

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

Corso di Campi Elettromagnetici
a.a. 2018-2019

27 Maggio 2019

Summary of the past lecture

Antenna Parameters

Parameters of the Tx Antenna

Parameters of the Rx Antenna

Summary of the past lectures

TX Antenna Parameters

- Effective length
 - Radiation pattern
 - Radiation pattern lobes
 - Beamwidth
- Directivity
- Gain
- Radiation Resistance
- Equivalent circuit of the tx antenna
- Input Impedance and Input Resistance



Summary of the past lecture

Antenna Parameters

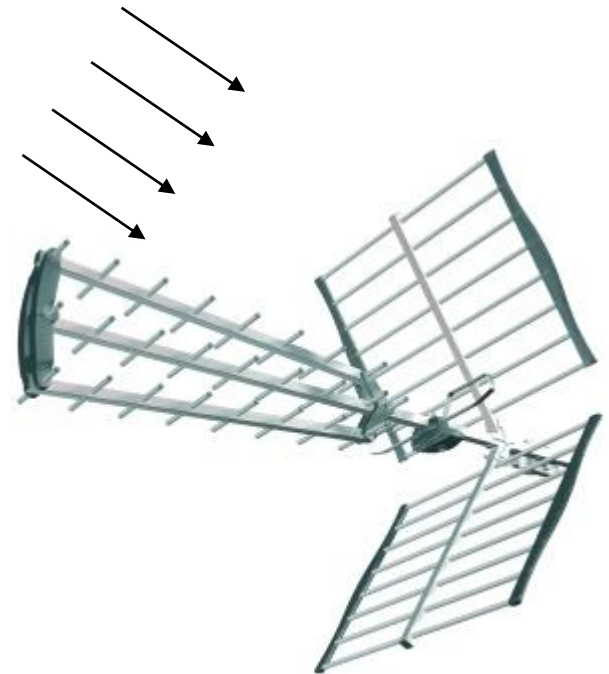
Parameters of the Tx Antenna

Parameters of the Rx Antenna

Receiving mode

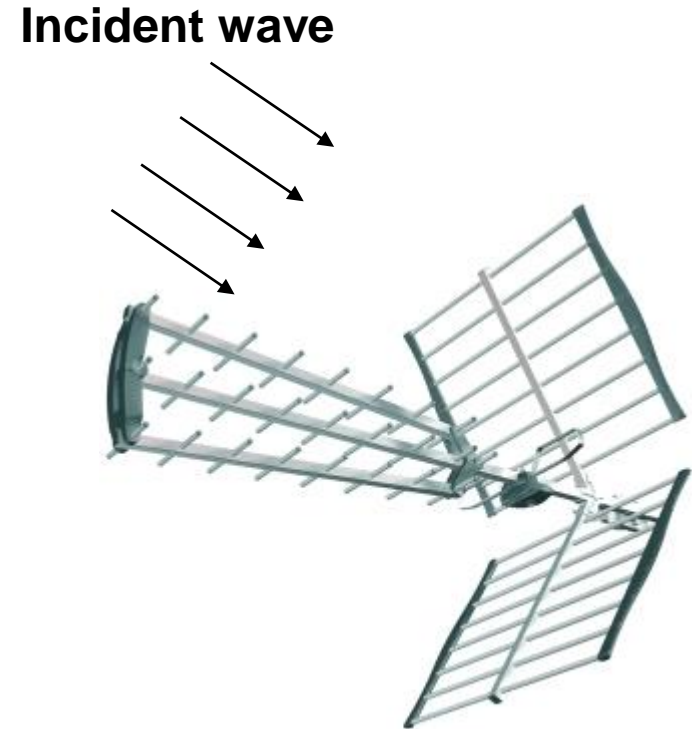
- When an antenna is operating as a receiving antenna, it extracts a certain amount of power from an incident electromagnetic wave.
- Since an incident wave comes from a far distance may be thought of as a uniform (local) plane wave being intercepted by the antenna.

Incident wave



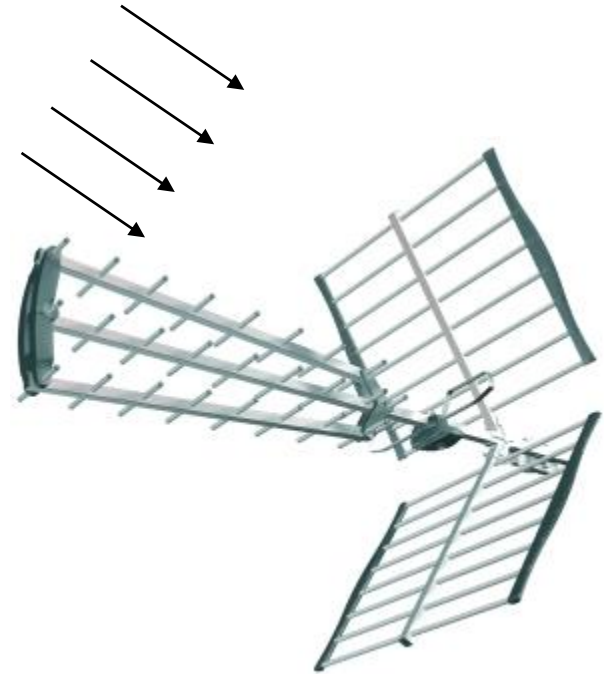
Receiving mode

- The use of the antenna in the receiving mode is shown in Figure.
- The incident wave impinges upon the antenna, and it induces a voltage V_0 at the input terminals .



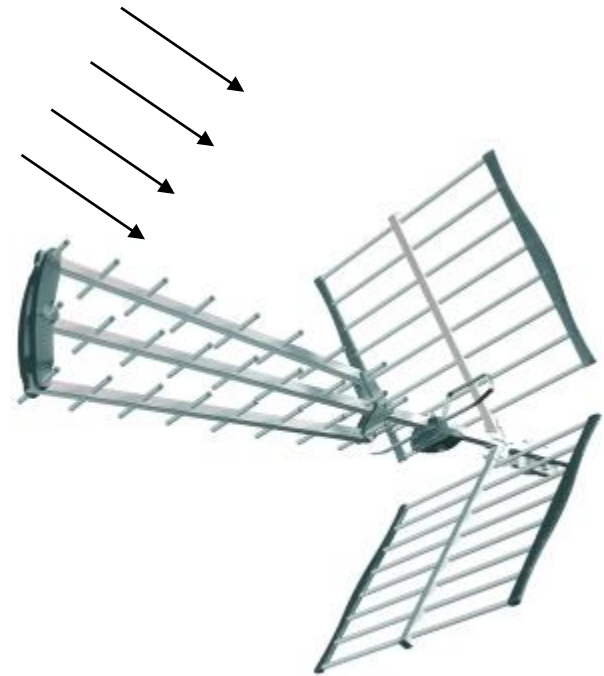
Rx Antenna parameters

- Rx effective length
- Equivalent circuit of the rx antenna
- Effective Area



Rx Antenna parameters

- Rx effective length
- Equivalent circuit of the rx antenna
- Effective Area



Rx effective length

Tx effective length

$$\mathbf{l}(\vartheta, \varphi) = l_{\vartheta}(\vartheta, \varphi) \hat{i}_{\vartheta} + l_{\varphi}(\vartheta, \varphi) \hat{i}_{\varphi}$$

Elementary electrical dipole

$$\mathbf{l}(\vartheta, \varphi) = \Delta z \sin \vartheta \hat{i}_{\vartheta}$$

Small loop antenna

$$\mathbf{l}(\vartheta, \varphi) = -j\beta\Delta S \sin \vartheta \hat{i}_{\varphi}$$

It can be shown that for an elementary electrical dipole or for a small loop antenna, the following property is valid:

$$|V_0| = |\mathbf{E}_i \cdot \mathbf{l}|$$

\mathbf{l} is the tx antenna effective length

\mathbf{E}_i is the incident, locally plane, field

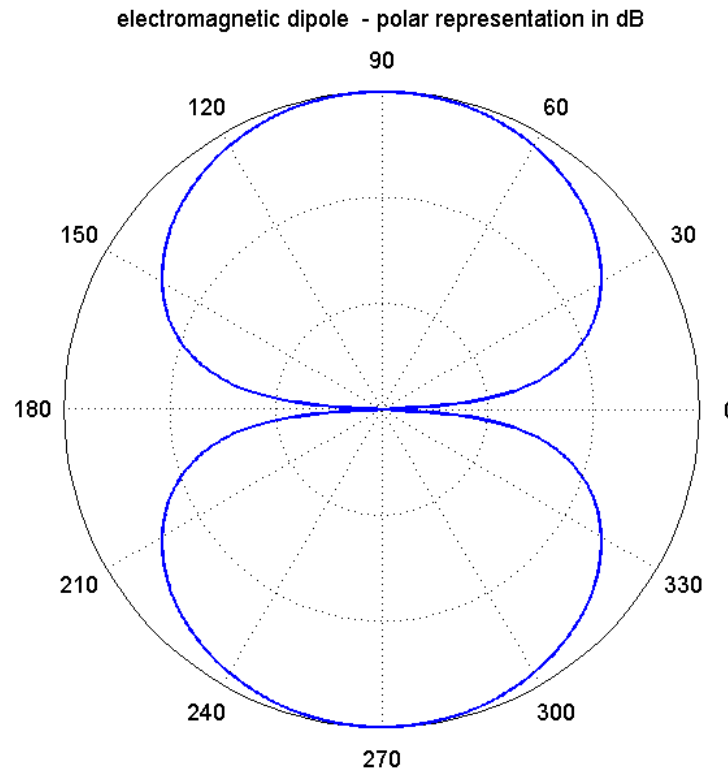
V_0 is the voltage induced at the antenna terminals, which are assumed open-circuited

..... MEMO

Radiation pattern of the electrical elementary dipole

$$I(\vartheta, \varphi) = \Delta z \sin \vartheta \hat{i}_\vartheta$$

Vertical plane ($\varphi=0$)



Rx effective length

Interestingly, this result can be extended to ALL the antennas by applying the RECIPROcity THEOREM.

It can be shown that for an elementary electrical dipole or for a small loop antenna, the following property is valid:

$$|V_0| = |\mathbf{E}_i \cdot \mathbf{l}|$$

\mathbf{l} is the tx antenna effective length

\mathbf{E}_i is the incident, locally plane, field

V_0 is the voltage induced at the antenna terminals, which are assumed open-circuited

Rx effective length

$$|V_0| = |\mathbf{E}_i \cdot \mathbf{l}|$$

Where

\mathbf{E}_i is the incident, locally plane, field

V_0 is the voltage induced at the antenna terminals, which are assumed open-circuited

$\mathbf{l}(\mathcal{G}, \varphi) = l_{\mathcal{G}}(\mathcal{G}, \varphi) \hat{i}_{\mathcal{G}} + l_{\varphi}(\mathcal{G}, \varphi) \hat{i}_{\varphi}$ can be referred to as **receiving effective length** of the antenna (and not only transmitting effective length)

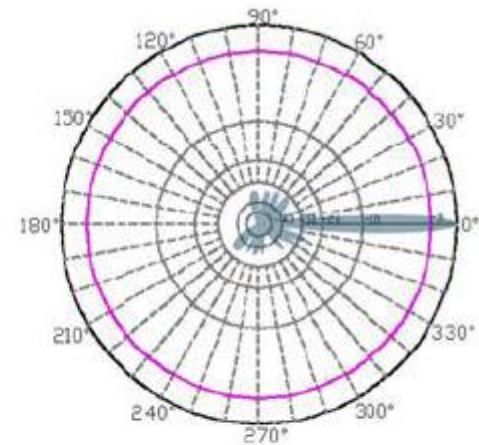
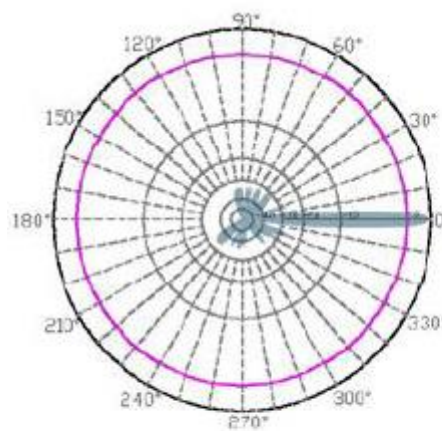
Note that this means that the behavior of an antenna when transmitting and when receiving are related.

Rx effective length

three examples from the real life



Rx effective length

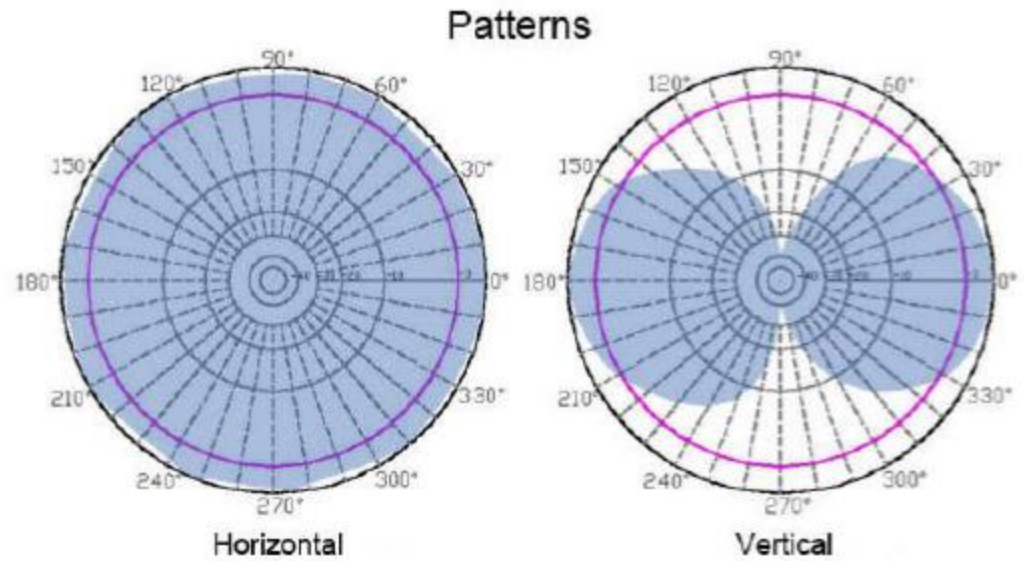


Horizontal

Vertical

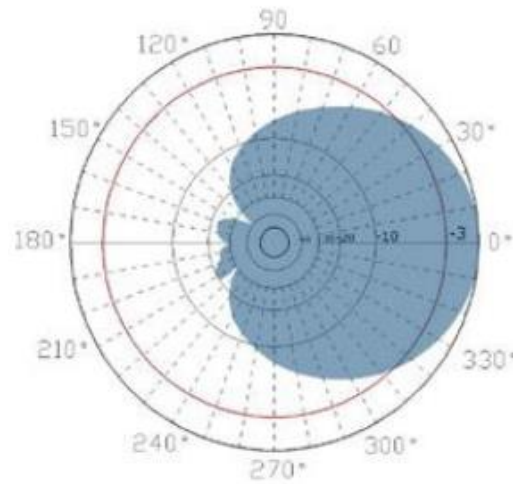
ZDASP5400-29-6 Patterns

Rx effective length

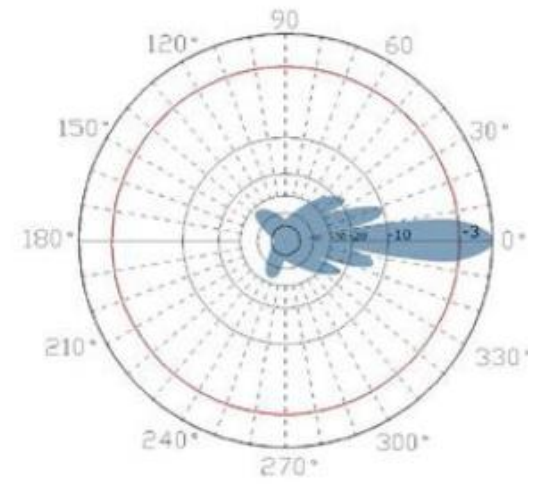


ZDAEW1900-3 Patterns

Rx effective length



Horizontal

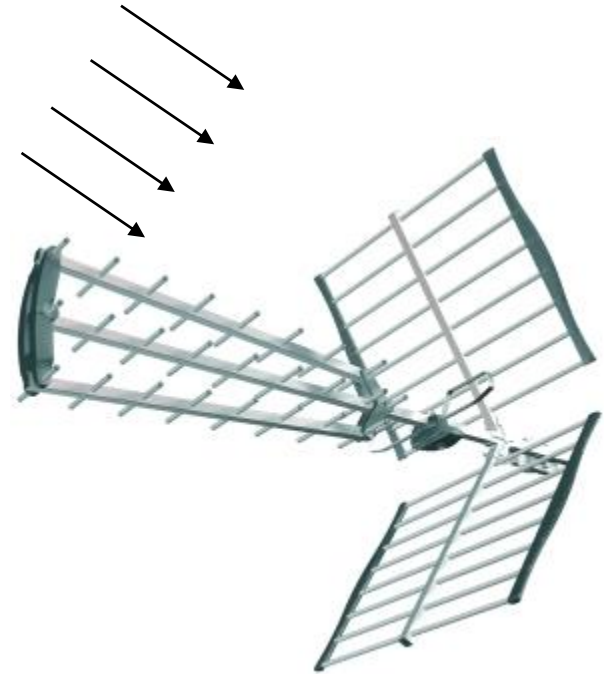


Vertical

ZDADJ800-13-90 Patterns

Rx Antenna parameters

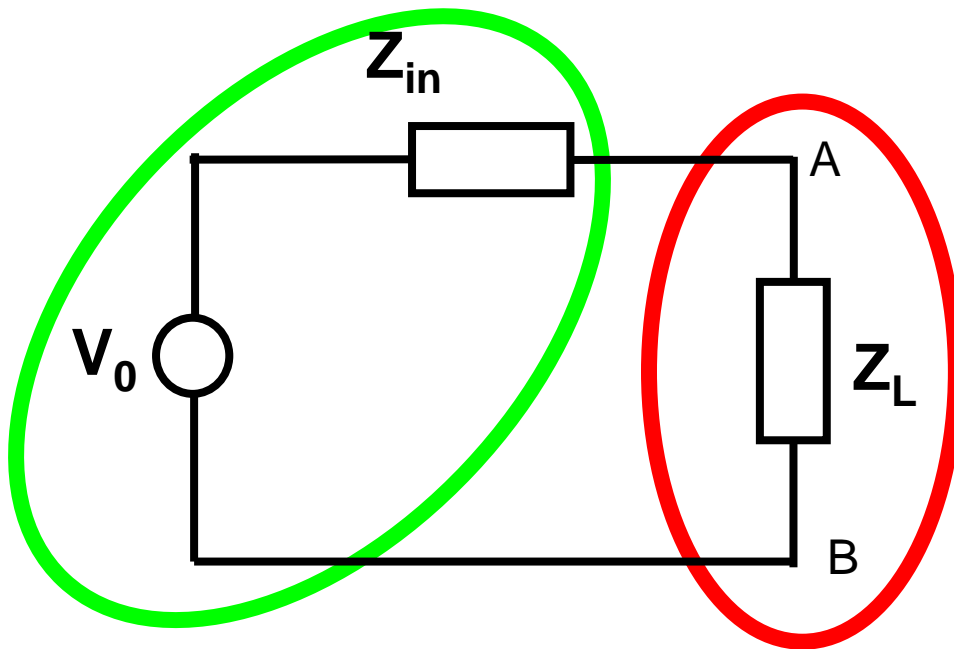
- Rx effective length
- Equivalent circuit of the rx antenna
- Effective Area



Equivalent circuit of the Rx antenna

- The incident electric field sets up currents on the antenna. Such currents may be represented by a Thevenin-equivalent generator, which delivers power to any connected receiving load impedance.

Equivalent circuit of the Rx antenna



Rx antenna

Load

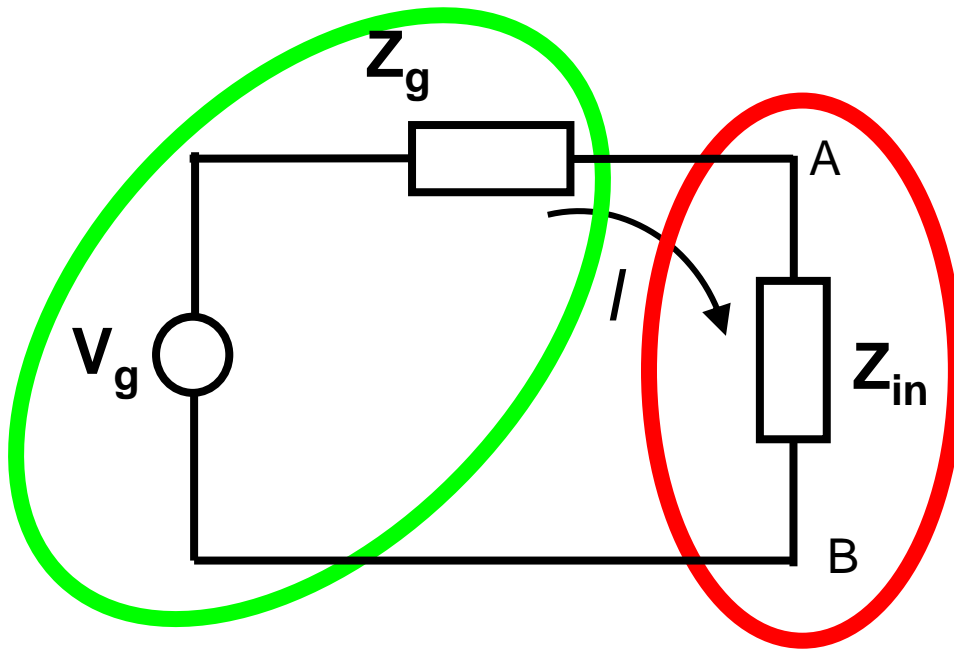
$$|V_0| = |\mathbf{E}_i \cdot \mathbf{l}|$$

$$Z_{in} = R_{in} + jX_{in}$$

$$Z_L = R_L + jX_L$$

....MEMO..

Equivalent circuit of the Tx antenna



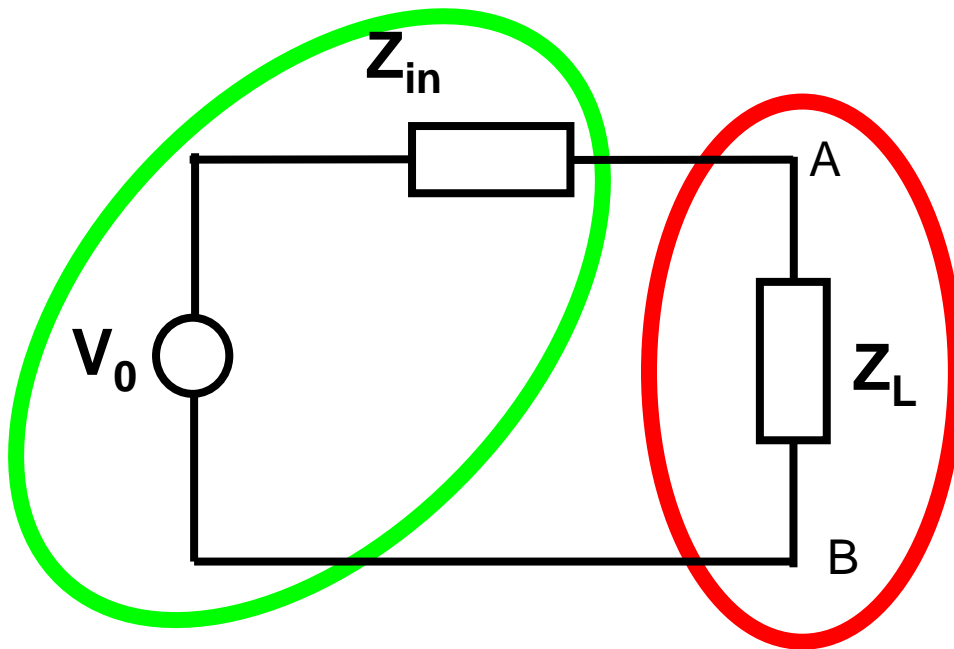
$$Z_{in} = R_{in} + jX_{in}$$

$$P_{in} = \frac{1}{2} R_{in} |I|^2$$

Tx circuit

Tx Antenna

Equivalent circuit of the Rx antenna



$$|V_0| = |\mathbf{E}_i \cdot \mathbf{l}|$$

$$Z_{in} = R_{in} + jX_{in}$$

$$Z_L = R_L + jX_L$$

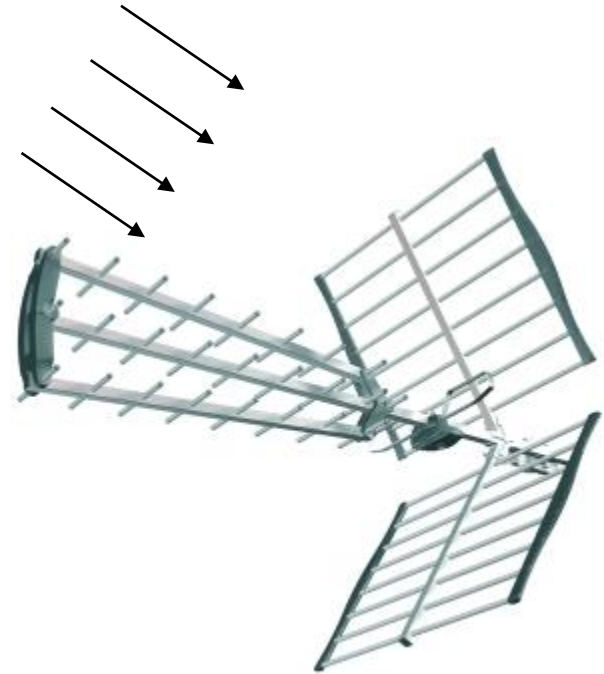
Rx antenna

Load

$$P_L = \frac{1}{2} R_L |I_L|^2 = \frac{1}{2} \frac{R_L}{|Z_{in} + Z_L|^2} |V_0|^2 = \frac{1}{2} \frac{R_L}{|Z_{in} + Z_L|^2} |\mathbf{E}_i \cdot \mathbf{l}|^2$$

Rx Antenna parameters

- Rx effective length
- Equivalent circuit of the rx antenna
- Effective Area

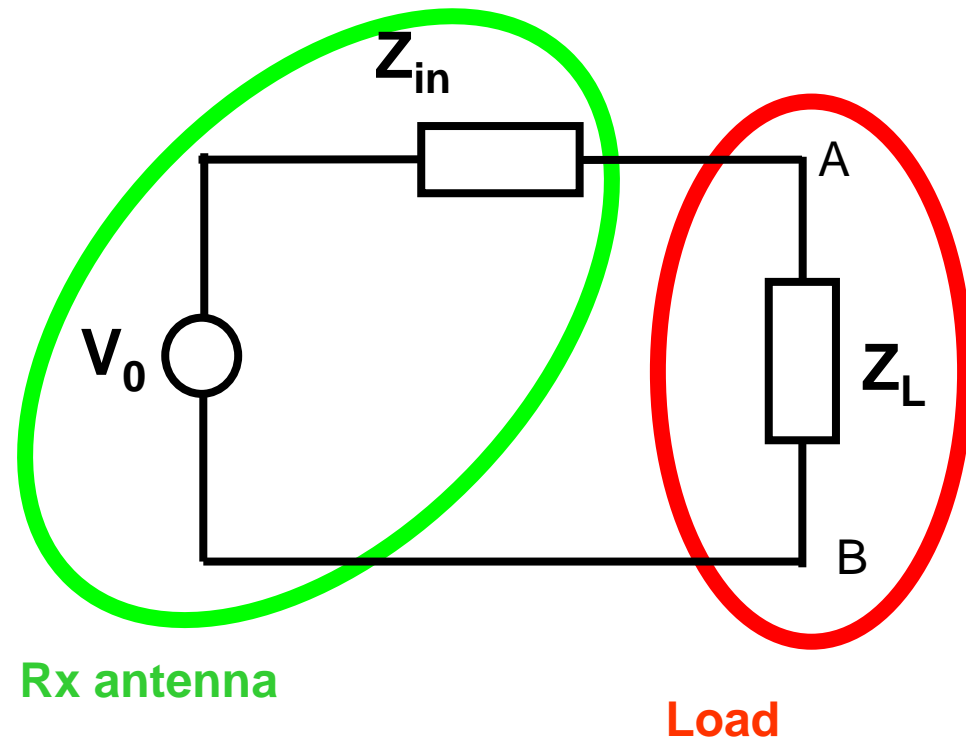


Equivalent circuit of the Rx antenna

$$|V_0| = |\mathbf{E}_i \cdot \mathbf{l}|$$

$$Z_{in} = R_{in} + jX_{in}$$

$$Z_L = R_L + jX_L$$

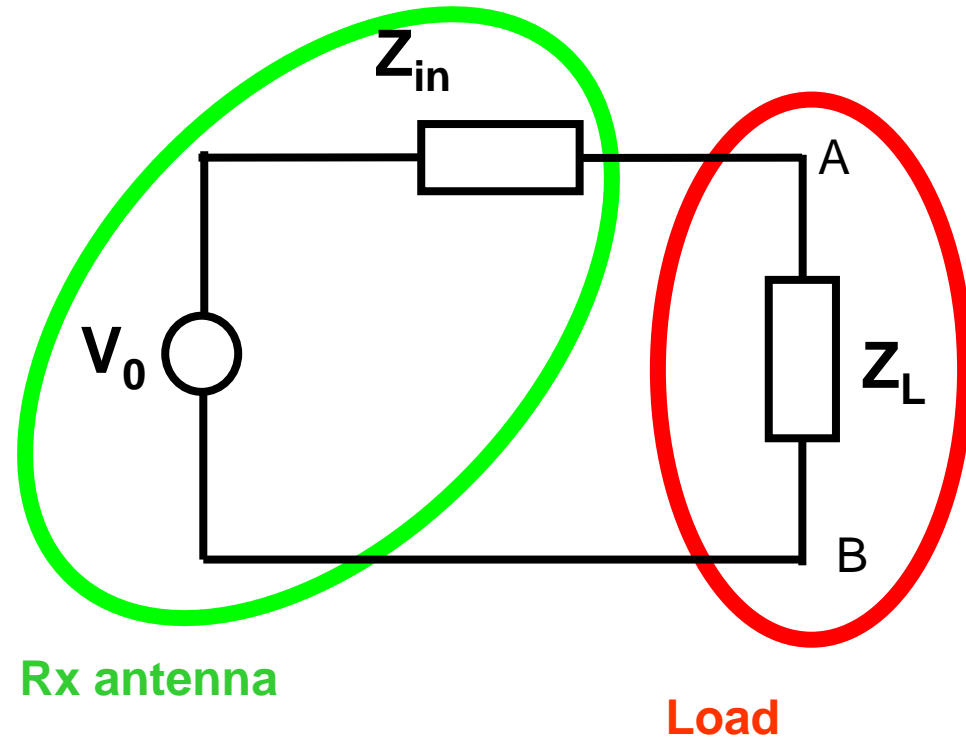


$$P_L = \frac{1}{2} \frac{R_L}{|Z_{in} + Z_L|^2} |\mathbf{E}_i \cdot \mathbf{l}|^2$$

Equivalent circuit of the Rx antenna

1) Polarization matching

$$|\mathbf{E}_i \cdot \mathbf{l}| = |\mathbf{E}_i| |\mathbf{l}|$$



$$P_L = \frac{1}{2} \frac{R_L}{|Z_{in} + Z_L|^2} |\mathbf{E}_i \cdot \mathbf{l}|^2$$

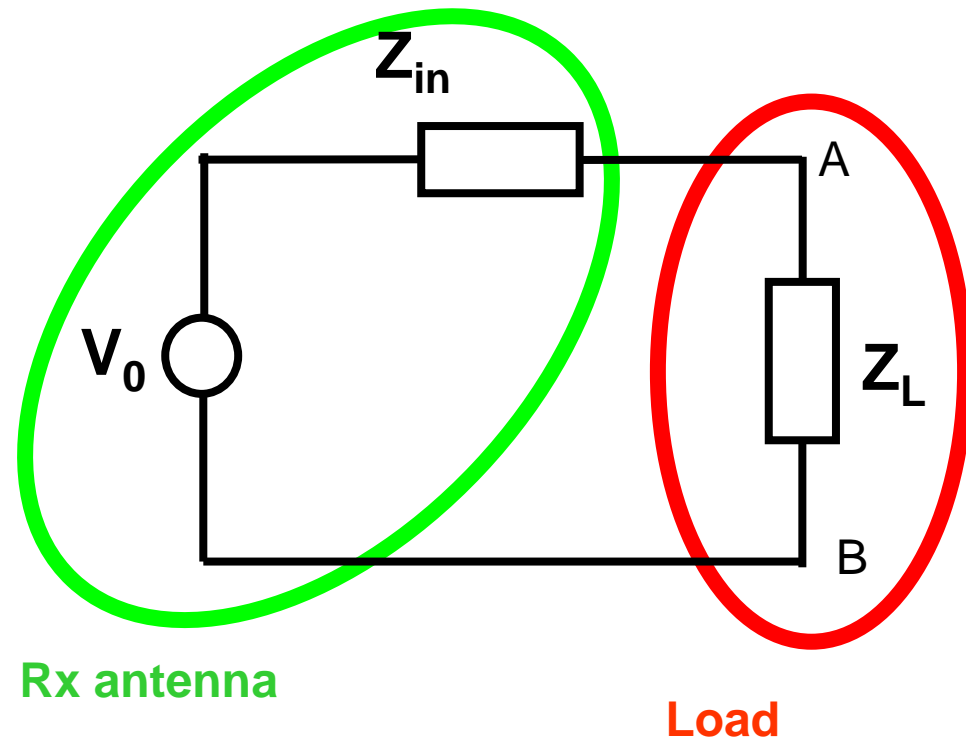
Equivalent circuit of the Rx antenna

1) Polarization matching

$$|\mathbf{E}_i \cdot \mathbf{l}| = |\mathbf{E}_i| |\mathbf{l}|$$

2) Power matching

$$Z_L = Z_{in}^* \Rightarrow \begin{cases} Z_{in} = R_{in} + jX_{in} \\ Z_L = R_{in} - jX_{in} \end{cases}$$
$$\Rightarrow \frac{R_L}{|Z_{in} + Z_L|^2} = \frac{R_{in}}{(2R_{in})^2} = \frac{1}{4R_{in}}$$



$$P_L = \frac{1}{2} \frac{R_L}{|Z_{in} + Z_L|^2} |\mathbf{E}_i \cdot \mathbf{l}|^2$$

Equivalent circuit of the Rx antenna

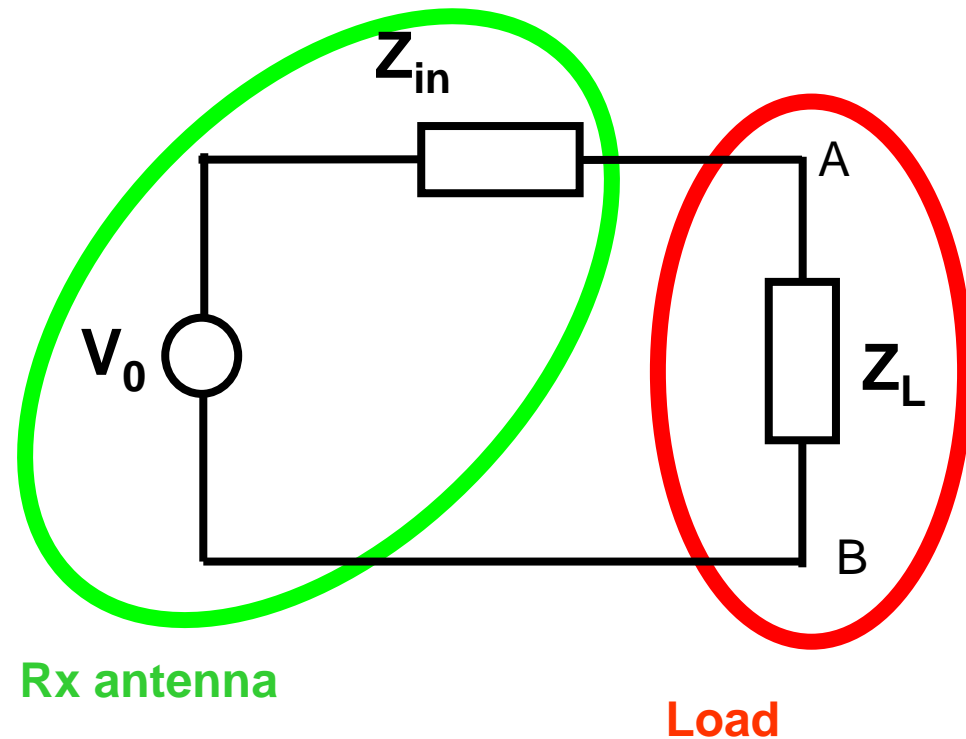
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$$P_L = \frac{1}{2} \frac{R_L}{|Z_{in} + Z_L|^2} |\mathbf{E}_i \cdot \mathbf{l}|^2 \Rightarrow P_{Lmax} = \frac{1}{2} \frac{1}{4R_{in}} |\mathbf{E}_i|^2 |\mathbf{l}|^2 = \frac{\zeta}{\zeta} \frac{1}{2} \frac{1}{4R_{in}} |\mathbf{E}_i|^2 |\mathbf{l}|^2$$

Equivalent circuit of the Rx antenna

1) Polarization matching

$$|\mathbf{E}_i \cdot \mathbf{l}| = |\mathbf{E}_i| |\mathbf{l}|$$

2) Power matching

$$Z_L = Z_{in}^* \Rightarrow \begin{cases} Z_{in} = R_{in} + jX_{in} \\ Z_L = R_{in} - jX_{in} \end{cases}$$
$$\Rightarrow \frac{R_L}{|Z_{in} + Z_L|^2} = \frac{R_{in}}{(2R_{in})^2} = \frac{1}{4R_{in}}$$

$$P_{Lmax} = \frac{1}{2} \frac{|\mathbf{E}_i|^2}{\zeta} \left[\frac{\zeta |\mathbf{l}(\vartheta, \varphi)|^2}{4R_{in}} \right]$$

Effective Area
 $A_{eff}(\vartheta, \varphi)$

$$P_L = \frac{1}{2} \frac{R_L}{|Z_{in} + Z_L|^2} |\mathbf{E}_i \cdot \mathbf{l}|^2 \Rightarrow P_{Lmax} = \frac{1}{2} \frac{1}{4R_{in}} |\mathbf{E}_i|^2 |\mathbf{l}|^2 = \frac{\zeta}{\zeta} \frac{1}{2} \frac{1}{4R_{in}} |\mathbf{E}_i|^2 |\mathbf{l}|^2$$

Effective area

- In general, the aperture of an antenna is not directly related to its physical size.
- However some types of antennas, for example parabolic dishes and horns, have a physical aperture (opening) which collects the radio waves.
- In these aperture antennas, the effective aperture A must always be less than the area of the antenna's physical aperture A_{phys} .
- The ratio of $A_{\text{eff}}/A_{\text{phys}}$ vary from 0.35 to 0.70 but can range up to 0.90.

Gain

$$G(\vartheta, \varphi) = \frac{\frac{1}{2} \frac{|\mathbf{E}|^2}{\zeta}}{\frac{1}{4\pi r^2} P_{in}}$$

Effective Area

$$A_{eff}(\vartheta, \varphi) = \frac{\zeta |\mathbf{l}(\vartheta, \varphi)|^2}{4R_{in}}$$

$$\mathbf{E}(r, \vartheta, \varphi) = \frac{j\zeta l e^{-j\beta r}}{2\lambda r} \mathbf{l}(\vartheta, \varphi)$$

$$P_{in} = \frac{1}{2} R_{in} |l|^2$$

$$G(\vartheta, \varphi) = \frac{4\pi r^2}{2\zeta} \frac{|\mathbf{E}|^2}{P_{in}} = \frac{4\pi r^2}{2\zeta} \left(\frac{\zeta^2 |l|^2}{4\lambda^2} \frac{|\mathbf{l}(\vartheta, \varphi)|^2}{r^2} \right) \frac{2}{R_{in} |l|^2} = \frac{4\pi}{\lambda^2} \left[\frac{\zeta |\mathbf{l}(\vartheta, \varphi)|^2}{4R_{in}} \right]$$

$$G(\vartheta, \varphi) = \frac{4\pi}{\lambda^2} A_{eff}(\vartheta, \varphi)$$

Effective Area

three examples from the real life



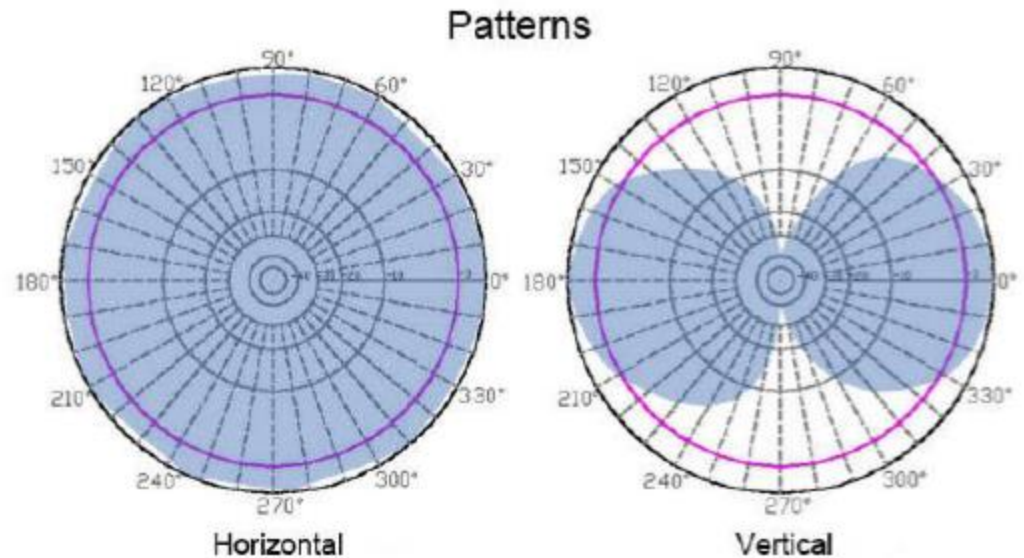
Effective Area

three examples from the real life



(maximum) Gain = 3 dB

$$G(\vartheta, \varphi) = \frac{4\pi}{\lambda^2} A_{eff}(\vartheta, \varphi)$$



ZDAEW1900-3 Patterns

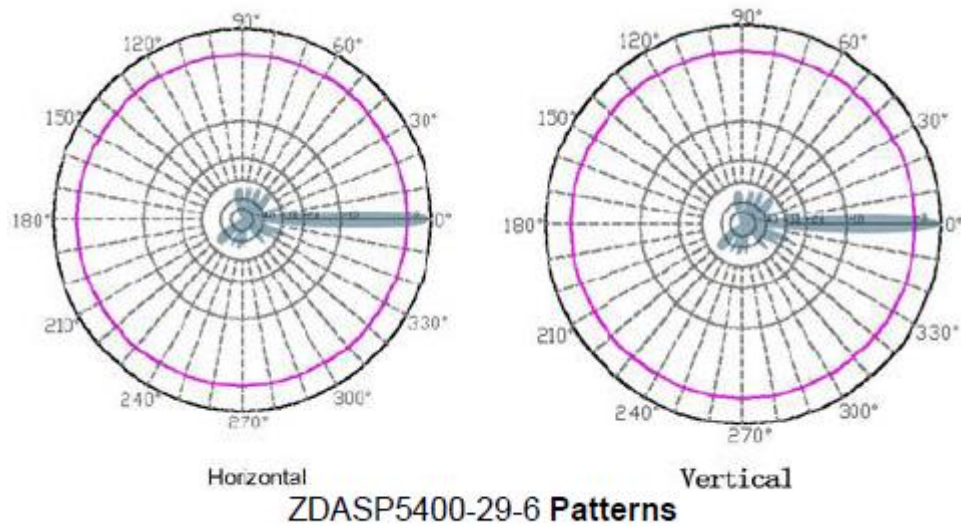
Effective Area

three examples from the real life



(maximum) Gain = 29 dB

$$G(\vartheta, \varphi) = \frac{4\pi}{\lambda^2} A_{eff}(\vartheta, \varphi)$$



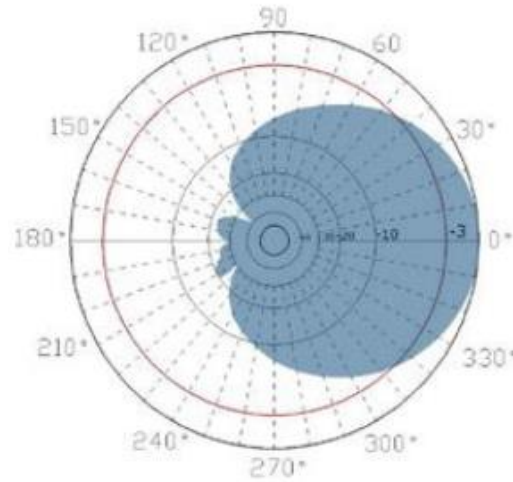
Effective Area

three examples from the real life

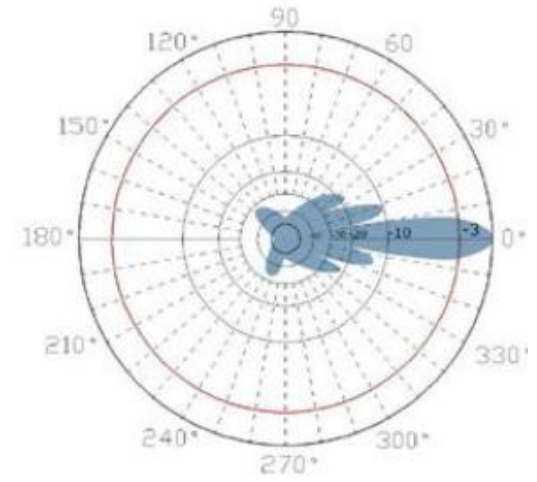


(maximum) Gain = 13 dB

$$G(\vartheta, \varphi) = \frac{4\pi}{\lambda^2} A_{eff}(\vartheta, \varphi)$$



Horizontal



Vertical

ZDADJ800-13-90 Patterns

Equivalent circuit of the Rx antenna

$$P_L = \frac{1}{2} \frac{R_L}{|Z_{in} + Z_L|^2} |\mathbf{E}_i \cdot \mathbf{l}|^2$$

$$P_{Lmax} = \frac{1}{2} \frac{|\mathbf{E}_i|^2 |\mathbf{l}|^2}{4R_{in}}$$

Maximum power transfer to the load is achieved when the two following conditions are simultaneously verified:

1) Polarization matching

$$\Rightarrow |\mathbf{E}_i \cdot \mathbf{l}| = |\mathbf{E}_i| |\mathbf{l}|$$

2) Power matching

$$\Rightarrow \frac{R_L}{|Z_{in} + Z_L|^2} = \frac{1}{4R_{in}}$$

$$P_{Lmax} = \frac{1}{2} \frac{|\mathbf{E}_i|^2}{\zeta} A_{eff}(\vartheta, \varphi)$$

Equivalent circuit of the Rx antenna

$$P_L = \frac{1}{2} \frac{R_L}{|Z_{in} + Z_L|^2} |\mathbf{E}_i \cdot \mathbf{l}|^2$$

$$P_{Lmax} = \frac{1}{2} \frac{|\mathbf{E}_i|^2 |\mathbf{l}|^2}{4R_{in}}$$

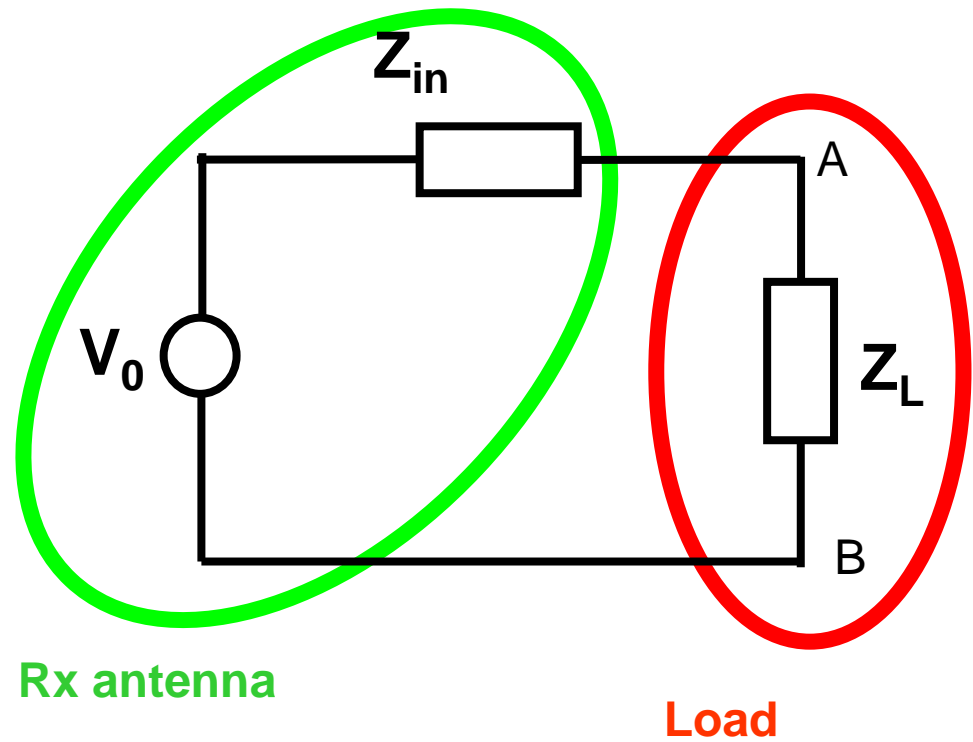
$$P_L = \underbrace{\frac{1}{2}}_{P_{Lmax}} \underbrace{\frac{R_L}{|Z_{in} + Z_L|^2}}_{\eta_B} |\mathbf{E}_i \cdot \mathbf{l}|^2 \left[\frac{4R_{in}}{4R_{in}} \frac{|\mathbf{E}_i|^2 |\mathbf{l}|^2}{|\mathbf{E}_i|^2 |\mathbf{l}|^2} \right] = \underbrace{\frac{1}{2} \frac{|\mathbf{E}_i|^2 |\mathbf{l}|^2}{4R_{in}}}_{P_{Lmax}} \underbrace{\frac{|\mathbf{E}_i \cdot \mathbf{l}|^2}{|\mathbf{E}_i|^2 |\mathbf{l}|^2}}_{\eta_A} \underbrace{\frac{4R_{in} R_L}{|Z_{in} + Z_L|^2}}_{\eta_B}$$

$$P_L = \eta_A \eta_B P_{Lmax}$$

Equivalent circuit of the Rx antenna

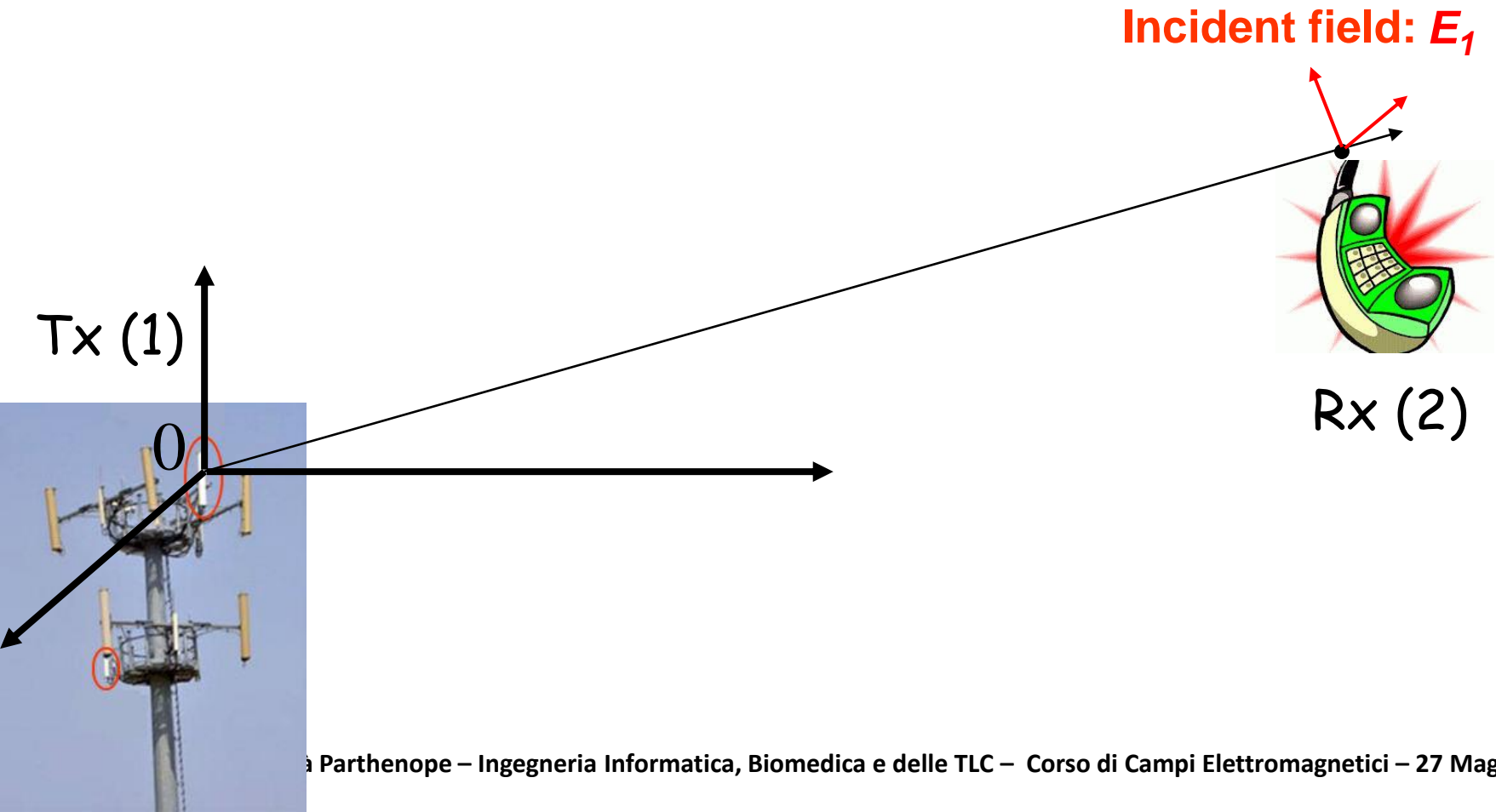
$$P_L = \eta_A \eta_B P_{Lmax}$$

$$P_{Lmax} = \frac{1}{2} \frac{|\mathbf{E}_i|^2}{\zeta} A_{eff}(\vartheta, \varphi)$$



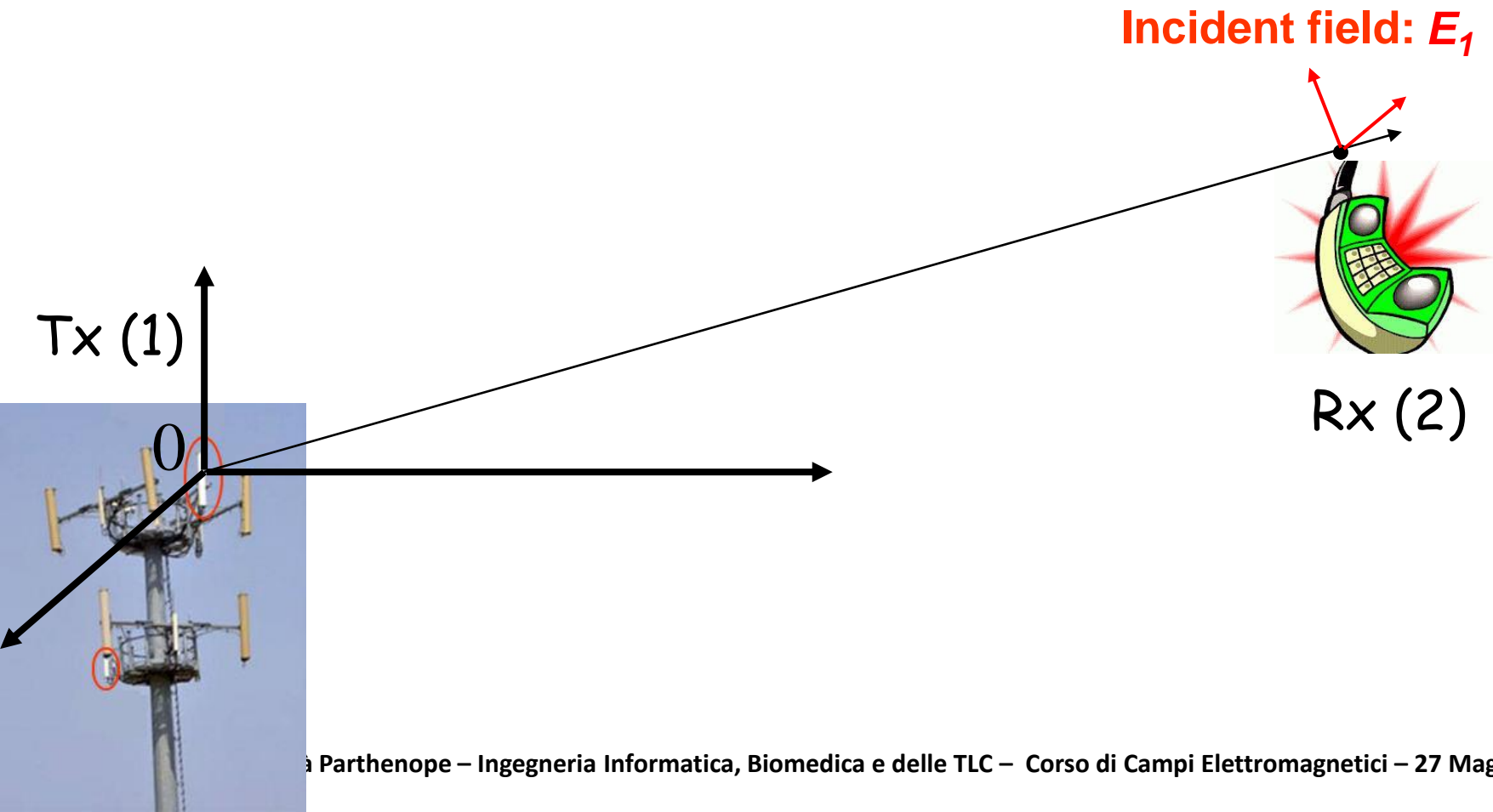
$$P_L = \frac{1}{2} \frac{|\mathbf{E}_i|^2}{\zeta} A_{eff}(\vartheta, \varphi) \eta_A \eta_B$$

Radio link equation



Radio link equation

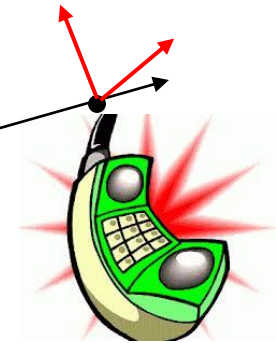
$$P_2 = \frac{1}{2} \frac{|\mathbf{E}_1|^2}{\zeta} A_{eff} \eta_A \eta_B$$



Radio link equation

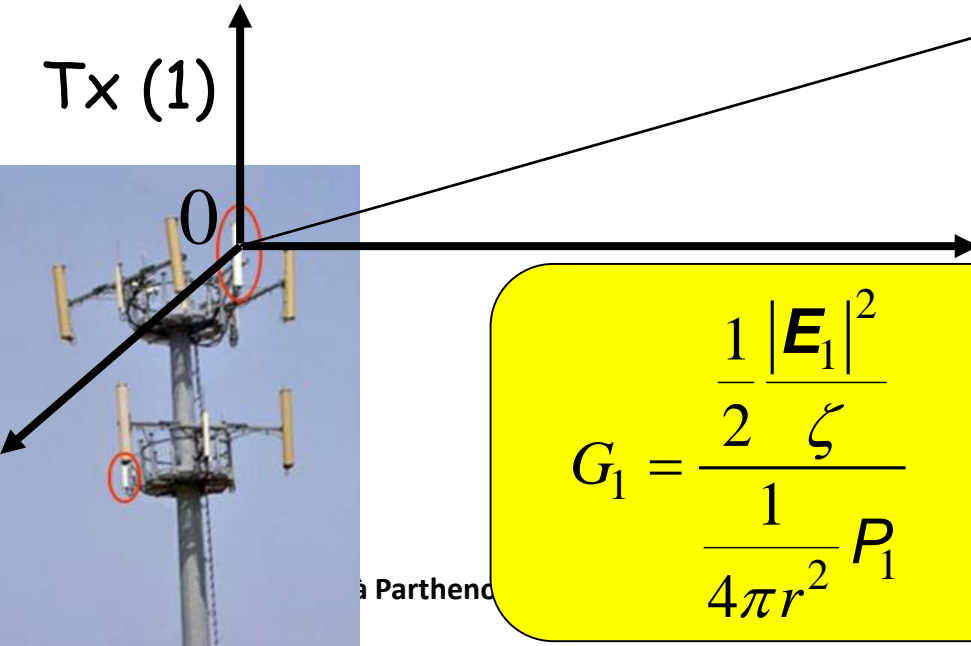
$$P_2 = \frac{1}{2} \frac{|\mathbf{E}_1|^2}{\zeta} A_{eff} \eta_A \eta_B$$

Incident field: \mathbf{E}_1



Rx (2)

Tx (1)

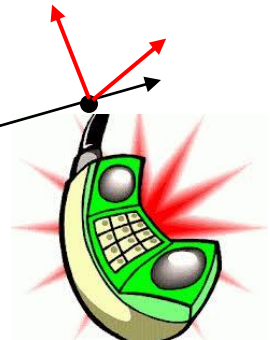


$$G_1 = \frac{\frac{1}{2} \frac{|\mathbf{E}_1|^2}{\zeta}}{\frac{1}{4\pi r^2} P_1}$$

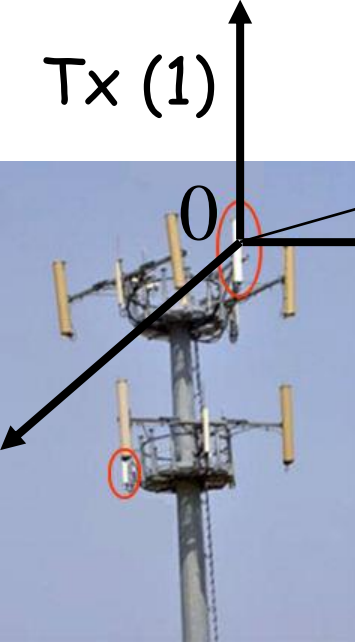
Radio link equation

$$P_2 = \frac{1}{2} \frac{|\mathbf{E}_1|^2}{\zeta} A_{eff} \eta_A \eta_B$$

Incident field: E_1



Rx (2)

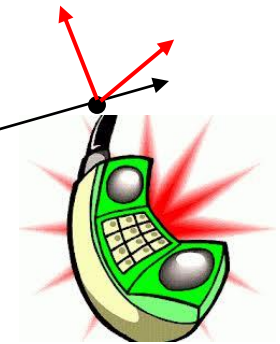


$$G_1 = \frac{\frac{1}{2} \frac{|\mathbf{E}_1|^2}{\zeta}}{\frac{1}{4\pi r^2} P_1} \longrightarrow \frac{1}{2} \frac{|\mathbf{E}_1|^2}{\zeta} = \frac{P_1 G_1}{4\pi r^2}$$

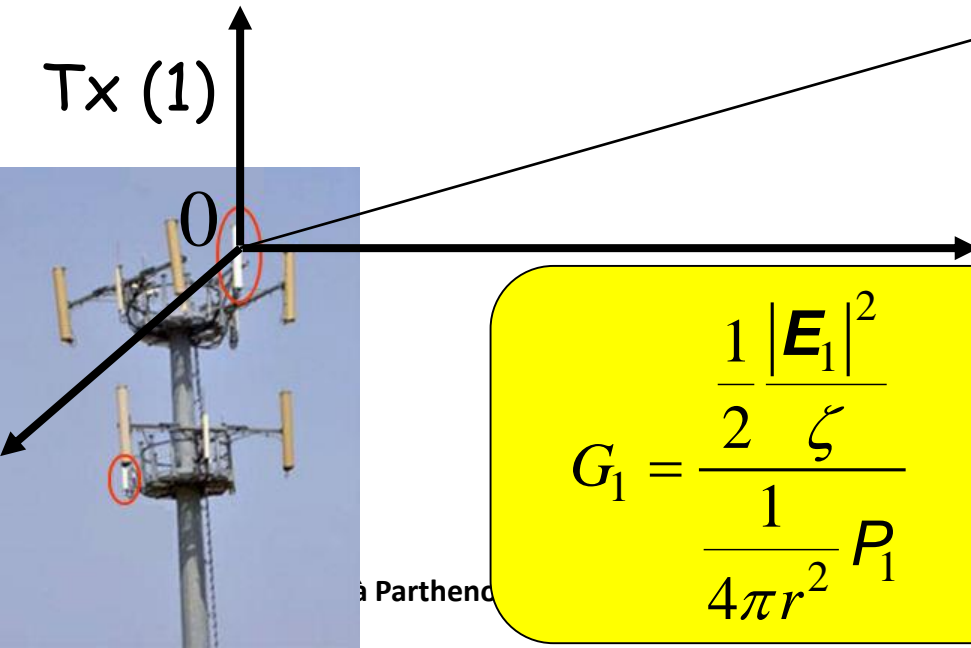
Radio link equation

$$P_2 = \frac{1}{2} \frac{|\mathbf{E}_1|^2}{\zeta} A_{eff2} \eta_A \eta_B = P_1 G_1 A_{eff2} \left(\frac{1}{4\pi r^2} \right) \eta_A \eta_B$$

Incident field: E_1



Rx (2)



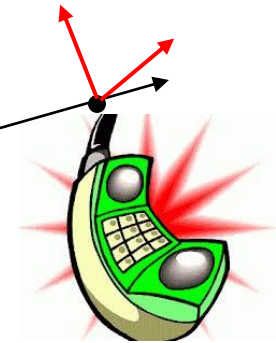
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Radio link equation

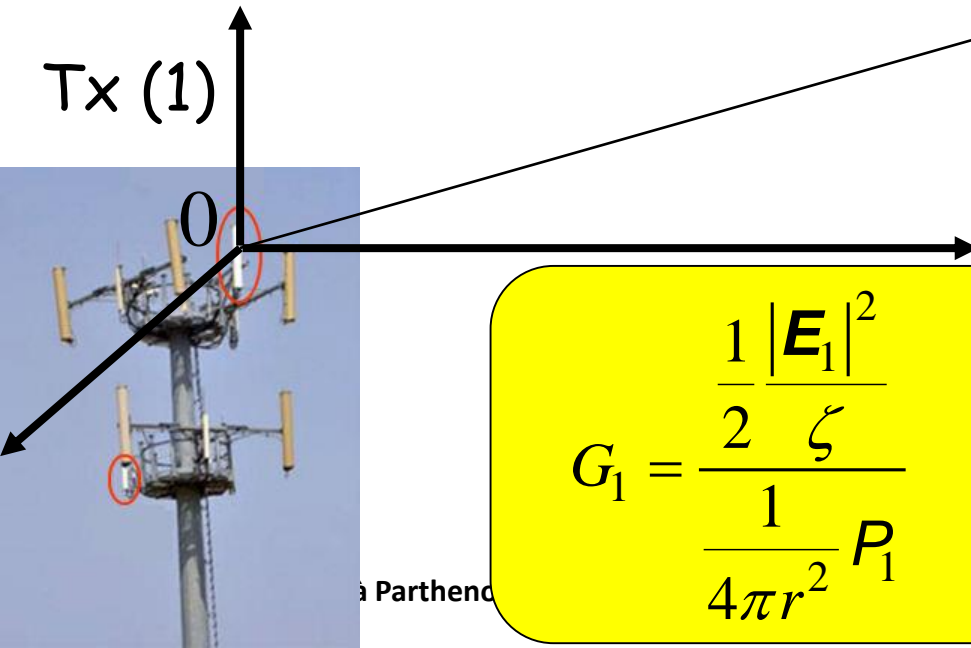
$$P_2 = \frac{1}{2} \frac{|\mathbf{E}_1|^2}{\zeta} A_{eff 2} \eta_A \eta_B = P_1 G_1 A_{eff 2} \left(\frac{1}{4\pi r^2} \right) \eta_A \eta_B$$

$$P_2 = P_1 G_1 A_{eff 2} \left(\frac{1}{4\pi r^2} \right) \eta_A \eta_B$$

Incident field: \mathbf{E}_1



Rx (2)

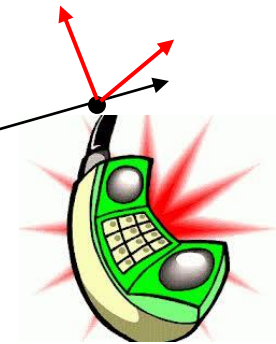


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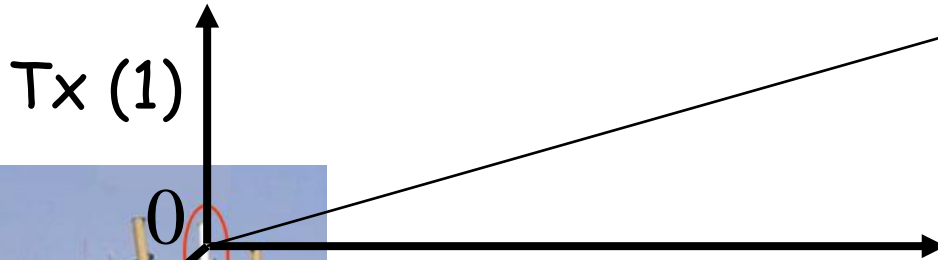
Radio link equation (Friis equation)

$$P_2 = P_1 G_1 A_{eff 2} \left(\frac{1}{4\pi r^2} \right) \eta_A \eta_B$$

Incident field: E_1



Rx (2)



Radio link equation



(maximum) Gain = 29 dB

$$G(\vartheta, \varphi) = \frac{4\pi}{\lambda^2} A_{eff}(\vartheta, \varphi)$$

