Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2020-2021 - Laurea "Triennale" – Secondo semestre - Secondo anno

Università degli Studi di Napoli "Parthenope"

Stefano Perna

THEOREMS

Poynting

Time domain – Phasor domain

Uniqueness (Interior problem – Exterior problem)

Time domain – Phasor domain

Equivalence

Phasor domain

Image Theory

Reciprocity

Phasor domain

Color legend

New formulas, important considerations, important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Mathematical tools that we will exploit today

$$\frac{\partial\left[\vec{\mathbf{A}}(t)\cdot\vec{\mathbf{B}}(t)\right]}{\partial t}=\vec{\mathbf{A}}(t)\cdot\left[\frac{\partial\vec{\mathbf{B}}(\vec{\mathbf{r}},t)}{\partial t}\right]+\vec{\mathbf{B}}(t)\cdot\left[\frac{\partial\vec{\mathbf{A}}(\vec{\mathbf{r}},t)}{\partial t}\right]$$

$$\frac{\partial\left[\left|\vec{\mathbf{A}}(t)\right|^2\right]}{\partial t}=\vec{\mathbf{A}}(t)\cdot\left[\frac{\partial\vec{\mathbf{A}}(\vec{\mathbf{r}},t)}{\partial t}\right]+\vec{\mathbf{A}}(t)\cdot\left[\frac{\partial\vec{\mathbf{A}}(\vec{\mathbf{r}},t)}{\partial t}\right]=2\vec{\mathbf{A}}(t)\cdot\left[\frac{\partial\vec{\mathbf{A}}(\vec{\mathbf{r}},t)}{\partial t}\right]$$

$$\frac{1}{2}\frac{\partial\left|\vec{\mathbf{A}}(t)\right|^2}{\partial t}=\vec{\mathbf{A}}(t)\cdot\left[\frac{\partial\vec{\mathbf{A}}(\vec{\mathbf{r}},t)}{\partial t}\right]$$

Mathematical tools that we will exploit today

$$\nabla \cdot \left[\vec{\mathbf{A}}(\vec{\mathbf{r}}) \times \vec{\mathbf{B}}(\vec{\mathbf{r}}) \right] = \vec{\mathbf{B}}(\vec{\mathbf{r}}) \cdot \left[\nabla \times \vec{\mathbf{A}}(\vec{\mathbf{r}}) \right] - \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \left[\nabla \times \vec{\mathbf{B}}(\vec{\mathbf{r}}) \right]$$

$$\iiint_V dV \, \nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \oint\!\!\!\oint_S dS \, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}$$

$$\frac{1}{2} \frac{\partial \left| \vec{\mathbf{A}}(t) \right|^2}{\partial t} = \vec{\mathbf{A}}(t) \cdot \left[\frac{\partial \vec{\mathbf{A}}(\vec{\mathbf{r}}, t)}{\partial t} \right]$$

Poynting theorem (TD)

$$\vec{\mathbf{s}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

Poynting vector

$$[\vec{\mathbf{e}}]:\frac{Volt}{m}$$

$$[\vec{\mathbf{h}}]:\frac{Ampere}{m}$$

$$[\vec{\mathbf{s}}]:\frac{Volt\times Ampere}{m^2}=\frac{Watt}{m^2}$$

$$p(t)=v(t)\times i(t)$$

$$Watt=Volt\times Ampere$$

$$[p(t)]:Watt$$

$$[v(t)]:Volt$$

$$[i(t)]:Ampere$$

Poynting theorem (TD)

$$\vec{\mathbf{s}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

Poynting vector

$$[\vec{\mathbf{s}}]: \frac{Watt}{m^2}$$

$$\nabla\cdot\vec{\mathbf{s}}(\vec{\mathbf{r}},t) = \nabla\cdot\left[\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right] = \vec{\mathbf{h}}(\vec{\mathbf{r}},t)\cdot\left[\nabla\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right] - \vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot\left[\nabla\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right] =$$

$$= \vec{\mathbf{h}}(\vec{\mathbf{r}},t)\cdot\left[-\frac{\partial\vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t}\right] - \vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot\left[\frac{\partial\vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) + \vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)\right]$$

$$= \vec{\mathbf{h}}(\vec{\mathbf{r}},t)\cdot\left[-\frac{\partial\vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t}\right] - \vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot\left[\frac{\partial\vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t}\right] - \vec{\mathbf{j}}(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t) - \vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$$

$$\nabla\cdot\left[\vec{\mathbf{A}}(\vec{\mathbf{r}})\times\vec{\mathbf{B}}(\vec{\mathbf{r}})\right] = \vec{\mathbf{B}}(\vec{\mathbf{r}})\cdot\left[\nabla\times\vec{\mathbf{A}}(\vec{\mathbf{r}})\right] - \vec{\mathbf{A}}(\vec{\mathbf{r}})\cdot\left[\nabla\times\vec{\mathbf{B}}(\vec{\mathbf{r}})\right]$$

Time domain - Differential form

$$\begin{cases} \nabla\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial\vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial\vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) + \vec{\mathbf{j}}_0(\vec{\mathbf{r}},t) \\ \nabla\cdot\vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) + \rho_0(\vec{\mathbf{r}},t) \\ \nabla\cdot\vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \end{cases}$$

Poynting theorem (TD)

$$\vec{\mathbf{s}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

Poynting vector

$$[\vec{\mathbf{s}}]:\frac{Watt}{m^2}$$

Hypotheses on the medium

- **Linear**
- **Local (TND & SND)**

$$\begin{cases}\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\boldsymbol{\varepsilon}(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\\ \vec{\mathbf{b}}(\vec{\mathbf{r}},t)=\boldsymbol{\mu}(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\\ \vec{\mathbf{j}}(\vec{\mathbf{r}},t)=\sigma\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\end{cases}$$

$$\nabla\cdot\vec{\mathbf{s}}(\vec{\mathbf{r}},t)=\nabla\cdot\left[\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right]=\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\cdot\left[\nabla\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right]-\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot\left[\nabla\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right]=$$

$$=\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\cdot\left[-\frac{\partial\vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t}\right]-\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot\left[\frac{\partial\vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t}+\vec{\mathbf{j}}(\vec{\mathbf{r}},t)+\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)\right]$$

$$=\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\cdot\left[-\frac{\partial\vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t}\right]-\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot\left[\frac{\partial\vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t}\right]-\vec{\mathbf{j}}(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)-\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\left|\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right|^2$$

$$=\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\cdot\left\{-\frac{\partial\left[\boldsymbol{\mu}(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right]}{\partial t}\right\}-\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot\left\{\frac{\partial\left[\boldsymbol{\varepsilon}(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right]}{\partial t}\right\}-\sigma\left|\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right|^2-\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$$

Poynting theorem (TD)

$$\vec{\mathbf{s}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

Poynting vector

$$[\vec{\mathbf{s}}]:\frac{Watt}{m^2}$$

Hypotheses on the medium

- **Linear**
- **Local (TND & SND)**

$$\nabla\cdot\vec{\mathbf{s}}(\vec{\mathbf{r}},t)=\nabla\cdot\left[\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right]=\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\cdot\left[\nabla\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right]-\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot\left[\nabla\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right]=$$

$$=\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\cdot\left\{-\frac{\partial\left[\boldsymbol{\mu}(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right]}{\partial t}\right\}-\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot\left\{\frac{\partial\left[\boldsymbol{\varepsilon}(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right]}{\partial t}\right\}-\sigma\left|\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right|^2-\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$$

Poynting theorem (TD)

$$\vec{\mathbf{s}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

Poynting vector

$$[\vec{\mathbf{s}}]:\frac{Watt}{m^2}$$

Hypotheses on the medium

- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Time-invariant**

$$\begin{cases}\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\\ \vec{\mathbf{b}}(\vec{\mathbf{r}},t)=\mu(\vec{\mathbf{r}})\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\\ \vec{\mathbf{j}}(\vec{\mathbf{r}},t)=\sigma\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\end{cases}$$

$$\nabla\cdot\vec{\mathbf{s}}(\vec{\mathbf{r}},t)=\nabla\cdot\left[\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right]=\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\cdot\left[\nabla\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right]-\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot\left[\nabla\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right]=$$

$$=\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\cdot\left\{-\frac{\partial\left[\boldsymbol{\mu}(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right]}{\partial t}\right\}-\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot\left\{\frac{\partial\left[\boldsymbol{\varepsilon}(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right]}{\partial t}\right\}-\sigma\left|\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right|^2-\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$$

$$=\mu(\vec{\mathbf{r}})\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\cdot\left[-\frac{\partial\vec{\mathbf{h}}(\vec{\mathbf{r}},t)}{\partial t}\right]-\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot\frac{\partial\vec{\mathbf{e}}(\vec{\mathbf{r}},t)}{\partial t}-\sigma\left|\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right|^2-\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$$

$$=-\frac{1}{2}\mu(\vec{\mathbf{r}})\frac{\partial\left|\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right|^2}{\partial t}-\frac{1}{2}\varepsilon(\vec{\mathbf{r}})\frac{\partial\left|\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right|^2}{\partial t}-\sigma\left|\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right|^2-\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{A}}(t)\cdot\frac{\partial\vec{\mathbf{A}}(\vec{\mathbf{r}},t)}{\partial t}=\frac{1}{2}\frac{\partial\left|\vec{\mathbf{A}}(t)\right|^2}{\partial t}$$

Poynting theorem (TD)

$$\vec{\mathbf{e}}\cdot\left[\frac{\partial(\varepsilon\vec{\mathbf{e}})}{\partial t}\right]=\vec{\mathbf{e}}\cdot\left[\varepsilon\frac{\partial\vec{\mathbf{e}}}{\partial t}\right]=\varepsilon\vec{\mathbf{e}}\cdot\left[\frac{\partial\vec{\mathbf{e}}}{\partial t}\right]=\frac{1}{2}\varepsilon\left[\frac{\partial|\vec{\mathbf{e}}|^2}{\partial t}\right]$$

Why isotropic?

$$\vec{\mathbf{e}}\cdot\left[\frac{\partial(\boldsymbol{\varepsilon}\cdot\vec{\mathbf{e}})}{\partial t}\right]=\vec{\mathbf{e}}\cdot\left[\boldsymbol{\varepsilon}\cdot\frac{\partial\vec{\mathbf{e}}}{\partial t}\right]\neq\boldsymbol{\varepsilon}\cdot\vec{\mathbf{e}}\cdot\left[\frac{\partial\vec{\mathbf{e}}}{\partial t}\right]\ldots\ldots\ldots$$

$$\vec{\mathbf{A}}^T\cdot\left[\underline{\underline{\mathbf{B}}}\cdot\vec{\mathbf{C}}\right]=\left[\underline{\underline{\mathbf{B}}}\cdot\vec{\mathbf{C}}\right]^T\cdot\vec{\mathbf{A}}$$

$$\vec{\mathbf{A}}^T\cdot\left[\underline{\underline{\mathbf{B}}}\cdot\vec{\mathbf{C}}\right]\neq\left[\underline{\underline{\mathbf{B}}}\cdot\vec{\mathbf{A}}\right]^T\cdot\vec{\mathbf{C}}$$

Hypotheses on the medium

- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Time-invariant**

$$\begin{cases}\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\\ \vec{\mathbf{b}}(\vec{\mathbf{r}},t)=\mu(\vec{\mathbf{r}})\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\\ \vec{\mathbf{j}}(\vec{\mathbf{r}},t)=\sigma\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\end{cases}$$

$$\nabla\cdot\vec{\mathbf{s}}(\vec{\mathbf{r}},t)=\nabla\cdot\left[\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right]=\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\cdot\left[\nabla\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right]-\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot\left[\nabla\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right]=$$

$$=\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\cdot\left\{-\frac{\partial\left[\boldsymbol{\mu}(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right]}{\partial t}\right\}-\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot\left\{\frac{\partial\left[\boldsymbol{\varepsilon}(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right]}{\partial t}\right\}-\sigma\left|\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right|^2-\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$$

$$=\mu(\vec{\mathbf{r}})\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\cdot\left[-\frac{\partial\vec{\mathbf{h}}(\vec{\mathbf{r}},t)}{\partial t}\right]-\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot\frac{\partial\vec{\mathbf{e}}(\vec{\mathbf{r}},t)}{\partial t}-\sigma\left|\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right|^2-\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$$

$$=-\frac{1}{2}\mu(\vec{\mathbf{r}})\frac{\partial\left|\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right|^2}{\partial t}-\frac{1}{2}\varepsilon(\vec{\mathbf{r}})\frac{\partial\left|\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right|^2}{\partial t}-\sigma\left|\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right|^2-\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{A}}(t)\cdot\frac{\partial\vec{\mathbf{A}}(\vec{\mathbf{r}},t)}{\partial t}=\frac{1}{2}\frac{\partial\left|\vec{\mathbf{A}}(t)\right|^2}{\partial t}$$

Poynting theorem (TD)

$$\vec{\mathbf{s}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

Poynting vector

$$[\vec{\mathbf{s}}]:\frac{Watt}{m^2}$$

Hypotheses on the medium

- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Time-invariant**

$$\nabla\cdot\vec{\mathbf{s}}(\vec{\mathbf{r}},t)=\nabla\cdot\left[\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right]=\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\cdot\left[\nabla\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right]-\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot\left[\nabla\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right]=$$

$$=\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\cdot\left\{-\frac{\partial\left[\boldsymbol{\mu}(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right]}{\partial t}\right\}-\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot\left\{\frac{\partial\left[\boldsymbol{\varepsilon}(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right]}{\partial t}\right\}-\sigma\left|\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right|^2-\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$$

$$=\mu(\vec{\mathbf{r}})\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\cdot\left[-\frac{\partial\vec{\mathbf{h}}(\vec{\mathbf{r}},t)}{\partial t}\right]-\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot\frac{\partial\vec{\mathbf{e}}(\vec{\mathbf{r}},t)}{\partial t}-\sigma\left|\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right|^2-\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$$

$$=-\frac{1}{2}\mu(\vec{\mathbf{r}})\frac{\partial\left|\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right|^2}{\partial t}-\frac{1}{2}\varepsilon(\vec{\mathbf{r}})\frac{\partial\left|\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right|^2}{\partial t}-\sigma\left|\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right|^2-\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$$

Poynting theorem (TD)

$$\vec{\mathbf{s}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

Poynting vector

$$[\vec{\mathbf{s}}]:\frac{Watt}{m^2}$$

$$\nabla\cdot\vec{\mathbf{s}}+\frac{\partial}{\partial t}\left(\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right)+\sigma\left|\vec{\mathbf{e}}\right|^2=-\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}$$

Hypotheses on the medium

- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Time-invariant**

$$\nabla\cdot\vec{\mathbf{s}}(\vec{\mathbf{r}},t)=-\frac{\partial}{\partial t}\left(\frac{1}{2}\mu(\vec{\mathbf{r}})\left|\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right|^2+\frac{1}{2}\varepsilon(\vec{\mathbf{r}})\left|\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right|^2\right)-\sigma\left|\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right|^2-\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$$

$$=-\frac{1}{2}\mu(\vec{\mathbf{r}})\frac{\partial\left|\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right|^2}{\partial t}-\frac{1}{2}\varepsilon(\vec{\mathbf{r}})\frac{\partial\left|\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right|^2}{\partial t}-\sigma\left|\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right|^2-\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$$

Poynting theorem (TD)

$$\vec{\mathbf{s}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

Poynting vector

$$[\vec{\mathbf{s}}]:\frac{Watt}{m^2}$$

$$\nabla\cdot\vec{\mathbf{s}}+\frac{\partial}{\partial t}\left(\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right)+\sigma\left|\vec{\mathbf{e}}\right|^2=-\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}$$

Hypotheses on the medium

- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Time-invariant**

Poynting theorem (TD)

$$\vec{\mathbf{s}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

Poynting vector

$$[\vec{\mathbf{s}}]:\frac{Watt}{m^2}$$

$$\nabla\cdot\vec{\mathbf{s}}+\frac{\partial}{\partial t}\left(\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right)+\sigma\left|\vec{\mathbf{e}}\right|^2=-\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}$$

Hypotheses on the medium

- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Time-invariant**

$$-\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}$$

$$\vec{\mathbf{F}}=q\vec{\mathbf{e}}+q\vec{\mathbf{v}}_0\times\vec{\mathbf{b}}\;\Rightarrow\;\frac{\vec{\mathbf{F}}}{dV}=\vec{\mathbf{f}}=\frac{q}{dV}\vec{\mathbf{e}}+\frac{q}{dV}\vec{\mathbf{v}}_0\times\vec{\mathbf{b}}\;\Rightarrow\;\boxed{\vec{\mathbf{f}}=\rho\vec{\mathbf{e}}+\rho\vec{\mathbf{v}}_0\times\vec{\mathbf{b}}}$$

$$\boxed{dL}=\vec{\mathbf{f}}\cdot d\vec{\mathbf{s}}=dt\,\vec{\mathbf{f}}\cdot\vec{\mathbf{v}}_0=dt\left[\rho\vec{\mathbf{e}}+\rho\left(\vec{\mathbf{v}}_0\times\vec{\mathbf{b}}\right)\right]\cdot\vec{\mathbf{v}}_0=dt\,\rho\vec{\mathbf{e}}\cdot\vec{\mathbf{v}}_0+dt\,\rho\left(\vec{\mathbf{v}}_0\times\vec{\mathbf{b}}\right)\cdot\vec{\mathbf{v}}_0=\boxed{dt\,\rho\vec{\mathbf{e}}\cdot\vec{\mathbf{v}}_0}$$

$$\frac{dL}{dt}=\rho\vec{\mathbf{e}}\cdot\vec{\mathbf{v}}_0=\vec{\mathbf{e}}\cdot\vec{\mathbf{j}}_0$$

$$\rho=\frac{q}{dV}\qquad\rho\vec{\mathbf{v}}_0=\vec{\mathbf{j}}_0\qquad\vec{\mathbf{v}}_0=\frac{d\vec{\mathbf{s}}}{dt}$$

$$\left[\vec{\mathbf{F}}\right]:Newton$$

$$\left[\vec{\mathbf{f}}\right]:\frac{Newton}{m^3}$$

$$[dL]:\frac{Joule}{m^3}$$

$$\left[\frac{dL}{dt}\right]:\frac{Joule}{s\times m^3}=\frac{Watt}{m^3}$$

$$\left[-\iiint_V dV\,\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}\right]:Watt$$

Poynting theorem (TD)

$$\vec{\mathbf{s}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(\vec{\mathbf{r}},t) \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

Poynting vector $[\vec{\mathbf{s}}] : \frac{Watt}{m^2}$

$$\nabla \cdot \vec{\mathbf{s}} + \frac{\partial}{\partial t}\left(\frac{1}{2}\mu|\vec{\mathbf{h}}|^2 + \frac{1}{2}\varepsilon|\vec{\mathbf{e}}|^2\right) + \sigma|\vec{\mathbf{e}}|^2 = -\vec{\mathbf{j}}_0 \cdot \vec{\mathbf{e}}$$

Hypotheses on the medium

- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Time-invariant**

$$\nabla \cdot \vec{\mathbf{s}} + \frac{\partial}{\partial t}\left(\frac{1}{2}\mu|\vec{\mathbf{h}}|^2 + \frac{1}{2}\varepsilon|\vec{\mathbf{e}}|^2\right) + \sigma|\vec{\mathbf{e}}|^2 = \underbrace{-\vec{\mathbf{j}}_0 \cdot \vec{\mathbf{e}}}_{p_0}$$

p_0 → **Power density delivered by the sources to the field**

Poynting theorem (TD)

$$\vec{\mathbf{s}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

Poynting vector

$$[\vec{\mathbf{s}}]:\frac{Watt}{m^2}$$

$$\nabla\cdot\vec{\mathbf{s}}+\frac{\partial}{\partial t}\left(\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right)+\sigma\left|\vec{\mathbf{e}}\right|^2=-\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}$$

Hypotheses on the medium

- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Time-invariant**

$$\sigma\left|\vec{\mathbf{e}}\right|^2$$

$$\left[\sigma\left|\vec{\mathbf{e}}\right|^2\right]:\frac{Siemens}{m}\frac{Volt^2}{m^2}=\frac{1}{m^3}\frac{Volt^2}{\Omega}=\frac{Watt}{m^3}$$

$$[\vec{\mathbf{e}}]:\frac{Volt}{m}$$

$$[\sigma]:\frac{Siemens}{m}=\frac{1}{\Omega m}$$

$$p(t)=v(t)\cdot i(t)=Ri^2(t)=\frac{v^2(t)}{R}$$

$$Watt=\Omega\,Ampere^2=\frac{Volt^2}{\Omega}$$

$$\left[\iiint_V dV\,\sigma\left|\vec{\mathbf{e}}\right|^2\right]:Watt$$

Poynting theorem (TD)

$$\vec{\mathbf{s}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

Poynting vector $[\vec{\mathbf{s}}]:\frac{Watt}{m^2}$

$$\nabla\cdot\vec{\mathbf{s}}+\frac{\partial}{\partial t}\left(\frac{1}{2}\mu|\vec{\mathbf{h}}|^2+\frac{1}{2}\varepsilon|\vec{\mathbf{e}}|^2\right)+\sigma|\vec{\mathbf{e}}|^2=-\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}$$

Hypotheses on the medium

- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Time-invariant**

$$\nabla\cdot\vec{\mathbf{s}}+\frac{\partial}{\partial t}\left(\frac{1}{2}\mu|\vec{\mathbf{h}}|^2+\frac{1}{2}\varepsilon|\vec{\mathbf{e}}|^2\right)+\underbrace{\sigma|\vec{\mathbf{e}}|^2}_{p_j}=\underbrace{-\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}}_{p_0}$$

p_j → **Power density dissipated in the conducting medium**

p_0 → **Power density delivered by the sources to the field**

Poynting theorem (TD)

$$\vec{\mathbf{s}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

Poynting vector

$$[\vec{\mathbf{s}}]:\frac{Watt}{m^2}$$

$$\nabla\cdot\vec{\mathbf{s}}+\frac{\partial}{\partial t}\left(\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right)+\sigma\left|\vec{\mathbf{e}}\right|^2=-\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}$$

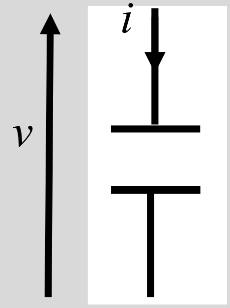
Hypotheses on the medium

- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Time-invariant**

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right)$$

$$\left[\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right]:\ \frac{Farad\times Volt^2}{m^3}=\sec\times\frac{Ampere\times Volt}{m^3}=\sec\times\frac{Watt}{m^3}=\frac{Joule}{m^3}$$

$$[\vec{\mathbf{e}}]:\frac{Volt}{m}\qquad[\varepsilon]:\frac{Farad}{m}$$



$$i(t)=C\frac{dv(t)}{dt}$$

$$Farad=\frac{Ampere\times\sec}{Volt}$$

Poynting theorem (TD)

$$\vec{\mathbf{s}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

Poynting vector

$$[\vec{\mathbf{s}}]:\frac{Watt}{m^2}$$

$$\nabla\cdot\vec{\mathbf{s}}+\frac{\partial}{\partial t}\left(\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right)+\sigma\left|\vec{\mathbf{e}}\right|^2=-\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}$$

Hypotheses on the medium

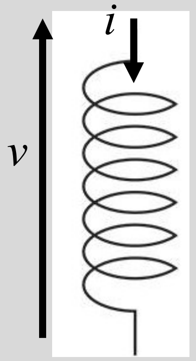
- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Time-invariant**

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right)$$

$$\left[\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right]:\frac{Joule}{m^3}$$

$$\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2\right]:\frac{Henry\times Ampere^2}{m^3}=\sec\times\frac{Volt\times Ampere}{m^3}=\sec\times\frac{Watt}{m^3}=\frac{Joule}{m^3}$$

$$[\vec{\mathbf{h}}]:\frac{Ampere}{m} \qquad [\mu]:\frac{Henry}{m}$$



$$v(t)=L\frac{di(t)}{dt}$$

$$Henry=\frac{Volt\times\sec}{Ampere}$$

Poynting theorem (TD)

$$\vec{\mathbf{s}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

Poynting vector

$$[\vec{\mathbf{s}}]:\frac{Watt}{m^2}$$

$$\nabla\cdot\vec{\mathbf{s}}+\frac{\partial}{\partial t}\left(\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right)+\sigma\left|\vec{\mathbf{e}}\right|^2=-\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}$$

Hypotheses on the medium

- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Time-invariant**

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right)$$

$$\left[\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right]:\frac{Joule}{m^3}$$

$$\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2\right]:\frac{Joule}{m^3}$$

Poynting theorem (TD)

$$\vec{\mathbf{s}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

Poynting vector

$$[\vec{\mathbf{s}}]:\frac{Watt}{m^2}$$

$$\nabla\cdot\vec{\mathbf{s}}+\frac{\partial}{\partial t}\left(\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right)+\sigma\left|\vec{\mathbf{e}}\right|^2=-\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}$$

Hypotheses on the medium

- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Time-invariant**

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right)$$

$$\left[\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right]:\frac{Joule}{m^3}$$

$$\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2\right]:\frac{Joule}{m^3}$$

Dimensional units of energy densities.
This is not surprising, since in the static case they are the electric energy density and the magnetic energy density, respectively.

Note in addition that the overall quantity:

$$\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2$$

is a state function

Poynting theorem (TD)

$$\vec{\mathbf{s}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(\vec{\mathbf{r}},t) \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

Poynting vector

$$[\vec{\mathbf{s}}] : \frac{Watt}{m^2}$$

$$\nabla \cdot \vec{\mathbf{s}} + \frac{\partial}{\partial t}\left(\frac{1}{2}\mu \left|\vec{\mathbf{h}}\right|^2 + \frac{1}{2}\varepsilon \left|\vec{\mathbf{e}}\right|^2 \right) + \sigma \left|\vec{\mathbf{e}}\right|^2 = -\vec{\mathbf{j}}_0 \cdot \vec{\mathbf{e}}$$

Hypotheses on the medium

- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Time-invariant**

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\mu \left|\vec{\mathbf{h}}\right|^2 + \frac{1}{2}\varepsilon \left|\vec{\mathbf{e}}\right|^2 \right) = w$$

$$\left[\frac{1}{2}\varepsilon \left|\vec{\mathbf{e}}\right|^2 \right] : \frac{Joule}{m^3}$$

$$\left[\frac{1}{2}\mu \left|\vec{\mathbf{h}}\right|^2 \right] : \frac{Joule}{m^3}$$

$$\nabla \cdot \vec{\mathbf{s}} + \frac{\partial}{\partial t} w + \sigma \left|\vec{\mathbf{e}}\right|^2 = -\vec{\mathbf{j}}_0 \cdot \vec{\mathbf{e}}$$

$$\int_{t_1}^{t_2} dt \nabla \cdot \vec{\mathbf{s}} + \int_{t_1}^{t_2} dt \frac{\partial}{\partial t} w + \int_{t_1}^{t_2} dt \sigma \left|\vec{\mathbf{e}}\right|^2 = -\int_{t_1}^{t_2} dt \vec{\mathbf{j}}_0 \cdot \vec{\mathbf{e}}$$

$$\int_{t_1}^{t_2} dt \frac{\partial}{\partial t} w = w(t_2) - w(t_1)$$

$$\int_{t_1}^{t_2} dt \sigma \left|\vec{\mathbf{e}}\right|^2 > 0 \qquad (\sigma > 0)$$

Poynting theorem (TD)

$$\vec{\mathbf{s}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

Poynting vector

$$[\vec{\mathbf{s}}]:\frac{Watt}{m^2}$$

$$\nabla\cdot\vec{\mathbf{s}}+\frac{\partial}{\partial t}\left(\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right)+\sigma\left|\vec{\mathbf{e}}\right|^2=-\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}$$

Hypotheses on the medium

- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Time-invariant**

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right)=w$$

$$\left[\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right]:\frac{Joule}{m^3}=w_e$$

$$\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2\right]:\frac{Joule}{m^3}=w_m$$

Dimensional units of energy densities.
This is not surprising, since in the static case they are the electric energy density and the magnetic energy density, respectively.

Note in addition that the overall quantity:

$$\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2$$

is a state function

It is thus reasonable to identify this state function as the energy density of the electromagnetic field

Poynting theorem (TD)

$$\vec{\mathbf{s}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(\vec{\mathbf{r}},t) \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

Poynting vector

$$[\vec{\mathbf{s}}] : \frac{Watt}{m^2}$$

$$\nabla \cdot \vec{\mathbf{s}} + \frac{\partial}{\partial t}\left(\frac{1}{2}\mu|\vec{\mathbf{h}}|^2 + \frac{1}{2}\varepsilon|\vec{\mathbf{e}}|^2\right) + \sigma|\vec{\mathbf{e}}|^2 = -\vec{\mathbf{j}}_0 \cdot \vec{\mathbf{e}}$$

Hypotheses on the medium

- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Time-invariant**

$$\nabla \cdot \vec{\mathbf{s}} + \underbrace{\frac{\partial}{\partial t}\left(\frac{1}{2}\mu|\vec{\mathbf{h}}|^2 + \frac{1}{2}\varepsilon|\vec{\mathbf{e}}|^2\right)}_{\frac{\partial}{\partial t}w} + \underbrace{\sigma|\vec{\mathbf{e}}|^2}_{p_j} = \underbrace{-\vec{\mathbf{j}}_0 \cdot \vec{\mathbf{e}}}_{p_0}$$

$\frac{\partial}{\partial t}w$ → **Time derivative of the energy density of the e.m. field**

p_j → **Power density dissipated in the conducting medium**

p_0 → **Power density delivered by the sources to the field**

Poynting theorem (TD)

$\vec{\mathbf{s}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(\vec{\mathbf{r}},t) \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t)$ **Poynting vector** $[\vec{\mathbf{s}}]: \frac{Watt}{m^2}$

$$\nabla \cdot \vec{\mathbf{s}} + \frac{\partial}{\partial t} w + p_j = p_0$$

$$w = \frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2 + \frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2$$

$$p_j = \sigma\left|\vec{\mathbf{e}}\right|^2$$

$$p_0 = -\vec{\mathbf{j}}_0 \cdot \vec{\mathbf{e}}$$

Hypotheses on the medium

- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Time-invariant**

$$\underbrace{\nabla \cdot \vec{\mathbf{s}} + \frac{\partial}{\partial t}\left(\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2 + \frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right)}_{\frac{\partial}{\partial t}w} + \underbrace{\sigma\left|\vec{\mathbf{e}}\right|^2}_{p_j} = \underbrace{-\vec{\mathbf{j}}_0 \cdot \vec{\mathbf{e}}}_{p_0}$$

$\frac{\partial}{\partial t}w$ → **Time derivative of the energy density of the e.m. field**

p_j → **Power density dissipated in the conducting medium**

p_0 → **Power density delivered by the sources to the field**

Poynting theorem (TD)

$$\vec{\mathbf{s}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

Poynting vector $\left[\vec{\mathbf{s}}\right]:\frac{Watt}{m^2}$

$$\nabla\cdot\vec{\mathbf{s}}+\frac{\partial}{\partial t}w+p_j=p_0$$

$$w=\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2$$

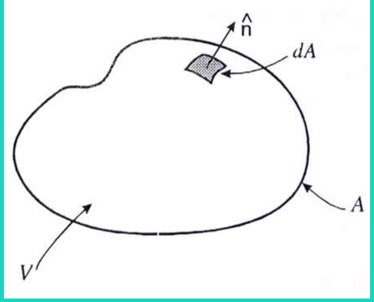
$$p_j=\sigma\left|\vec{\mathbf{e}}\right|^2$$

$$p_0=-\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}$$

Hypotheses on the medium

- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Time-invariant**

$$\iiint_V dV\,\nabla\cdot\vec{\mathbf{s}}(\vec{\mathbf{r}},t)+\iiint_V dV\frac{\partial w(\vec{\mathbf{r}},t)}{\partial t}+\iiint_V dV p_j(\vec{\mathbf{r}},t)=\iiint_V dV\,p_0(\vec{\mathbf{r}},t)$$



$$\oiint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}+\frac{d}{dt}\iiint_V dV\,w(\vec{\mathbf{r}},t)+\iiint_V dV p_j(\vec{\mathbf{r}},t)=\iiint_V dV\,p_0(\vec{\mathbf{r}},t)$$

$$\iiint_V dV\,\nabla\cdot\vec{\mathbf{A}}(\vec{\mathbf{r}})=\oiint_S dS\,\vec{\mathbf{A}}(\vec{\mathbf{r}})\cdot\hat{\mathbf{n}}$$

Poynting theorem (TD)

$$\vec{\mathbf{s}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

Poynting vector $[\vec{\mathbf{s}}]:\frac{Watt}{m^2}$

$$\oint\!\!\!\oint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}+\frac{d}{dt}\iiint_V dV\,w(\vec{\mathbf{r}},t)+\iiint_V dV p_j(\vec{\mathbf{r}},t)=\iiint_V dV\,p_0(\vec{\mathbf{r}},t)$$

Hypotheses on the medium
- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Time-invariant**

This equation states that the conservation of energy if we read $\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}$ as the electromagnetic flux per unit area.

The energy associated with the electromagnetic field is distributed within, and propagated through the medium. In any given space and time interval, the energy delivered by the sources, equals the sum of the energy dissipated, transferred outside and stored inside the volume

$w=\frac{1}{2}\mu|\vec{\mathbf{h}}|^2+\frac{1}{2}\varepsilon|\vec{\mathbf{e}}|^2$ **Energy density of the e.m. field** ➡ $\iiint_V dV\,w(\vec{\mathbf{r}},t)$ **Energy of the e.m. field**

$p_j=\sigma|\vec{\mathbf{e}}|^2$ **Power density dissipated in the conducting medium** ➡ $\iiint_V dV p_j(\vec{\mathbf{r}},t)$ **Power dissipated in the conducting medium**

$p_0=-\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}$ **Power density delivered by the sources to the field** ➡ $\iiint_V dV\,p_0(\vec{\mathbf{r}},t)$ **Power delivered by the sources to the field**

Poynting theorem (TD)

$$\vec{\mathbf{s}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

Poynting vector

$$[\vec{\mathbf{s}}]:\frac{Watt}{m^2}$$

$$\nabla\cdot\vec{\mathbf{s}}(\vec{\mathbf{r}},t)+\frac{\partial}{\partial t}w(\vec{\mathbf{r}},t)+p_j(\vec{\mathbf{r}},t)=p_0(\vec{\mathbf{r}},t)$$

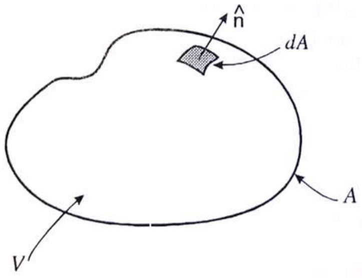
$$\oiint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}+\frac{d}{dt}\iiint_V dV\,w(\vec{\mathbf{r}},t)+\iiint_V dV p_j(\vec{\mathbf{r}},t)=\iiint_V dV\,p_0(\vec{\mathbf{r}},t)$$

Electromagnetic power flux

$$P_S(t)+\frac{\partial}{\partial t}W(t)+P_j(t)=P_0(t)$$

Hypotheses on the medium

- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Time-invariant**



$$w(\vec{\mathbf{r}},t)=\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2$$ **Energy density of the e.m. field** ➡ $$\iiint_V dV\,w(\vec{\mathbf{r}},t)=W(t)$$ **Energy of the e.m. field**

$$p_j(\vec{\mathbf{r}},t)=\sigma\left|\vec{\mathbf{e}}\right|^2$$ **Power density dissipated in the conducting medium** ➡ $$\iiint_V dV p_j(\vec{\mathbf{r}},t)=P_j(t)$$ **Power dissipated in the conducting medium**

$$p_0(\vec{\mathbf{r}},t)=-\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}$$ **Power density delivered by the sources to the field** ➡ $$\iiint_V dV\,p_0(\vec{\mathbf{r}},t)=P(t)$$ **Power delivered by the sources to the field**

Poynting theorem (TD)

$$\vec{\mathbf{s}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$ **Poynting vector** $$[\vec{\mathbf{s}}]:\frac{Watt}{m^2}$$

$$\oint\!\!\!\oint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}$$

Electromagnetic power flux

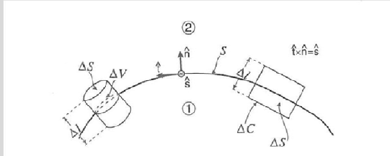
Hypotheses on the medium

- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Time-invariant**

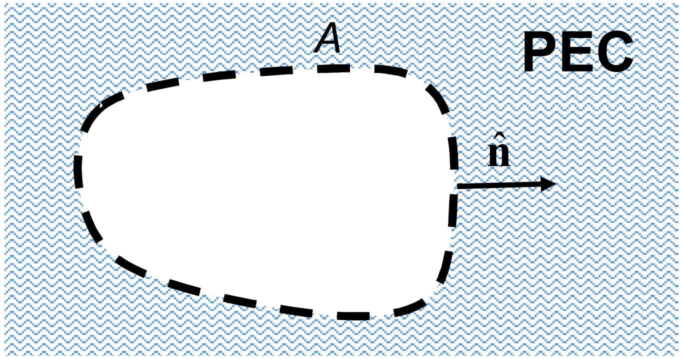
MEMO: Fields at boundaries

One example: the medium 1 is a PEC

$$\hat{\mathbf{n}}\times(\vec{\mathbf{e}}_2-\vec{\mathbf{e}}_1)=\mathbf{0}$$
$$\hat{\mathbf{n}}\times(\vec{\mathbf{h}}_2-\vec{\mathbf{h}}_1)=\vec{\mathbf{j}}_s$$
$$(\vec{\mathbf{d}}_2-\vec{\mathbf{d}}_1)\cdot\hat{\mathbf{n}}=\rho_s$$
$$(\vec{\mathbf{b}}_2-\vec{\mathbf{b}}_1)\cdot\hat{\mathbf{n}}=0$$
$$(\vec{\mathbf{j}}_2-\vec{\mathbf{j}}_1)\cdot\hat{\mathbf{n}}=-\frac{\partial\rho_s}{\partial t}$$



$$\vec{\mathbf{e}}_1=\mathbf{0}\qquad \hat{\mathbf{n}}\times\vec{\mathbf{e}}_2=\mathbf{0}$$
$$\vec{\mathbf{h}}_1=\mathbf{0}\qquad \hat{\mathbf{n}}\times\vec{\mathbf{h}}_2=\vec{\mathbf{j}}_s$$
$$\vec{\mathbf{d}}_1=\mathbf{0}\qquad \vec{\mathbf{d}}_2\cdot\hat{\mathbf{n}}=\rho_s$$
$$\vec{\mathbf{b}}_1=\mathbf{0}\qquad \vec{\mathbf{b}}_2\cdot\hat{\mathbf{n}}=0$$



$$\oint\!\!\!\oint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}=\oint\!\!\!\oint_A dA\left[\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right]\cdot\hat{\mathbf{n}}=\oint\!\!\!\oint_A dA\left[\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right]\cdot\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=0$$

$$\vec{\mathbf{A}}\cdot\left[\vec{\mathbf{B}}\times\vec{\mathbf{C}}\right]=\vec{\mathbf{C}}\cdot\left[\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right]=\vec{\mathbf{B}}\cdot\left[\vec{\mathbf{C}}\times\vec{\mathbf{A}}\right]$$