

# **Campi Elettromagnetici**

**Corso di Laurea in Ingegneria Informatica,  
Biomedica e delle Telecomunicazioni**

**a.a. 2020-2021 - Laurea “Triennale” – Secondo semestre - Secondo anno**

**Università degli Studi di Napoli “Parthenope”**

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# Color legend

New formulas, important considerations,  
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

# Poynting theorem (TD)

$$\vec{\mathbf{s}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t)$$

**Poynting vector**

$$[\vec{\mathbf{s}}]: \frac{\text{Watt}}{m^2}$$

# Poynting theorem (TD)

$$\vec{s}(\vec{r}, t) = \vec{e}(\vec{r}, t) \times \vec{h}(\vec{r}, t)$$

Poynting vector

$$[\vec{s}]: \frac{Watt}{m^2}$$

$$\nabla \cdot \vec{s}(\vec{r}, t) + \frac{\partial}{\partial t} w(\vec{r}, t) + p_j(\vec{r}, t) = p_0(\vec{r}, t)$$

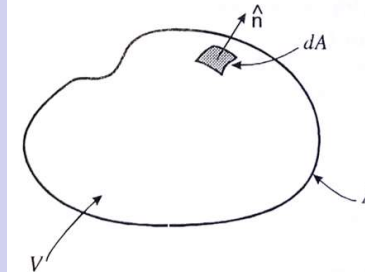
$$\oiint_A dA \vec{s}(\vec{r}, t) \cdot \hat{n} + \frac{d}{dt} \iiint_V dV w(\vec{r}, t) + \iiint_V dV p_j(\vec{r}, t) = \iiint_V dV p_0(\vec{r}, t)$$

Electromagnetic  
power flux

$$P_S(t) + \frac{d}{dt} W(t) + P_j(t) = P_0(t)$$

Hypotheses on the medium

- Linear
- Local (TND & SND)
- Isotropic
- Time-invariant



$$w(\vec{r}, t) = \frac{1}{2} \mu |\vec{h}|^2 + \frac{1}{2} \varepsilon |\vec{e}|^2 \quad \text{Energy density of the e.m. field} \quad \Rightarrow \quad \iiint_V dV w(\vec{r}, t) = W(t) \quad \text{Energy of the e.m. field}$$

$$p_j(\vec{r}, t) = \sigma |\vec{e}|^2 \quad \text{Power density dissipated in the conducting medium} \quad \Rightarrow \quad \iiint_V dV p_j(\vec{r}, t) = P_j(t) \quad \text{Power dissipated in the conducting medium}$$

$$p_0(\vec{r}, t) = -\vec{j}_0 \cdot \vec{e} \quad \text{Power density delivered by the sources to the field} \quad \Rightarrow \quad \iiint_V dV p_0(\vec{r}, t) = P(t) \quad \text{Power delivered by the sources to the field}$$

# THEOREMS

## **Poynting**

Time domain – Phasor domain

## **Uniqueness** (Interior problem – Exterior problem)

Time domain – Phasor domain

## **Equivalence**

Phasor domain

## **Image Theory**

## **Reciprocity**

Phasor domain

# Mathematical tools that we will exploit today

$$\nabla \cdot [\vec{\mathbf{A}}(\vec{\mathbf{r}}) \times \vec{\mathbf{B}}(\vec{\mathbf{r}})] = \vec{\mathbf{B}}(\vec{\mathbf{r}}) \cdot [\nabla \times \vec{\mathbf{A}}(\vec{\mathbf{r}})] - \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot [\nabla \times \vec{\mathbf{B}}(\vec{\mathbf{r}})]$$

$$\iiint_V dV \nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \oiint_S dS \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}$$

$$\frac{1}{2} \frac{\partial |\vec{\mathbf{A}}(t)|^2}{\partial t} = \vec{\mathbf{A}}(t) \cdot \left[ \frac{\partial \vec{\mathbf{A}}(\vec{\mathbf{r}}, t)}{\partial t} \right]$$

# Poynting theorem (PD)

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \frac{1}{2} [\vec{\mathbf{E}}(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^*(\vec{\mathbf{r}})]$$

Poynting vector

$$[\vec{\mathbf{S}}]: \frac{\text{Watt}}{\text{m}^2}$$

$$\begin{aligned} \nabla \cdot \vec{\mathbf{S}}(\vec{\mathbf{r}}) &= \frac{1}{2} \nabla \cdot [\vec{\mathbf{E}}(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^*(\vec{\mathbf{r}})] = \frac{1}{2} \vec{\mathbf{H}}^*(\vec{\mathbf{r}}) \cdot [\nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}})] - \frac{1}{2} \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot [\nabla \times \vec{\mathbf{H}}^*(\vec{\mathbf{r}})] = \\ &= \frac{1}{2} \vec{\mathbf{H}}^*(\vec{\mathbf{r}}) \cdot [\nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}})] - \frac{1}{2} \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot [\nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}})]^* = \\ &= \frac{1}{2} \vec{\mathbf{H}}^*(\vec{\mathbf{r}}) \cdot [-j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}})] - \frac{1}{2} \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot [j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}})]^* \\ &= -\frac{1}{2} j\omega_0 \vec{\mathbf{H}}^*(\vec{\mathbf{r}}) \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) + \frac{1}{2} j\omega_0 \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{D}}^*(\vec{\mathbf{r}}) - \frac{1}{2} \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{J}}^*(\vec{\mathbf{r}}) - \frac{1}{2} \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{J}}_0^*(\vec{\mathbf{r}}) \end{aligned}$$

$$\nabla \cdot [\vec{\mathbf{A}}(\vec{\mathbf{r}}) \times \vec{\mathbf{B}}(\vec{\mathbf{r}})] = \vec{\mathbf{B}}(\vec{\mathbf{r}}) \cdot [\nabla \times \vec{\mathbf{A}}(\vec{\mathbf{r}})] - \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot [\nabla \times \vec{\mathbf{B}}(\vec{\mathbf{r}})]$$

Phasor domain

$$\nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}})$$

$$\nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}})$$

$$\nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) + \rho_0(\vec{\mathbf{r}})$$

$$\nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) = 0$$

# Poynting theorem (PD)

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \frac{1}{2} [\vec{\mathbf{E}}(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^*(\vec{\mathbf{r}})]$$

Poynting vector

$$[\vec{\mathbf{S}}]: \frac{\text{Watt}}{\text{m}^2}$$

Hypotheses on the medium (TD)

- Linear
- Isotropic
- Time-invariant
- Local (TND & SND)

$$\nabla \cdot \vec{\mathbf{S}}(\vec{\mathbf{r}}) = -\frac{1}{2} j\omega_0 \vec{\mathbf{H}}^*(\vec{\mathbf{r}}) \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) + \frac{1}{2} j\omega_0 \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{D}}^*(\vec{\mathbf{r}}) - \frac{1}{2} \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{J}}^*(\vec{\mathbf{r}}) - \frac{1}{2} \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{J}}_0^*(\vec{\mathbf{r}})$$

$$\begin{cases} \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \varepsilon(\vec{\mathbf{r}}) \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) \\ \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = \mu(\vec{\mathbf{r}}) \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) \\ \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) = \sigma \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) \end{cases}$$



# Poynting theorem (PD)

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \frac{1}{2} [\vec{\mathbf{E}}(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^*(\vec{\mathbf{r}})]$$

Poynting vector

$$[\vec{\mathbf{S}}]: \frac{\text{Watt}}{\text{m}^2}$$

Hypotheses on the medium (TD)

- Linear
- Isotropic
- Time-invariant
- Local (~~TD~~ & SND)

$$\nabla \cdot \vec{\mathbf{S}}(\vec{\mathbf{r}}) = -\frac{1}{2} j\omega_0 \vec{\mathbf{H}}^*(\vec{\mathbf{r}}) \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) + \frac{1}{2} j\omega_0 \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{D}}^*(\vec{\mathbf{r}}) - \frac{1}{2} \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{J}}^*(\vec{\mathbf{r}}) - \frac{1}{2} \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{J}}_0^*(\vec{\mathbf{r}})$$

$$\begin{cases} \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \varepsilon(\vec{\mathbf{r}}) \vec{\mathbf{E}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{B}}(\vec{\mathbf{r}}) = \mu(\vec{\mathbf{r}}) \vec{\mathbf{H}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{J}}(\vec{\mathbf{r}}) = \sigma \vec{\mathbf{E}}(\vec{\mathbf{r}}) \end{cases}$$

$$\begin{cases} \varepsilon: \text{real} \\ \mu: \text{real} \\ \sigma: \text{real} \end{cases}$$

# Poynting theorem (PD)

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \frac{1}{2} [\vec{\mathbf{E}}(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^*(\vec{\mathbf{r}})]$$

Poynting vector

$$[\vec{\mathbf{S}}]: \frac{Watt}{m^2}$$

Hypotheses on the medium (PD)

- Linear
- Isotropic
- Time-invariant
- Time-Dispersive
- Space-Nondispersive

$$\nabla \cdot \vec{\mathbf{S}}(\vec{\mathbf{r}}) = -\frac{1}{2} j\omega_0 \vec{\mathbf{H}}^*(\vec{\mathbf{r}}) \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) + \frac{1}{2} j\omega_0 \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{D}}^*(\vec{\mathbf{r}}) - \frac{1}{2} \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{J}}^*(\vec{\mathbf{r}}) - \frac{1}{2} \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{J}}_0^*(\vec{\mathbf{r}})$$

$$\begin{cases} \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \varepsilon(\vec{\mathbf{r}}) \vec{\mathbf{E}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{B}}(\vec{\mathbf{r}}) = \mu(\vec{\mathbf{r}}) \vec{\mathbf{H}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{J}}(\vec{\mathbf{r}}) = \sigma \vec{\mathbf{E}}(\vec{\mathbf{r}}) \end{cases}$$

$$\begin{cases} \varepsilon: real \\ \mu: real \\ \sigma: real \end{cases}$$

# Poynting theorem (PD)

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \frac{1}{2} [\vec{\mathbf{E}}(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^*(\vec{\mathbf{r}})]$$

Poynting vector

$$[\vec{\mathbf{S}}]: \frac{\text{Watt}}{\text{m}^2}$$

$$\nabla \cdot \vec{\mathbf{S}}(\vec{\mathbf{r}}) = -\frac{1}{2} j\omega_0 \vec{\mathbf{H}}^*(\vec{\mathbf{r}}) \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) + \frac{1}{2} j\omega_0 \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{D}}^*(\vec{\mathbf{r}}) - \frac{1}{2} \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{J}}^*(\vec{\mathbf{r}}) - \frac{1}{2} \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{J}}_0^*(\vec{\mathbf{r}})$$

$$\nabla \cdot \vec{\mathbf{S}} = -\frac{1}{2} j\omega_0 \vec{\mathbf{H}}^* \cdot \vec{\mathbf{B}} + \frac{1}{2} j\omega_0 \vec{\mathbf{E}} \cdot \vec{\mathbf{D}}^* - \frac{1}{2} \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}^* - \frac{1}{2} \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^*$$

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$$\begin{cases} \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \varepsilon(\vec{\mathbf{r}}) \vec{\mathbf{E}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{B}}(\vec{\mathbf{r}}) = \mu(\vec{\mathbf{r}}) \vec{\mathbf{H}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{J}}(\vec{\mathbf{r}}) = \sigma \vec{\mathbf{E}}(\vec{\mathbf{r}}) \end{cases}$$

$$\begin{cases} \varepsilon = \varepsilon_1 - j\varepsilon_2 \\ \mu = \mu_1 - j\mu_2 \\ \sigma: \text{real} \end{cases}$$

# Poynting theorem (PD)

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \frac{1}{2} [\vec{\mathbf{E}}(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^*(\vec{\mathbf{r}})]$$

Poynting vector

$$[\vec{\mathbf{S}}]: \frac{Watt}{m^2}$$

$$\begin{aligned} \nabla \cdot \vec{\mathbf{S}} &= -\frac{1}{2} j\omega_0 \vec{\mathbf{H}}^* \cdot \vec{\mathbf{B}} + \frac{1}{2} j\omega_0 \vec{\mathbf{E}} \cdot \vec{\mathbf{D}}^* - \frac{1}{2} \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}^* - \frac{1}{2} \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0 \\ &= -\frac{1}{2} j\omega_0 \mu_1 |\vec{\mathbf{H}}|^2 - \frac{1}{2} \omega_0 \mu_2 |\vec{\mathbf{H}}|^2 \end{aligned}$$

$$-j\omega_0 \vec{\mathbf{H}}^* \cdot \vec{\mathbf{B}} = -j\omega_0 \vec{\mathbf{H}}^* \cdot [(\mu_1 - j\mu_2) \vec{\mathbf{H}}] = -j\omega_0 \mu_1 |\vec{\mathbf{H}}|^2 - \omega_0 \mu_2 |\vec{\mathbf{H}}|^2$$

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Poynting vector

$$[\vec{\mathbf{S}}]: \frac{Watt}{m^2}$$

$$\begin{aligned} \nabla \cdot \vec{\mathbf{S}} &= -\frac{1}{2} j\omega_0 \vec{\mathbf{H}}^* \cdot \vec{\mathbf{B}} + \frac{1}{2} j\omega_0 \vec{\mathbf{E}} \cdot \vec{\mathbf{D}}^* - \frac{1}{2} \vec{\mathbf{E}} \cdot \vec{\mathbf{J}} - \frac{1}{2} \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0 \\ &= -\frac{1}{2} j\omega_0 \mu_1 |\vec{\mathbf{H}}|^2 - \frac{1}{2} \omega_0 \mu_2 |\vec{\mathbf{H}}|^2 + \frac{1}{2} j\omega_0 \varepsilon_1 |\vec{\mathbf{E}}|^2 - \frac{1}{2} \omega_0 \varepsilon_2 |\vec{\mathbf{E}}|^2 \end{aligned}$$

$$j\omega_0 \vec{\mathbf{E}} \cdot \vec{\mathbf{D}}^* = j\omega_0 \vec{\mathbf{E}} \cdot [(\varepsilon_1 - j\varepsilon_2) \vec{\mathbf{E}}]^* = j\omega_0 \vec{\mathbf{E}} \cdot [(\varepsilon_1 + j\varepsilon_2) \vec{\mathbf{E}}^*] = j\omega_0 \varepsilon_1 |\vec{\mathbf{E}}|^2 - \omega_0 \varepsilon_2 |\vec{\mathbf{E}}|^2$$

## Hypotheses on the medium (PD)

- Linear
- Isotropic
- Time-invariant
- Time-Dispersive
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$$\begin{cases} \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \varepsilon(\vec{\mathbf{r}}) \vec{\mathbf{E}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{B}}(\vec{\mathbf{r}}) = \mu(\vec{\mathbf{r}}) \vec{\mathbf{H}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{J}}(\vec{\mathbf{r}}) = \sigma \vec{\mathbf{E}}(\vec{\mathbf{r}}) \end{cases}$$

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Poynting vector

$$[\vec{\mathbf{S}}]: \frac{Watt}{m^2}$$

$$\nabla \cdot \vec{\mathbf{S}} = -\frac{1}{2} j\omega_0 \vec{\mathbf{H}}^* \cdot \vec{\mathbf{B}} + \frac{1}{2} j\omega_0 \vec{\mathbf{E}} \cdot \vec{\mathbf{D}}^* - \frac{1}{2} \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}^* - \frac{1}{2} \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0$$

$$= -\frac{1}{2} j\omega_0 \mu_1 |\vec{\mathbf{H}}|^2 - \frac{1}{2} \omega_0 \mu_2 |\vec{\mathbf{H}}|^2 + \frac{1}{2} j\omega_0 \varepsilon_1 |\vec{\mathbf{E}}|^2 - \frac{1}{2} \omega_0 \varepsilon_2 |\vec{\mathbf{E}}|^2 - \frac{1}{2} \sigma |\vec{\mathbf{E}}|^2 - \frac{1}{2} \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0$$

$$-\vec{\mathbf{E}} \cdot \vec{\mathbf{J}}^* = -\vec{\mathbf{E}} \cdot \sigma \vec{\mathbf{E}}^* = -\sigma |\vec{\mathbf{E}}|^2$$

## Hypotheses on the medium (PD)

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$$\begin{cases} \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \varepsilon(\vec{\mathbf{r}}) \vec{\mathbf{E}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{B}}(\vec{\mathbf{r}}) = \mu(\vec{\mathbf{r}}) \vec{\mathbf{H}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{J}}(\vec{\mathbf{r}}) = \sigma \vec{\mathbf{E}}(\vec{\mathbf{r}}) \end{cases}$$

$$\begin{cases} \varepsilon = \varepsilon_1 - j\varepsilon_2 \\ \mu = \mu_1 - j\mu_2 \\ \sigma: \text{real} \end{cases}$$

# Poynting theorem (PD)

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \frac{1}{2} [\vec{\mathbf{E}}(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^*(\vec{\mathbf{r}})]$$

Poynting vector

$$[\vec{\mathbf{S}}]: \frac{\text{Watt}}{\text{m}^2}$$

Hypotheses on the medium (PD)

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$$\nabla \cdot \vec{\mathbf{S}} = -\frac{1}{2} j\omega_0 \vec{\mathbf{H}}^* \cdot \vec{\mathbf{B}} + \frac{1}{2} j\omega_0 \vec{\mathbf{E}} \cdot \vec{\mathbf{D}}^* - \frac{1}{2} \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}^* - \frac{1}{2} \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^*$$

$$= -\frac{1}{2} j\omega_0 \mu_1 |\vec{\mathbf{H}}|^2 - \frac{1}{2} \omega_0 \mu_2 |\vec{\mathbf{H}}|^2 + \frac{1}{2} j\omega_0 \varepsilon_1 |\vec{\mathbf{E}}|^2 - \frac{1}{2} \omega_0 \varepsilon_2 |\vec{\mathbf{E}}|^2 - \frac{1}{2} \sigma |\vec{\mathbf{E}}|^2 - \frac{1}{2} \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^*$$

$$\begin{cases} \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \varepsilon(\vec{\mathbf{r}}) \vec{\mathbf{E}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{B}}(\vec{\mathbf{r}}) = \mu(\vec{\mathbf{r}}) \vec{\mathbf{H}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{J}}(\vec{\mathbf{r}}) = \sigma \vec{\mathbf{E}}(\vec{\mathbf{r}}) \end{cases}$$

$$\nabla \cdot \vec{\mathbf{S}}(\vec{\mathbf{r}}) = \nabla \cdot \vec{\mathbf{S}}_1(\vec{\mathbf{r}}) + j\nabla \cdot \vec{\mathbf{S}}_2(\vec{\mathbf{r}})$$

$$\nabla \cdot \vec{\mathbf{S}}_1 = -\frac{1}{2} \omega_0 \mu_2 |\vec{\mathbf{H}}|^2 - \frac{1}{2} \omega_0 \varepsilon_2 |\vec{\mathbf{E}}|^2 - \frac{1}{2} \sigma |\vec{\mathbf{E}}|^2 - \frac{1}{2} \text{Re} \left\{ \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^* \right\}$$

$$\nabla \cdot \vec{\mathbf{S}}_2 = -\frac{1}{2} \omega_0 \mu_1 |\vec{\mathbf{H}}|^2 + \frac{1}{2} \omega_0 \varepsilon_1 |\vec{\mathbf{E}}|^2 - \frac{1}{2} \text{Im} \left\{ \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^* \right\}$$

$$\begin{cases} \varepsilon = \varepsilon_1 - j\varepsilon_2 \\ \mu = \mu_1 - j\mu_2 \\ \sigma: \text{real} \end{cases}$$

# Poynting theorem (PD)

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \frac{1}{2} [\vec{\mathbf{E}}(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^*(\vec{\mathbf{r}})]$$

Poynting vector

$$[\vec{\mathbf{S}}]: \frac{Watt}{m^2}$$

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \vec{\mathbf{S}}_1(\vec{\mathbf{r}}) + j\vec{\mathbf{S}}_2(\vec{\mathbf{r}})$$

$$\nabla \cdot \vec{\mathbf{S}}_1 + \frac{1}{2} \omega_0 \mu_2 |\vec{\mathbf{H}}|^2 + \frac{1}{2} \omega_0 \varepsilon_2 |\vec{\mathbf{E}}|^2 + \frac{1}{2} \sigma |\vec{\mathbf{E}}|^2 = -\frac{1}{2} \text{Re} \{ \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^* \}$$

$$\nabla \cdot \vec{\mathbf{S}}_2 + \frac{1}{2} \omega_0 \mu_1 |\vec{\mathbf{H}}|^2 - \frac{1}{2} \omega_0 \varepsilon_1 |\vec{\mathbf{E}}|^2 = -\frac{1}{2} \text{Im} \{ \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^* \}$$

$$\nabla \cdot \vec{\mathbf{S}}(\vec{\mathbf{r}}) = \nabla \cdot \vec{\mathbf{S}}_1(\vec{\mathbf{r}}) + j \nabla \cdot \vec{\mathbf{S}}_2(\vec{\mathbf{r}})$$

$$\nabla \cdot \vec{\mathbf{S}}_1 = -\frac{1}{2} \omega_0 \mu_2 |\vec{\mathbf{H}}|^2 - \frac{1}{2} \omega_0 \varepsilon_2 |\vec{\mathbf{E}}|^2 - \frac{1}{2} \sigma |\vec{\mathbf{E}}|^2 - \frac{1}{2} \text{Re} \{ \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^* \}$$

$$\nabla \cdot \vec{\mathbf{S}}_2 = -\frac{1}{2} \omega_0 \mu_1 |\vec{\mathbf{H}}|^2 + \frac{1}{2} \omega_0 \varepsilon_1 |\vec{\mathbf{E}}|^2 - \frac{1}{2} \text{Im} \{ \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^* \}$$

## Hypotheses on the medium (PD)

- Linear
- Isotropic
- Time-invariant
- Time-Dispersive
- Space-Nondispersive

$$\begin{cases} \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \varepsilon(\vec{\mathbf{r}}) \vec{\mathbf{E}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{B}}(\vec{\mathbf{r}}) = \mu(\vec{\mathbf{r}}) \vec{\mathbf{H}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{J}}(\vec{\mathbf{r}}) = \sigma \vec{\mathbf{E}}(\vec{\mathbf{r}}) \end{cases}$$

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# Poynting theorem (PD)

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \frac{1}{2} [\vec{\mathbf{E}}(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^*(\vec{\mathbf{r}})]$$

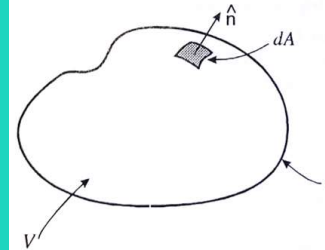
Poynting vector

$$[\vec{\mathbf{S}}]: \frac{\text{Watt}}{\text{m}^2}$$

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \vec{\mathbf{S}}_1(\vec{\mathbf{r}}) + j\vec{\mathbf{S}}_2(\vec{\mathbf{r}})$$

$$\nabla \cdot \vec{\mathbf{S}}_1 + \frac{1}{2} \omega_0 \mu_2 |\vec{\mathbf{H}}|^2 + \frac{1}{2} \omega_0 \varepsilon_2 |\vec{\mathbf{E}}|^2 + \frac{1}{2} \sigma |\vec{\mathbf{E}}|^2 = -\frac{1}{2} \text{Re} \{ \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^* \}$$

$$\nabla \cdot \vec{\mathbf{S}}_2 + \frac{1}{2} \omega_0 \mu_1 |\vec{\mathbf{H}}|^2 - \frac{1}{2} \omega_0 \varepsilon_1 |\vec{\mathbf{E}}|^2 = -\frac{1}{2} \text{Im} \{ \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^* \}$$



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$$\begin{cases} \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \varepsilon(\vec{\mathbf{r}}) \vec{\mathbf{E}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{B}}(\vec{\mathbf{r}}) = \mu(\vec{\mathbf{r}}) \vec{\mathbf{H}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{J}}(\vec{\mathbf{r}}) = \sigma \vec{\mathbf{E}}(\vec{\mathbf{r}}) \end{cases}$$

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$$\iiint_V dV \nabla \cdot \vec{\mathbf{S}}_1 + \iiint_V dV \left[ \frac{1}{2} \omega_0 \mu_2 |\vec{\mathbf{H}}|^2 + \frac{1}{2} \omega_0 \varepsilon_2 |\vec{\mathbf{E}}|^2 + \frac{1}{2} \sigma |\vec{\mathbf{E}}|^2 \right] = \iiint_V dV \left[ -\frac{1}{2} \text{Re} \{ \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^* \} \right]$$

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$$\iiint_V dV \nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \oiint_S dS \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}$$

# Poynting theorem (PD)

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \frac{1}{2} [\vec{\mathbf{E}}(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^*(\vec{\mathbf{r}})]$$

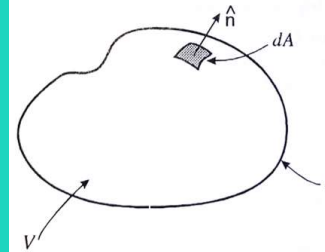
Poynting vector

$$[\vec{\mathbf{S}}]: \frac{\text{Watt}}{\text{m}^2}$$

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \vec{\mathbf{S}}_1(\vec{\mathbf{r}}) + j\vec{\mathbf{S}}_2(\vec{\mathbf{r}})$$

$$\nabla \cdot \vec{\mathbf{S}}_1 + \frac{1}{2} \omega_0 \mu_2 |\vec{\mathbf{H}}|^2 + \frac{1}{2} \omega_0 \varepsilon_2 |\vec{\mathbf{E}}|^2 + \frac{1}{2} \sigma |\vec{\mathbf{E}}|^2 = -\frac{1}{2} \text{Re} \{ \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^* \}$$

$$\nabla \cdot \vec{\mathbf{S}}_2 + \frac{1}{2} \omega_0 \mu_1 |\vec{\mathbf{H}}|^2 - \frac{1}{2} \omega_0 \varepsilon_1 |\vec{\mathbf{E}}|^2 = -\frac{1}{2} \text{Im} \{ \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^* \}$$



Hypotheses on the medium (PD)

- Linear
- Isotropic
- Time-invariant
- Time-Dispersive
- Space-Nondispersive

$$\begin{cases} \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \varepsilon(\vec{\mathbf{r}}) \vec{\mathbf{E}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{B}}(\vec{\mathbf{r}}) = \mu(\vec{\mathbf{r}}) \vec{\mathbf{H}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{J}}(\vec{\mathbf{r}}) = \sigma \vec{\mathbf{E}}(\vec{\mathbf{r}}) \end{cases}$$

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TD

Power flux associated to the e.m. field

Time derivative of the energy of the e.m. field

Power dissipated in the conducting medium

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# Poynting theorem

...MEMO : phasors and time averages

$$\begin{aligned} \dot{\mathbf{f}}_1(\vec{r}, t) &\longrightarrow \dot{\mathbf{F}}_1(\vec{r}) \\ \dot{\mathbf{f}}_2(\vec{r}, t) &\longrightarrow \dot{\mathbf{F}}_2(\vec{r}) \end{aligned}$$

$$\begin{aligned} \vec{\mathbf{S}}_1(\vec{r}) &= \text{Re}\{\vec{\mathbf{S}}(\vec{r})\} = \text{Re}\left\{\frac{1}{2}\vec{\mathbf{E}}(\vec{r})\times\vec{\mathbf{H}}^*(\vec{r})\right\} = \frac{1}{2}\text{Re}\{\vec{\mathbf{E}}(\vec{r})\times\vec{\mathbf{H}}^*(\vec{r})\} \\ &= \langle\vec{\mathbf{e}}(\vec{r}, t)\times\vec{\mathbf{h}}(\vec{r}, t)\rangle = \langle\vec{\mathbf{s}}(\vec{r}, t)\rangle \end{aligned}$$

$$\langle\vec{\mathbf{f}}_1(\vec{r}, t)\cdot\vec{\mathbf{f}}_2(\vec{r}, t)\rangle = \frac{1}{T}\int_0^T\vec{\mathbf{f}}_1(\vec{r}, t)\cdot\vec{\mathbf{f}}_2(\vec{r}, t)dt = \frac{1}{2}\text{Re}\{\vec{\mathbf{F}}_1(\vec{r})\cdot\vec{\mathbf{F}}_2^*(\vec{r})\}$$

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Time averaged power flux associated to the e.m. field

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TD

Power flux associated to the e.m. field

Time derivative of the energy of the e.m. field

Power dissipated in the conducting medium

Power delivered by the sources to the field

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$$\text{Re} \left\{ -\frac{1}{2} \vec{\mathbf{E}}(\vec{r}) \cdot \vec{\mathbf{J}}_0^*(\vec{r}) \right\} = \langle -\vec{\mathbf{j}}_0(\vec{r}, t) \cdot \vec{\mathbf{e}}(\vec{r}, t) \rangle$$

$$\langle \vec{\mathbf{f}}_1(\vec{r}, t) \cdot \vec{\mathbf{f}}_2(\vec{r}, t) \rangle = \frac{1}{T} \int_0^T \vec{\mathbf{f}}_1(\vec{r}, t) \cdot \vec{\mathbf{f}}_2(\vec{r}, t) dt = \frac{1}{2} \text{Re} \{ \vec{\mathbf{F}}_1(\vec{r}) \cdot \vec{\mathbf{F}}_2^*(\vec{r}) \}$$

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TD

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$$\begin{aligned} \frac{1}{2} \sigma |\vec{E}(\vec{r})|^2 &= \frac{1}{2} \text{Re} \left\{ \sigma |\vec{E}(\vec{r})|^2 \right\} = \frac{1}{2} \text{Re} \left\{ \sigma \vec{E}(\vec{r}) \cdot \vec{E}^*(\vec{r}) \right\} = \langle \sigma \vec{e}(\vec{r}, t) \cdot \vec{e}(\vec{r}, t) \rangle \\ &= \langle \sigma |\vec{e}(\vec{r}, t)|^2 \rangle \end{aligned}$$

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Time averaged power flux associated to the e.m. field

Time averaged power dissipated in the conducting medium

Time averaged power delivered by the sources to the field

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TD

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Where is the energy?

Remember that the energy is a state function

$$\int_{t_1}^{t_2} dt \frac{\partial}{\partial t} w(t) = w(t_2) - w(t_1)$$

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**TD**

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Time averaged power flux associated to the e.m. field

**LOSSES**  
( $\varepsilon_2$ ) electric losses  
( $\mu_2$ ) magnetic losses

Time averaged power dissipated in the conducting medium

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$$\begin{cases} \vec{D}(\vec{r}) = \varepsilon(\vec{r}) \vec{E}(\vec{r}) \\ \vec{B}(\vec{r}) = \mu(\vec{r}) \vec{H}(\vec{r}) \\ \vec{J}(\vec{r}) = \sigma \vec{E}(\vec{r}) \end{cases}$$

$$\varepsilon_2 > 0; \mu_2 > 0; \sigma > 0$$

Dispersion and losses are related each other: a (time) dispersive medium presents losses

$$\begin{cases} \varepsilon = \varepsilon_1 - j\varepsilon_2 \\ \mu = \mu_1 - j\mu_2 \\ \sigma : \text{real} \end{cases}$$