Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2020-2021 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

Stefano Perna

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Poynting theorem (TD)

$$\vec{\mathbf{s}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

Poynting vector

$$[\vec{\mathbf{s}}]:\frac{Watt}{m^2}$$

$$\nabla\cdot\vec{\mathbf{s}}(\vec{\mathbf{r}},t)+\frac{\partial}{\partial t}w(\vec{\mathbf{r}},t)+p_j(\vec{\mathbf{r}},t)=p_0(\vec{\mathbf{r}},t)$$

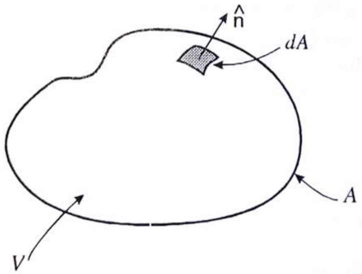
$$\boxed{\oiint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}}+\frac{d}{dt}\iiint_V dV\,w(\vec{\mathbf{r}},t)+\iiint_V dV p_j(\vec{\mathbf{r}},t)=\iiint_V dV\,p_0(\vec{\mathbf{r}},t)$$

Electromagnetic power flux

$$P_S(t)+\frac{d}{dt}W(t)+P_j(t)=P_0(t)$$

Hypotheses on the medium

- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Time-invariant**



$$w(\vec{\mathbf{r}},t)=\frac{1}{2}\mu|\vec{\mathbf{h}}|^2+\frac{1}{2}\varepsilon|\vec{\mathbf{e}}|^2$$ **Energy density of the e.m. field** ➡ $\iiint_V dV\,w(\vec{\mathbf{r}},t)=W(t)$ **Energy of the e.m. field**

$$p_j(\vec{\mathbf{r}},t)=\sigma|\vec{\mathbf{e}}|^2$$ **Power density dissipated in the conducting medium** ➡ $\iiint_V dV p_j(\vec{\mathbf{r}},t)=P_j(t)$ **Power dissipated in the conducting medium**

$$p_0(\vec{\mathbf{r}},t)=-\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}$$ **Power density delivered by the sources to the field** ➡ $\iiint_V dV\,p_0(\vec{\mathbf{r}},t)=P(t)$ **Power delivered by the sources to the field**

THEOREMS

Poynting

Time domain – Phasor domain

Uniqueness (Interior problem – Exterior problem)

Time domain – Phasor domain

Equivalence

Phasor domain

Image Theory

Reciprocity

Phasor domain

Poynting theorem (PD)

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \frac{1}{2}\left[\vec{\mathbf{E}}(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^*(\vec{\mathbf{r}})\right]$$

Poynting vector

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$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \vec{\mathbf{S}}_1(\vec{\mathbf{r}}) + j\vec{\mathbf{S}}_2(\vec{\mathbf{r}})$$

$$\oiint_A dA\,\vec{\mathbf{S}}_1 \cdot \hat{\mathbf{n}} + \iiint_V dV\left[\frac{1}{2}\omega_0\mu_2\left|\vec{\mathbf{H}}\right|^2 + \frac{1}{2}\omega_0\varepsilon_2\left|\vec{\mathbf{E}}\right|^2 + \frac{1}{2}\sigma\left|\vec{\mathbf{E}}\right|^2\right] = \iiint_V dV\left[-\frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^*\right\}\right]$$

$$\oiint_A dA\,\vec{\mathbf{S}}_2 \cdot \hat{\mathbf{n}} + 2\omega_0\iiint_V dV\left[\frac{1}{4}\mu_1\left|\vec{\mathbf{H}}\right|^2 - \frac{1}{4}\varepsilon_1\left|\vec{\mathbf{E}}\right|^2\right] = \iiint_V dV\left[-\frac{1}{2}\mathrm{Im}\left\{\vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^*\right\}\right]$$

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- **Time-Dispersive**
- **Space-Nondispersive**

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TD

$$\underbrace{\oiint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}}_{\text{Power flux associated to the e.m. field}} + \underbrace{\frac{d}{dt}\iiint_V dV\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2 + \frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right]}_{\text{Time derivative of the energy of the e.m. field}} + \underbrace{\iiint_V dV\,\sigma\left|\vec{\mathbf{e}}\right|^2}_{\text{Power dissipated in the conducting medium}} = \underbrace{-\iiint_V dV\,\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}}_{\text{Power delivered by the sources to the field}}$$

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Power flux associated to the e.m. field

Time derivative of the energy of the e.m. field

Power dissipated in the conducting medium

Power delivered by the sources to the field

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Poynting theorem

…MEMO : phasors and time averages

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Time averaged power flux associated to the e.m. field

Time averaged power dissipated in the conducting medium

Time averaged power delivered by the sources to the field

TD

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Time averaged power flux associated to the e.m. field

LOSSES
(ε_2) electric losses
(μ_2) magnetic losses

Time averaged power dissipated in the conducting medium

Time averaged power delivered by the sources to the field

$$\varepsilon_2 > 0;\ \mu_2 > 0;\ \sigma > 0$$

Dispersion and losses are related each other: a (time) dispersive medium presents losses

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$$\vec{f}_1(\vec{r},t) = \sqrt{\frac{\mu}{2}}\,\vec{h}(\vec{r},t) \rightarrow \sqrt{\frac{\mu}{2}}\,\vec{H}(\vec{r}) = \vec{F_1}(\vec{r}) \qquad \vec{f}_2(\vec{r},t) = \sqrt{\frac{\mu}{2}}\,\vec{h}(\vec{r},t) \rightarrow \sqrt{\frac{\mu}{2}}\,\vec{H}(\vec{r}) = \vec{F_2}(\vec{r})$$

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$$\left[\vec{\mathbf{S}}\right] : \frac{Watt}{m^2}$$

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \vec{\mathbf{S}}_1(\vec{\mathbf{r}}) + j\vec{\mathbf{S}}_2(\vec{\mathbf{r}})$$

$$\oint\!\!\!\oint_A dA\,\vec{\mathbf{S}}_2 \cdot \hat{\mathbf{n}} + 2\omega_0 \iiint_V dV \left[\boxed{\frac{1}{4}\mu_1\left|\vec{\mathbf{H}}\right|^2} - \boxed{\frac{1}{4}\varepsilon_1\left|\vec{\mathbf{E}}\right|^2}\right] = \iiint_V dV \left[-\frac{1}{2}\mathrm{Im}\left\{\vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^*\right\}\right]$$

Time averaged magnetic energy density

Time averaged electric energy density

$$\frac{1}{4}\mu_1\left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2 = \frac{1}{2}\frac{1}{2}\mu\left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2 = \frac{1}{2}\mathrm{Re}\left\{\frac{1}{2}\mu\left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2\right\} = \frac{1}{2}\mathrm{Re}\left\{\frac{1}{2}\mu\vec{\mathbf{H}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{H}}(\vec{\mathbf{r}})^*\right\} = \left\langle \frac{1}{2}\mu\left|\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right|^2 \right\rangle$$

$$\frac{1}{4}\varepsilon_1\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2 = \frac{1}{2}\frac{1}{2}\varepsilon\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2 = \frac{1}{2}\mathrm{Re}\left\{\frac{1}{2}\varepsilon\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2\right\} = \frac{1}{2}\mathrm{Re}\left\{\frac{1}{2}\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{E}}^*(\vec{\mathbf{r}})\right\} = \left\langle \frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right|^2 \right\rangle$$

Hypotheses on the medium (PD)

- **Linear**
- **Isotropic**
- **Time-invariant**
- **Time-Nondispersive**
- **Space-Nondispersive**

$$\begin{cases} \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \varepsilon(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{B}}(\vec{\mathbf{r}}) = \mu(\vec{\mathbf{r}})\vec{\mathbf{H}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{J}}(\vec{\mathbf{r}}) = \sigma\vec{\mathbf{E}}(\vec{\mathbf{r}}) \end{cases}$$

$$\begin{cases} \varepsilon = \varepsilon_1 - j\varepsilon_2 \\ \mu = \mu_1 - j\mu_2 \\ \sigma : real \end{cases} \qquad \begin{cases} \varepsilon_2 = 0 \\ \mu_2 = 0 \end{cases}$$

THEOREMS

Poynting

Time domain – Phasor domain

Uniqueness (Interior problem – Exterior problem)

Time domain – Phasor domain

Equivalence

Phasor domain

Image Theory

Reciprocity

Phasor domain

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Poynting theorem

TD

$$\vec{\mathbf{s}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

$$\oiint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}+\frac{d}{dt}\iiint_V dV\,w(\vec{\mathbf{r}},t)+\iiint_V dV p_j(\vec{\mathbf{r}},t)=\iiint_V dV\,p_0(\vec{\mathbf{r}},t)$$

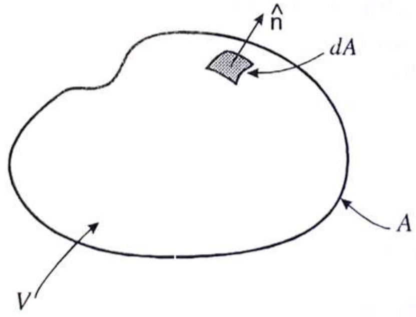
$$P_S(t)+\frac{d}{dt}W(t)+P_j(t)=P_0(t)$$

$$w(\vec{\mathbf{r}},t)=\frac{1}{2}\mu\left|\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right|^2$$

$$p_j(\vec{\mathbf{r}},t)=\sigma\left|\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right|^2$$

$$p_0(\vec{\mathbf{r}},t)=-\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$$

Linear **Isotropic** **Space- Nondispersive** **Time-Nondispersive** **Time-invariant**



PD

$$\vec{\mathbf{S}}(\vec{\mathbf{r}})=\frac{1}{2}\left[\vec{\mathbf{E}}(\vec{\mathbf{r}})\times\vec{\mathbf{H}}^*(\vec{\mathbf{r}})\right]=\vec{\mathbf{S}}_1(\vec{\mathbf{r}})+j\vec{\mathbf{S}}_2(\vec{\mathbf{r}})$$

$$\oiint_A dA\,\vec{\mathbf{S}}_1(\vec{\mathbf{r}})\cdot\hat{\mathbf{n}}+\iiint_V dV\left[\frac{1}{2}\omega_0\mu_2\left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2+\frac{1}{2}\omega_0\varepsilon_2\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2+\frac{1}{2}\sigma\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2\right]=\iiint_V dV\left[-\frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{E}}(\vec{\mathbf{r}})\cdot\vec{\mathbf{J}}_0^*(\vec{\mathbf{r}})\right\}\right]$$

$$\oiint_A dA\,\vec{\mathbf{S}}_2(\vec{\mathbf{r}})\cdot\hat{\mathbf{n}}+2\omega_0\iiint_V dV\left[\frac{1}{4}\mu_1\left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2-\frac{1}{4}\varepsilon_1\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2\right]=\iiint_V dV\left[-\frac{1}{2}\mathrm{Im}\left\{\vec{\mathbf{E}}(\vec{\mathbf{r}})\cdot\vec{\mathbf{J}}_0^*(\vec{\mathbf{r}})\right\}\right]$$

Linear **Isotropic** **Space- Nondispersive** **Time-Dispersive** **Time-invariant**

Mathematical tools that we will exploit today

$$\vec{\mathbf{A}}(\vec{\mathbf{r}})\cdot\left[\vec{\mathbf{B}}(\vec{\mathbf{r}})\times\vec{\mathbf{C}}(\vec{\mathbf{r}})\right]=\vec{\mathbf{C}}(\vec{\mathbf{r}})\cdot\left[\vec{\mathbf{A}}(\vec{\mathbf{r}})\times\vec{\mathbf{B}}(\vec{\mathbf{r}})\right]=\vec{\mathbf{B}}(\vec{\mathbf{r}})\cdot\left[\vec{\mathbf{C}}(\vec{\mathbf{r}})\times\vec{\mathbf{A}}(\vec{\mathbf{r}})\right]$$

Uniqueness (TD)

$\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t),\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$

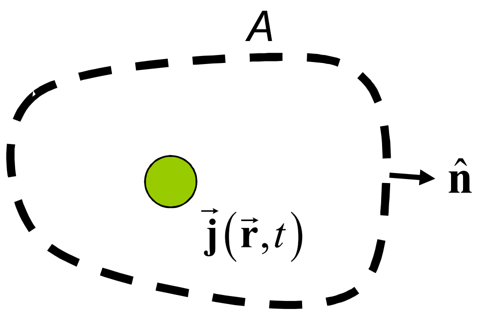
- Consider a source distribution $\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$ with its associated electromagnetic field $(\vec{\mathbf{e}},\vec{\mathbf{h}})$

Uniqueness (TD)

$\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$ $\qquad\qquad$ $\vec{\mathbf{e}}(\vec{\mathbf{r}},t),\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$

- Consider a source distribution $\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$ with its associated electromagnetic field $\left(\vec{\mathbf{e}},\vec{\mathbf{h}}\right)$

Uniqueness (TD)

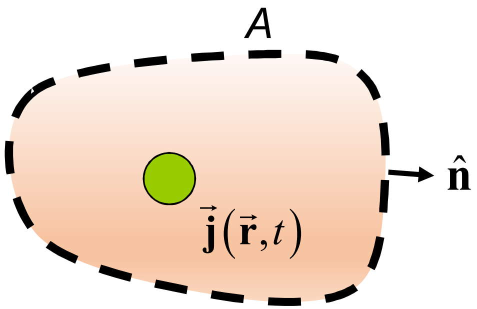


$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t),\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

I Consider a source distribution $\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$ with its associated electromagnetic field $\left(\vec{\mathbf{e}},\vec{\mathbf{h}}\right)$

II Consider a (smooth) surface *A* with an everywhere defined unit normal $\hat{\mathbf{n}}$

Uniqueness (TD)



$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$$

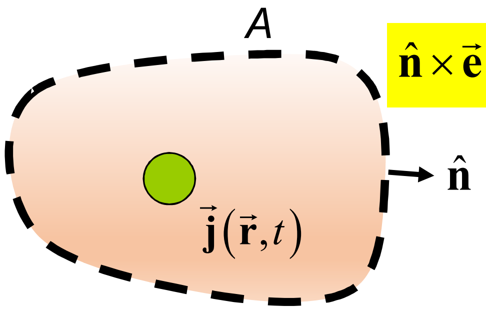
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t),\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

I Consider a source distribution $\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$ with its associated electromagnetic field $(\vec{\mathbf{e}},\vec{\mathbf{h}})$

II Consider a (smooth) surface *A* with an everywhere defined unit normal $\hat{\mathbf{n}}$

III Consider the values of the electromagnetic field everywhere in **the finite volume V** bounded by the surface A **at the initial time**; that is, consider $\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$

Uniqueness (TD)



$\hat{\mathbf{n}}\times\vec{\mathbf{e}}\ \left(\text{or}\ \ \hat{\mathbf{n}}\times\vec{\mathbf{h}}\right)$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t),\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$

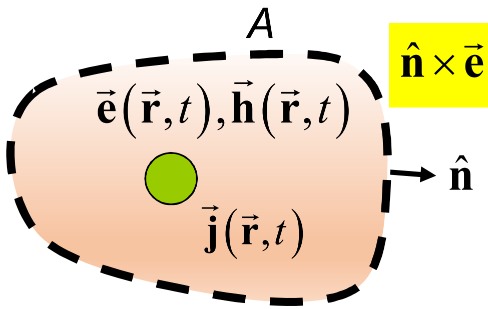
I Consider a source distribution $\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$ with its associated electromagnetic field $\left(\vec{\mathbf{e}},\vec{\mathbf{h}}\right)$

II Consider a (smooth) surface *A* with an everywhere defined unit normal $\hat{\mathbf{n}}$

III Consider the values of the electromagnetic field everywhere in **the finite volume V** bounded by the surface A **at the initial time**; that is, consider $\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$

IV Consider the values of the tangential component of the electric (**or** magnetic) field upon the surface A at any time after the initial one; that is, consider $\hat{\mathbf{n}}\times\vec{\mathbf{e}}$ (**or** $\hat{\mathbf{n}}\times\vec{\mathbf{h}}$) **on the boundary at any time**

Uniqueness (TD)



$\hat{n}\times\vec{e}\ (\text{or}\ \ \hat{n}\times\vec{h})$

$\vec{e}(\vec{r},t_0),\vec{h}(\vec{r},t_0)$

Interior Problem

I Consider a source distribution $\vec{j}(\vec{r},t)$ with its associated electromagnetic field $(\vec{e},\vec{h})$

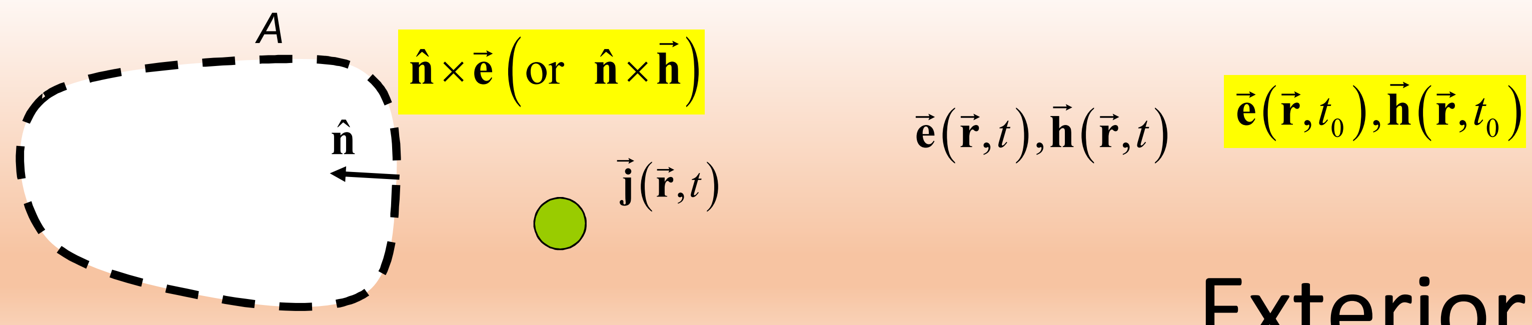
II Consider a (smooth) surface *A* with an everywhere defined unit normal $\hat{n}$

III Consider the values of the electromagnetic field everywhere in **the finite volume V** bounded by the surface A **at the initial time**; that is, consider $\vec{e}(\vec{r},t_0),\vec{h}(\vec{r},t_0)$

IV Consider the values of the tangential component of the electric (**or** magnetic) field upon the surface A at any time after the initial one; that is, consider $\hat{n}\times\vec{e}$ (**or** $\hat{n}\times\vec{h}$) **on the boundary at any time**

The Uniqueness Theorem states that the electromagnetic field produced by the source in (I) within the **finite volume V bounded** by the surface A in (II), enforcing **the initial condition** in (III) and **the boundary condition** in (IV) is unique.

Uniqueness (TD)



Exterior Problem

I Consider a source distribution $\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$ with its associated electromagnetic field $\left(\vec{\mathbf{e}},\vec{\mathbf{h}}\right)$

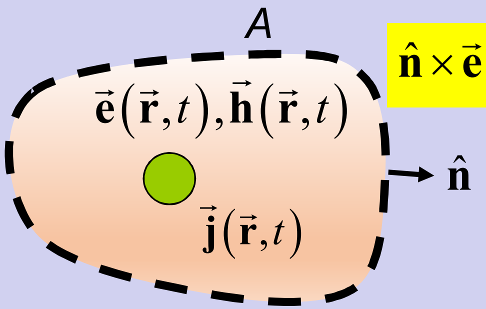
II Consider a (smooth) surface *A* with an everywhere defined unit normal $\hat{\mathbf{n}}$

III Consider the values of the electromagnetic field everywhere in **the infinite volume outside** the surface A **at the initial time**; that is, consider $\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$

IV Consider the values of the tangential component of the electric (**or** magnetic) field upon the surface A at any time after the initial one; that is, consider $\hat{\mathbf{n}} \times \vec{\mathbf{e}}$ (**or** $\hat{\mathbf{n}} \times \vec{\mathbf{h}}$) **on the boundary at any time**

The Uniqueness Theorem states that the electromagnetic field produced by the source in (I) within the **infinite volume V outside** the surface A in (II), enforcing **the initial condition** in (III) and **the boundary condition** in (IV) is unique.

Uniqueness (TD-Interior Problem)



$\hat{\mathbf{n}}\times\vec{\mathbf{e}}\ \left(\text{or}\ \ \hat{\mathbf{n}}\times\vec{\mathbf{h}}\right)$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$

Source distribution: $\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$

$$\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t),\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t) \qquad \vec{\mathbf{e}}_2(\vec{\mathbf{r}},t),\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t_0)=\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t_0)$$

$$\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t_0)=\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t_0)$$

$$\hat{\mathbf{n}}\times\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)=\hat{\mathbf{n}}\times\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)\ \text{on the boundary}$$

Field difference: source distribution $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)$ **= 0**

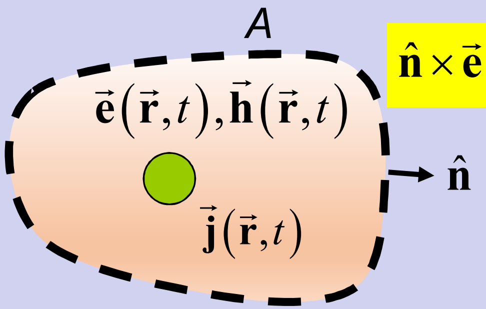
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t) \qquad \vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t_0)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t_0)=0$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t_0)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t_0)=0$$

$$\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\hat{\mathbf{n}}\times\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\hat{\mathbf{n}}\times\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)=0\ \ \text{on the boundary}$$

Uniqueness (TD-Interior Problem)



$\hat{\mathbf{n}}\times\vec{\mathbf{e}}\ \left(\text{or}\ \ \hat{\mathbf{n}}\times\vec{\mathbf{h}}\right)$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$

Field difference: source distribution $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)$ **= 0**

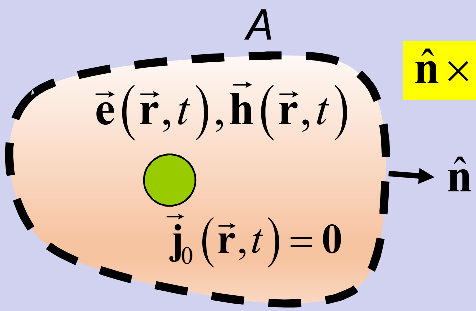
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t) \qquad \vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t_0)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t_0)=0$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t_0)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t_0)=0$$

$$\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\hat{\mathbf{n}}\times\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\hat{\mathbf{n}}\times\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)=0 \quad \text{on the boundary}$$

Uniqueness (TD-Interior Problem)



$\hat{\mathbf{n}}\times\vec{\mathbf{e}}=\mathbf{0}$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

Let's apply the Poynting theorem (TD)

Medium
- Linear
- Isotropic
- Space-Nondispersive
- Time-Nondispersive
- Time-invariant

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$

Source distribution $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)$ = 0

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

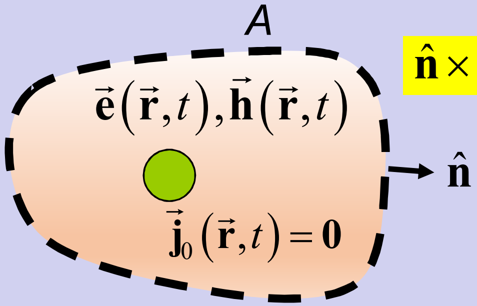
$\boxed{\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\mathbf{0}\quad \text{on the boundary}}$

$$\cancel{\oiint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}}+\frac{d}{dt}\iiint_V dV\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right]+\iiint_V dV\,\sigma\left|\vec{\mathbf{e}}\right|^2=-\iiint_V dV\,\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}$$

$$\oiint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}=\oiint_A dA\left[\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right]\cdot\hat{\mathbf{n}}=\oiint_A dA\boxed{\left[\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right]}\cdot\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=0$$

$$\vec{\mathbf{A}}\cdot\left[\vec{\mathbf{B}}\times\vec{\mathbf{C}}\right]=\vec{\mathbf{C}}\cdot\left[\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right]=\vec{\mathbf{B}}\cdot\left[\vec{\mathbf{C}}\times\vec{\mathbf{A}}\right]$$

Uniqueness (TD-Interior Problem)



$\hat{\mathbf{n}}\times\vec{\mathbf{e}}=\mathbf{0}$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

Let's apply the Poynting theorem (TD)

Medium
- **Linear**
- **Isotropic**
- **Space-Nondispersive**
- **Time-Nondispersive**
- **Time-invariant**

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

Source distribution $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)$ = 0

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

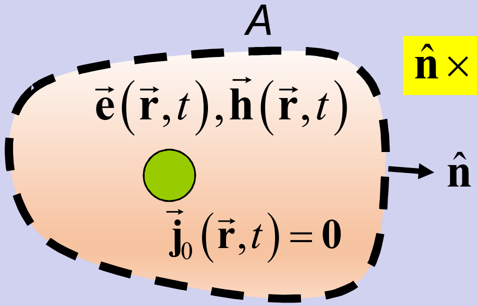
$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\mathbf{0}$ on the boundary

$$\cancel{\oiint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}}+\frac{d}{dt}\iiint_V dV\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right]+\iiint_V dV\,\sigma\left|\vec{\mathbf{e}}\right|^2=\cancel{-\iiint_V dV\,\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}}$$

$$\iiint_V dV\,\boxed{\vec{\mathbf{j}}_0}\cdot\vec{\mathbf{e}}=0$$

Uniqueness (TD-Interior Problem)



$\hat{\mathbf{n}}\times\vec{\mathbf{e}}=\mathbf{0}$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

Let's apply the Poynting theorem (TD)

Medium
- **Linear**
- **Isotropic**
- **Space-Nondispersive**
- **Time-Nondispersive**
- **Time-invariant**

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$

Source distribution $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)$ **= 0**

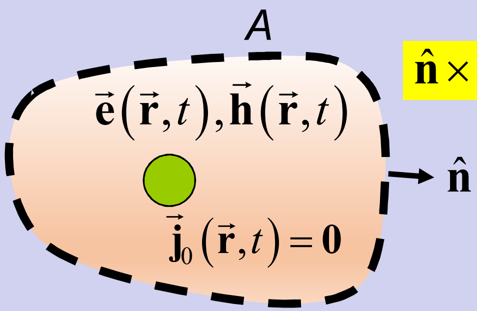
$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\mathbf{0}$ on the boundary

$$\cancel{\oint\!\!\!\oint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}}+\frac{d}{dt}\iiint_V dV\left[\frac{1}{2}\mu|\vec{\mathbf{h}}|^2+\frac{1}{2}\varepsilon|\vec{\mathbf{e}}|^2\right]+\iiint_V dV\,\sigma|\vec{\mathbf{e}}|^2=\cancel{-\iiint_V dV\,\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}}$$

Uniqueness (TD-Interior Problem)



$\hat{\mathbf{n}}\times\vec{\mathbf{e}}=\mathbf{0}$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

Let's apply the Poynting theorem (TD)

Medium
- **Linear**
- **Isotropic**
- **Space-Nondispersive**
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$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$

Source distribution $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)$ **= 0**

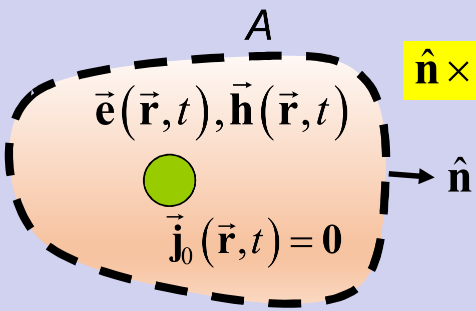
$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\mathbf{0}$ on the boundary

$$\frac{d}{dt}\iiint_V dV\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right]+\iiint_V dV\,\sigma\left|\vec{\mathbf{e}}\right|^2=\mathbf{0}$$

Uniqueness (TD-Interior Problem)



$\hat{\mathbf{n}}\times\vec{\mathbf{e}}=\mathbf{0}$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

Let's apply the Poynting theorem (TD)

Medium
- **Linear**
- **Isotropic**
- **Space-Nondispersive**
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$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$

Source distribution $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)$ **= 0**

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\mathbf{0}$ on the boundary

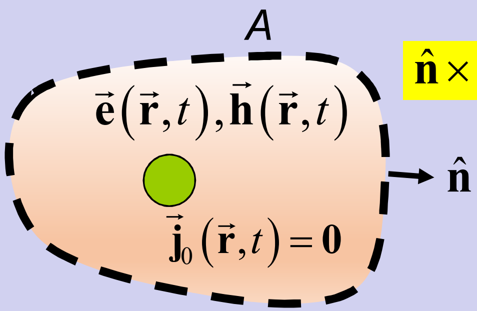
$$\frac{d}{dt}\iiint_V dV\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right]+\iiint_V dV\,\sigma\left|\vec{\mathbf{e}}\right|^2=0$$

$$\iiint_V dV\,\sigma\left|\vec{\mathbf{e}}\right|^2=P_j(t)\geq 0$$

$$\iiint_V dV\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right|^2\right]=W(t)\geq 0$$

$$\iiint_V dV\boxed{\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)\right|^2\right]}=W(t_0)=0$$

Uniqueness (TD-Interior Problem)



$\hat{\mathbf{n}}\times\vec{\mathbf{e}}=\mathbf{0}$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

Let's apply the Poynting theorem (TD)

Medium
- **Linear**
- **Isotropic**
- **Space-Nondispersive**
- **Time-Nondispersive**
- **Time-invariant**

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

Source distribution $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)$ **= 0**

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

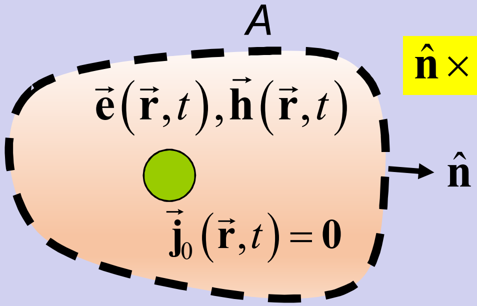
$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\mathbf{0}$ on the boundary

$$\frac{d}{dt}\iiint_V dV\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right]+\iiint_V dV\sigma\left|\vec{\mathbf{e}}\right|^2=0 \qquad \frac{d}{dt}W(t)+P_j(t)=0$$

$$\iiint_V dV\sigma\left|\vec{\mathbf{e}}\right|^2=P_j(t)\geq 0 \qquad \iiint_V dV\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right|^2\right]=W(t)\geq 0 \qquad \iiint_V dV\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)\right|^2\right]=W(t_0)=0$$

Uniqueness (TD-Interior Problem)



$\hat{\mathbf{n}}\times\vec{\mathbf{e}}=\mathbf{0}$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$$

Let's apply the Poynting theorem (TD)

Medium
- **Linear**
- **Isotropic**
- **Space-Nondispersive**
- **Time-Nondispersive**
- **Time-invariant**

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

Source distribution $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)$ **= 0**

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$$

$\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\mathbf{0}$ on the boundary

$$\frac{d}{dt}W(t)+P_j(t)=0 \;\Rightarrow\; \frac{d}{dt}W(t)=-P_j(t) \;\Rightarrow\; \begin{matrix} W(t_0)=0 \\ \dfrac{d}{dt}W(t)\le 0 \\ W(t)\ge 0 \end{matrix} \;\Rightarrow\; W(t)=0 \;\Rightarrow\; \begin{matrix} \vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\mathbf{0} \\ \vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\mathbf{0} \end{matrix}$$

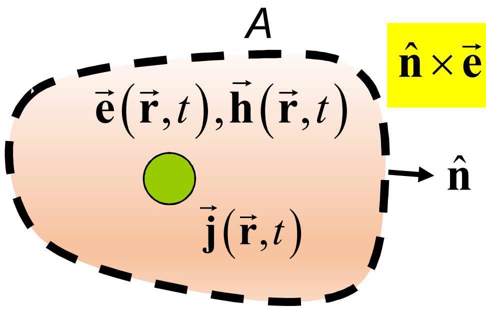
cvd

$$\iiint_V dV\,\sigma\left|\vec{\mathbf{e}}\right|^2=P_j(t)\ge 0$$

$$\boxed{\iiint_V dV\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right|^2\right]=}\boxed{W(t)\ge 0}$$

$$\iiint_V dV\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)\right|^2\right]=\boxed{W(t_0)=0}$$

Uniqueness (TD-Interior Problem)



$\hat{\mathbf{n}}\times\vec{\mathbf{e}}\ \left(\text{or}\ \ \hat{\mathbf{n}}\times\vec{\mathbf{h}}\right)$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$

I Consider a source distribution $\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$ with its associated electromagnetic field $\left(\vec{\mathbf{e}},\vec{\mathbf{h}}\right)$

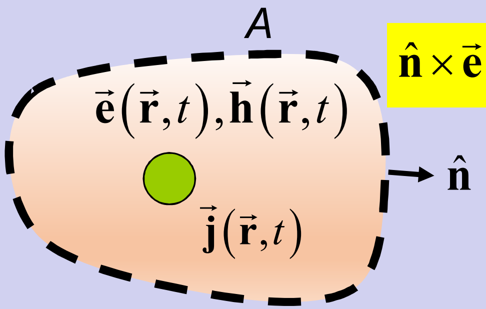
II Consider a (smooth) surface *A* with an everywhere defined unit normal $\hat{\mathbf{n}}$

III Consider the values of the electromagnetic field everywhere in **the finite volume V** bounded by the surface A **at the initial time**; that is, consider $\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$

IV Consider the values of the tangential component of the electric (**or** magnetic) field upon the surface A at any time after the initial one; that is, consider $\hat{\mathbf{n}}\times\vec{\mathbf{e}}$ (**or** $\hat{\mathbf{n}}\times\vec{\mathbf{h}}$) **on the boundary at any time**

The Uniqueness Theorem states that the electromagnetic field produced by the source in (I) within the **finite volume V bounded** by the surface A in (II), enforcing **the initial condition** in (III) and **the boundary condition** in (IV) is unique.

Uniqueness (TD-Interior Problem)



$\hat{\mathbf{n}}\times\vec{\mathbf{e}}\ \left(\text{or}\ \ \hat{\mathbf{n}}\times\vec{\mathbf{h}}\right)$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$

Source distribution: $\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$

$$\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t),\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t) \qquad \vec{\mathbf{e}}_2(\vec{\mathbf{r}},t),\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t_0)=\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t_0)$$

$$\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t_0)=\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t_0)$$

$$\hat{\mathbf{n}}\times\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)=\hat{\mathbf{n}}\times\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)\ \text{on the boundary}$$

Field difference: source distribution= 0

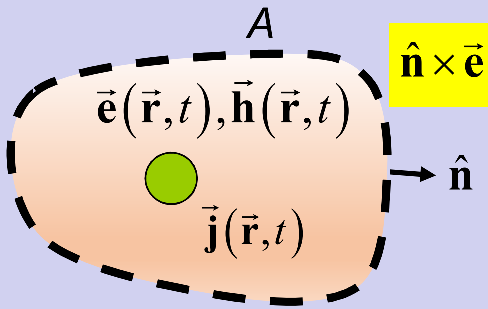
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t) \qquad \vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t_0)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t_0)=0$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t_0)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t_0)=0$$

$$\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\hat{\mathbf{n}}\times\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\hat{\mathbf{n}}\times\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)=0\ \ \text{on the boundary}$$

Uniqueness (TD-Interior Problem)



$\hat{\mathbf{n}}\times\vec{\mathbf{e}}\left(\text{or}\;\;\hat{\mathbf{n}}\times\vec{\mathbf{h}}\right)$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$

Source distribution: $\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$

$$\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t),\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t) \qquad \vec{\mathbf{e}}_2(\vec{\mathbf{r}},t),\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t_0)=\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t_0)$$

$$\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t_0)=\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t_0)$$

$$\hat{\mathbf{n}}\times\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)=\hat{\mathbf{n}}\times\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)\text{ on the boundary}$$

Field difference: source distribution= 0

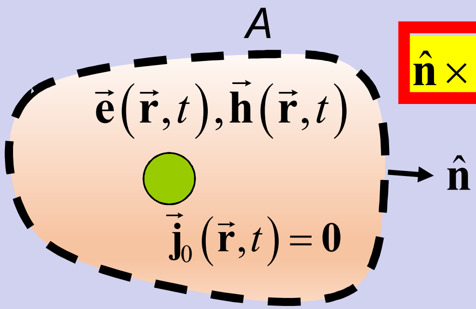
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t) \qquad \vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t_0)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t_0)=0$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t_0)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t_0)=0$$

$$\hat{\mathbf{n}}\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\hat{\mathbf{n}}\times\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\hat{\mathbf{n}}\times\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)=0\text{ on the boundary}$$

Uniqueness (TD-Interior Problem)



$\boxed{\hat{\mathbf{n}}\times\vec{\mathbf{h}}=\mathbf{0}}$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$$

Let's apply the Poynting theorem (TD)

Medium
- **Linear**
- **Isotropic**
- **Space-Nondispersive**
- **Time-Nondispersive**
- **Time-invariant**

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

Source distribution $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)$ **= 0**

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$$

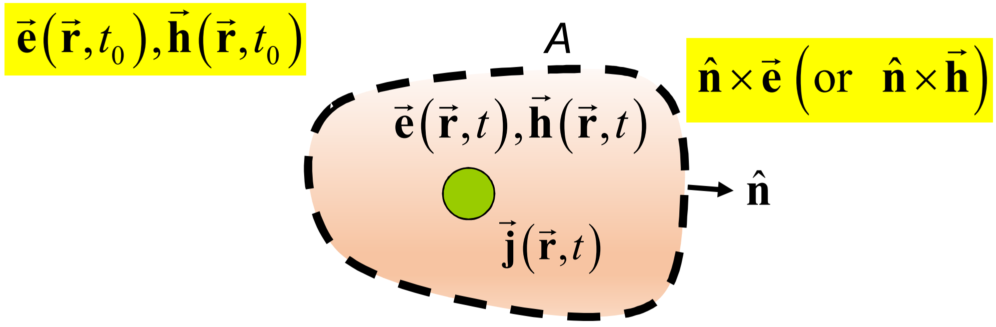
$\boxed{\hat{\mathbf{n}}\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\mathbf{0}\ \text{ on the boundary}}$

$$\cancel{\oint\!\!\!\oint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}}+\frac{d}{dt}\iiint_V dV\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right]+\iiint_V dV\,\sigma\left|\vec{\mathbf{e}}\right|^2=-\iiint_V dV\,\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}$$

$$\oint\!\!\!\oint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}=\oint\!\!\!\oint_A dA\left[\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right]\cdot\hat{\mathbf{n}}=\oint\!\!\!\oint_A dA\left[\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right]\cdot\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\oint\!\!\!\oint_A dA\boxed{\left[\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\times\hat{\mathbf{n}}\right]}\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=0$$

$$\vec{\mathbf{A}}\cdot\left[\vec{\mathbf{B}}\times\vec{\mathbf{C}}\right]=\vec{\mathbf{C}}\cdot\left[\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right]=\vec{\mathbf{B}}\cdot\left[\vec{\mathbf{C}}\times\vec{\mathbf{A}}\right]$$

Uniqueness (TD-Interior Problem)



I Consider a source distribution $\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$ with its associated electromagnetic field $(\vec{\mathbf{e}},\vec{\mathbf{h}})$

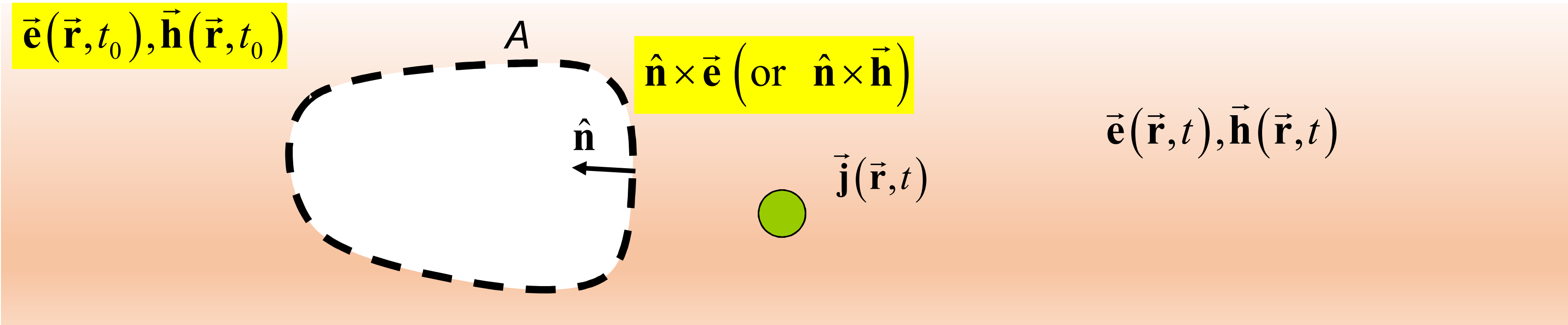
II Consider a (smooth) surface *A* with an everywhere defined unit normal $\hat{\mathbf{n}}$

III Consider the values of the electromagnetic field everywhere in **the finite volume V** bounded by the surface A **at the initial time**; that is, consider $\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$

IV Consider the values of the tangential component of the electric (**or** magnetic) field upon the surface A at any time after the initial one; that is, consider $\hat{\mathbf{n}}\times\vec{\mathbf{e}}$ (**or** $\hat{\mathbf{n}}\times\vec{\mathbf{h}}$) **on the boundary at any time**

The Uniqueness Theorem states that the electromagnetic field produced by the source in (I) within the **finite volume V bounded** by the surface A in (II), enforcing **the initial condition** in (III) and **the boundary condition** in (IV) is unique.

Uniqueness (TD-Exterior Problem)



I Consider a source distribution $\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$ with its associated electromagnetic field $(\vec{\mathbf{e}},\vec{\mathbf{h}})$

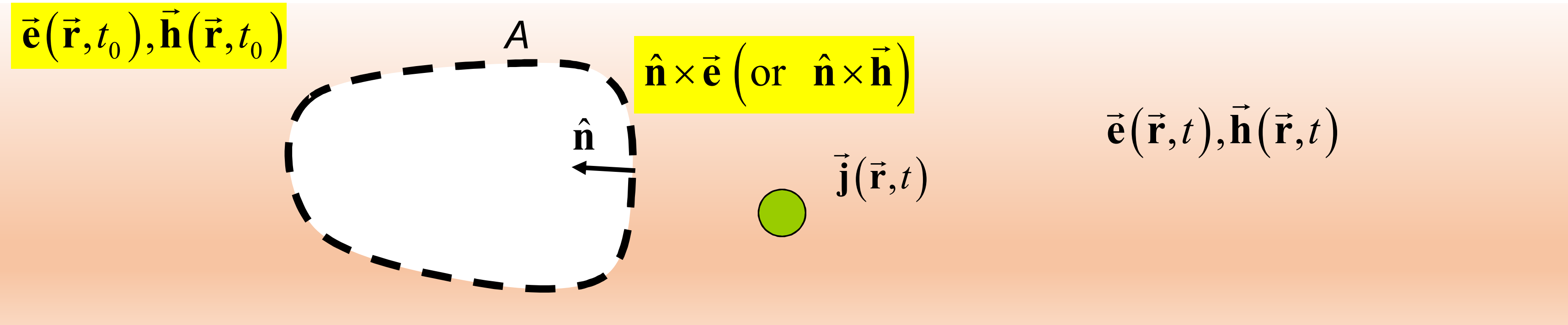
II Consider a (smooth) surface *A* with an everywhere defined unit normal $\hat{\mathbf{n}}$

III Consider the values of the electromagnetic field everywhere in **the infinite volume outside** the surface A **at the initial time**; that is, consider $\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$

IV Consider the values of the tangential component of the electric (**or** magnetic) field upon the surface A at any time after the initial one; that is, consider $\hat{\mathbf{n}}\times\vec{\mathbf{e}}$ (**or** $\hat{\mathbf{n}}\times\vec{\mathbf{h}}$) **on the boundary at any time**

The Uniqueness Theorem states that the electromagnetic field produced by the source in (I) within the **infinite volume V outside** the surface A in (II), enforcing **the initial condition** in (III) and **the boundary condition** in (IV) is unique.

Uniqueness (TD-Exterior Problem)



Source distribution: $\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$

$$\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t),\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t) \qquad \vec{\mathbf{e}}_2(\vec{\mathbf{r}},t),\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t_0)=\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t_0)$$

$$\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t_0)=\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t_0)$$

$$\hat{\mathbf{n}}\times\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)=\hat{\mathbf{n}}\times\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)\ \text{on the boundary}$$

Field difference: source distribution= 0

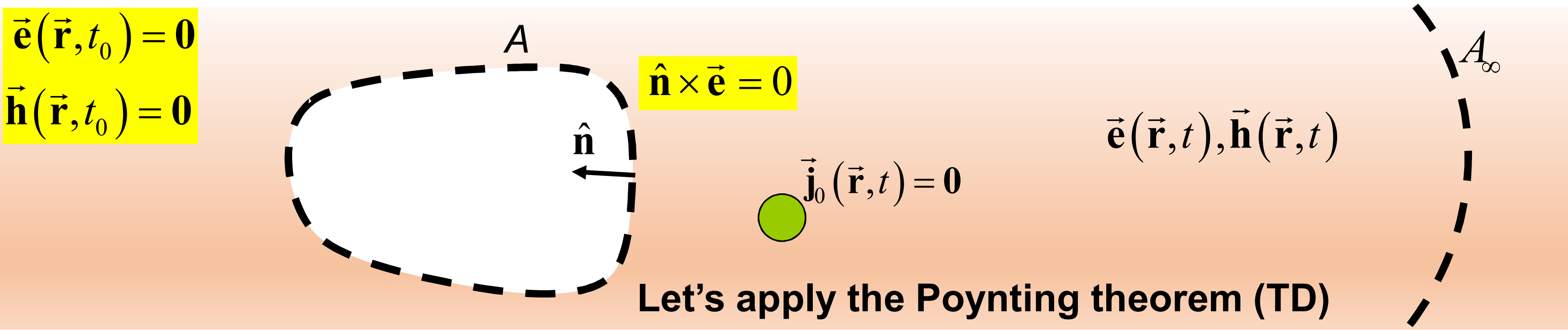
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t) \qquad \vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t_0)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t_0)=0$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t_0)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t_0)=0$$

$$\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\hat{\mathbf{n}}\times\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\hat{\mathbf{n}}\times\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)=0\ \ \text{on the boundary}$$

Uniqueness (TD-Exterior Problem)



Medium
- Linear
- Isotropic
- Space-Nondispersive
- Time-Nondispersive
- Time-invariant

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

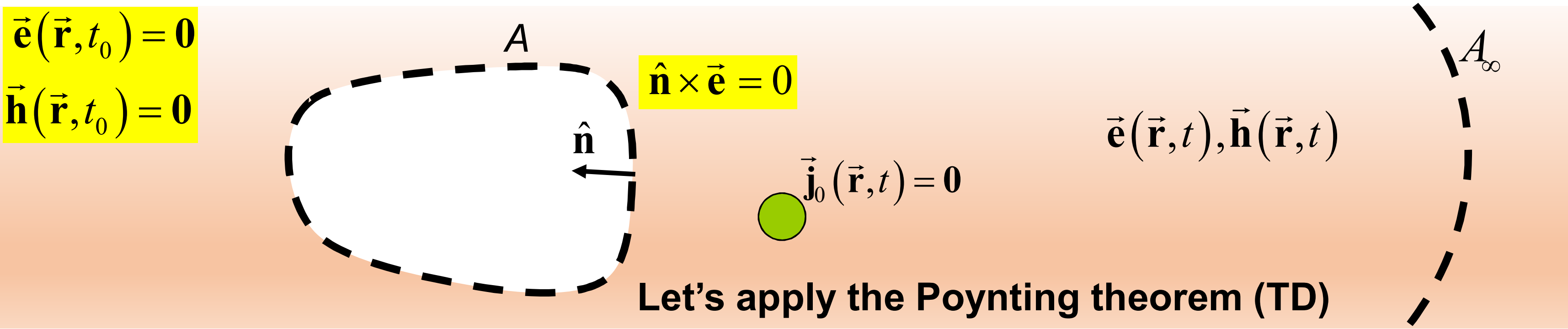
Source distribution $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)$ **= 0**

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$$

$\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\mathbf{0}$ on the boundary

Uniqueness (TD-Exterior Problem)



Medium
- **Linear**
- **Isotropic**
- **Space-Nondispersive**
- **Time-Nondispersive**
- **Time-invariant**

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

Source distribution $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)$ = 0

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$$

$$\boxed{\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\mathbf{0}\quad \text{on the boundary}}$$

$$\cancel{\oint\!\!\oint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)}\cdot\hat{\mathbf{n}}+\oint\!\!\oint_{A_\infty} dA_\infty\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}+\frac{d}{dt}\iiint_V dV\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right]+\iiint_V dV\,\sigma\left|\vec{\mathbf{e}}\right|^2=-\iiint_V dV\,\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}$$

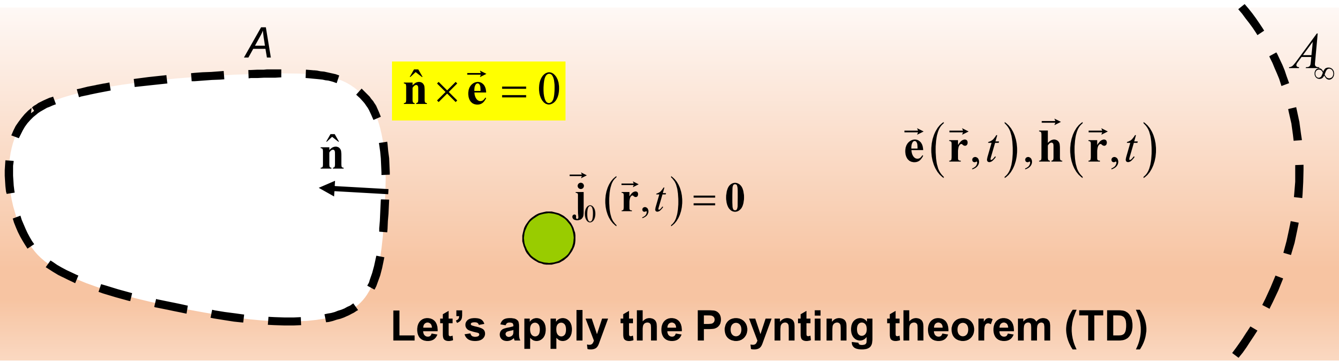
$$\oint\!\!\oint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}=\oint\!\!\oint_A dA\left[\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right]\cdot\hat{\mathbf{n}}=\oint\!\!\oint_A dA\boxed{\left[\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right]}\cdot\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=0$$

$$\vec{\mathbf{A}}\cdot\left[\vec{\mathbf{B}}\times\vec{\mathbf{C}}\right]=\vec{\mathbf{C}}\cdot\left[\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right]=\vec{\mathbf{B}}\cdot\left[\vec{\mathbf{C}}\times\vec{\mathbf{A}}\right]$$

Uniqueness (TD-Exterior Problem)

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$



$\hat{\mathbf{n}}\times\vec{\mathbf{e}}=0$

$\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)=\mathbf{0}$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t),\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$

Let's apply the Poynting theorem (TD)

Medium
- **Linear**
- **Isotropic**
- **Space-Nondispersive**
- **Time-Nondispersive**
- **Time-invariant**

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$

Source distribution $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)$ **= 0**

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

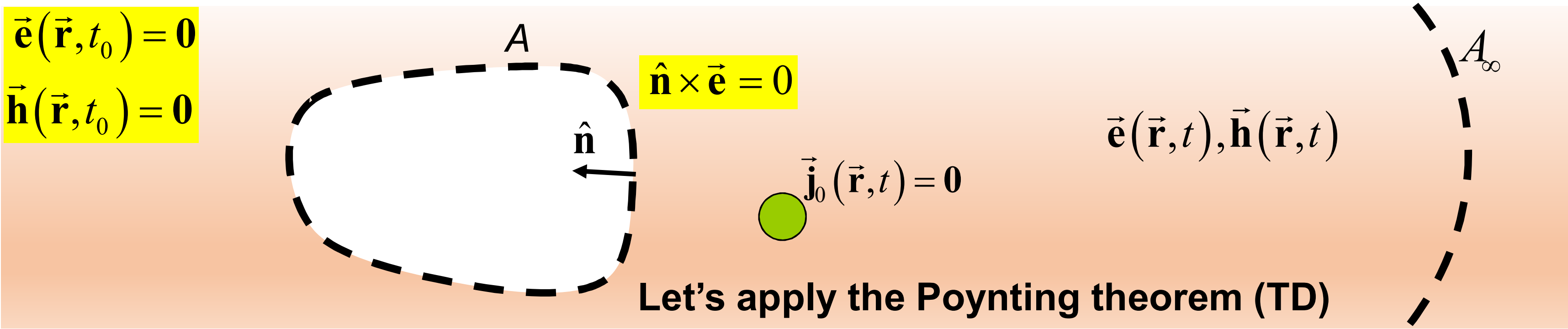
$\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\mathbf{0}$ on the boundary

$$\cancel{\oint\!\!\!\oint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}}+\cancel{\oint\!\!\!\oint_{A_\infty} dA_\infty\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}}+\frac{d}{dt}\iiint_V dV\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right]+\iiint_V dV\,\sigma\left|\vec{\mathbf{e}}\right|^2=-\iiint_V dV\,\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}$$

$$\oint\!\!\!\oint_{A_\infty} dA_\infty\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}=0$$

A_∞ is a large sphere whose radius $R>ct$, c being the speed of the light

Uniqueness (TD-Exterior Problem)



Medium
- **Linear**
- **Isotropic**
- **Space-Nondispersive**
- **Time-Nondispersive**
- **Time-invariant**

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

Source distribution $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)$ **= 0**

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$$

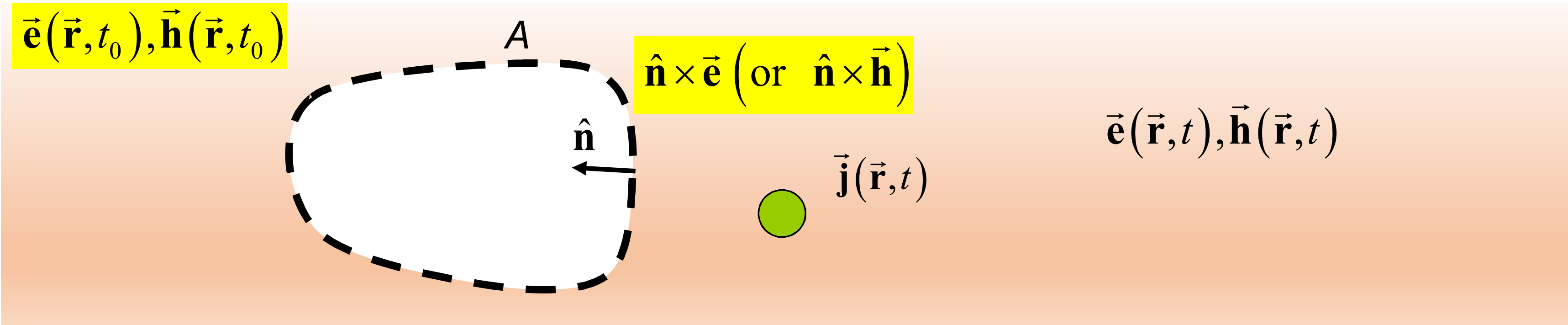
$\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\mathbf{0}$ on the boundary

$$\cancel{\oiint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}}+\cancel{\oiint_{A_\infty} dA_\infty\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}}+\frac{d}{dt}\iiint_V dV\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right]+\iiint_V dV\,\sigma\left|\vec{\mathbf{e}}\right|^2=\cancel{-\iiint_V dV\,\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}}$$

$$\Rightarrow\quad \begin{array}{l} W(t_0)=0\\ \dfrac{d}{dt}W(t)\le 0\\ W(t)\ge 0\end{array}\quad\Rightarrow\quad \begin{array}{l}\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\mathbf{0}\\ \vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\mathbf{0}\end{array}$$

cvd

Uniqueness (TD-Exterior Problem)



I Consider a source distribution $\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$ with its associated electromagnetic field $(\vec{\mathbf{e}},\vec{\mathbf{h}})$

II Consider a (smooth) surface *A* with an everywhere defined unit normal $\hat{\mathbf{n}}$

III Consider the values of the electromagnetic field everywhere in **the infinite volume outside** the surface A **at the initial time**; that is, consider $\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$

IV Consider the values of the tangential component of the electric (**or** magnetic) field upon the surface A at any time after the initial one; that is, consider $\hat{\mathbf{n}}\times\vec{\mathbf{e}}$ (**or** $\hat{\mathbf{n}}\times\vec{\mathbf{h}}$) **on the boundary at any time**

The Uniqueness Theorem states that the electromagnetic field produced by the source in (I) within the **infinite volume V outside** the surface A in (II), enforcing **the initial condition** in (III) and **the boundary condition** in (IV) is unique.