Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2020-2021 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

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Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Constitutive relationships

Time domain - Differential form

$$
\begin{cases}
\nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\
\nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) + \vec{\mathbf{j}}_0(\vec{\mathbf{r}},t) \\
\nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) + \rho_0(\vec{\mathbf{r}},t) \\
\nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0
\end{cases}
$$

Even assuming knowledge of the impressed sources $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t);\rho_0(\vec{\mathbf{r}},t)$

Number of independent scalar equations: 7

Number of unknown scalar quantities: 16

Maxwell equations involve a number of unknowns larger than the number of equations!

The additional missing equations are provided by the **constitutive relationships**, which describe interaction of fields and matter from a macroscopic point of view

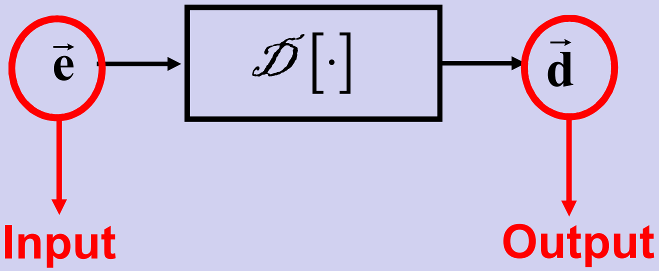
Constitutive relationships

Linear & Anisotropic media

$$\vec{\mathbf{d}} = \mathscr{D}\left[\vec{\mathbf{e}}\right]$$

$$\vec{\mathbf{b}} = \mathscr{B}\left[\vec{\mathbf{h}}\right]$$

$$\vec{\mathbf{j}} = \mathscr{J}\left[\vec{\mathbf{e}}\right]$$



Constitutive relationships

Linear & Anisotropic & Dispersive media

$$\vec{\mathbf{d}} = \mathscr{D}\left[\vec{\mathbf{e}}\right] \qquad \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \int d\vec{\mathbf{r}}' \int dt' \, \mathbf{g}_d\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t,t'\right) \cdot \vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$$

$$\vec{\mathbf{b}} = \mathscr{B}\left[\vec{\mathbf{h}}\right] \qquad \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = \int d\vec{\mathbf{r}}' \int dt' \, \mathbf{g}_b\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t,t'\right) \cdot \vec{\mathbf{h}}\left(\vec{\mathbf{r}}',t'\right)$$

$$\vec{\mathbf{j}} = \mathscr{J}\left[\vec{\mathbf{e}}\right] \qquad \vec{\mathbf{j}}(\vec{\mathbf{r}},t) = \int d\vec{\mathbf{r}}' \int dt' \, \mathbf{g}_j\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t,t'\right) \cdot \vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$$

Linear & Isotropic & Dispersive media

$$\vec{\mathbf{d}} = \mathscr{D}\left[\vec{\mathbf{e}}\right] \qquad \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \int d\vec{\mathbf{r}}' \int dt' \, g_d\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t,t'\right) \vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$$

$$\vec{\mathbf{b}} = \mathscr{B}\left[\vec{\mathbf{h}}\right] \qquad \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = \int d\vec{\mathbf{r}}' \int dt' \, g_b\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t,t'\right) \vec{\mathbf{h}}\left(\vec{\mathbf{r}}',t'\right)$$

$$\vec{\mathbf{j}} = \mathscr{J}\left[\vec{\mathbf{e}}\right] \qquad \vec{\mathbf{j}}(\vec{\mathbf{r}},t) = \int d\vec{\mathbf{r}}' \int dt' \, g_j\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t,t'\right) \vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$$

Constitutive relationships

In the following, just for the sake of simplicity, we will consider isotropic media

Time: dispersive
Space: dispersive

	Space-Dispersive (SD) Time-Dispersive (TD)
SV-TV	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g_d\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t,t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$
SV-TI	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g_d\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$
SI-TV	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g_d\left(\vec{\mathbf{r}}-\vec{\mathbf{r}}',t,t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$
SI-TI	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g_d\left(\vec{\mathbf{r}}-\vec{\mathbf{r}}',t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$

Time: nondispersive
Space: dispersive

	Space-Dispersive (SD) Time-Dispersive (TD)	Space-Dispersive (SD) Time-Nondispersive (TND)
SV-TV	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g_d\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t,t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}' g_d\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t\right)$
SV-TI	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g_d\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}' g_d\left(\vec{\mathbf{r}},\vec{\mathbf{r}}'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t\right)$
SI-TV	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g_d\left(\vec{\mathbf{r}}-\vec{\mathbf{r}}',t,t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}' g_d\left(\vec{\mathbf{r}}-\vec{\mathbf{r}}',t\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t\right)$
SI-TI	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g_d\left(\vec{\mathbf{r}}-\vec{\mathbf{r}}',t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}' g_d\left(\vec{\mathbf{r}}-\vec{\mathbf{r}}'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t\right)$

Time: dispersive
Space: nondispersive

	Space-Dispersive (SD) Time-Dispersive (TD)	Space-Nondispersive (SND) Time-Dispersive (TD)
SV-TV	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g_d\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t,t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d\left(\vec{\mathbf{r}},t,t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t'\right)$
SV-TI	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g_d\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d\left(\vec{\mathbf{r}},t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t'\right)$
SI-TV	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g_d\left(\vec{\mathbf{r}}-\vec{\mathbf{r}}',t,t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d\left(t,t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t'\right)$
SI-TI	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g_d\left(\vec{\mathbf{r}}-\vec{\mathbf{r}}',t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d\left(t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t'\right)$

Dispersive (time & space) vs. nondispersive (time & space)

	Space-Dispersive (SD) **Time-Dispersive (TD)**	**Space-Nondispersive (SND)** **Time-Nondispersive (TND)**
SV-TV	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g_d\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t,t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon\left(\vec{\mathbf{r}},t\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t\right)$
SV-TI	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g_d\left(\vec{\mathbf{r}},\vec{\mathbf{r}}',t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon\left(\vec{\mathbf{r}}\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t\right)$
SI-TV	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g_d\left(\vec{\mathbf{r}}-\vec{\mathbf{r}}',t,t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon\left(t\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t\right)$
SI-TI	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int d\vec{\mathbf{r}}'\int dt'\, g_d\left(\vec{\mathbf{r}}-\vec{\mathbf{r}}',t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon\,\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t\right)$

Permittivity

	SND+TND: Local
SV-TV	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon(\vec{\mathbf{r}},t)\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$
SV-TI	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$
SI-TV	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon(t)\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$
SI-TI	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon\,\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$

$$[\varepsilon]=?$$

$$[\varepsilon]=\frac{Coulomb}{m^2}\frac{m}{Volt}=\frac{Coulomb}{Volt}\frac{1}{m}=\frac{Farad}{m}$$

$$\left[\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right]=\frac{Volt}{m} \qquad \left[\vec{\mathbf{d}}(\vec{\mathbf{r}},t)\right]=\frac{Coulomb}{m^2}$$

$$C=\frac{q}{\Delta \mathrm{v}} \qquad Farad=\frac{Coulomb}{Volt}$$

Permeability

	SND+TND: Local
SV-TV	$\vec{\mathbf{b}}(\vec{\mathbf{r}},t)=\mu(\vec{\mathbf{r}},t)\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$
SV-TI	$\vec{\mathbf{b}}(\vec{\mathbf{r}},t)=\mu(\vec{\mathbf{r}})\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$
SI-TV	$\vec{\mathbf{b}}(\vec{\mathbf{r}},t)=\mu(t)\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$
SI-TI	$\vec{\mathbf{b}}(\vec{\mathbf{r}},t)=\mu\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$

$$[\mu]=?$$

$$[\mu]=\frac{Weber}{m^2}\frac{m}{Ampere}=\frac{Weber}{Ampere}\frac{1}{m}=\frac{Henry}{m}$$

$$\left[\vec{\mathbf{b}}(\vec{\mathbf{r}},t)\right]=\frac{Weber}{m^2}\qquad\left[\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right]=\frac{Ampere}{m}$$

$$L=\frac{\Phi_{\vec{\mathbf{b}}}}{i}\qquad Henry=\frac{Weber}{Ampere}$$

Conductivity

	SND+TND: Local
SV-TV	$\vec{\mathbf{j}}(\vec{\mathbf{r}},t)=\sigma(\vec{\mathbf{r}},t)\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$
SV-TI	$\vec{\mathbf{j}}(\vec{\mathbf{r}},t)=\sigma(\vec{\mathbf{r}})\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$
SI-TV	$\vec{\mathbf{j}}(\vec{\mathbf{r}},t)=\sigma(t)\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$
SI-TI	$\vec{\mathbf{j}}(\vec{\mathbf{r}},t)=\sigma\,\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$

$$[\sigma]=?$$

$$[\sigma]=\frac{Ampere}{m^2}\frac{m}{Volt}=\frac{Ampere}{Volt}\frac{1}{m}=\frac{1}{\Omega\, m}=\frac{Siemens}{m}$$

$$\left[\vec{\mathbf{j}}(\vec{\mathbf{r}},t)\right]=\frac{Ampere}{m^2} \qquad \left[\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right]=\frac{Volt}{m}$$

$$\Delta \mathrm{v}=R\,i \qquad \Omega=\frac{Volt}{Ampere}=\frac{1}{Siemens}$$

Conductors

$$\vec{\mathbf{j}}(\vec{\mathbf{r}},t)=\sigma\,\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$$

Metal	Conductivity σ [siemens/m]
Silver, 99.98% pure	6.14×10^7
Copper, annealed	5.80×10^7
Copper, hard drawn	5.65×10^7
Gold, pure drawn	4.10×10^7
Aluminum, commercial hard drawn	3.54×10^7
Magnesium	2.17×10^7
Tungsten	1.81×10^7
Zinc	1.74×10^7
Nickel	1.28×10^7
Iron, 99.98% pure	1.00×10^7
Steel	$1.00–0.5 \times 10^7$
Lead	0.48×10^7
Mercury	0.10×10^7

From G.Franceschetti, ‘Electromagnetics, Theory, Techniques, and Engineering paradigms’, Plenum Press

Perfect Electric Conductors (PEC)

$$\sigma \to \infty \quad \Rightarrow \quad \vec{\mathbf{e}} = 0 \quad \Rightarrow \quad \vec{\mathbf{h}} = 0$$

Vacuo

$$\vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \varepsilon_0 \vec{\mathbf{e}}(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{b}}(\vec{\mathbf{r}},t) = \mu_0 \vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

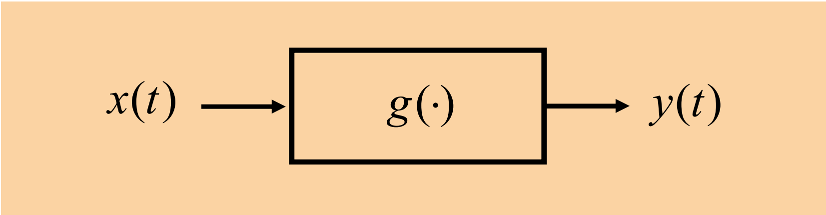
$$\sigma = 0$$

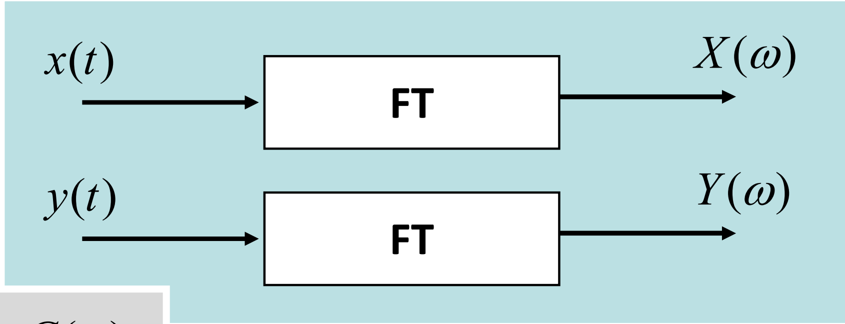
$$\mu_0 = 4\pi \times 10^{-7} \quad Henry / m$$

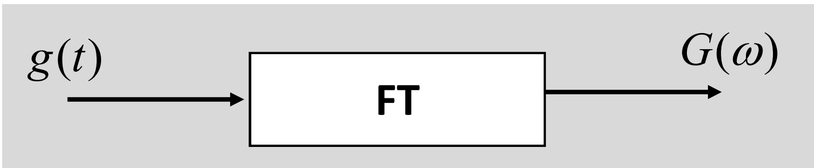
$$\varepsilon_0 = 8.8 \times 10^{-12} \quad Farad / m$$

Fourier and Phasor domains

Memo: linear time-invariant (LTI) systems

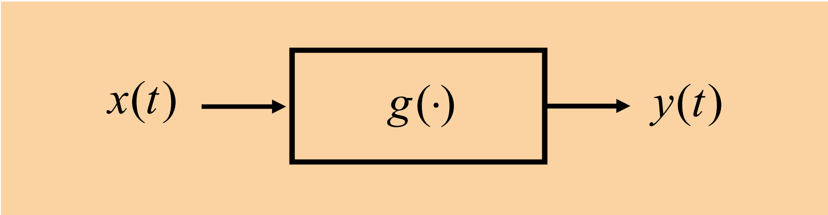


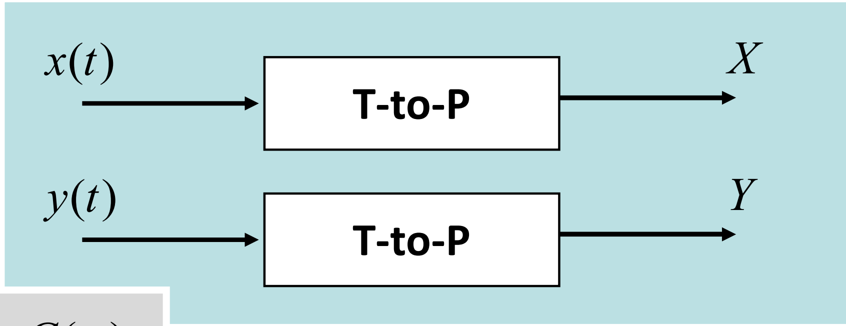


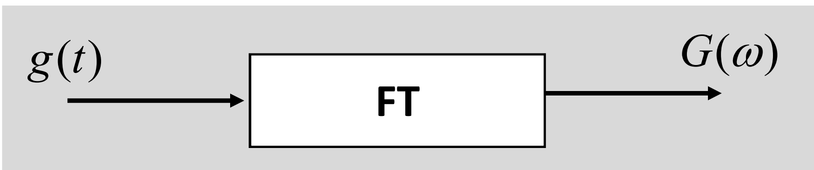


	Time domain	Frequency domain
Time-dispersive	$y(t)=\int dt'\, g(t-t')\,x(t')$	$Y(\omega)=G(\omega)X(\omega)$
Time-nondispersive	$y(t)=\tilde{g}\,x(t)$	$Y(\omega)=\tilde{g}X(\omega)$

Memo: linear time-invariant (LTI) systems







	Time domain	Frequency domain	Phasor domain
Time-dispersive	$y(t)=\int dt'\, g(t-t')\,x(t')$	$Y(\omega)=G(\omega)X(\omega)$	$Y=G(\omega_0)X$
Time-nondispersive	$y(t)=\tilde{g}\,x(t)$	$Y(\omega)=\tilde{g}X(\omega)$	$Y=\tilde{g}X$

Fourier and Phasor domains

	Time domain	Frequency domain	Phasor domain
Time-nondispersive Time-invariant Space-nondispersive Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$		
Time-nondispersive Time-invariant Space-nondispersive Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$		
Time-dispersive Time-invariant Space-nondispersive Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$		
Time-dispersive Time-invariant Space-nondispersive Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(\vec{\mathbf{r}},t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$		

Time: nondispersive & invariant
Space: nondispersive

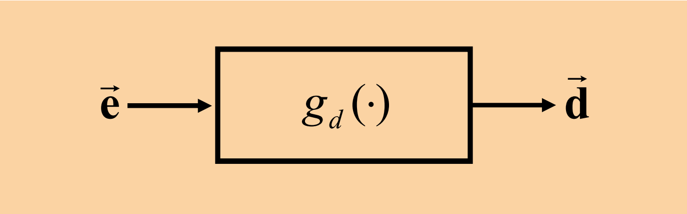
$$\vec{\mathbf{e}} \longrightarrow \boxed{g_d(\cdot)} \longrightarrow \vec{\mathbf{d}}$$

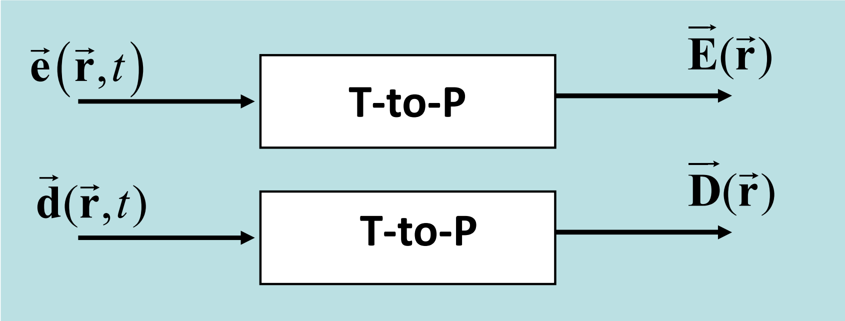
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) \longrightarrow \boxed{\mathrm{FT}} \longrightarrow \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$$

$$\vec{\mathbf{d}}(\vec{\mathbf{r}},t) \longrightarrow \boxed{\mathrm{FT}} \longrightarrow \vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)$$

	Time domain	Frequency domain	Phasor domain
Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	
Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	

Time: nondispersive & invariant
Space: nondispersive





	Time domain	Frequency domain	Phasor domain
Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}})$

Time: nondispersive & invariant
Space: nondispersive

$$\vec{\mathbf{e}} \longrightarrow \boxed{g_d(\cdot)} \longrightarrow \vec{\mathbf{d}}$$

Real quantities, all equal each other

	Time domain	Frequency domain	Phasor domain
Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \varepsilon\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) = \varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}}) = \varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \varepsilon(\vec{\mathbf{r}})\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) = \varepsilon(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}}) = \varepsilon(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}})$

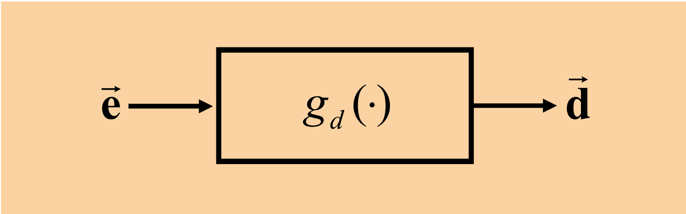
Time: nondispersive & invariant
Space: nondispersive

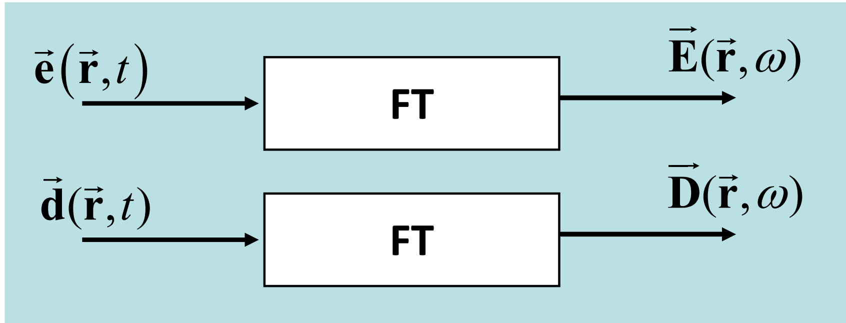
$$\vec{\mathbf{e}} \longrightarrow \boxed{g_d(\cdot)} \longrightarrow \vec{\mathbf{d}}$$

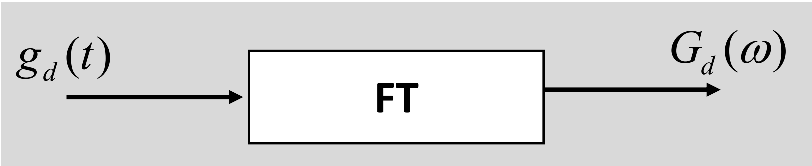
Real quantities, all equal each other

	Time domain	Frequency domain	Phasor domain
Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \varepsilon\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) = \varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}}) = \varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \varepsilon(\vec{\mathbf{r}})\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) = \varepsilon(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}}) = \varepsilon(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}})$

Time: dispersive & invariant
Space: nondispersive







	Time domain	Frequency domain	Phasor domain
Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d\left(t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t'\right)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=G_d(\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	
Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d\left(\vec{\mathbf{r}},t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t'\right)$		

Time: dispersive & invariant
Space: nondispersive

$$\vec{\mathbf{e}} \longrightarrow \boxed{g_d(\cdot)} \longrightarrow \vec{\mathbf{d}}$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) \xrightarrow{\boxed{\text{FT}}} \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$$

$$\vec{\mathbf{d}}(\vec{\mathbf{r}},t) \xrightarrow{\boxed{\text{FT}}} \vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)$$

$$g_d(\vec{\mathbf{r}},t) \xrightarrow{\boxed{\text{FT}}} G_d(\vec{\mathbf{r}},\omega)$$

	Time domain	Frequency domain	Phasor domain
Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d\left(t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t'\right)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=G_d(\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	
Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d\left(\vec{\mathbf{r}},t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t'\right)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=G_d(\vec{\mathbf{r}},\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	

Time: dispersive & invariant
Space: nondispersive

$$\vec{\mathbf{e}} \longrightarrow \boxed{g_d(\cdot)} \longrightarrow \vec{\mathbf{d}}$$

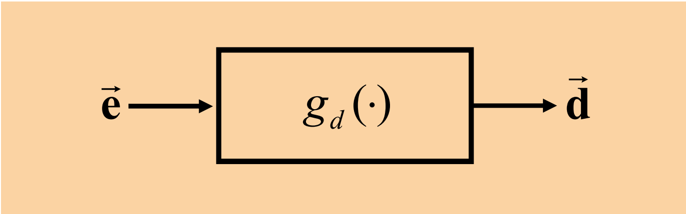
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) \longrightarrow \boxed{\text{T-to-P}} \longrightarrow \vec{\mathbf{E}}(\vec{\mathbf{r}})$$

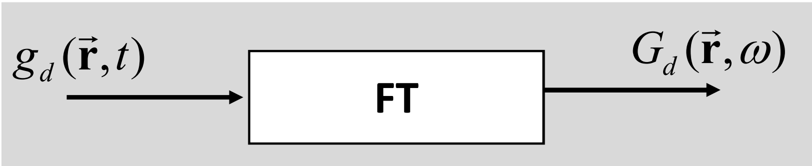
$$\vec{\mathbf{d}}(\vec{\mathbf{r}},t) \longrightarrow \boxed{\text{T-to-P}} \longrightarrow \vec{\mathbf{D}}(\vec{\mathbf{r}})$$

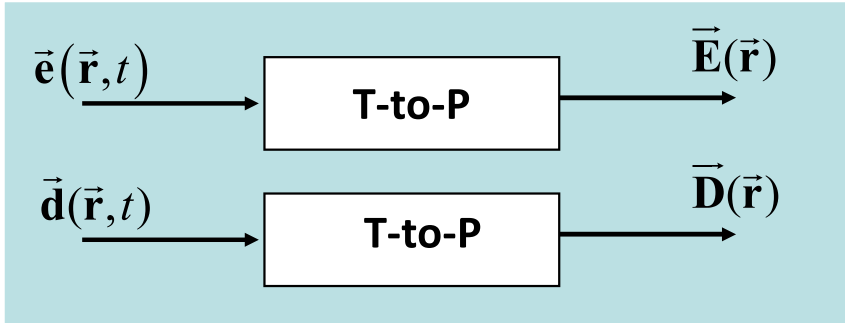
$$g_d(t) \longrightarrow \boxed{\text{FT}} \longrightarrow G_d(\omega)$$

	Time domain	Frequency domain	Phasor domain
Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=G_d(\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=G_d(\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(\vec{\mathbf{r}},t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=G_d(\vec{\mathbf{r}},\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	

Time: dispersive & invariant
Space: nondispersive







	Time domain	Frequency domain	Phasor domain
Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=G_d(\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=G_d(\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(\vec{\mathbf{r}},t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=G_d(\vec{\mathbf{r}},\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=G_d(\vec{\mathbf{r}},\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$

Time: dispersive & invariant
Space: nondispersive

$$\vec{\mathbf{e}} \longrightarrow \boxed{g_d(\cdot)} \longrightarrow \vec{\mathbf{d}}$$

	Real	Complex	Complex
	Time domain	**Frequency domain**	**Phasor domain**
Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(t-t')\,\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=G_d(\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=G_d(\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(\vec{\mathbf{r}},t-t')\,\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=G_d(\vec{\mathbf{r}},\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=G_d(\vec{\mathbf{r}},\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$

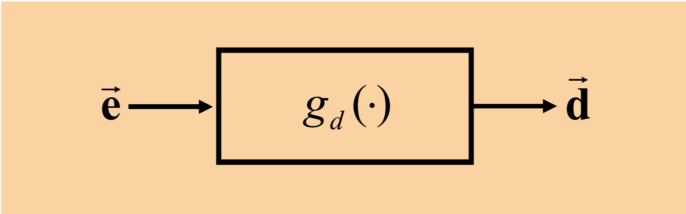
Time: dispersive & invariant
Space: nondispersive

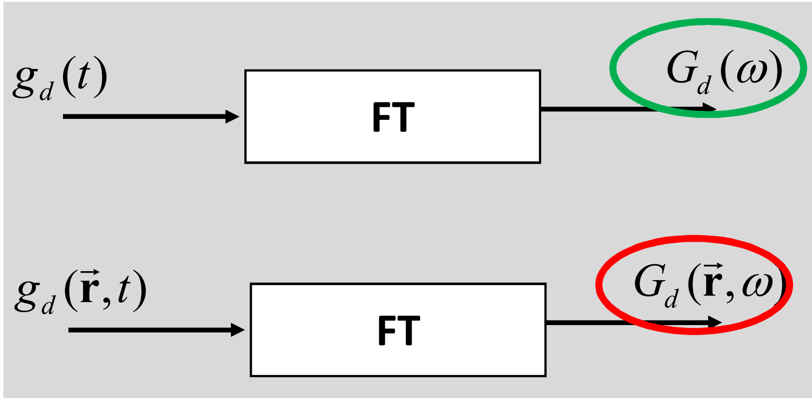
$$\vec{\mathbf{e}} \longrightarrow \boxed{g_d(\cdot)} \longrightarrow \vec{\mathbf{d}}$$

Real | **Complex** | **Complex**

	Time domain	Frequency domain	Phasor domain
Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d\left(t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t'\right)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=G_d(\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=G_d(\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d\left(\vec{\mathbf{r}},t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t'\right)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=G_d(\vec{\mathbf{r}},\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=G_d(\vec{\mathbf{r}},\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$

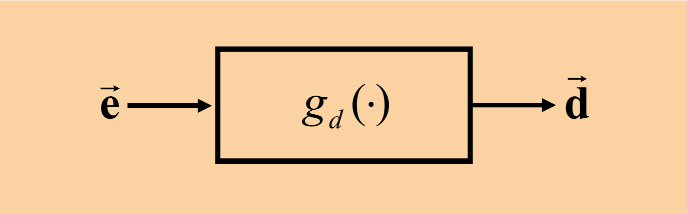
Time: dispersive & invariant
Space: nondispersive

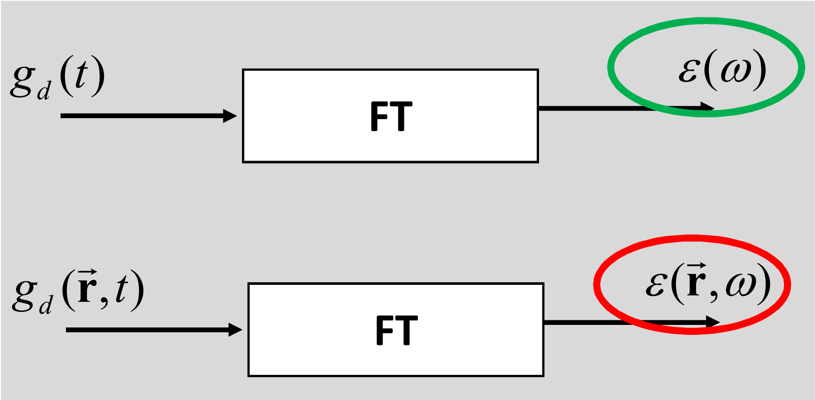




	Time domain	Frequency domain	Phasor domain
Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d\left(t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t'\right)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=G_d(\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=G_d(\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d\left(\vec{\mathbf{r}},t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t'\right)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=G_d(\vec{\mathbf{r}},\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=G_d(\vec{\mathbf{r}},\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$

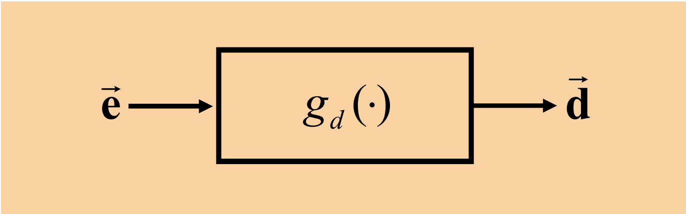
Time: dispersive & invariant
Space: nondispersive

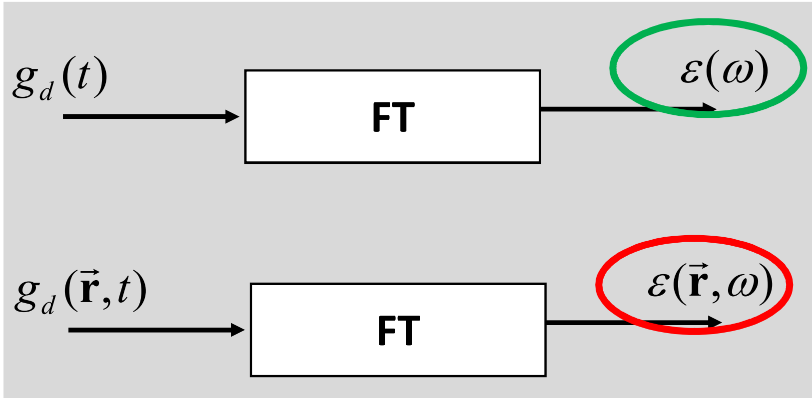




	Time domain	Frequency domain	Phasor domain
Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d\left(t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t'\right)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=G_d(\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=G_d(\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d\left(\vec{\mathbf{r}},t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t'\right)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=G_d(\vec{\mathbf{r}},\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=G_d(\vec{\mathbf{r}},\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$

Time: dispersive & invariant
Space: nondispersive





	Time domain	Frequency domain	Phasor domain
Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d\left(t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t'\right)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon(\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon(\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d\left(\vec{\mathbf{r}},t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t'\right)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon(\vec{\mathbf{r}},\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon(\vec{\mathbf{r}},\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$

Fourier and Phasor domains

	Time domain	Frequency domain	Phasor domain
Time-nondispersive **Time-invariant** **Space-nondispersive** **Space-invariant**	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Time-nondispersive **Time-invariant** **Space-nondispersive** **Space-variant**	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Time-dispersive **Time-invariant** **Space-nondispersive** **Space-invariant**	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon(\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon(\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Time-dispersive **Time-invariant** **Space-nondispersive** **Space-variant**	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(\vec{\mathbf{r}},t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon(\vec{\mathbf{r}},\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon(\vec{\mathbf{r}},\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$

Fourier and Phasor domains

	Time domain	Frequency domain	Phasor domain
Time-nondispersive Time-invariant Space-nondispersive Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Time-nondispersive Time-invariant Space-nondispersive Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Time-dispersive Time-invariant Space-nondispersive Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon(\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon(\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Time-dispersive Time-invariant Space-nondispersive Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(\vec{\mathbf{r}},t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon(\vec{\mathbf{r}},\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon(\vec{\mathbf{r}},\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$

Fourier and Phasor domains

	Time domain	Frequency domain	Phasor domain
Time-nondispersive Time-invariant Space-nondispersive Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Time-nondispersive Time-invariant Space-nondispersive Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Time-dispersive Time-invariant Space-nondispersive Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon(\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon(\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Normal media	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(\vec{\mathbf{r}},t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon(\vec{\mathbf{r}},\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon(\vec{\mathbf{r}},\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$

Fourier and Phasor domains

	Time domain	Frequency domain	Phasor domain
Normal media	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d\left(\vec{\mathbf{r}},t-t'\right)\vec{\mathbf{e}}\left(\vec{\mathbf{r}},t'\right)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon(\vec{\mathbf{r}},\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon(\vec{\mathbf{r}},\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$
	$\vec{\mathbf{b}}(\vec{\mathbf{r}},t)=\int dt'\, g_b\left(\vec{\mathbf{r}},t-t'\right)\vec{\mathbf{h}}\left(\vec{\mathbf{r}},t'\right)$	$\vec{\mathbf{B}}(\vec{\mathbf{r}},\omega)=\mu(\vec{\mathbf{r}},\omega)\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{B}}(\vec{\mathbf{r}})=\mu(\vec{\mathbf{r}},\omega_0)\vec{\mathbf{H}}(\vec{\mathbf{r}})$
Conductors	$\vec{\mathbf{j}}(\vec{\mathbf{r}},t)=\sigma\,\vec{\mathbf{e}}\left(\vec{\mathbf{r}}',t'\right)$	$\vec{\mathbf{J}}(\vec{\mathbf{r}},\omega)=\sigma\,\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{J}}(\vec{\mathbf{r}})=\sigma\,\vec{\mathbf{E}}(\vec{\mathbf{r}})$

Fourier and Phasor domains

	Time domain	Frequency domain	Phasor domain
Time-nondispersive Time-invariant Space-nondispersive Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Time-nondispersive Time-invariant Space-nondispersive Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Time-dispersive Time-invariant Space-nondispersive Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon(\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon(\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$
Normal media	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(\vec{\mathbf{r}},t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	$\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)=\varepsilon(\vec{\mathbf{r}},\omega)\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)$	$\vec{\mathbf{D}}(\vec{\mathbf{r}})=\varepsilon(\vec{\mathbf{r}},\omega_0)\vec{\mathbf{E}}(\vec{\mathbf{r}})$

Fourier and Phasor domains

	Time domain	Frequency domain Phasor domain
Time-nondispersive Time-invariant Space-nondispersive Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$	$\vec{\mathbf{D}}=\varepsilon\vec{\mathbf{E}}$ $\vec{\mathbf{B}}=\mu\vec{\mathbf{H}}$ $\vec{\mathbf{J}}=\sigma\vec{\mathbf{E}}$
Time-nondispersive Time-invariant Space-nondispersive Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$	
Time-dispersive Time-invariant Space-nondispersive Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\,g_d(t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	
Normal media	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\,g_d(\vec{\mathbf{r}},t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$	

Fourier and Phasor domains

	Time domain
Time-nondispersive Time-invariant Space-nondispersive Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$
Time-nondispersive Time-invariant Space-nondispersive Space-variant	
Time-dispersive Time-invariant Space-nondispersive Space-invariant	
Normal media	

$$\begin{cases} \nabla\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=-\dfrac{\partial\vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\dfrac{\partial\vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t}+\vec{\mathbf{j}}(\vec{\mathbf{r}},t)+\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t) \\ \nabla\cdot\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\rho(\vec{\mathbf{r}},t)+\rho_0(\vec{\mathbf{r}},t) \\ \nabla\cdot\vec{\mathbf{b}}(\vec{\mathbf{r}},t)=0 \end{cases}$$

$$\Downarrow$$

$$\begin{cases} \nabla\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=-\mu\dfrac{\partial\vec{\mathbf{h}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\varepsilon\dfrac{\partial\vec{\mathbf{e}}(\vec{\mathbf{r}},t)}{\partial t}+\sigma\vec{\mathbf{e}}(\vec{\mathbf{r}},t)+\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t) \\ \varepsilon\nabla\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\rho(\vec{\mathbf{r}},t)+\rho_0(\vec{\mathbf{r}},t) \\ \nabla\cdot\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=0 \end{cases}$$

Fourier and Phasor domains

	Time domain
Time-nondispersive Time-invariant Space-nondispersive Space-invariant	
Time-nondispersive Time-invariant Space-nondispersive Space-variant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$
Time-dispersive Time-invariant Space-nondispersive Space-invariant	
Normal media	

$$\begin{cases} \nabla\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=-\dfrac{\partial\vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\dfrac{\partial\vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t}+\vec{\mathbf{j}}(\vec{\mathbf{r}},t)+\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t) \\ \nabla\cdot\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\rho(\vec{\mathbf{r}},t)+\rho_0(\vec{\mathbf{r}},t) \\ \nabla\cdot\vec{\mathbf{b}}(\vec{\mathbf{r}},t)=0 \end{cases}$$

$$\Downarrow$$

$$\begin{cases} \nabla\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=-\mu(\vec{\mathbf{r}})\dfrac{\partial\vec{\mathbf{h}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\varepsilon(\vec{\mathbf{r}})\dfrac{\partial\vec{\mathbf{e}}(\vec{\mathbf{r}},t)}{\partial t}+\sigma\vec{\mathbf{e}}(\vec{\mathbf{r}},t)+\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t) \\ \nabla\cdot\varepsilon(\vec{\mathbf{r}})\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\rho(\vec{\mathbf{r}},t)+\rho_0(\vec{\mathbf{r}},t) \\ \nabla\cdot\mu(\vec{\mathbf{r}})\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=0 \end{cases}$$

Fourier and Phasor domains

	Time domain
Time-nondispersive Time-invariant Space-nondispersive Space-invariant	
Time-nondispersive Time-invariant Space-nondispersive Space-variant	
Time-dispersive Time-invariant Space-nondispersive Space-invariant	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$
Normal media	$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\int dt'\, g_d(\vec{\mathbf{r}},t-t')\vec{\mathbf{e}}(\vec{\mathbf{r}},t')$

$$
\begin{cases}
\nabla\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=-\dfrac{\partial\vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t}\\
\nabla\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\dfrac{\partial\vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t}+\vec{\mathbf{j}}(\vec{\mathbf{r}},t)+\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)\\
\nabla\cdot\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\rho(\vec{\mathbf{r}},t)\\
\nabla\cdot\vec{\mathbf{b}}(\vec{\mathbf{r}},t)=0
\end{cases}
$$

Much more convenient to work in the frequency/phasor domains!

Fourier and Phasor domains

Time-nondispersive
Time-invariant
Space-nondispersive
Space-invariant

Time-nondispersive
Time-invariant
Space-nondispersive
Space-variant

Time-dispersive
Time-invariant
Space-nondispersive
Space-invariant

Normal media

Frequency domain

$$\begin{cases} \nabla\times\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega\vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) \\ \nabla\times\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = \boxed{j\omega\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega)+\vec{\mathbf{J}}(\vec{\mathbf{r}},\omega)}+\vec{\mathbf{J}}_0(\vec{\mathbf{r}},\omega) \\ \nabla\cdot\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) = \rho(\vec{\mathbf{r}},\omega)+\rho_0(\vec{\mathbf{r}},\omega) \\ \nabla\cdot\vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) = 0 \end{cases}$$

Phasor domain

$$\begin{cases} \nabla\times\vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0\vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla\times\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \boxed{j\omega_0\vec{\mathbf{D}}(\vec{\mathbf{r}})+\vec{\mathbf{J}}(\vec{\mathbf{r}})}+\vec{\mathbf{J}}_0(\vec{\mathbf{r}}) \\ \nabla\cdot\vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}})+\rho_0(\vec{\mathbf{r}}) \\ \nabla\cdot\vec{\mathbf{B}}(\vec{\mathbf{r}}) = 0 \end{cases}$$

$$\vec{\mathbf{D}}=\varepsilon\vec{\mathbf{E}};\quad \vec{\mathbf{B}}=\mu\vec{\mathbf{H}};\quad \vec{\mathbf{J}}=\sigma\vec{\mathbf{E}};$$

$$\begin{cases} \nabla\times\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega\mu\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) \\ \nabla\times\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = \boxed{j\omega\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)+\sigma\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)}+\vec{\mathbf{J}}_0(\vec{\mathbf{r}},\omega) \\ \nabla\cdot\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = \rho(\vec{\mathbf{r}},\omega)+\rho_0(\vec{\mathbf{r}},\omega) \\ \nabla\cdot\mu\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = 0 \end{cases}$$

$$\begin{cases} \nabla\times\vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0\mu\vec{\mathbf{H}}(\vec{\mathbf{r}}) \\ \nabla\times\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \boxed{j\omega_0\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}})+\sigma\vec{\mathbf{E}}(\vec{\mathbf{r}})}+\vec{\mathbf{J}}_0(\vec{\mathbf{r}}) \\ \nabla\cdot\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}})+\rho_0(\vec{\mathbf{r}}) \\ \nabla\cdot\mu\vec{\mathbf{H}}(\vec{\mathbf{r}}) = 0 \end{cases}$$

Fourier and Phasor domains

Time-nondispersive
Time-invariant
Space-nondispersive
Space-invariant

Time-nondispersive
Time-invariant
Space-nondispersive
Space-variant

Time-dispersive
Time-invariant
Space-nondispersive
Space-invariant

Normal media

Frequency domain

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega\mu\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = \boxed{j\omega\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) + \sigma\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega)} + \vec{\mathbf{J}}_0(\vec{\mathbf{r}},\omega) \\ \nabla \cdot \varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = \rho(\vec{\mathbf{r}},\omega) + \rho_0(\vec{\mathbf{r}},\omega) \\ \nabla \cdot \mu\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = 0 \end{cases}$$

Phasor domain

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0\mu\vec{\mathbf{H}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = \boxed{j\omega_0\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}}) + \sigma\vec{\mathbf{E}}(\vec{\mathbf{r}})} + \vec{\mathbf{J}}_0(\vec{\mathbf{r}}) \\ \nabla \cdot \varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) + \rho_0(\vec{\mathbf{r}}) \\ \nabla \cdot \mu\vec{\mathbf{H}}(\vec{\mathbf{r}}) = 0 \end{cases}$$

$$\vec{\mathbf{D}} = \varepsilon\vec{\mathbf{E}} \qquad \vec{\mathbf{J}} = \sigma\vec{\mathbf{E}}$$

$$j\omega\varepsilon\vec{\mathbf{E}} + \sigma\vec{\mathbf{E}} = j\omega\varepsilon\left[1 + \frac{\sigma}{j\omega\varepsilon}\right]\vec{\mathbf{E}} = j\omega\varepsilon\left[1 - j\frac{\sigma}{\omega\varepsilon}\right]\vec{\mathbf{E}} = j\omega\varepsilon_{eq}\vec{\mathbf{E}}$$

$$\boxed{\varepsilon_{eq} = \varepsilon\left[1 - j\frac{\sigma}{\omega\varepsilon}\right]}$$

One consideration

- Conducting media

$$\varepsilon_{eq} = \varepsilon\left[1 - j\frac{\sigma}{\omega\varepsilon}\right]$$

- Highly conducting media

$$\sigma >> \omega\varepsilon \quad \Longrightarrow \quad \varepsilon_{eq} = \varepsilon\left[1 - j\frac{\sigma}{\omega\varepsilon}\right] \approx \frac{\sigma}{j\omega}$$

Another consideration

- In time-dispersive media, when working in the Fourier/Phasor domain, we have:

$$\varepsilon = \varepsilon_1 - j\varepsilon_2$$

$$\mu = \mu_1 - j\mu_2$$

It can be shown that the real and imaginary parts of these quantities are not independent each other: they are related by the Kramers- Kröning relations

…. two last considerations

- Note that causality and finite velocity of propagation must be enforced when writing the impulse response that describes the medium

- Note that, due to the finite velocity of propagation, space-dispersive media are time-dispersive too