

Campi Elettromagnetici

**Corso di Laurea in Ingegneria Informatica,
Biomedica e delle Telecomunicazioni**

a.a. 2020-2021 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

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Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Constitutive relationships

Time domain - Differential form

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) + \vec{\mathbf{j}}_0(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) + \rho_0(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

Even assuming knowledge of the impressed sources $\vec{\mathbf{j}}_0(\vec{\mathbf{r}}, t); \rho_0(\vec{\mathbf{r}}, t)$

Number of independent scalar equations: **7**

Number of unknown scalar quantities: **16**

Maxwell equations involve a number of unknowns larger than the number of equations!

The additional missing equations are provided by the **constitutive relationships**, which describe interaction of fields and matter from a macroscopic point of view

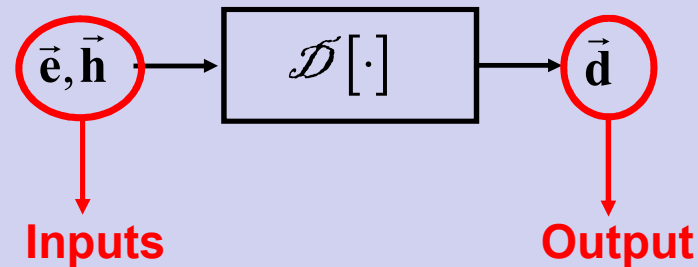
Constitutive relationships

Inductions and currents must be represented in terms of fields

$$\vec{\mathbf{d}} = \mathcal{D}[\vec{\mathbf{e}}, \vec{\mathbf{h}}]$$

$$\vec{\mathbf{b}} = \mathcal{B}[\vec{\mathbf{e}}, \vec{\mathbf{h}}]$$

$$\vec{\mathbf{j}} = \mathcal{J}[\vec{\mathbf{e}}, \vec{\mathbf{h}}]$$



$\mathcal{D}[\cdot]$, $\mathcal{B}[\cdot]$ and $\mathcal{J}[\cdot]$ are functionals which depend upon the medium in which the electromagnetic field is considered and upon the fields themselves.

Constitutive relationships

Linear media

$$\vec{\mathbf{d}}_1 = \mathcal{D}[\vec{\mathbf{e}}_1, \vec{\mathbf{h}}_1]$$

$$\vec{\mathbf{d}}_2 = \mathcal{D}[\vec{\mathbf{e}}_2, \vec{\mathbf{h}}_2]$$



$$\vec{\mathbf{d}}_1 + \vec{\mathbf{d}}_2 = \mathcal{D}[\vec{\mathbf{e}}_1 + \vec{\mathbf{e}}_2, \vec{\mathbf{h}}_1 + \vec{\mathbf{h}}_2]$$

Constitutive relationships

Linear media

■ Local (non-dispersive) media

■ Anisotropic media

$$\vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \boldsymbol{\varepsilon}(\vec{\mathbf{r}}, t) \cdot \vec{\mathbf{e}}(\vec{\mathbf{r}}, t)$$

$$\vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = \boldsymbol{\mu}(\vec{\mathbf{r}}, t) \cdot \vec{\mathbf{h}}(\vec{\mathbf{r}}, t)$$

$$\vec{\mathbf{j}}(\vec{\mathbf{r}}, t) = \boldsymbol{\sigma}(\vec{\mathbf{r}}, t) \cdot \vec{\mathbf{e}}(\vec{\mathbf{r}}, t)$$

$\boldsymbol{\varepsilon}(\vec{\mathbf{r}}, t); \boldsymbol{\mu}(\vec{\mathbf{r}}, t); \boldsymbol{\sigma}(\vec{\mathbf{r}}, t)$: 3x3 matrices

■ Local (non-dispersive) media

■ Isotropic media

$$\vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \varepsilon(\vec{\mathbf{r}}, t) \vec{\mathbf{e}}(\vec{\mathbf{r}}, t)$$

$$\vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = \mu(\vec{\mathbf{r}}, t) \vec{\mathbf{h}}(\vec{\mathbf{r}}, t)$$

$$\vec{\mathbf{j}}(\vec{\mathbf{r}}, t) = \sigma(\vec{\mathbf{r}}, t) \vec{\mathbf{e}}(\vec{\mathbf{r}}, t)$$

$\varepsilon(\vec{\mathbf{r}}, t); \mu(\vec{\mathbf{r}}, t); \sigma(\vec{\mathbf{r}}, t)$: scalar functions

Constitutive relationships

Linear media

Class

Time-dispersive

Time-nondispersive

Time-variant

Time-invariant

Property

Effect on the I-O relation

Constitutive relationships

Linear media

Class

Space-dispersive

Space-nondispersive

Space-variant

Space-invariant

Property

Effect on the I-O relation

Constitutive relationships

Linear media

Class

Time-dispersive

Time-nondispersive

Time-variant

Time-invariant

Property

The output **at time t depends only** on the value of the input **at the same time t**

Constitutive relationships

Linear media

Class

Time-dispersive

Time-nondispersive

Time-variant

Time-invariant

Property

The output **at time t depends** on the values of the input **throughout a time-interval.**

Constitutive relationships

Linear media

Class

Time-dispersive

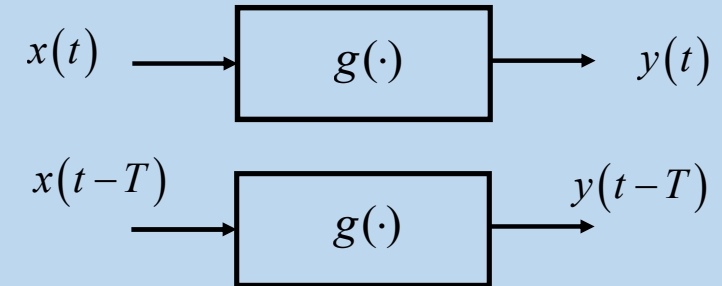
Time-nondispersive

Time-variant

Time-invariant

Property

A **time translation** of the input implies **the same translation** of the output



Constitutive relationships

Linear media

Class

Time-dispersive

Time-nondispersive

Time-variant

Time-invariant

Property

A **time translation** of the input **does not** imply **the same translation** of the output

Constitutive relationships

Linear media

Class

Space-dispersive

Space-nondispersive

Space-variant

Space-invariant

Property

The output **at space \vec{r}**
depends only on the value of
the input **at the same space \vec{r}**

Constitutive relationships

Linear media

Class

Space-dispersive

Space-nondispersive

Space-variant

Space-invariant

Property

The output **at space \vec{r}**
depends on the values of
the input **throughout a**
space-interval

Constitutive relationships

Linear media

Class

Space-dispersive

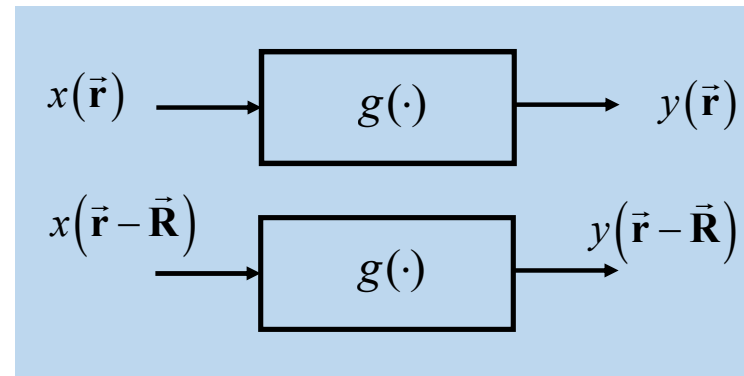
Space-nondispersive

Space-variant

Space-invariant

Property

A **space translation** of the input implies **the same translation** of the output



Constitutive relationships

Linear media

Class

Space-dispersive

Space-nondispersive

Space-variant

Space-invariant

Property

A **space translation** of the input **does not** imply **the same translation** of the output

Constitutive relationships

Linear media

Class

Property

Effect on the I-O relation

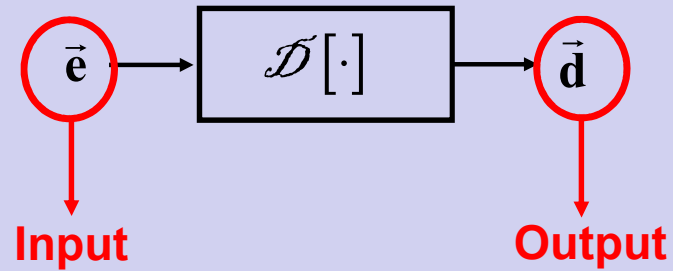
Constitutive relationships

Linear & Anisotropic media

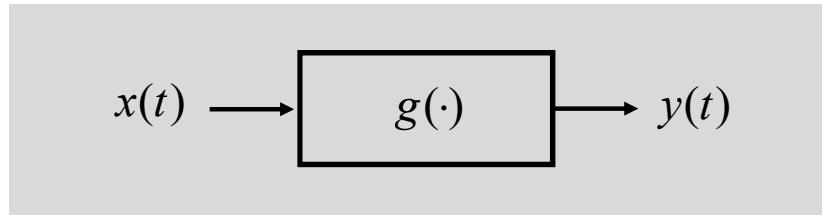
$$\vec{\mathbf{d}} = \mathcal{D}[\vec{\mathbf{e}}]$$

$$\vec{\mathbf{b}} = \mathcal{B}[\vec{\mathbf{h}}]$$

$$\vec{\mathbf{j}} = \mathcal{J}[\vec{\mathbf{e}}]$$



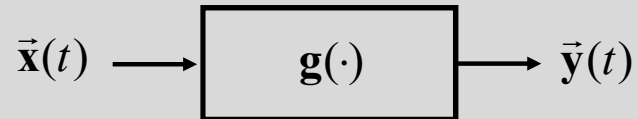
Memo: linear systems



$x(t)$ and $y(t)$ are scalar

	Time-dispersive (TD)	Time-nondispersive (TND)
Time-variant (TV)	$y(t) = \int dt' g(t, t') x(t')$	$y(t) = \tilde{g}(t) x(t)$
Time-invariant (TI)	$y(t) = \int dt' g(t - t') x(t')$	$y(t) = \tilde{g} x(t)$

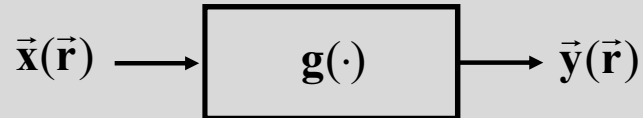
Linear systems



$\vec{x}(t)$ and $\vec{y}(t)$ are vectors

	Time-dispersive (TD)	Time-nondispersive (TND)
Time-variant (TV)	$\vec{y}(t) = \int dt' \mathbf{g}(t, t') \cdot \vec{x}(t')$	$\vec{y}(t) = \tilde{\mathbf{g}}(t) \cdot \vec{x}(t)$
Time-invariant (TI)	$\vec{y}(t) = \int dt' \mathbf{g}(t - t') \cdot \vec{x}(t')$	$\vec{y}(t) = \tilde{\mathbf{g}} \cdot \vec{x}(t)$

Linear systems



$\vec{x}(\vec{r})$ and $\vec{y}(\vec{r})$ are vectors

	Space-dispersive (SD)	Space-nondispersive (SND)
Space-variant (SV)	$\vec{y}(\vec{r}) = \int d\vec{r}' g(\vec{r}, \vec{r}') \cdot \vec{x}(\vec{r}')$	$\vec{y}(\vec{r}) = \tilde{g}(\vec{r}) \cdot \vec{x}(\vec{r})$
Space-invariant (SI)	$\vec{y}(\vec{r}) = \int d\vec{r}' g(\vec{r} - \vec{r}') \cdot \vec{x}(\vec{r}')$	$\vec{y}(\vec{r}) = \tilde{g} \cdot \vec{x}(\vec{r})$

	Time-dispersive (TD)	Time-nondispersive (TND)
Time-variant (TV)	$\vec{y}(t) = \int dt' \mathbf{g}(t, t') \cdot \vec{x}(t')$	$\vec{y}(t) = \tilde{\mathbf{g}}(t) \cdot \vec{x}(t)$
Time-invariant (TI)	$\vec{y}(t) = \int dt' \mathbf{g}(t - t') \cdot \vec{x}(t')$	$\vec{y}(t) = \tilde{\mathbf{g}} \cdot \vec{x}(t)$
	Space-dispersive (SD)	Space-nondispersive (SND)
Space-variant (SV)	$\vec{y}(\vec{r}) = \int d\vec{r}' \mathbf{g}(\vec{r}, \vec{r}') \cdot \vec{x}(\vec{r}')$	$\vec{y}(\vec{r}) = \tilde{\mathbf{g}}(\vec{r}) \cdot \vec{x}(\vec{r})$
Space-invariant (SI)	$\vec{y}(\vec{r}) = \int d\vec{r}' \mathbf{g}(\vec{r} - \vec{r}') \cdot \vec{x}(\vec{r}')$	$\vec{y}(\vec{r}) = \tilde{\mathbf{g}} \cdot \vec{x}(\vec{r})$

Constitutive relationships

Linear & Anisotropic & Dispersive media

$$\vec{\mathbf{d}} = \mathcal{D}[\vec{\mathbf{e}}] \quad \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \int d\vec{\mathbf{r}}' \int dt' \mathbf{g}_d(\vec{\mathbf{r}}, \vec{\mathbf{r}}', t, t') \cdot \vec{\mathbf{e}}(\vec{\mathbf{r}}', t')$$

$$\vec{\mathbf{b}} = \mathcal{B}[\vec{\mathbf{h}}] \quad \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = \int d\vec{\mathbf{r}}' \int dt' \mathbf{g}_b(\vec{\mathbf{r}}, \vec{\mathbf{r}}', t, t') \cdot \vec{\mathbf{h}}(\vec{\mathbf{r}}', t')$$

$$\vec{\mathbf{j}} = \mathcal{J}[\vec{\mathbf{e}}] \quad \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) = \int d\vec{\mathbf{r}}' \int dt' \mathbf{g}_j(\vec{\mathbf{r}}, \vec{\mathbf{r}}', t, t') \cdot \vec{\mathbf{e}}(\vec{\mathbf{r}}', t')$$

Linear & Isotropic & Dispersive media

$$\vec{\mathbf{d}} = \mathcal{D}[\vec{\mathbf{e}}] \quad \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \int d\vec{\mathbf{r}}' \int dt' g_d(\vec{\mathbf{r}}, \vec{\mathbf{r}}', t, t') \vec{\mathbf{e}}(\vec{\mathbf{r}}', t')$$

$$\vec{\mathbf{b}} = \mathcal{B}[\vec{\mathbf{h}}] \quad \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = \int d\vec{\mathbf{r}}' \int dt' g_b(\vec{\mathbf{r}}, \vec{\mathbf{r}}', t, t') \vec{\mathbf{h}}(\vec{\mathbf{r}}', t')$$

$$\vec{\mathbf{j}} = \mathcal{J}[\vec{\mathbf{e}}] \quad \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) = \int d\vec{\mathbf{r}}' \int dt' g_j(\vec{\mathbf{r}}, \vec{\mathbf{r}}', t, t') \vec{\mathbf{e}}(\vec{\mathbf{r}}', t')$$

Constitutive relationships

In the following, just for the sake of simplicity,
we will consider isotropic media

Time: dispersive
Space: dispersive

	Space-Dispersive (SD) Time-Dispersive (TD)
SV-TV	$\vec{d}(\vec{r}, t) = \int d\vec{r}' \int dt' g_d(\vec{r}, \vec{r}', t, t') \vec{e}(\vec{r}', t')$
SV-TI	$\vec{d}(\vec{r}, t) = \int d\vec{r}' \int dt' g_d(\vec{r}, \vec{r}', t - t') \vec{e}(\vec{r}', t')$
SI-TV	$\vec{d}(\vec{r}, t) = \int d\vec{r}' \int dt' g_d(\vec{r} - \vec{r}', t, t') \vec{e}(\vec{r}', t')$
SI-TI	$\vec{d}(\vec{r}, t) = \int d\vec{r}' \int dt' g_d(\vec{r} - \vec{r}', t - t') \vec{e}(\vec{r}', t')$

Time: nondispersive
Space: dispersive

	Space-Dispersive (SD) Time-Dispersive (TD)	Space-Dispersive (SD) Time-Nondispersive (TND)
SV-TV	$\vec{d}(\vec{r}, t) = \int d\vec{r}' \int dt' g_d(\vec{r}, \vec{r}', t, t') \vec{e}(\vec{r}', t')$	$\vec{d}(\vec{r}, t) = \int d\vec{r}' g_d(\vec{r}, \vec{r}', t) \vec{e}(\vec{r}', t)$
SV-TI	$\vec{d}(\vec{r}, t) = \int d\vec{r}' \int dt' g_d(\vec{r}, \vec{r}', t - t') \vec{e}(\vec{r}', t')$	$\vec{d}(\vec{r}, t) = \int d\vec{r}' g_d(\vec{r}, \vec{r}') \vec{e}(\vec{r}', t)$
SI-TV	$\vec{d}(\vec{r}, t) = \int d\vec{r}' \int dt' g_d(\vec{r} - \vec{r}', t, t') \vec{e}(\vec{r}', t')$	$\vec{d}(\vec{r}, t) = \int d\vec{r}' g_d(\vec{r} - \vec{r}', t) \vec{e}(\vec{r}', t)$
SI-TI	$\vec{d}(\vec{r}, t) = \int d\vec{r}' \int dt' g_d(\vec{r} - \vec{r}', t - t') \vec{e}(\vec{r}', t')$	$\vec{d}(\vec{r}, t) = \int d\vec{r}' g_d(\vec{r} - \vec{r}') \vec{e}(\vec{r}', t)$

Time: dispersive
Space: nondispersive

	Space-Dispersive (SD) Time-Dispersive (TD)	Space-Nondispersive (SND) Time-Dispersive (TD)
SV-TV	$\vec{d}(\vec{r}, t) = \int d\vec{r}' \int dt' g_d(\vec{r}, \vec{r}', t, t') \vec{e}(\vec{r}', t')$	$\vec{d}(\vec{r}, t) = \int dt' g_d(\vec{r}, t, t') \vec{e}(\vec{r}, t')$
SV-TI	$\vec{d}(\vec{r}, t) = \int d\vec{r}' \int dt' g_d(\vec{r}, \vec{r}', t - t') \vec{e}(\vec{r}', t')$	$\vec{d}(\vec{r}, t) = \int dt' g_d(\vec{r}, t - t') \vec{e}(\vec{r}, t')$
SI-TV	$\vec{d}(\vec{r}, t) = \int d\vec{r}' \int dt' g_d(\vec{r} - \vec{r}', t, t') \vec{e}(\vec{r}', t')$	$\vec{d}(\vec{r}, t) = \int dt' g_d(t, t') \vec{e}(\vec{r}, t')$
SI-TI	$\vec{d}(\vec{r}, t) = \int d\vec{r}' \int dt' g_d(\vec{r} - \vec{r}', t - t') \vec{e}(\vec{r}', t')$	$\vec{d}(\vec{r}, t) = \int dt' g_d(t - t') \vec{e}(\vec{r}, t')$

Dispersive (time & space) vs. nondispersive (time & space)

	Space-Dispersive (SD) Time-Dispersive (TD)	Space-Nondispersive (SND) Time-Nondispersive (TND)
SV-TV	$\vec{d}(\vec{r}, t) = \int d\vec{r}' \int dt' g_d(\vec{r}, \vec{r}', t, t') \vec{e}(\vec{r}', t')$	$\vec{d}(\vec{r}, t) = \varepsilon(\vec{r}, t) \vec{e}(\vec{r}, t)$
SV-TI	$\vec{d}(\vec{r}, t) = \int d\vec{r}' \int dt' g_d(\vec{r}, \vec{r}', t - t') \vec{e}(\vec{r}', t')$	$\vec{d}(\vec{r}, t) = \varepsilon(\vec{r}) \vec{e}(\vec{r}, t)$
SI-TV	$\vec{d}(\vec{r}, t) = \int d\vec{r}' \int dt' g_d(\vec{r} - \vec{r}', t, t') \vec{e}(\vec{r}', t')$	$\vec{d}(\vec{r}, t) = \varepsilon(t) \vec{e}(\vec{r}, t)$
SI-TI	$\vec{d}(\vec{r}, t) = \int d\vec{r}' \int dt' g_d(\vec{r} - \vec{r}', t - t') \vec{e}(\vec{r}', t')$	$\vec{d}(\vec{r}, t) = \varepsilon \vec{e}(\vec{r}, t)$

Permittivity

	SND+TND: Local	
SV-TV	$\vec{d}(\vec{r},t) = \varepsilon(\vec{r},t)\vec{e}(\vec{r},t)$	$[\varepsilon] = ?$
		$[\varepsilon] = \frac{\text{Coulomb}}{m^2} \frac{m}{\text{Volt}} = \frac{\text{Coulomb}}{\text{Volt}} \frac{1}{m} = \frac{\text{Farad}}{m}$
SV-TI	$\vec{d}(\vec{r},t) = \varepsilon(\vec{r})\vec{e}(\vec{r},t)$	
SI-TV	$\vec{d}(\vec{r},t) = \varepsilon(t)\vec{e}(\vec{r},t)$	$[\vec{e}(\vec{r},t)] = \frac{\text{Volt}}{m}$ $[\vec{d}(\vec{r},t)] = \frac{\text{Coulomb}}{m^2}$
SI-TI	$\vec{d}(\vec{r},t) = \varepsilon\vec{e}(\vec{r},t)$	$C = \frac{q}{\Delta v}$ $\text{Farad} = \frac{\text{Coulomb}}{\text{Volt}}$

Permeability

	SND+TND: Local	
SV-TV	$\vec{\mathbf{b}}(\vec{\mathbf{r}},t) = \mu(\vec{\mathbf{r}},t)\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$	$[\mu] = ?$
SV-TI	$\vec{\mathbf{b}}(\vec{\mathbf{r}},t) = \mu(\vec{\mathbf{r}})\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$	$[\mu] = \frac{\text{Weber}}{m^2} \frac{m}{\text{Ampere}} = \frac{\text{Weber}}{\text{Ampere m}} = \frac{\text{Henry}}{m}$
SI-TV	$\vec{\mathbf{b}}(\vec{\mathbf{r}},t) = \mu(t)\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$	
SI-TI	$\vec{\mathbf{b}}(\vec{\mathbf{r}},t) = \mu\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$	$[\vec{\mathbf{b}}(\vec{\mathbf{r}},t)] = \frac{\text{Weber}}{m^2} \quad [\vec{\mathbf{h}}(\vec{\mathbf{r}},t)] = \frac{\text{Ampere}}{m}$ $L = \frac{\Phi_{\vec{\mathbf{b}}}}{i} \quad \text{Henry} = \frac{\text{Weber}}{\text{Ampere}}$

Conductivity

	SND+TND: Local
SV-TV	$\vec{\mathbf{j}}(\vec{\mathbf{r}},t) = \sigma(\vec{\mathbf{r}},t)\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$
SV-TI	$\vec{\mathbf{j}}(\vec{\mathbf{r}},t) = \sigma(\vec{\mathbf{r}})\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$
SI-TV	$\vec{\mathbf{j}}(\vec{\mathbf{r}},t) = \sigma(t)\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$
SI-TI	$\vec{\mathbf{j}}(\vec{\mathbf{r}},t) = \sigma\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$

$$[\sigma] = ?$$

$$[\sigma] = \frac{\text{Ampere}}{m^2} \frac{m}{\text{Volt}} = \frac{\text{Ampere}}{\text{Volt}} \frac{1}{m} = \frac{1}{\Omega m} = \frac{\text{Siemens}}{m}$$

$$[\vec{\mathbf{j}}(\vec{\mathbf{r}},t)] = \frac{\text{Ampere}}{m^2} \quad [\vec{\mathbf{e}}(\vec{\mathbf{r}},t)] = \frac{\text{Volt}}{m}$$

$$\Delta v = Ri \quad \Omega = \frac{\text{Volt}}{\text{Ampere}} = \frac{1}{\text{Siemens}}$$

Exercise n.1

Linear+ Isotropic + Space-Variant (SV) + Time-Variant (TV)

Space-dispersive (SD) +Time-dispersive (TD)

$$\vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \int d\vec{\mathbf{r}}' \int dt' g_d(\vec{\mathbf{r}}, \vec{\mathbf{r}}', t, t') \vec{\mathbf{e}}(\vec{\mathbf{r}}', t')$$

$$[g_d] = \frac{\text{Farad}}{s \cdot m^4}$$

Space-dispersive (SD) +Time-nondispersive (TD)

$$\vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \int d\vec{\mathbf{r}}' g_d(\vec{\mathbf{r}}, \vec{\mathbf{r}}', t) \vec{\mathbf{e}}(\vec{\mathbf{r}}', t)$$

$$[g_d] = \frac{\text{Farad}}{m^4}$$

Space-nondispersive (SND) +Time-dispersive (TD)

$$\vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \int dt' g_d(\vec{\mathbf{r}}, t, t') \vec{\mathbf{e}}(\vec{\mathbf{r}}, t')$$

$$[g_d] = \frac{\text{Farad}}{s \cdot m}$$

Space-nondispersive (SND) +Time-nondispersive (TD)

$$\vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \varepsilon(\vec{\mathbf{r}}, t) \vec{\mathbf{e}}(\vec{\mathbf{r}}, t)$$

$$[\varepsilon] = \frac{\text{Farad}}{m}$$

Exercise n.2

Linear+ Isotropic + Space-Variant (SV) + Time-Invariant (TI)

Space-Dispersive (SD) +Time-Dispersive (TD)

$$\vec{\mathbf{b}}(\vec{\mathbf{r}},t) = \int d\vec{\mathbf{r}}' \int dt' g_b(\vec{\mathbf{r}},\vec{\mathbf{r}}',t-t') \vec{\mathbf{h}}(\vec{\mathbf{r}}',t')$$

$$[g_b] = \frac{\text{Henry}}{s \cdot m^4}$$

Space-Dispersive (SD) +Time-Nondispersive (TD)

$$\vec{\mathbf{b}}(\vec{\mathbf{r}},t) = \int d\vec{\mathbf{r}}' g_b(\vec{\mathbf{r}},\vec{\mathbf{r}}') \vec{\mathbf{h}}(\vec{\mathbf{r}}',t)$$

$$[g_b] = \frac{\text{Henry}}{m^4}$$

Space-Nondispersive (SND) +Time-Dispersive (TD)

$$\vec{\mathbf{b}}(\vec{\mathbf{r}},t) = \int dt' g_b(\vec{\mathbf{r}},t-t') \vec{\mathbf{h}}(\vec{\mathbf{r}},t')$$

$$[g_b] = \frac{\text{Henry}}{s \cdot m}$$

Space-Nondispersive (SND) +Time-Nondispersive (TD)

$$\vec{\mathbf{b}}(\vec{\mathbf{r}},t) = \mu(\vec{\mathbf{r}}) \vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

$$[\mu] = \frac{\text{Henry}}{m}$$

Conductors

$$\vec{j}(\vec{r}, t) = \sigma \vec{e}(\vec{r}', t')$$

Metal	Conductivity σ [siemens/m]
Silver, 99.98% pure	6.14×10^7
Copper, annealed	5.80×10^7
Copper, hard drawn	5.65×10^7
Gold, pure drawn	4.10×10^7
Aluminum, commercial hard drawn	3.54×10^7
Magnesium	2.17×10^7
Tungsten	1.81×10^7
Zinc	1.74×10^7
Nickel	1.28×10^7
Iron, 99.98% pure	1.00×10^7
Steel	$1.00-0.5 \times 10^7$
Lead	0.48×10^7
Mercury	0.10×10^7

From G.Franceschetti, 'Electromagnetics, Theory, Techniques, and Engineering paradigms', Plenum Press

Perfect Electric Conductors (PEC)

$$\sigma \rightarrow \infty \quad \Rightarrow \quad \vec{e} = 0 \quad \Rightarrow \quad \vec{h} = 0$$

Vacuo

$$\vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \varepsilon_0 \vec{\mathbf{e}}(\vec{\mathbf{r}}, t)$$

$$\vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = \mu_0 \vec{\mathbf{h}}(\vec{\mathbf{r}}, t)$$

$$\sigma = 0$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ Henry / m}$$

$$\varepsilon_0 = 8.8 \times 10^{-12} \text{ Farad / m}$$