{\mathbf{J}}_0^*(\vec{\mathbf{r}})\right\}\right]}$$

$$\oint\!\!\!\oint_{A_\infty} dA_\infty\frac{1}{2\zeta}\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2+\iiint_V dV\left[\frac{1}{2}\omega_0\mu_2\left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2+\frac{1}{2}\omega_0\varepsilon_2\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2+\frac{1}{2}\sigma\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2\right]=0$$

Uniqueness (PD-Exterior Problem)



Medium

- **Linear**
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- **Space-Nondispersive**
- **Time-Dispersive**
- **Time-invariant**

$$\vec{\mathbf{E}}(\vec{\mathbf{r}})=\vec{\mathbf{E}}_1(\vec{\mathbf{r}})-\vec{\mathbf{E}}_2(\vec{\mathbf{r}})$$
$$\vec{\mathbf{H}}(\vec{\mathbf{r}})=\vec{\mathbf{H}}_1(\vec{\mathbf{r}})-\vec{\mathbf{H}}_2(\vec{\mathbf{r}})$$

Source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}})=\mathbf{0}$

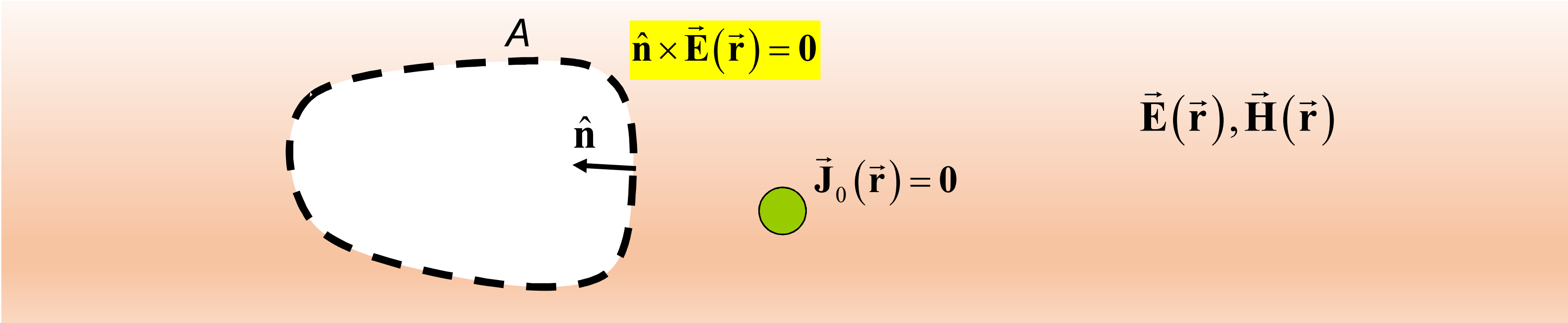
$\hat{\mathbf{n}}\times\vec{\mathbf{E}}(\vec{\mathbf{r}})=\mathbf{0}$ on the boundary

Radiation condition at infinity

$$\oint\!\!\oint_{A_\infty} dA_\infty \frac{1}{2\zeta}\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2+\iiint_V dV\left[\frac{1}{2}\omega_0\mu_2\left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2+\frac{1}{2}\omega_0\varepsilon_2\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2+\frac{1}{2}\sigma\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2\right]=0 \quad\Rightarrow\quad \begin{aligned}\vec{\mathbf{E}}(\vec{\mathbf{r}})&=\mathbf{0}\\ \vec{\mathbf{H}}(\vec{\mathbf{r}})&=\mathbf{0}\end{aligned}$$

cvd

Uniqueness (PD-Exterior Problem)



Medium

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$$\vec{\mathbf{H}}(\vec{\mathbf{r}})=\vec{\mathbf{H}}_1(\vec{\mathbf{r}})-\vec{\mathbf{H}}_2(\vec{\mathbf{r}})$$

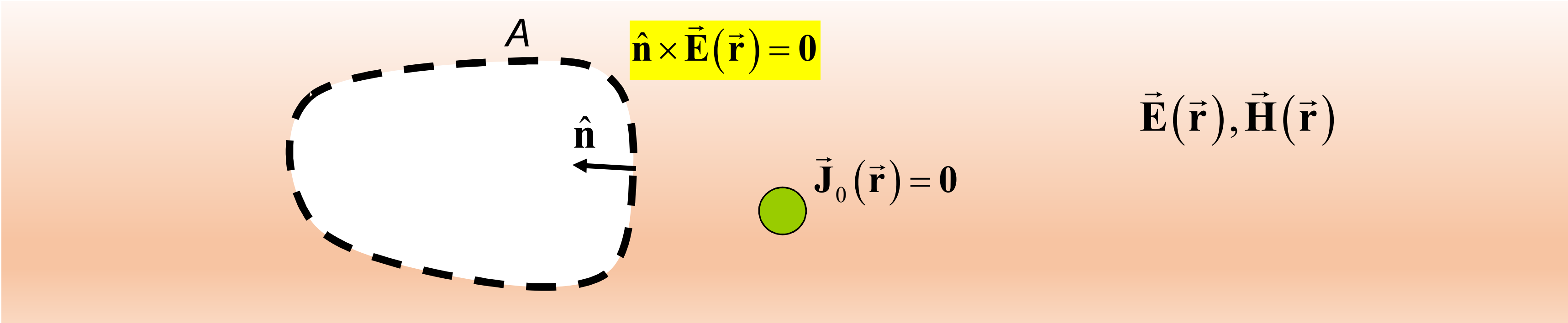
Source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}})$ **= 0**

$\hat{\mathbf{n}}\times\vec{\mathbf{E}}(\vec{\mathbf{r}})=\mathbf{0}$ on the boundary

Radiation condition at infinity

$$\oint\!\!\!\oint_{A_\infty} dA_\infty \frac{1}{2\zeta}\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2+\iiint_V dV\left[\frac{1}{2}\omega_0\mu_2\left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2+\frac{1}{2}\omega_0\varepsilon_2\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2+\frac{1}{2}\sigma\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2\right]=0$$

Uniqueness (PD-Exterior Problem)



Medium
- **Linear**
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$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{E}}_2(\vec{\mathbf{r}})$$
$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{H}}_2(\vec{\mathbf{r}})$$

Source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}})$ **= 0**

$\hat{\mathbf{n}} \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathbf{0}$ on the boundary

Radiation condition at infinity

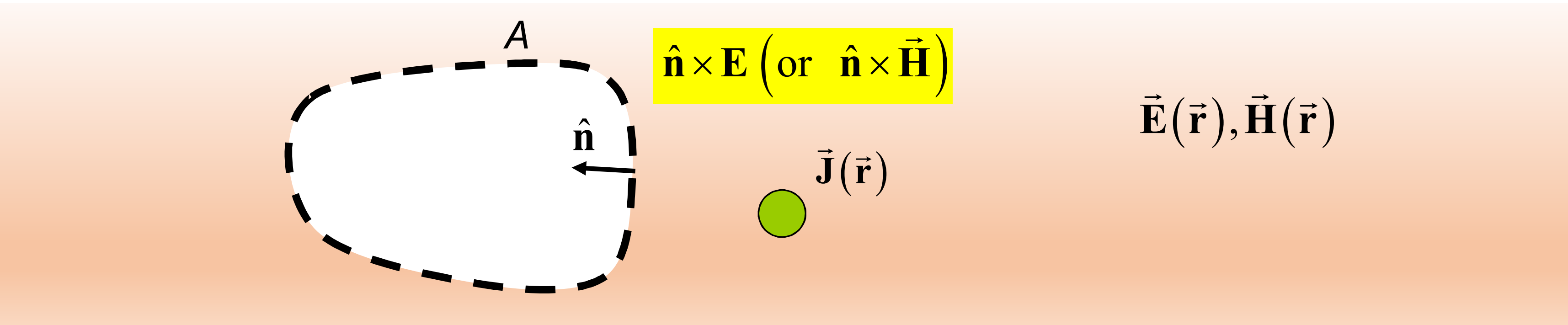
$$\boxed{\begin{cases} \varepsilon_2 = 0 \\ \mu_2 = 0 \end{cases}} \quad + \quad \boxed{\text{No Homic losses} \quad \sigma = 0}$$

Uniqueness is still ensured!

$$\oint\!\!\!\oint_{A_\infty} dA_\infty \frac{1}{2\zeta}\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2 + \iiint_V dV \cancel{\left[\frac{1}{2}\omega_0\mu_2\left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2 + \frac{1}{2}\omega_0\varepsilon_2\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2 + \frac{1}{2}\sigma\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2\right]} = 0 \quad \Rightarrow \quad \begin{matrix} \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathbf{0} \\ \vec{\mathbf{H}}(\vec{\mathbf{r}}) = \mathbf{0} \end{matrix}$$

cvd

Uniqueness (PD-Exterior Problem)



I Consider a source distribution $\vec{\mathbf{J}}(\vec{\mathbf{r}})$ with its associated electromagnetic field $\vec{\mathbf{E}}(\vec{\mathbf{r}}),\vec{\mathbf{H}}(\vec{\mathbf{r}})$

II Consider a (smooth) surface *A* with an everywhere defined unit normal $\hat{\mathbf{n}}$

IV Consider the values of the tangential component of the electric (**or** magnetic) field upon the surface A; that is, consider $\hat{\mathbf{n}}\times\mathbf{E}\left(\text{or}\ \ \hat{\mathbf{n}}\times\vec{\mathbf{H}}\right)$ **on the boundary**

The Uniqueness Theorem states that the electromagnetic field produced by the source in (I) within the **infinite volume V outside** the surface A in (II), enforcing **the boundary condition** in (IV) **as well as the radiation condition at infinity** is unique.