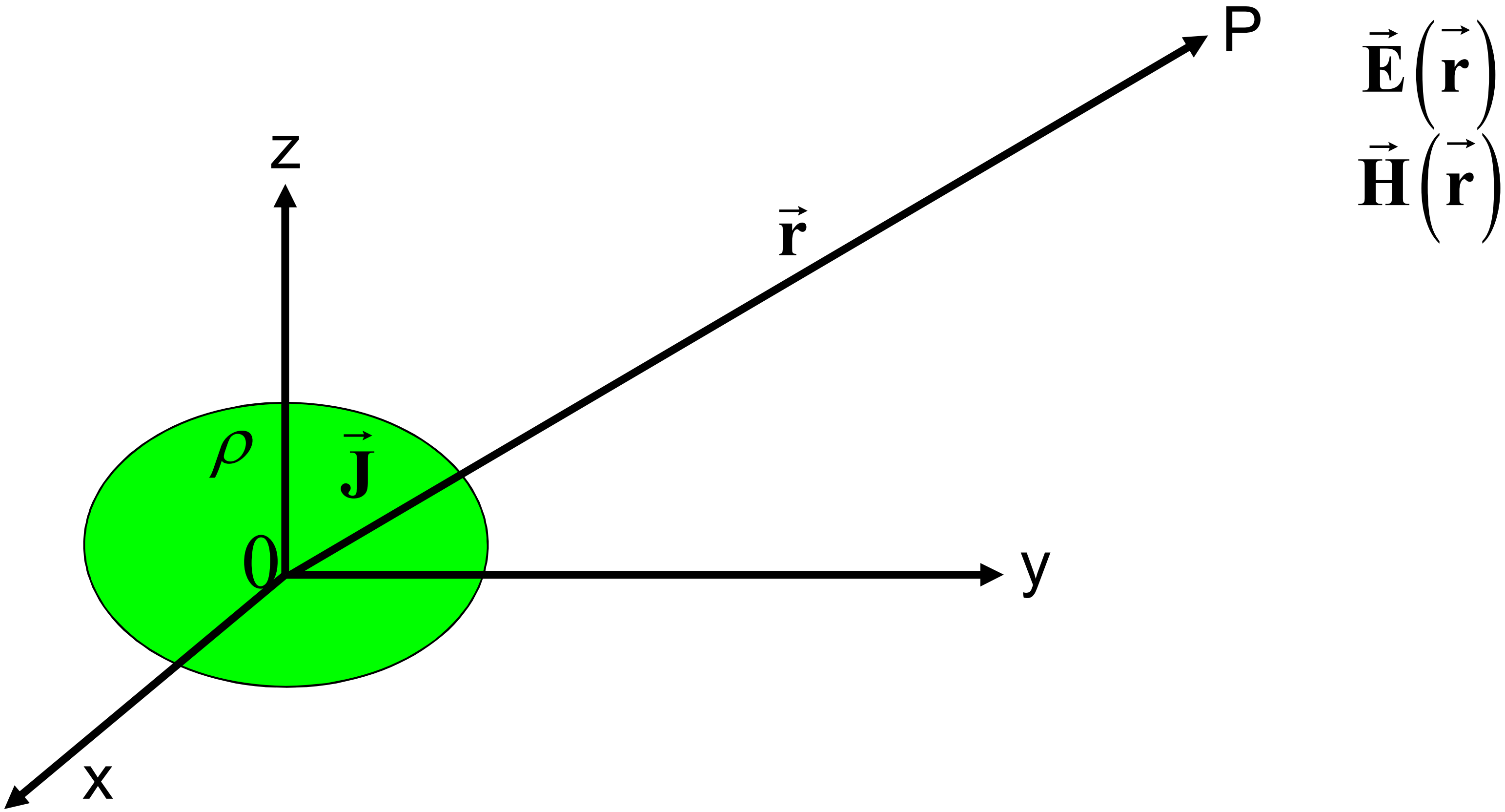
Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

Corso di Campi Elettromagnetici
a.a. 2018-2019

16 Maggio 2019

Summary of the past lecture

Radiation problem



Summary of the past lecture

Potentials

$\boldsymbol{J}(\boldsymbol{r})$

$$\nabla^2 \boldsymbol{A} + k^2 \boldsymbol{A} = -\mu \boldsymbol{J}$$ **Vector Helmholtz equation**

$$\mathbf{A}(\mathbf{r}) = \frac{\mu}{4\pi} \int \mathbf{J}(\mathbf{r}') \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r}'$$

$$\boldsymbol{H} = \frac{1}{\mu} \nabla \times \boldsymbol{A} \quad \text{and} \quad \boldsymbol{E} = -j\omega \boldsymbol{A} + \frac{\nabla\nabla \cdot \boldsymbol{A}}{j\omega\varepsilon\mu}$$

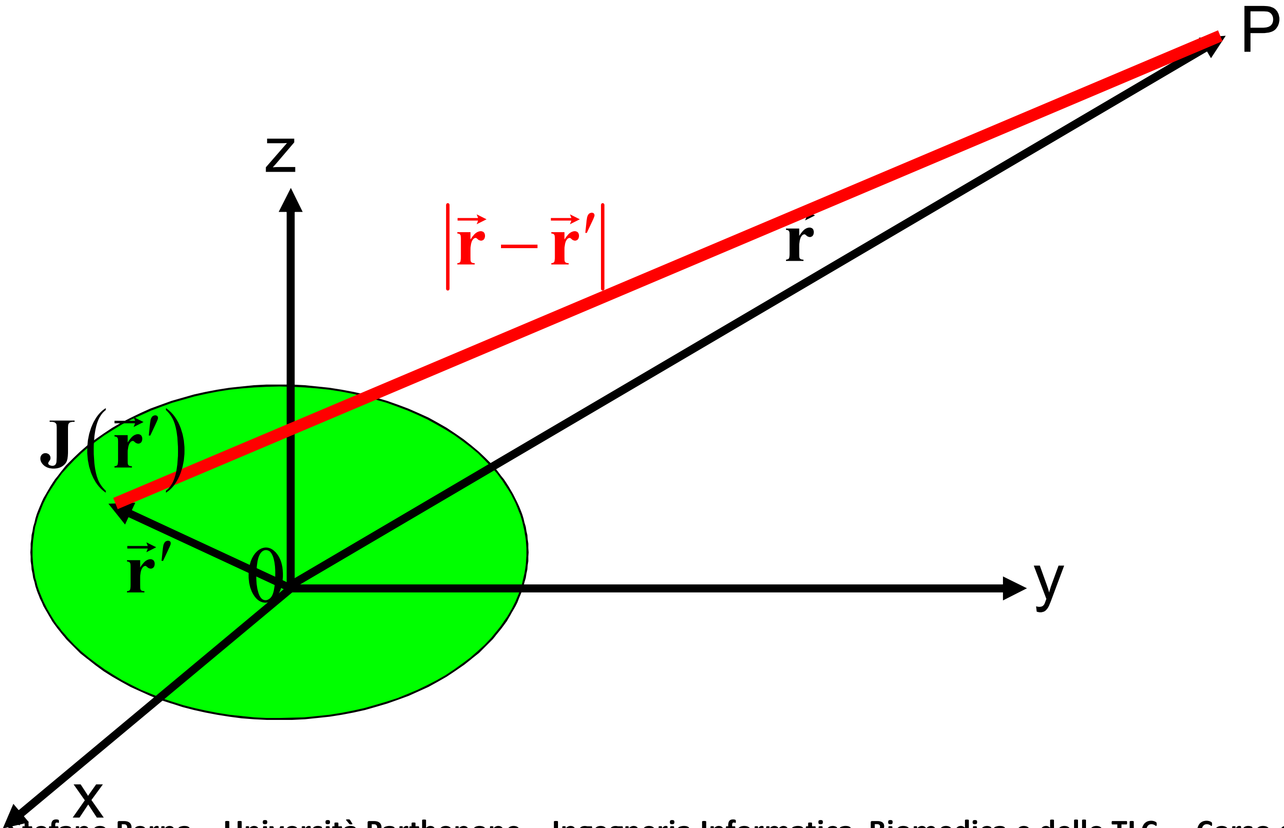
$\boldsymbol{E}(\boldsymbol{r})$

$\boldsymbol{H}(\boldsymbol{r})$

Summary of the past lecture

Potentials

$$\mathbf{A}(\vec{\mathbf{r}}) = \frac{\mu}{4\pi}\int \mathbf{J}(\vec{\mathbf{r}}')\frac{e^{-jk|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}d\,\vec{\mathbf{r}}'$$

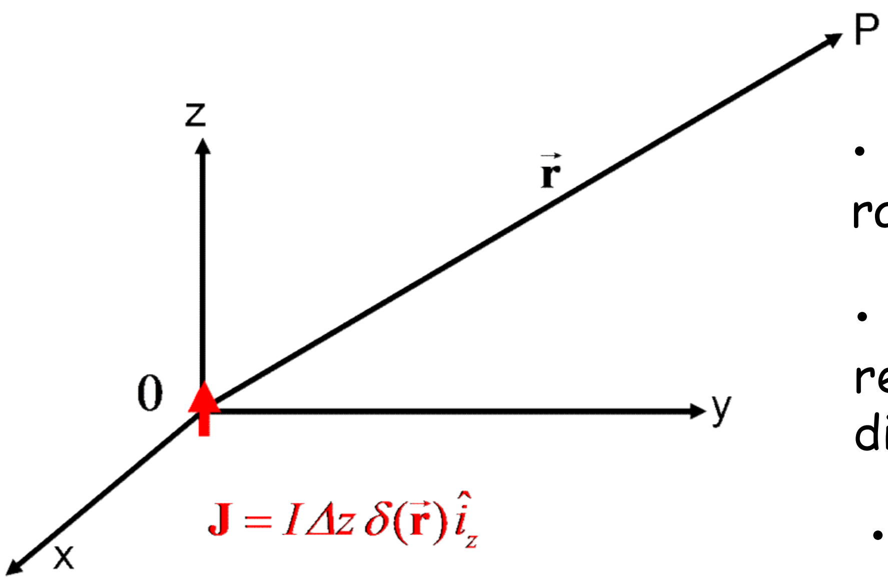


Summary of the past lecture

Elementary electrical dipole

- A δ-source radiating element is also known as elementary electrical dipole.

$$\mathbf{J} = I \Delta z\, \delta(\vec{\mathbf{r}})\, \hat{i}_z = I \Delta z\, \delta(x)\delta(y)\delta(z)\, \hat{i}_z$$



- Why are we interested in such a radiating element?
- Why is such a radiating element referred to as elementary electrical dipole?
- How can we physically approximate an elementary electrical dipole?

Summary of the past lecture
Elementary electrical dipole

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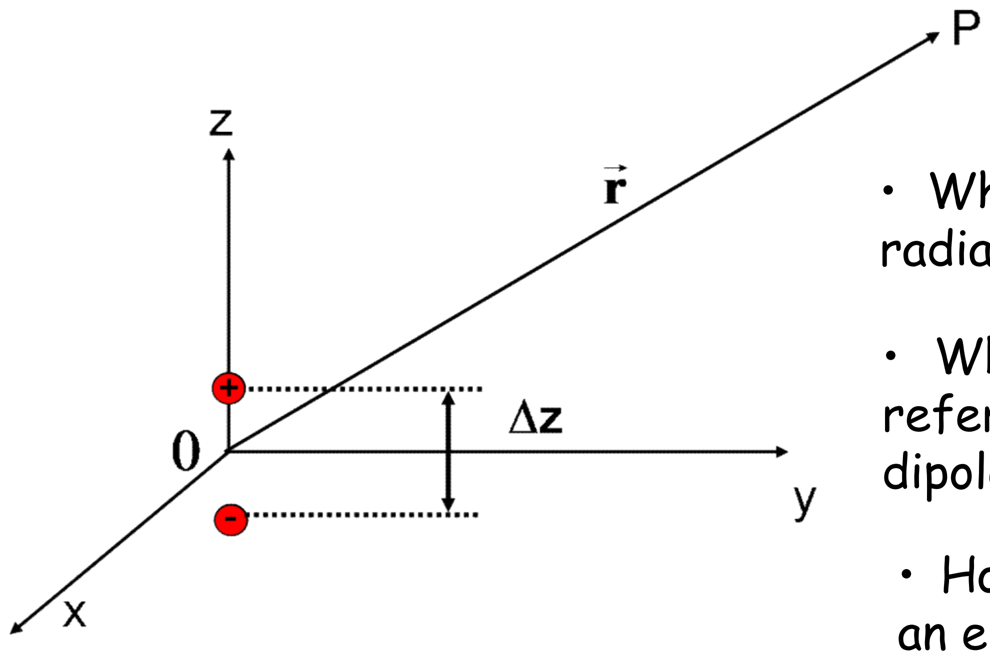

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Summary of the past lecture
Elementary electrical dipole

$$\mathbf{J} = I\Delta z\,\delta(\vec{\mathbf{r}})\hat{i}_z = I\Delta z\,\delta(x)\delta(y)\delta(z)\hat{i}_z$$

↓ J

$$\nabla^2 \mathbf{A} + k^2 \mathbf{A} = -\mu \mathbf{J}$$ Vector Helmholtz equation

↓ $$\mathbf{A} = \frac{\mu}{4\pi} I\Delta z \frac{e^{-jkr}}{r}\hat{i}_z$$

$$\mathbf{H} = \frac{1}{\mu}\nabla \times \mathbf{A} \quad \text{and} \quad \mathbf{E} = -j\omega \mathbf{A} + \frac{\nabla\nabla \cdot \mathbf{A}}{j\omega\varepsilon\mu}$$

↓ E, H

Elementary electrical dipole

The E.M. field radiated by the elementary electrical dipole

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathrm{E}_r(r,\vartheta)\hat{i}_r + \mathrm{E}_\vartheta(r,\vartheta)\hat{i}_\vartheta$$

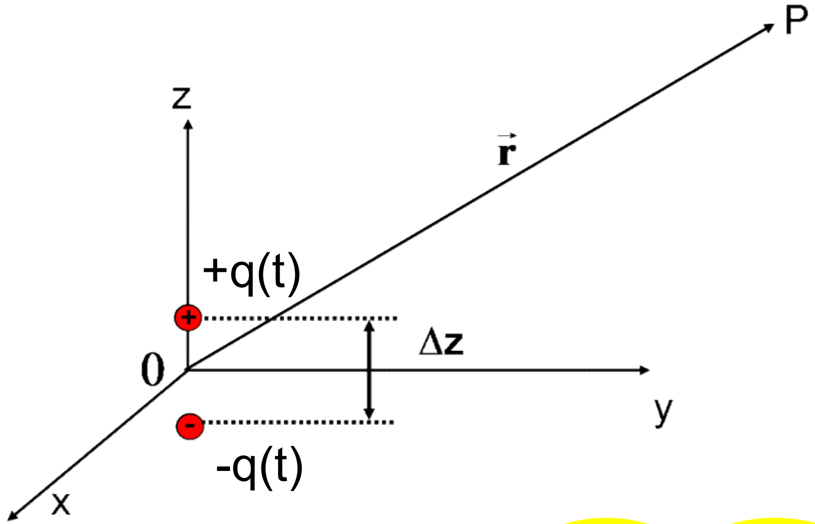
$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \mathrm{H}_\varphi(r,\vartheta)\hat{i}_\varphi$$

$$\begin{cases} E_r = \zeta \dfrac{I\Delta z}{2\pi}\left(\dfrac{1}{r^2} + \dfrac{1}{j\beta r^3}\right)\cos\vartheta \exp(-j\beta r) \\ E_\vartheta = \zeta \dfrac{I\Delta z}{4\pi}\left(\dfrac{j\beta}{r} + \dfrac{1}{r^2} + \dfrac{1}{j\beta r^3}\right)\sin\vartheta \exp(-j\beta r) \\ H_\varphi = \dfrac{I\Delta z}{4\pi}\left(\dfrac{j\beta}{r} + \dfrac{1}{r^2}\right)\sin\vartheta \exp(-j\beta r) \end{cases}$$

Because of the problem symmetry there is no dependence on the azimuth angle φ.

…. Memo

$$\mathbf{J} = I\,\Delta z\,\delta(\vec{\mathbf{r}})\,\hat{i}_z = I\,\Delta z\,\delta(x)\delta(y)\delta(z)\,\hat{i}_z$$



It can be easily shown that this electric current density is the same as that of an electrical dipole such that:

1) the two charges, of opposite sign, have equal time variation;

2) in the spectral domain, the relation between I and the time-varying charge Q is:

$$j\omega Q = I$$

All the quantities, included the expressions of the fields, can be provided in terms of dipole moment U

$$I\,\Delta z = j\omega Q\Delta z = j\omega U$$

Elementary electrical dipole

The E.M. field radiated by the elementary electrical dipole

$$\begin{cases} E_r = \zeta \dfrac{I\Delta z}{2\pi}\left(\dfrac{1}{r^2}+\dfrac{1}{j\beta r^3}\right)\cos\vartheta \exp(-j\beta r) \\ E_\vartheta = \zeta \dfrac{I\Delta z}{4\pi}\left(\dfrac{j\beta}{r}+\dfrac{1}{r^2}+\dfrac{1}{j\beta r^3}\right)\sin\vartheta \exp(-j\beta r) \\ H_\varphi = \dfrac{I\Delta z}{4\pi}\left(\dfrac{j\beta}{r}+\dfrac{1}{r^2}\right)\sin\vartheta \exp(-j\beta r) \end{cases}$$

… for $\omega=0$ simplifies as

$$\begin{cases} E_r = \dfrac{Q\Delta z}{2\pi}\dfrac{1}{\varepsilon r^3}\cos\vartheta \\ E_\vartheta = \dfrac{Q\Delta z}{4\pi}\dfrac{1}{\varepsilon r^3}\sin\vartheta \\ H_\varphi = 0 \end{cases}$$

Elementary electrical dipole

The E.M. field radiated by the elementary electrical dipole

$$\begin{cases} E_r = \zeta \dfrac{j\omega Q \Delta z}{2\pi} \left(\dfrac{1}{r^2} + \dfrac{1}{j\beta r^3} \right) \cos\vartheta \exp(-j\beta r) \\ E_\vartheta = \zeta \dfrac{j\omega Q \Delta z}{4\pi} \left(\dfrac{j\beta}{r} + \dfrac{1}{r^2} + \dfrac{1}{j\beta r^3} \right) \sin\vartheta \exp(-j\beta r) \\ H_\varphi = \dfrac{j\omega Q \Delta z}{4\pi} \left(\dfrac{j\beta}{r} + \dfrac{1}{r^2} \right) \sin\vartheta \exp(-j\beta r) \end{cases}$$

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Elementary electrical dipole

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… for r>>λ simplifies as

Elementary electrical dipole

- The distance-dependent terms can be written as follows:

$$\left(1+\frac{1}{j\beta r}+\frac{1}{(j\beta r)^2}\right)$$

- When $\beta r = 2\pi r/\lambda >> 1$ only the first term can be considered, i.e. $r >> \lambda$.
- The remaining term is known as far-field component while the neglected ones are near-field ones.

Elementary electrical dipole

…the E.M. field radiated by the elementary electrical dipole..

$$\begin{cases} E_r = \zeta \dfrac{I\Delta z}{2\pi}\left(\dfrac{1}{r^2} + \dfrac{1}{j\beta r^3}\right)\cos\vartheta \exp(-j\beta r) \\ E_\vartheta = \zeta \dfrac{I\Delta z}{4\pi}\left(\dfrac{j\beta}{r} + \dfrac{1}{r^2} + \dfrac{1}{j\beta r^3}\right)\sin\vartheta \exp(-j\beta r) \\ H_\varphi = \dfrac{I\Delta z}{4\pi}\left(\dfrac{j\beta}{r} + \dfrac{1}{r^2}\right)\sin\vartheta \exp(-j\beta r) \end{cases}$$

… for r>>λ simplifies as

$$\begin{cases} E_r = 0 \\ E_\vartheta = j\zeta \dfrac{I\Delta z}{2\lambda r}\sin\vartheta \exp(-j\beta r) \\ H_\varphi = j\dfrac{I\Delta z}{2\lambda r}\sin\vartheta \exp(-j\beta r) = \dfrac{E_\vartheta}{\zeta} \end{cases}$$

Elementary electrical dipole

- In the far-field case (r>>λ) the elementary electrical dipole behaves as follows:

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathrm{E}_{\vartheta}(r,\vartheta)\hat{i}_{\vartheta}$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \mathrm{H}_{\varphi}(r,\vartheta)\hat{i}_{\varphi}$$

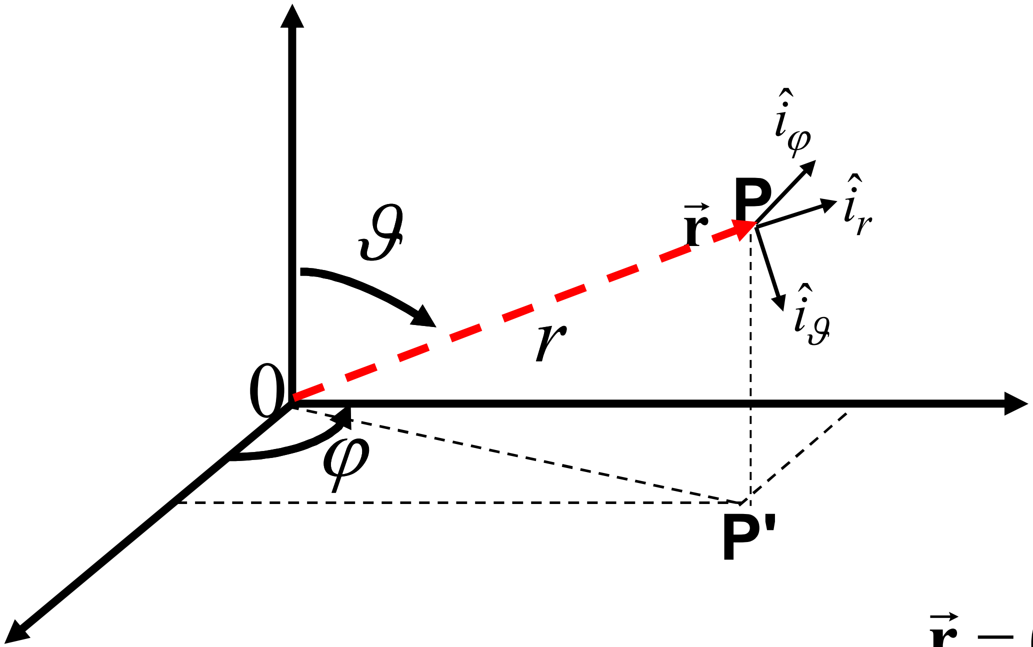
$$\begin{cases} E_{\vartheta} = j\zeta \dfrac{I\Delta z}{2\lambda r}\sin\vartheta \exp(-j\beta r) \\ H_{\varphi} = j\dfrac{I\Delta z}{2\lambda r}\sin\vartheta \exp(-j\beta r) = \dfrac{E_{\vartheta}}{\zeta} \end{cases}$$

- Note the far-field relationship between ***E*** and ***H***

…memo….

spherical coordinate system

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathrm{E}_r(\vec{\mathbf{r}})\hat{i}_r + \mathrm{E}_\varphi(\vec{\mathbf{r}})\hat{i}_\varphi + \mathrm{E}_\vartheta(\vec{\mathbf{r}})\hat{i}_\vartheta$$



$$\hat{i}_\varphi = \hat{i}_r \times \hat{i}_\vartheta$$

$$\hat{i}_\vartheta = \hat{i}_\varphi \times \hat{i}_r$$

$$\hat{i}_r = \hat{i}_\vartheta \times \hat{i}_\varphi$$

$$\vec{\mathbf{r}} = (r, \vartheta, \varphi)$$

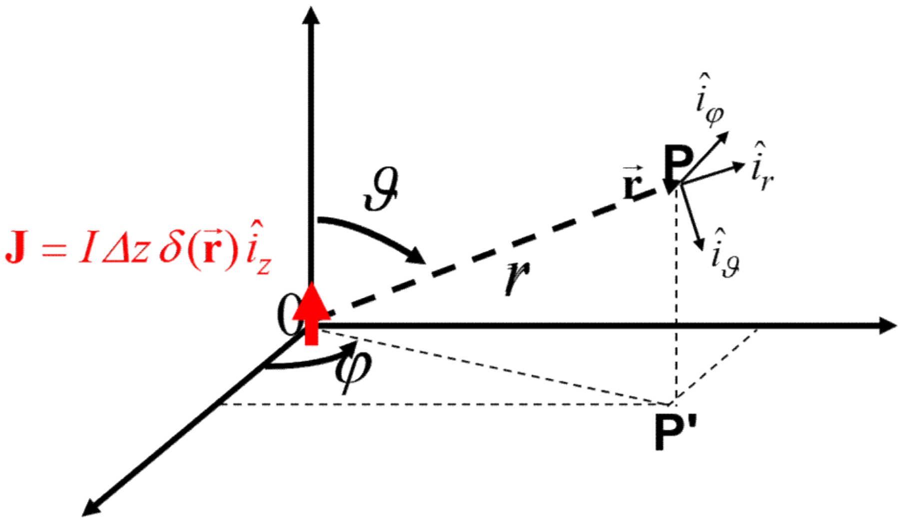
Elementary electrical dipole

In the far-field case (r>>λ)

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathrm{E}_{\vartheta}(r,\vartheta)\hat{i}_{\vartheta}$$

$$\vec{\mathbf{H}} = \vec{\mathbf{H}}(\vec{\mathbf{r}}) = \mathrm{H}_{\varphi}(r,\vartheta)\hat{i}_{\varphi}$$

$$\begin{cases} E_{\vartheta} = j\zeta \dfrac{I\Delta z}{2\lambda r}\sin\vartheta \exp(-j\beta r) \\ H_{\varphi} = j\dfrac{I\Delta z}{2\lambda r}\sin\vartheta \exp(-j\beta r) = \dfrac{E_{\vartheta}}{\zeta} \end{cases}$$



- the e.m. field propagates along $\hat{i}_r$
- the e.m. field lies on the plane orthogonal to the propagation direction
- |E| and |H| exhibit the decaying factor 1/r
- |E| and |H| are proportional through ζ

Elementary electrical dipole

In the far-field case (r>>λ), we have

$$\vec{\mathbf{E}} = \mathrm{E}_{\vartheta}\hat{i}_{\vartheta}$$

$$\vec{\mathbf{H}} = \mathrm{H}_{\varphi}\hat{i}_{\varphi} = \frac{E_{\vartheta}}{\zeta}\hat{i}_{\varphi}$$

$$\Rightarrow \quad \zeta\mathbf{H} = \hat{i}_r \times \mathbf{E}$$

$$\hat{i}_{\varphi} = \hat{i}_r \times \hat{i}_{\vartheta}$$

$$\hat{i}_{\vartheta} = \hat{i}_{\varphi} \times \hat{i}_r$$

$$\hat{i}_r = \hat{i}_{\vartheta} \times \hat{i}_{\varphi}$$

and the Poynting vector:

$$\mathbf{S} = \frac{1}{2}\mathbf{E}\times\mathbf{H}^* = \frac{1}{2}\frac{|E_{\vartheta}|^2}{\zeta}\hat{i}_r = \frac{1}{2}\frac{|\mathbf{E}|^2}{\zeta}\hat{i}_r$$

Elementary electrical dipole

- In order to further characterize its behavior one can evaluate the Poynting vector and the associated power for the overall e.m. field (over a sphere centered in the origin):

$$P = \frac{1}{2} \oiint_S \left[\mathbf{E} \times \mathbf{H}^* \right] \cdot \hat{i}_r dS = \frac{1}{2} \int_0^{2\pi} d\varphi \int_0^{\pi} d\vartheta r^2 \sin\vartheta E_\vartheta H_\varphi^*$$

I) $dS = r^2 \sin\vartheta \, d\vartheta \, d\varphi$

II) $\left[\mathbf{E} \times \mathbf{H}^* \right] \cdot \hat{i}_r = \left[\left(E_\vartheta \hat{i}_\vartheta + E_r \hat{i}_r \right) \times \left(H_\varphi^* \hat{i}_\varphi \right) \right] \cdot \hat{i}_r = E_\vartheta H_\varphi^*$

$$P = \frac{1}{2} \oiint_S \left[\mathbf{E} \times \mathbf{H}^* \right] \cdot \hat{i}_r dS = \frac{1}{2} \int_0^{2\pi} d\varphi \int_0^{\pi} d\vartheta r^2 \sin\vartheta E_\vartheta H_\varphi^* = \frac{1}{2} \frac{2\pi}{3} \zeta \left(\frac{\Delta z}{\lambda} \right)^2 \left[1 - j \frac{1}{(\beta r)^3} \right] |I|^2$$

$$\begin{cases} E_r = \zeta \dfrac{I \Delta z}{2\pi} \left(\dfrac{1}{r^2} + \dfrac{1}{j\beta r^3} \right) \cos\vartheta \exp(-j\beta r) \\ E_\vartheta = \zeta \dfrac{I \Delta z}{4\pi} \left(\dfrac{j\beta}{r} + \dfrac{1}{r^2} + \dfrac{1}{j\beta r^3} \right) \sin\vartheta \exp(-j\beta r) \\ H_\varphi = \dfrac{I \Delta z}{4\pi} \left(\dfrac{j\beta}{r} + \dfrac{1}{r^2} \right) \sin\vartheta \exp(-j\beta r) \end{cases}$$

$$P = P_1 + jP_2$$

$$P_1 = \frac{1}{2} \frac{2\pi}{3} \zeta \left(\frac{\Delta z}{\lambda} \right)^2 |I|^2$$

$$P_2 = -\frac{1}{2} \frac{2\pi}{3} \zeta \left(\frac{\Delta z}{\lambda} \right)^2 \frac{1}{(\beta r)^3} |I|^2$$

$$\left(\frac{j\beta}{r} + \frac{1}{r^2} + \frac{1}{j\beta r^3} \right) \left(\frac{-j\beta}{r} + \frac{1}{r^2} \right) = \left(\frac{\beta}{r} \right)^2 - j \frac{1}{\beta r^5} = \frac{\beta^2}{r^2} \left[1 - j \frac{1}{(\beta r)^3} \right]$$

- *Note that in the far-field case only the first active power term exists and it does not depend on r.*

Elementary electrical dipole

- Note that the real part of the power, in lossless medium, is independent of *r*, therefore if one consider two different spherical surfaces one gets the same result. Only the so-called radiative terms contribute.
- The reactive part depends on *r*. Its sign is negative showing that there is an excess of stored **<u>electric</u>** energy in the neighbor of the electrical dipole (see Poynting's theorem)