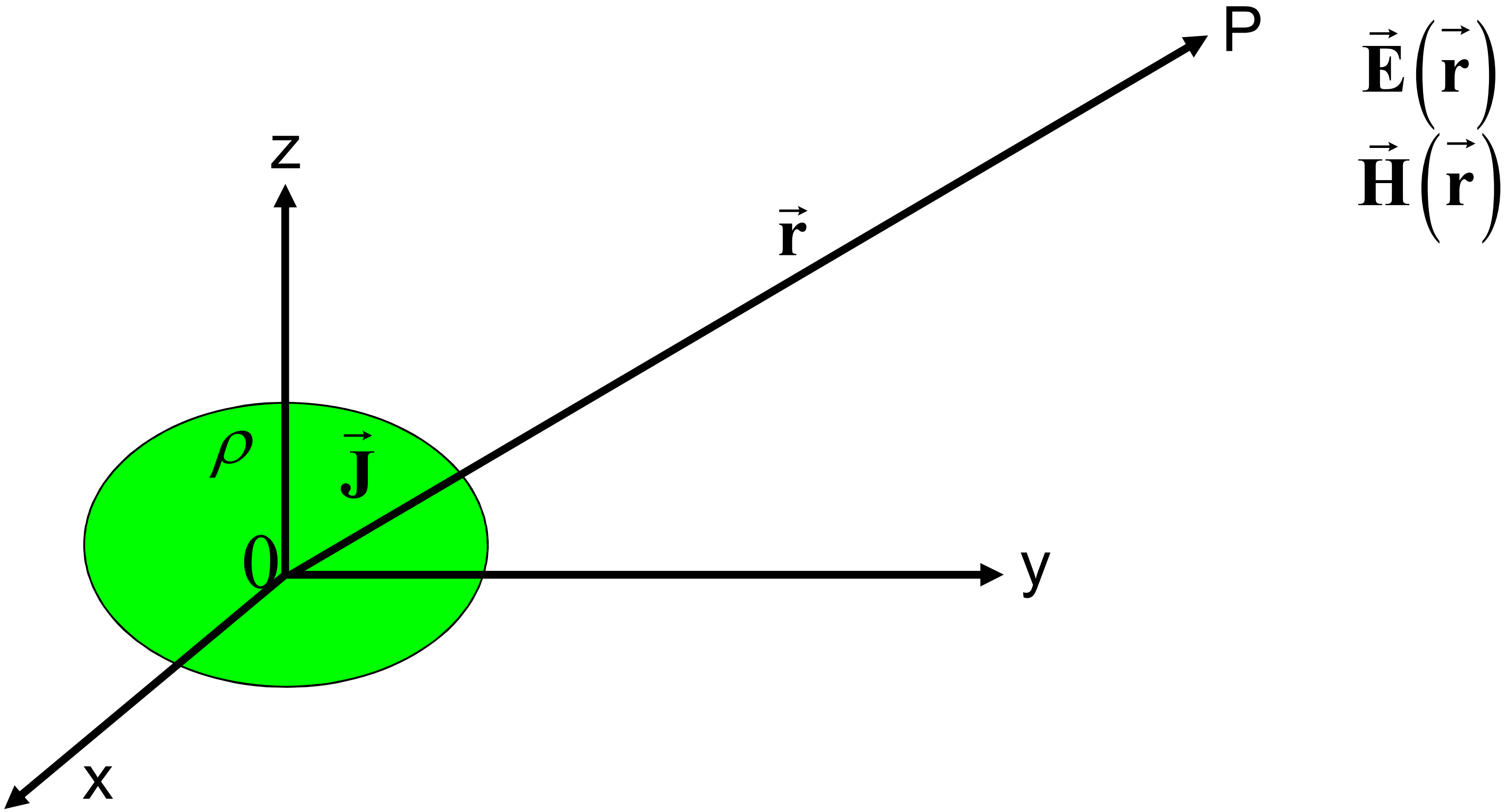
Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

Corso di Campi Elettromagnetici
a.a. 2018-2019

17 Maggio 2019

Summary of the past lecture

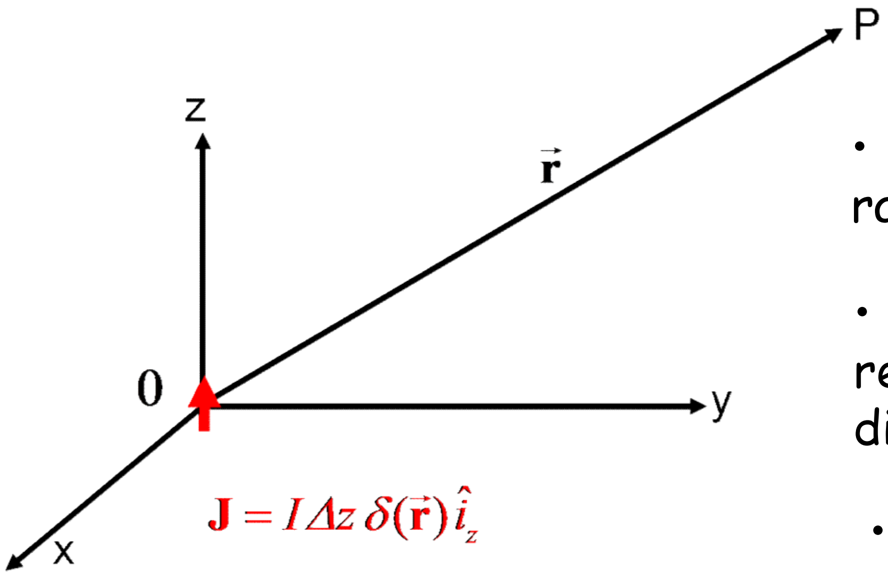
Radiation problem



Summary of the past lecture

Elementary electrical dipole

$$\mathbf{J} = I\Delta z\,\delta(\vec{\mathbf{r}})\,\hat{i}_z = I\Delta z\,\delta(x)\delta(y)\delta(z)\,\hat{i}_z$$



- Why are we interested in such a radiating element?
- Why is such a radiating element referred to as elementary electrical dipole?
- How can we physically approximate an elementary electrical dipole?

Summary of the past lecture

Elementary electrical dipole

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathrm{E}_r(r,\vartheta)\hat{i}_r + \mathrm{E}_\vartheta(r,\vartheta)\hat{i}_\vartheta$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \mathrm{H}_\varphi(r,\vartheta)\hat{i}_\varphi$$

$$\begin{cases} E_r = \zeta \dfrac{I\Delta z}{2\pi}\left(\dfrac{1}{r^2} + \dfrac{1}{j\beta r^3}\right)\cos\vartheta \exp(-j\beta r) \\ E_\vartheta = \zeta \dfrac{I\Delta z}{4\pi}\left(\dfrac{j\beta}{r} + \dfrac{1}{r^2} + \dfrac{1}{j\beta r^3}\right)\sin\vartheta \exp(-j\beta r) \\ H_\varphi = \dfrac{I\Delta z}{4\pi}\left(\dfrac{j\beta}{r} + \dfrac{1}{r^2}\right)\sin\vartheta \exp(-j\beta r) \end{cases}$$

Summary of the past lecture

Elementary electrical dipole

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathrm{E}_r(r,\vartheta)\hat{i}_r + \mathrm{E}_\vartheta(r,\vartheta)\hat{i}_\vartheta$$

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… for $\omega = 0$ simplifies as

$$\begin{cases} E_r = \dfrac{\mathrm{Q}\Delta z}{2\pi}\dfrac{1}{\varepsilon r^3}\cos\vartheta \\ E_\vartheta = \dfrac{\mathrm{Q}\Delta z}{4\pi}\dfrac{1}{\varepsilon r^3}\sin\vartheta \\ H_\varphi = 0 \end{cases}$$

Summary of the past lecture

Elementary electrical dipole

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathrm{E}_r(r,\vartheta)\hat{i}_r + \mathrm{E}_\vartheta(r,\vartheta)\hat{i}_\vartheta$$

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… for r>>λ simplifies as

$$\begin{cases} E_r = 0 \\ E_\vartheta = j\zeta \dfrac{I\Delta z}{2\lambda r}\sin\vartheta \exp(-j\beta r) \\ H_\varphi = j\dfrac{I\Delta z}{2\lambda r}\sin\vartheta \exp(-j\beta r) = \dfrac{E_\vartheta}{\zeta} \end{cases}$$

Summary of the past lecture

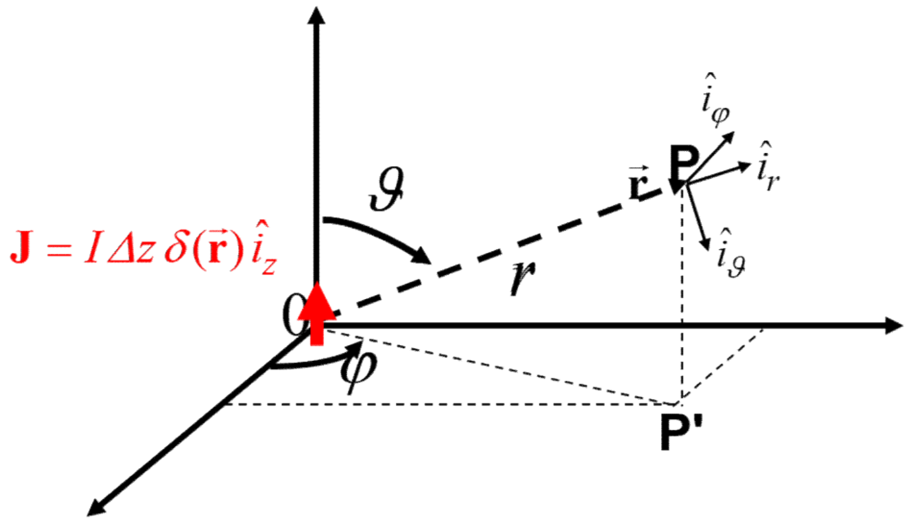
Elementary electrical dipole

In the far-field case (r>>λ)

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathrm{E}_\vartheta(r,\vartheta)\hat{i}_\vartheta$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \mathrm{H}_\varphi(r,\vartheta)\hat{i}_\varphi$$

$$\begin{cases} E_\vartheta = j\zeta \dfrac{I\Delta z}{2\lambda r}\sin\vartheta \exp(-j\beta r) \\ H_\varphi = j\dfrac{I\Delta z}{2\lambda r}\sin\vartheta \exp(-j\beta r) = \dfrac{E_\vartheta}{\zeta} \end{cases}$$



- the e.m. field propagates along $\hat{i}_r$
- the e.m. field lies on the plane orthogonal to the propagation direction
- |E| and |H| exhibit the decaying factor 1/r
- |E| and |H| are proportional through ζ

Summary of the past lecture

Elementary electrical dipole

In the far-field case (r>>λ)

$$\vec{\mathbf{E}} = \mathrm{E}_{\vartheta}\hat{i}_{\vartheta}$$

$$\vec{\mathbf{H}} = \mathrm{H}_{\varphi}\hat{i}_{\varphi} = \frac{E_{\vartheta}}{\zeta}\hat{i}_{\varphi}$$

$$\Rightarrow \quad \zeta\mathbf{H} = \hat{i}_r \times \mathbf{E}$$

and the Poynting vector:

$$\mathbf{S} = \frac{1}{2}\mathbf{E}\times\mathbf{H}^* = \frac{1}{2}\frac{\left|E_{\vartheta}\right|^2}{\zeta}\hat{i}_r = \frac{1}{2}\frac{\left|\mathbf{E}\right|^2}{\zeta}\hat{i}_r$$

Summary of the past lecture

Elementary electrical dipole

$$P = \frac{1}{2} \oiint_S \left[\mathbf{E} \times \mathbf{H}^* \right] \cdot \hat{i}_r dS$$

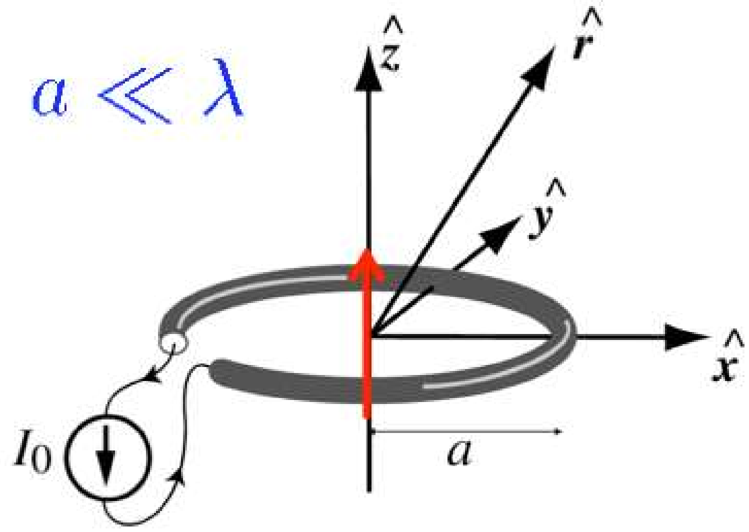
$$P = P_1 + jP_2$$

$$P_1 = \frac{1}{2} \frac{2\pi}{3} \zeta \left(\frac{\Delta z}{\lambda} \right)^2 |I|^2$$

$$P_2 = -\frac{1}{2} \frac{2\pi}{3} \zeta \left(\frac{\Delta z}{\lambda} \right)^2 \frac{1}{(\beta r)^3} |I|^2$$

Small loop antenna

- A simple and inexpensive antenna type is the loop antenna.



$$\Delta S = \pi a^2$$

Small loop antenna

Electrically small antennas are those whose overall length (circumference) is usually less than about one-tenth of a wavelength ($C < \lambda/10$).

Small loop antenna

$$\mathbf{J} = I\delta(z)\,\delta(r-a)\hat{i}'_{\varphi}$$

$\downarrow$ J

$$\nabla^2 \boldsymbol{A} + k^2 \boldsymbol{A} = -\mu \boldsymbol{J}$$ Vector Helmholtz equation

$\downarrow$

$$\mathbf{A}(\mathbf{r}) = \frac{\mu}{4\pi}\int \mathbf{J}(\mathbf{r}')\frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}d\mathbf{r}'$$

$$\boldsymbol{H} = \frac{1}{\mu}\nabla\times\boldsymbol{A} \quad \text{and} \quad \boldsymbol{E} = -j\omega\boldsymbol{A} + \frac{\nabla\nabla\cdot\boldsymbol{A}}{j\omega\varepsilon\mu}$$

$\downarrow$

$\boldsymbol{E}(\boldsymbol{r})$

$\boldsymbol{H}(\boldsymbol{r})$

Small loop antenna

$$\mathbf{J} = I\delta(z)\,\delta(r-a)\hat{i}'_\varphi$$

$$\mathbf{A}(\vec{\mathbf{r}}) = \frac{\mu}{4\pi}\int \mathbf{J}(\vec{\mathbf{r}}')\frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}d\,\vec{\mathbf{r}}' \quad = \frac{\mu}{4\pi}\int_0^{2\pi} d\varphi'\ I\ \frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}a\ \hat{i}'_\varphi$$

$$\approx \frac{j\beta I\mu\,\Delta S}{4\pi}\frac{e^{-j\beta r}}{r}\left[1+\frac{1}{j\beta r}\right]\sin\vartheta\ \hat{i}_\varphi$$

.. by assuming that the current I in the small loop is constant and that the radius of the loop a<<λ

Small loop antenna

$$\mathbf{J} = I\delta(z)\,\delta(r-a)\hat{i}'_\varphi$$

J ↓

$\nabla^2 \mathbf{A} + k^2 \mathbf{A} = -\mu \mathbf{J}$ Vector Helmholtz equation

↓ $\mathbf{A} \approx \dfrac{j\beta I \mu \Delta S}{4\pi} \dfrac{e^{-j\beta r}}{r} \left[1 + \dfrac{1}{j\beta r}\right] \sin\vartheta \, \hat{i}_\varphi$

$$\mathbf{H} = \frac{1}{\mu} \nabla \times \mathbf{A} \quad \text{and} \quad \mathbf{E} = -j\omega \mathbf{A} + \frac{\nabla\nabla \cdot \mathbf{A}}{j\omega\varepsilon\mu}$$

↓

E H

Small loop antenna

The E.M. field radiated by the small loop antenna

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \mathrm{H}_r(r,\vartheta)\hat{i}_r + \mathrm{H}_\vartheta(r,\vartheta)\hat{i}_\vartheta$$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathrm{E}_\varphi(r,\vartheta)\hat{i}_\varphi$$

$$\begin{cases} H_r = \dfrac{I\Delta S}{2\pi}\left(\dfrac{j\beta}{r^2} + \dfrac{1}{r^3}\right)\cos\vartheta\exp(-j\beta r) \\ H_\vartheta = \dfrac{I\Delta S}{4\pi}\left(\dfrac{(j\beta)^2}{r} + \dfrac{j\beta}{r^2} + \dfrac{1}{r^3}\right)\sin\vartheta\exp(-j\beta r) \\ E_\varphi = -\dfrac{\zeta I\Delta S}{4\pi}\left(\dfrac{(j\beta)^2}{r} + \dfrac{j\beta}{r^2}\right)\sin\vartheta\exp(-j\beta r) \end{cases}$$

Because of the problem symmetry there is no dependence on the azimuth angle φ.

Small loop antenna

The E.M. field radiated by the small loop antenna

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \mathrm{H}_r(r,\vartheta)\hat{i}_r + \mathrm{H}_\vartheta(r,\vartheta)\hat{i}_\vartheta$$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathrm{E}_\varphi(r,\vartheta)\hat{i}_\varphi$$

$$\begin{cases} H_r = \dfrac{I\Delta S}{2\pi}\left(\dfrac{j\beta}{r^2} + \dfrac{1}{r^3}\right)\cos\vartheta\exp(-j\beta r) \\ H_\vartheta = \dfrac{I\Delta S}{4\pi}\left(\dfrac{(j\beta)^2}{r} + \dfrac{j\beta}{r^2} + \dfrac{1}{r^3}\right)\sin\vartheta\exp(-j\beta r) \\ E_\varphi = -\dfrac{\zeta I\Delta S}{4\pi}\left(\dfrac{(j\beta)^2}{r} + \dfrac{j\beta}{r^2}\right)\sin\vartheta\exp(-j\beta r) \end{cases}$$

… for r>>λ simplifies as

$$\begin{cases} H_r = 0 \\ H_\vartheta = -\dfrac{\beta\Delta sI}{2\lambda r}\sin\vartheta\exp(-j\beta r) = \dfrac{-E_\varphi}{\zeta} \\ E_\varphi = \dfrac{\zeta\beta\Delta sI}{2\lambda r}\sin\vartheta\exp(-j\beta r) \end{cases}$$

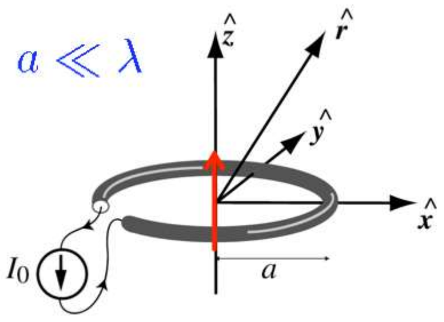
Small loop antenna

In the far-field case (r>>λ)

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathrm{E}_{\varphi}(r,\vartheta)\hat{i}_{\varphi}$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \mathrm{H}_{\vartheta}(r,\vartheta)\hat{i}_{\vartheta}$$

$$\begin{cases} E_{\varphi} = \dfrac{\zeta\beta\Delta sI}{2\lambda r}\sin\vartheta\exp(-j\beta r) \\ H_{\vartheta} = -\dfrac{\beta\Delta sI}{2\lambda r}\sin\vartheta\exp(-j\beta r) = \dfrac{-E_{\varphi}}{\zeta} \end{cases}$$



$$\Delta S = \pi a^2$$

Small loop antenna

In the far-field case (r>>λ)

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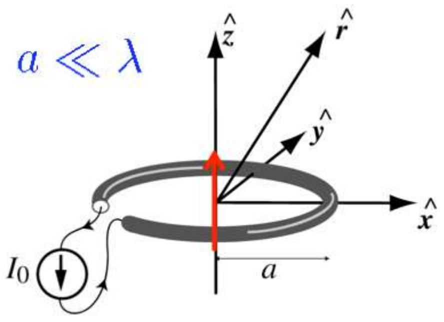

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- the e.m. field propagates along $\hat{i}_r$
- the e.m. field lies on the plane orthogonal to the propagation direction
- |E| and |H| exhibit the decaying factor 1/r
- |E| and |H| are proportional through ζ

Small loop antenna

In the far-field case (r>>λ), we have

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathrm{E}_\varphi \hat{i}_\varphi$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \mathrm{H}_\vartheta \hat{i}_\vartheta = \frac{-E_\varphi}{\zeta}\hat{i}_\vartheta$$

$\Rightarrow$

$$\zeta \mathbf{H} = \hat{i}_r \times \mathbf{E}$$

and the Poynting vector is:

$$\mathbf{S} = \frac{1}{2}\mathbf{E}\times\mathbf{H}^* = \frac{1}{2}\frac{\left|E_\varphi\right|^2}{\zeta}\hat{i}_r = \frac{1}{2}\frac{\left|\mathbf{E}\right|^2}{\zeta}\hat{i}_r$$

$$\hat{i}_\varphi = \hat{i}_r \times \hat{i}_\vartheta$$

$$\hat{i}_\vartheta = \hat{i}_\varphi \times \hat{i}_r$$

$$\hat{i}_r = \hat{i}_\vartheta \times \hat{i}_\varphi$$

Small loop antenna

- Similarly to the elementary electrical dipole, to further characterize the small loop anntenna behavior one can evaluate the Poynting vector and the associated power over a sphere centered in the origin:

$$P = \frac{1}{2}\oiint_S \left[\mathbf{E}\times\mathbf{H}^*\right]\cdot\hat{i}_r dS = -\frac{1}{2}\int_0^{2\pi} d\varphi \int_0^{\pi} d\vartheta r^2 \sin\vartheta E_\varphi H_\vartheta^*$$

$$\hat{i}_\varphi = \hat{i}_r \times \hat{i}_\vartheta$$
$$\hat{i}_\vartheta = \hat{i}_\varphi \times \hat{i}_r$$
$$\hat{i}_r = \hat{i}_\vartheta \times \hat{i}_\varphi$$

I) $dS = r^2 \sin\vartheta d\vartheta d\varphi$

II) $\left[\mathbf{E}\times\mathbf{H}^*\right]\cdot\hat{i}_r = \left[\left(E_\varphi\hat{i}_\varphi\right)\times\left(H_\vartheta^*\hat{i}_\vartheta + H_r^*\hat{i}_r\right)\right]\cdot\hat{i}_r = -E_\varphi H_\vartheta^*$

Small loop antenna

$$P=\frac{1}{2}\oiint_S\left[\mathbf{E}\times\mathbf{H}^*\right]\cdot\hat{i}_r dS=-\frac{1}{2}\int_0^{2\pi}d\varphi\int_0^{\pi}d\vartheta r^2\sin\vartheta E_\varphi H_\vartheta^*=\frac{1}{2}\frac{2\pi}{3}\zeta\left(\frac{\beta\Delta S}{\lambda}\right)^2\left[1+j\frac{1}{(\beta r)^3}\right]|I|^2$$

$$\begin{cases} H_r=\dfrac{I\Delta S}{2\pi}\left(\dfrac{j\beta}{r^2}+\dfrac{1}{r^3}\right)\cos\vartheta\exp(-j\beta r) \\ H_\vartheta=\dfrac{I\Delta S}{4\pi}\left(\dfrac{(j\beta)^2}{r}+\dfrac{j\beta}{r^2}+\dfrac{1}{r^3}\right)\sin\vartheta\exp(-j\beta r) \\ E_\varphi=-\dfrac{\zeta I\Delta S}{4\pi}\left(\dfrac{(j\beta)^2}{r}+\dfrac{j\beta}{r^2}\right)\sin\vartheta\exp(-j\beta r) \end{cases}$$

$$P=P_1+jP_2$$

$$P_1=\frac{1}{2}\frac{2\pi}{3}\zeta\left(\frac{\beta\Delta S}{\lambda}\right)^2|I|^2$$

$$P_2=\frac{1}{2}\frac{2\pi}{3}\zeta\left(\frac{\beta\Delta S}{\lambda}\right)^2\frac{1}{(\beta r)^3}|I|^2$$

Elementary electrical dipole vs. Small loop antenna

Elementary electrical dipole

$$P = P_1 + jP_2$$

$$P_1 = \frac{1}{2}\frac{2\pi}{3}\zeta\left(\frac{\Delta z}{\lambda}\right)^2 |I|^2$$

$$P_2 = -\frac{1}{2}\frac{2\pi}{3}\zeta\left(\frac{\Delta z}{\lambda}\right)^2 \frac{1}{(\beta r)^3}|I|^2$$

Small loop antenna

$$P = P_1 + jP_2$$

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$$P_2 = \frac{1}{2}\frac{2\pi}{3}\zeta\left(\frac{\beta \Delta S}{\lambda}\right)^2 \frac{1}{(\beta r)^3}|I|^2$$

Small loop antenna

The reactive part depends on r. Its sign is positive showing that there is an excess of stored **magnetic** energy in the neighbor of the magnetic dipole (see Poynting's theorem)

$$P_r = P_1 + jP_2$$

$$P_1 = \frac{1}{2}\frac{2\pi}{3}\zeta\left(\frac{\beta \Delta S}{\lambda}\right)^2 |I|^2$$

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Small loop antenna

WHY?

Elementary electrical dipole vs. Small loop antenna

Elementary electrical dipole

$$\mathbf{J} = I \Delta z\, \delta(\vec{\mathbf{r}})\, \hat{i}_z$$

- Why are we interested in such a radiating element?
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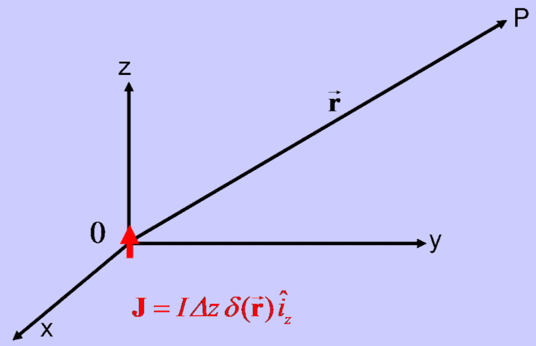
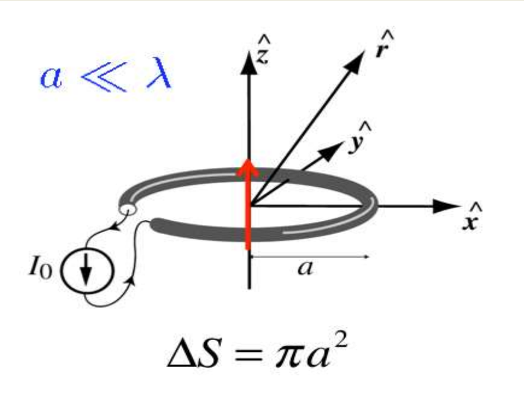


Small loop antenna

$$\mathbf{J} = I \delta(z)\, \delta(r - a) \hat{i}'_\varphi$$



Elementary electrical dipole vs. Small loop antenna

Elementary electrical dipole

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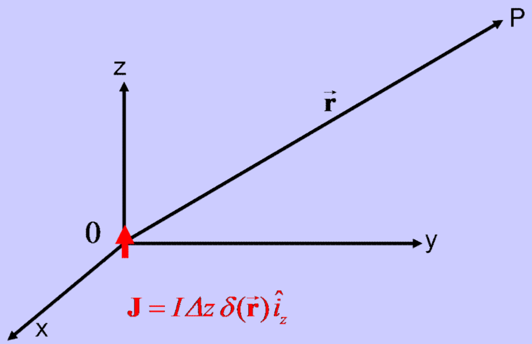

Small loop antenna

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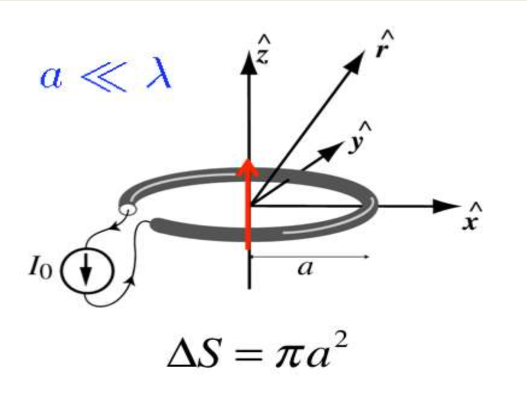

Elementary electrical dipole vs. Small loop antenna

Elementary electrical dipole

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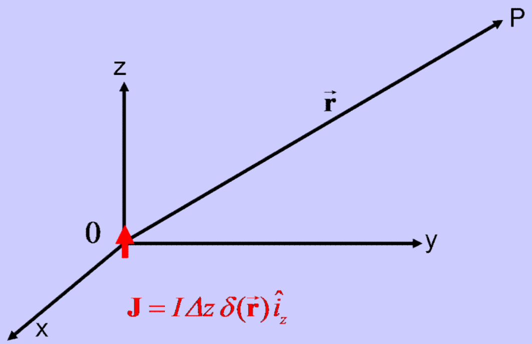
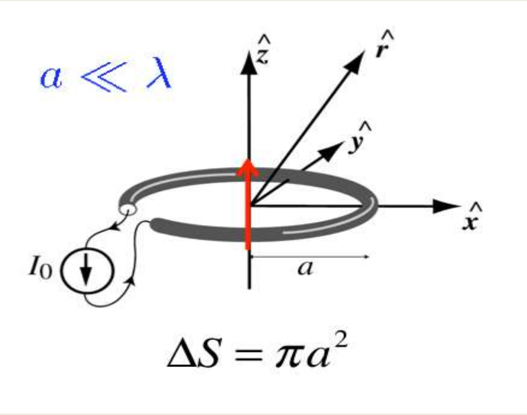


Small loop antenna

$$\mathbf{J} = I\,\delta(z)\,\delta(r-a)\,\hat{i}'_\varphi$$

$$\begin{cases} H_r = \dfrac{I\Delta S}{2\pi}\left(\dfrac{j\beta}{r^2} + \dfrac{1}{r^3}\right)\cos\vartheta \exp(-j\beta r) \\ H_\vartheta = \dfrac{I\Delta S}{4\pi}\left(\dfrac{(j\beta)^2}{r} + \dfrac{j\beta}{r^2} + \dfrac{1}{r^3}\right)\sin\vartheta \exp(-j\beta r) \\ E_\varphi = -\dfrac{\zeta I\Delta S}{4\pi}\left(\dfrac{(j\beta)^2}{r} + \dfrac{j\beta}{r^2}\right)\sin\vartheta \exp(-j\beta r) \end{cases}$$



Elementary electrical dipole vs. Small loop antenna

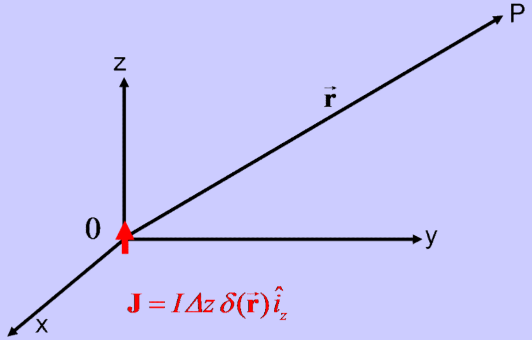
Elementary electrical dipole

$$\mathbf{J} = I\Delta z\,\delta(\vec{\mathbf{r}})\,\hat{i}_z$$

for $r >> \lambda$

$$\mathbf{E} = \frac{j\zeta I}{2\lambda}\frac{\exp(-j\beta r)}{r}\Delta z \sin\vartheta\ \hat{i}_\vartheta$$

$$\zeta\mathbf{H} = \hat{i}_r \times \mathbf{E}$$



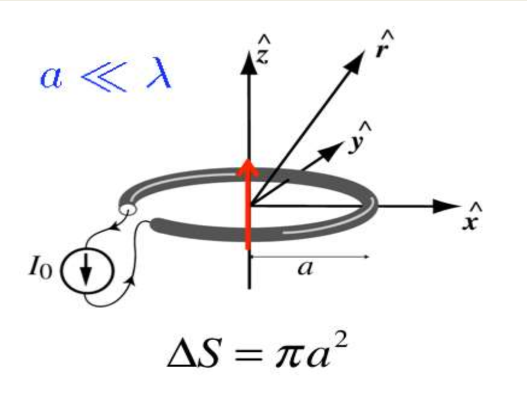
Small loop antenna

$$\mathbf{J} = I\delta(z)\,\delta(r-a)\hat{i}'_\varphi$$

for $r >> \lambda$

$$\mathbf{E} = \frac{\zeta\beta\Delta S I}{2\lambda}\frac{\exp(-j\beta r)}{r}\sin\vartheta\ \hat{i}_\varphi$$

$$\zeta\mathbf{H} = \hat{i}_r \times \mathbf{E}$$



Elementary electrical dipole vs. Small loop antenna

$$P = \frac{1}{2} \oiint_S \left[\mathbf{E} \times \mathbf{H}^* \right] \cdot \hat{i}_r dS$$

Elementary electrical dipole

$$P = P_1 + jP_2$$

$$P_1 = \frac{1}{2} \frac{2\pi}{3} \zeta \left(\frac{\Delta z}{\lambda} \right)^2 |I|^2$$

$$P_2 = -\frac{1}{2} \frac{2\pi}{3} \zeta \left(\frac{\Delta z}{\lambda} \right)^2 \frac{1}{(\beta r)^3} |I|^2$$

Small loop antenna

$$P = P_1 + jP_2$$

$$P_1 = \frac{1}{2} \frac{2\pi}{3} \zeta \left(\frac{\beta \Delta S}{\lambda} \right)^2 |I|^2$$

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Small loop antenna

WHY?

Magnetic Sources

James Clerk Maxwell 1831-1879

$$\begin{cases} \nabla \times \mathbf{E} = -j\omega\mu\mathbf{H} \\ \nabla \times \mathbf{H} = j\omega\varepsilon\mathbf{E} + \mathbf{J} \\ \nabla \cdot \varepsilon\mathbf{E} = \rho \\ \nabla \cdot \mu\mathbf{H} = 0 \end{cases}$$

Magnetic Sources

James Clerk Maxwell 1831-1879

$$\begin{cases} \nabla \times \mathbf{E} = -j\omega\mu\mathbf{H} - \mathbf{J}_{\mathrm{m}} \\ \nabla \times \mathbf{H} = j\omega\varepsilon\mathbf{E} + \mathbf{J} \\ \nabla \cdot \varepsilon\mathbf{E} = \rho \\ \nabla \cdot \mu\mathbf{H} = \rho_{\mathrm{m}} \end{cases}$$

Magnetic Sources

What is the relation between sources and fields in this case?

$$
\begin{cases}
\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H} - \mathbf{J}_m \\
\nabla \times \mathbf{H} = j\omega\varepsilon\mathbf{E} + \mathbf{J} \\
\nabla \cdot \varepsilon\mathbf{E} = \rho \\
\nabla \cdot \mu\mathbf{H} = \rho_m
\end{cases}
$$

Magnetic Sources

What is the relation between sources and fields in this case?

Let's simplify the question. What is the relation between sources and fields in this case?

$$\begin{cases} \nabla \times \mathbf{E} = -j\omega\mu\mathbf{H} - \mathbf{J}_m \\ \nabla \times \mathbf{H} = j\omega\varepsilon\mathbf{E} \\ \nabla \cdot \varepsilon\mathbf{E} = 0 \\ \nabla \cdot \mu\mathbf{H} = \rho_m \end{cases}$$

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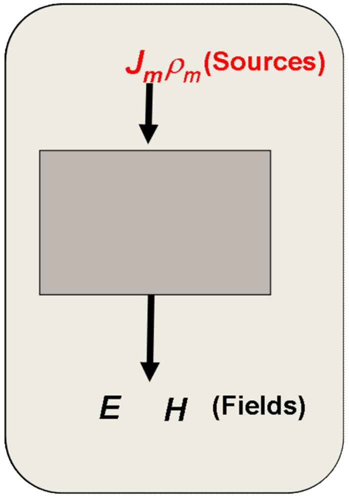

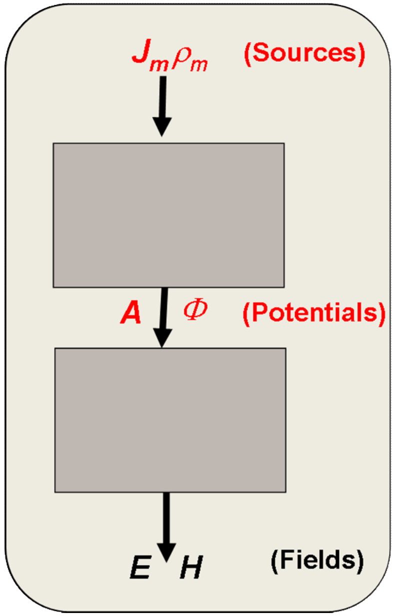

In principle, we could replace the same approach as that exploited for the electric sources

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In practice, we follow an easier way, provided by the **duality theorem**

$$\mathbf{J} \;\; \rho \;\longrightarrow\; \mathbf{E}, \mathbf{H}$$

$$\Longrightarrow$$

$$\mathbf{J}_{\mathrm{m}} = -\zeta\mathbf{J} \quad \rho_{\mathrm{m}} = -\zeta\rho \;\longrightarrow\; \mathbf{E}' = \zeta\mathbf{H} \;, \mathbf{H}' = -\frac{1}{\zeta}\mathbf{E}$$

Duality Theorem

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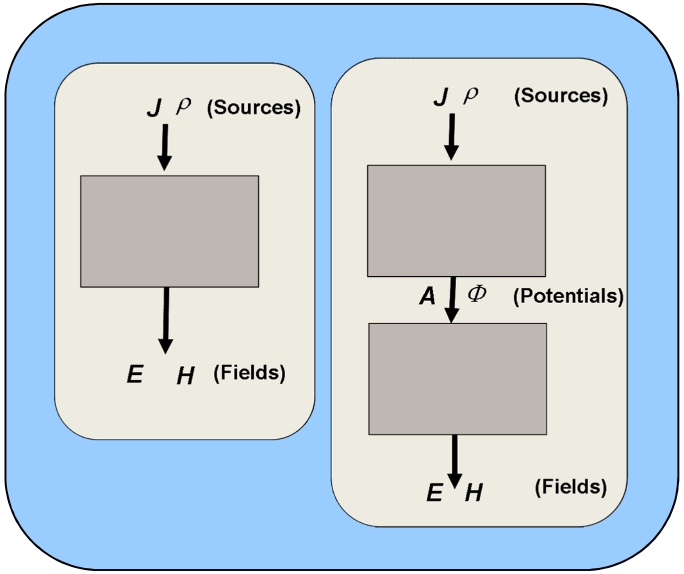
$$\mathbf{J}_m \quad \rho_m$$

$$\mathbf{J} = -\frac{1}{\zeta}\mathbf{J}_m$$

$$\rho = -\frac{1}{\zeta}\rho_m$$

$$\rightarrow$$



$$\rightarrow$$

$$\mathbf{E}' = \zeta\mathbf{H}$$

$$\mathbf{H}' = -\frac{1}{\zeta}\mathbf{E}$$

Elementary electrical and magnetic dipoles

Elementary electrical dipole

$$\mathbf{J} = I\Delta z\,\delta(\vec{\mathbf{r}})\,\hat{i}_z = I\Delta z\,\delta(x)\delta(y)\delta(z)\,\hat{i}_z$$

$$I\Delta z = j\omega Q\Delta z = j\omega U$$

Elementary magnetic dipole

$$\mathbf{J}_m = I_m\Delta z\,\delta(\vec{\mathbf{r}})\,\hat{i}_z = I_m\Delta z\,\delta(x)\delta(y)\delta(z)\,\hat{i}_z$$

$$I_m\Delta z = j\omega U_m$$

Elementary electrical and magnetic dipoles

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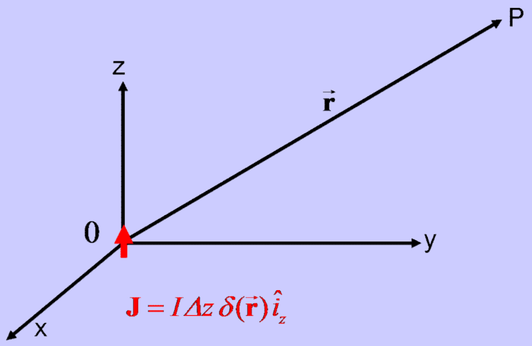

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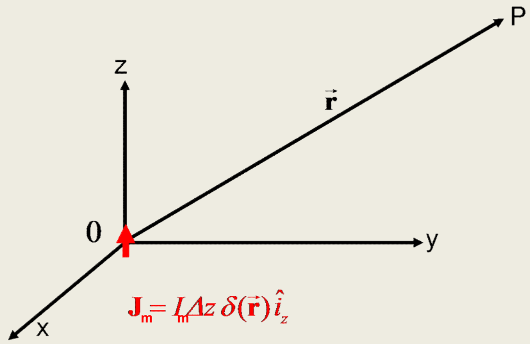

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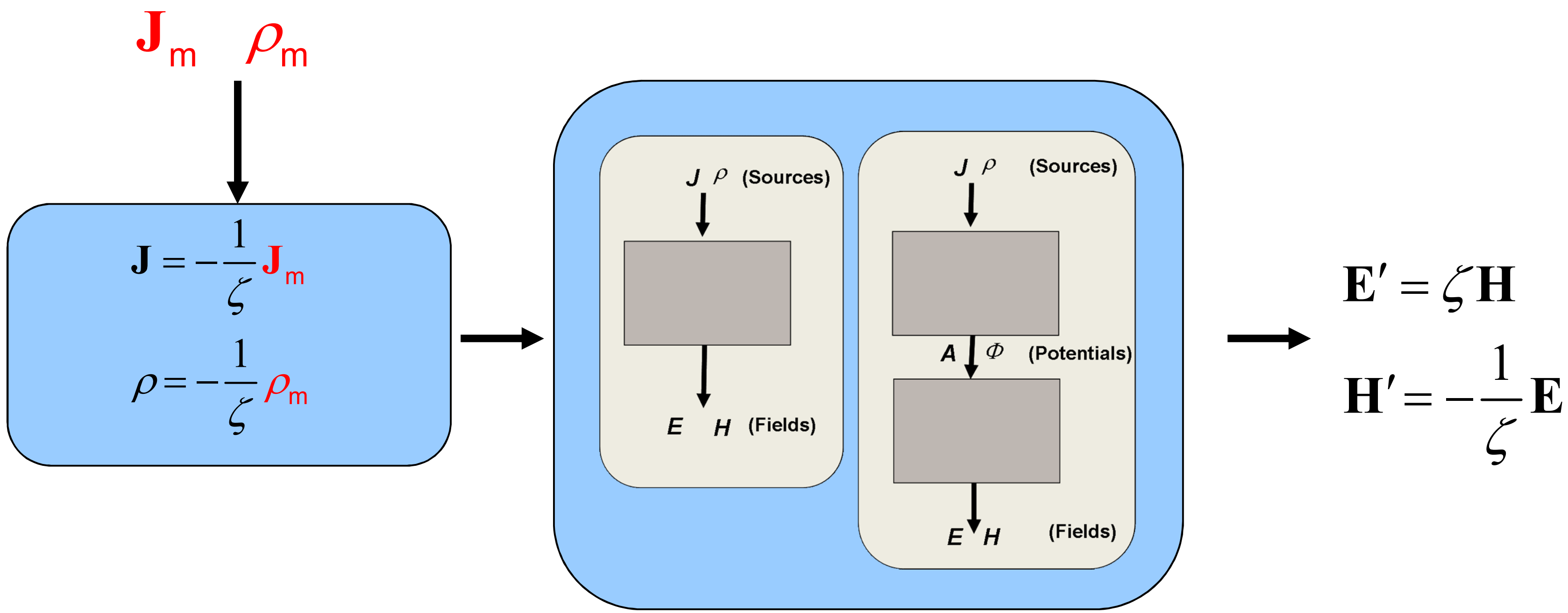
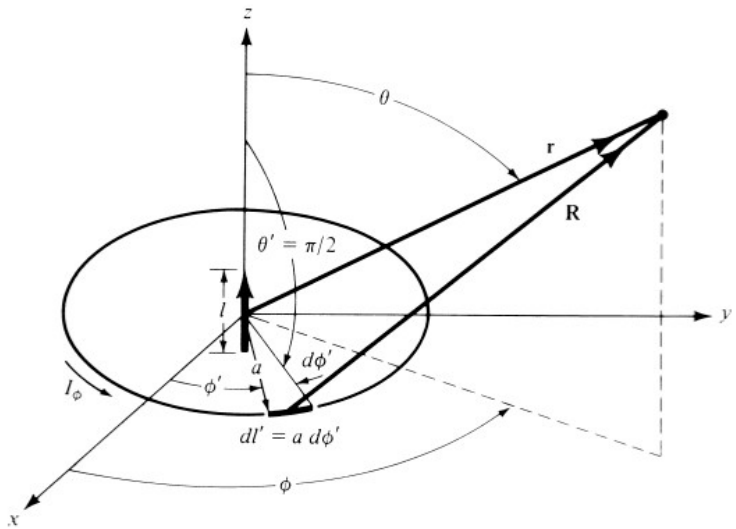


Elementary electrical and magnetic dipoles

Ampere equivalence theorem

By invoking the Duality theorem it is possible to demonstrate that the small loop antenna is equivalent to an elementary magnetic dipole, provided that:



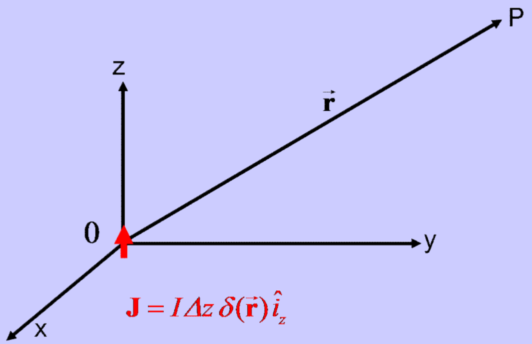
Geometry for circular loop

$$U_m = \mu I \Delta S$$

Elementary electrical and magnetic dipoles

Elementary electrical dipole

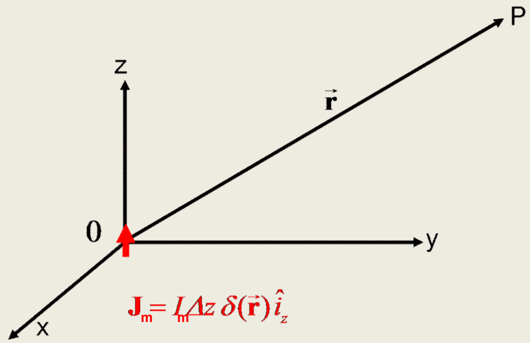
$$\mathbf{J} = I\,\Delta z\,\delta(\vec{\mathbf{r}})\,\hat{i}_z$$



- Why are we interested in such a radiating element?
- Why is such a radiating element referred to as elementary electrical dipole?
- How can we physically approximate an elementary electrical dipole?

Elementary magnetic dipole

$$\mathbf{J}_m = I_m\,\Delta z\,\delta(\vec{\mathbf{r}})\,\hat{i}_z$$



- Why are we interested in such a radiating element?
- Why is such a radiating element referred to as **elementary magnetic** dipole?
- How can we physically approximate an **elementary magnetic** dipole?

References

- C.H.Papas, *Theory of Electromagnetic Wave Propagation*, McGraw-Hill, 1965.
- C.A.Balanis, *Antenna Theory*, John Wiley, 1982.
- G.Franceschetti, *Campi Elettromagnetici*, Boringhieri, 1983.
- G.Conciauro, *Introduzione alle Onde Elettromagnetiche*, McGraw-Hill, 1993.
- C.G.Someda, *Electromagnetic Waves*, Chapman & Hall, 1998.