

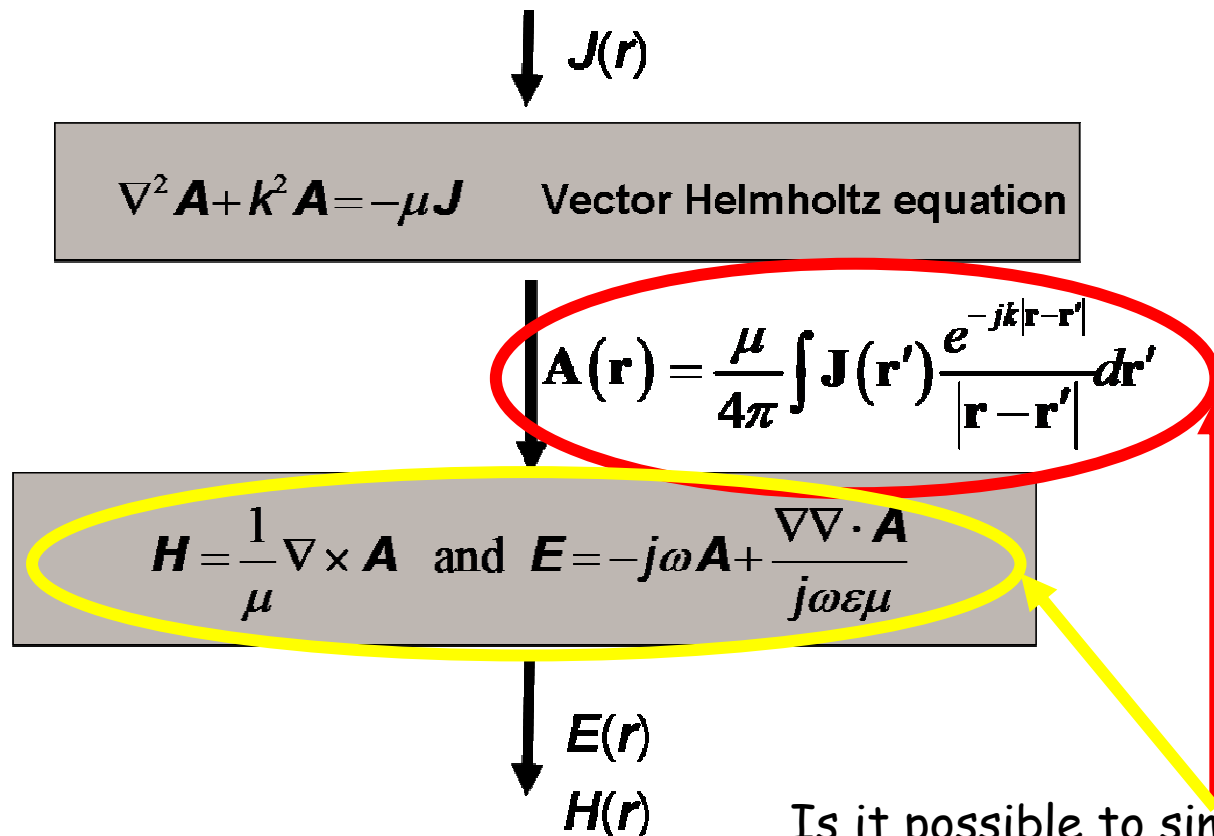
# Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

Corso di Campi Elettromagnetici  
a.a. 2017-2018

17 Maggio 2018

# Extended Antennas

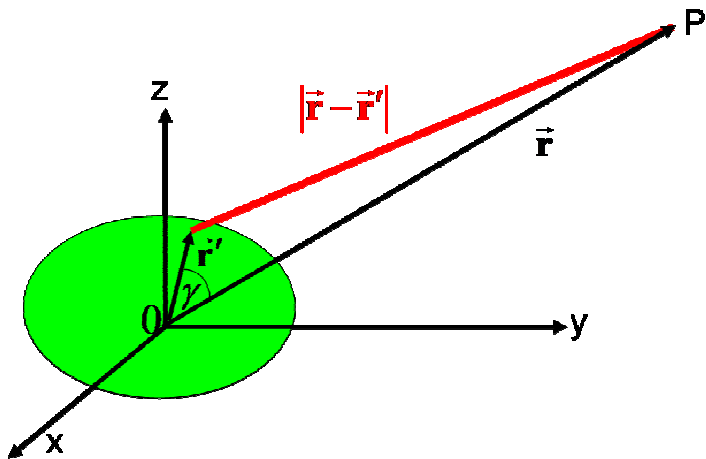
# Extended antennas



Is it possible to simplify the expressions of the fields, possibly via proper approximation of the vector potential  $\mathbf{A}$ ?

# Extended antennas

$$\mathbf{A}(\vec{\mathbf{r}}) = \frac{\mu}{4\pi} \int \mathbf{J}(\vec{\mathbf{r}}') \frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} d\vec{\mathbf{r}}' \quad |\vec{\mathbf{r}}-\vec{\mathbf{r}}'| = r - r' \cos \gamma + \frac{(r')^2}{2r} \sin^2 \gamma + \dots$$



$$\frac{1}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} = \frac{1}{r - r' \cos \gamma + \frac{(r')^2}{2r} \sin^2 \gamma \dots}$$

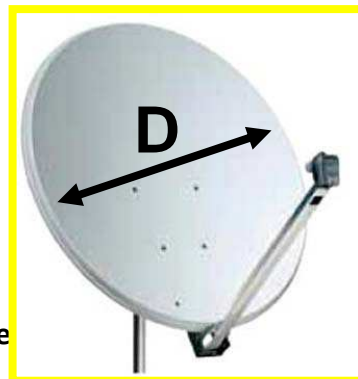
$$e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} = e^{-j\beta r} e^{j\beta r' \cos \gamma} e^{-j\beta \frac{(r')^2}{2r} \sin^2 \gamma} \dots$$

# Extended antennas

$$\mathbf{A}(\vec{\mathbf{r}}) = \frac{\mu}{4\pi} \int \mathbf{J}(\vec{\mathbf{r}}') \frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} d\vec{\mathbf{r}}' \quad |\vec{\mathbf{r}}-\vec{\mathbf{r}}'| = r - r' \cos \gamma + \frac{(r')^2}{2r} \sin^2 \gamma + \dots$$

$$\frac{1}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} = \frac{1}{r - r' \cos \gamma + \frac{(r')^2}{2r} \sin^2 \gamma + \dots} \approx \frac{1}{r} \quad \text{if } r' \ll r$$

$$e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} = e^{-j\beta r} e^{j\beta r' \cos \gamma} e^{-j\beta \frac{(r')^2}{2r} \sin^2 \gamma} \dots \approx e^{-j\beta r} \quad \text{if } r' \ll \lambda$$

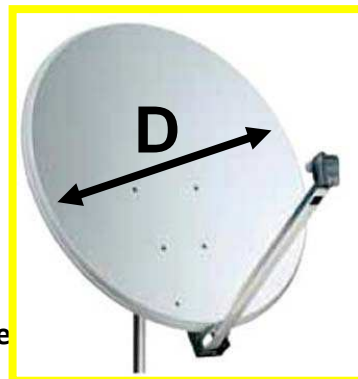


# Extended antennas

$$\mathbf{A}(\vec{\mathbf{r}}) = \frac{\mu}{4\pi} \int \mathbf{J}(\vec{\mathbf{r}}') \frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} d\vec{\mathbf{r}}' \quad |\vec{\mathbf{r}}-\vec{\mathbf{r}}'| = r - r' \cos \gamma + \frac{(r')^2}{2r} \sin^2 \gamma + \dots$$

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$$e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} = e^{-j\beta r} e^{j\beta r' \cos \gamma} e^{-j\beta \frac{(r')^2}{2r} \sin^2 \gamma} \dots \approx e^{-j\beta r} \quad \text{if } D \ll \lambda$$



# Extended antennas

$$\mathbf{A}(\vec{\mathbf{r}}) = \frac{\mu}{4\pi} \int \mathbf{J}(\vec{\mathbf{r}}') \frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} d\vec{\mathbf{r}}' \quad |\vec{\mathbf{r}}-\vec{\mathbf{r}}'| = r - r' \cos \gamma + \frac{(r')^2}{2r} \sin^2 \gamma + \dots$$

$$\frac{1}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} = \frac{1}{r - r' \cos \gamma + \frac{(r')^2}{2r} \sin^2 \gamma + \dots} \approx \frac{1}{r} \quad \text{if } D \ll r$$

$$e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} = e^{-j\beta r} e^{j\beta r' \cos \gamma} e^{-j\beta \frac{(r')^2}{2r} \sin^2 \gamma} \dots \approx e^{-j\beta r} \quad \text{if } D \ll \lambda$$

When the antennas are small with respect to the wavelength and to the distance from the observation point

$$\frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} \approx \frac{e^{-j\beta r}}{r} \quad \longrightarrow \quad \mathbf{A}(\vec{\mathbf{r}}) \approx \frac{\mu}{4\pi} \frac{e^{-j\beta r}}{r} \int \mathbf{J}(\vec{\mathbf{r}}') d\vec{\mathbf{r}}'$$



# Extended antennas

$$\mathbf{A}(\vec{\mathbf{r}}) = \frac{\mu}{4\pi} \int \mathbf{J}(\vec{\mathbf{r}}') \frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} d\vec{\mathbf{r}}' \quad |\vec{\mathbf{r}}-\vec{\mathbf{r}}'| = r - r' \cos \gamma + \frac{(r')^2}{2r} \sin^2 \gamma + \dots$$

$$\frac{1}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} = \frac{1}{r - r' \cos \gamma + \frac{(r')^2}{2r} \sin^2 \gamma + \dots} \approx \frac{1}{r} \quad \text{if } D \ll r$$

$$e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} = e^{-j\beta r} e^{j\beta r' \cos \gamma} e^{-j\beta \frac{(r')^2}{2r} \sin^2 \gamma} \dots \approx e^{-j\beta r} e^{j\beta \vec{\mathbf{r}}' \cdot \hat{\mathbf{i}}_r} \quad \text{if } r > \frac{2D^2}{\lambda}$$

For all the antennas, if the distance from the observation point is sufficiently large

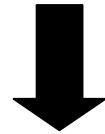
$$\frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} \approx \frac{e^{-j\beta r} e^{j\beta \vec{\mathbf{r}}' \cdot \hat{\mathbf{i}}_r}}{r} \quad \longrightarrow \quad \mathbf{A}(\vec{\mathbf{r}}) \approx \frac{\mu}{4\pi} \frac{e^{-j\beta r}}{r} \int \mathbf{J}(\vec{\mathbf{r}}') e^{j\beta \vec{\mathbf{r}}' \cdot \hat{\mathbf{i}}_r} d\vec{\mathbf{r}}'$$

# Extended antennas

$$r \gg D$$

$$r > \frac{2D^2}{\lambda}$$

- $\int \mathbf{J}(\vec{\mathbf{r}}') e^{j\beta \vec{\mathbf{r}}' \cdot \hat{\mathbf{i}}_r} d\vec{\mathbf{r}}' = \mathbf{M}(\vartheta, \varphi)$



$$\mathbf{A}(\vec{\mathbf{r}}) \approx \frac{\mu}{4\pi} \frac{e^{-j\beta r}}{r} \mathbf{M}(\vartheta, \varphi)$$

For all the antennas, if the distance from the observation point is **sufficiently large**

$$\frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} \approx \frac{e^{-j\beta r} e^{j\beta \vec{\mathbf{r}}' \cdot \hat{\mathbf{i}}_r}}{r} \quad \longrightarrow \quad \mathbf{A}(\vec{\mathbf{r}}) \approx \frac{\mu}{4\pi} \frac{e^{-j\beta r}}{r} \int \mathbf{J}(\vec{\mathbf{r}}') e^{j\beta \vec{\mathbf{r}}' \cdot \hat{\mathbf{i}}_r} d\vec{\mathbf{r}}'$$

# Extended antennas

$$\begin{aligned} r &\gg D \\ r &> \frac{2D^2}{\lambda} \\ r &\gg \lambda \end{aligned}$$

Fraunhofer region

$$\mathbf{M}(\vartheta, \varphi) = \int \mathbf{J}(\vec{\mathbf{r}}') e^{j\beta \vec{\mathbf{r}}' \cdot \hat{\mathbf{i}}_r} d\vec{\mathbf{r}}'$$

$$\mathbf{A}(\vec{\mathbf{r}}) \approx \frac{\mu}{4\pi} \frac{e^{-j\beta r}}{r} \mathbf{M}(\vartheta, \varphi)$$

$$\left\{ \begin{aligned} \nabla \nabla \cdot \mathbf{A}(\vec{\mathbf{r}}) &\approx -\beta^2 A_r(\vec{\mathbf{r}}) \hat{\mathbf{i}}_r \\ \nabla \times \mathbf{A}(\vec{\mathbf{r}}) &\approx -j\beta \hat{\mathbf{i}}_r \times \mathbf{A}(\vec{\mathbf{r}}) \end{aligned} \right.$$

# Radiation problem for extended antennas

↓  $\mathbf{J}(\mathbf{r})$

$$\nabla^2 \mathbf{A} + k^2 \mathbf{A} = -\mu \mathbf{J} \quad \text{Vector Helmholtz equation}$$

↓

$$\mathbf{A}(\mathbf{r}) \approx \frac{\mu}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}') e^{-j\beta r'}}{r} \int \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r}' d\mathbf{r}'$$

$$\mathbf{H} = \frac{1}{\mu} \nabla \times \mathbf{A} \quad \text{and} \quad \mathbf{E} = -j\omega \mathbf{A} + \frac{\nabla \nabla \cdot \mathbf{A}}{j\omega \epsilon \mu}$$

↙

$$-j\beta \hat{i}_r \times \mathbf{A}$$

↓  $\mathbf{E}(\mathbf{r})$   
 $\mathbf{H}(\mathbf{r})$

↘

$$-\beta^2 A_r \hat{i}_r$$

$$\nabla\nabla \cdot \mathbf{A}(\vec{\mathbf{r}}) \approx -\beta^2 A_r(\vec{\mathbf{r}})\hat{i}_r$$



$$\mathbf{E} = -j\omega\mathbf{A} + \frac{\nabla\nabla \cdot \mathbf{A}}{j\omega\epsilon\mu} \approx -j\omega\mathbf{A} - \frac{\beta^2 A_r \hat{i}_r}{j\omega\epsilon\mu} = -j\omega\mathbf{A} + j\omega A_r \hat{i}_r = -j\omega[\mathbf{A} - A_r \hat{i}_r]$$

$$\mathbf{H} = \frac{1}{\mu} \nabla \times \mathbf{A} \quad \text{and} \quad \mathbf{E} = -j\omega\mathbf{A} + \frac{\nabla\nabla \cdot \mathbf{A}}{j\omega\epsilon\mu}$$

$$\nabla\nabla \cdot \mathbf{A}(\vec{\mathbf{r}}) \approx -\beta^2 A_r(\vec{\mathbf{r}})\hat{i}_r$$

$$\nabla \times \mathbf{A}(\vec{\mathbf{r}}) \approx -j\beta \hat{i}_r \times \mathbf{A}(\vec{\mathbf{r}})$$

$$\mathbf{E}(\vec{\mathbf{r}}) = -j\omega \left[ \mathbf{A}(\vec{\mathbf{r}}) - A_r(\vec{\mathbf{r}}) \hat{i}_r \right]$$

$$\nabla \nabla \cdot \mathbf{A}(\vec{\mathbf{r}}) \approx -\beta^2 A_r(\vec{\mathbf{r}}) \hat{i}_r$$



$$\mathbf{E} = -j\omega \mathbf{A} + \frac{\nabla \nabla \cdot \mathbf{A}}{j\omega \epsilon \mu} \approx -j\omega \mathbf{A} - \frac{\beta^2 A_r \hat{i}_r}{j\omega \epsilon \mu} = -j\omega \mathbf{A} + j\omega A_r \hat{i}_r = -j\omega \left[ \mathbf{A} - A_r \hat{i}_r \right]$$

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$$\mathbf{E}(\vec{\mathbf{r}}) = -j\omega \left[ \mathbf{A}(\vec{\mathbf{r}}) - A_r(\vec{\mathbf{r}}) \hat{i}_r \right]$$

$$\nabla \times \mathbf{A}(\vec{\mathbf{r}}) \approx -j\beta \hat{i}_r \times \mathbf{A}(\vec{\mathbf{r}})$$



$$\begin{aligned} \zeta \mathbf{H} &= \frac{\zeta}{\mu} \nabla \times \mathbf{A} \approx \frac{\zeta}{\mu} (-j\beta) \hat{i}_r \times \mathbf{A} = \frac{\zeta \sqrt{\mu} \sqrt{\epsilon}}{\mu} \hat{i}_r \times [-j\omega \mathbf{A}] = \\ &= \hat{i}_r \times [-j\omega \mathbf{A}] = \hat{i}_r \times \left[ -j\omega (\mathbf{A} - A_r \hat{i}_r) \right] = \hat{i}_r \times \mathbf{E} \end{aligned}$$

$$\mathbf{H} = \frac{1}{\mu} \nabla \times \mathbf{A} \quad \text{and} \quad \mathbf{E} = -j\omega \mathbf{A} + \frac{\nabla \nabla \cdot \mathbf{A}}{j\omega \epsilon \mu}$$

$$\nabla \nabla \cdot \mathbf{A}(\vec{\mathbf{r}}) \approx -\beta^2 A_r(\vec{\mathbf{r}}) \hat{i}_r$$

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$$\mathbf{E}(\vec{\mathbf{r}}) = -j\omega \left[ \mathbf{A}(\vec{\mathbf{r}}) - A_r(\vec{\mathbf{r}}) \hat{i}_r \right]$$

$$\zeta \mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

$$\nabla \times \mathbf{A}(\vec{\mathbf{r}}) \approx -j\beta \hat{i}_r \times \mathbf{A}(\vec{\mathbf{r}})$$



$$\begin{aligned} \zeta \mathbf{H} &= \frac{\zeta}{\mu} \nabla \times \mathbf{A} \approx \frac{\zeta}{\mu} (-j\beta) \hat{i}_r \times \mathbf{A} = \frac{\zeta \sqrt{\mu} \sqrt{\epsilon}}{\mu} \hat{i}_r \times [-j\omega \mathbf{A}] = \\ &= \hat{i}_r \times [-j\omega \mathbf{A}] = \hat{i}_r \times \left[ -j\omega \left( \mathbf{A} - A_r \hat{i}_r \right) \right] = \hat{i}_r \times \mathbf{E} \end{aligned}$$

$$\mathbf{H} = \frac{1}{\mu} \nabla \times \mathbf{A} \quad \text{and} \quad \mathbf{E} = -j\omega \mathbf{A} + \frac{\nabla \nabla \cdot \mathbf{A}}{j\omega \epsilon \mu}$$

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# Extended antennas

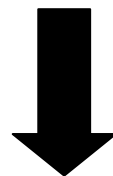
$$\begin{aligned}
 r &\gg D \\
 r &> \frac{2D^2}{\lambda} \\
 r &\gg \lambda
 \end{aligned}$$

Fraunhofer region

$$\begin{aligned}
 \mathbf{M}(\vartheta, \varphi) &= \int \mathbf{J}(\vec{\mathbf{r}}') e^{j\beta \vec{\mathbf{r}}' \cdot \hat{\mathbf{i}}_r} d\vec{\mathbf{r}}' \\
 \mathbf{A}(\vec{\mathbf{r}}) &\approx \frac{\mu}{4\pi} \frac{e^{-j\beta r}}{r} \mathbf{M}(\vartheta, \varphi)
 \end{aligned}$$



$$\left\{ \begin{aligned}
 \nabla \nabla \cdot \mathbf{A}(\vec{\mathbf{r}}) &\approx -\beta^2 A_r(\vec{\mathbf{r}}) \hat{\mathbf{i}}_r \\
 \nabla \times \mathbf{A}(\vec{\mathbf{r}}) &\approx -j\beta \hat{\mathbf{i}}_r \times \mathbf{A}(\vec{\mathbf{r}})
 \end{aligned} \right.$$



$$\left\{ \begin{aligned}
 \mathbf{E}(\vec{\mathbf{r}}) &= -j\omega \left[ \mathbf{A}(\vec{\mathbf{r}}) - A_r(\vec{\mathbf{r}}) \hat{\mathbf{i}}_r \right] = \frac{-j\omega\mu}{4\pi} \frac{e^{-j\beta r}}{r} \left[ \mathbf{M}(\vartheta, \varphi) - M_r(\vartheta, \varphi) \hat{\mathbf{i}}_r \right] \\
 \zeta \mathbf{H}(\vec{\mathbf{r}}) &= \hat{\mathbf{i}}_r \times \mathbf{E}(\vec{\mathbf{r}})
 \end{aligned} \right.$$

# Extended antennas & Fraunhofer region

$$r \gg D$$

$$r > \frac{2D^2}{\lambda}$$

$$r \gg \lambda$$

- the e.m. field propagates along  $\hat{i}_r$
- the e.m. field lies on the plane orthogonal to the propagation direction
- $|E|$  and  $|H|$  exhibit the decaying factor  $1/r$
- $|E|$  and  $|H|$  are proportional through  $\zeta$

Fraunhofer region

$$\begin{cases} \mathbf{E}(\vec{\mathbf{r}}) = -j\omega \left[ \mathbf{A}(\vec{\mathbf{r}}) - A_r(\vec{\mathbf{r}})\hat{i}_r \right] = \frac{-j\omega\mu}{4\pi} \frac{e^{-j\beta r}}{r} \left[ \mathbf{M}(\vartheta, \varphi) - M_r(\vartheta, \varphi)\hat{i}_r \right] \\ \zeta \mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}}) \end{cases}$$

# Field regions

- *Far-field (Fraunhofer) region* is defined as “that region of the field of an antenna where the angular field distribution is essentially independent of the distance from the antenna. If the antenna has a maximum overall dimension  $D$  ( $D > \lambda$ ), the far-field region is commonly taken to exist at distances greater than  $2D^2/\lambda$  from the antenna,  $\lambda$  being the wavelength”.
- In this region, the field components are essentially transverse

# Fraunhofer region

$$r \gg D$$

$$r > \frac{2D^2}{\lambda}$$

$$r \gg \lambda$$

$$\begin{cases} \mathbf{E}(\vec{\mathbf{r}}) = \frac{-j\omega\mu}{4\pi} \frac{e^{-j\beta r}}{r} [\mathbf{M}(\vartheta, \varphi) - M_r(\vartheta, \varphi) \hat{i}_r] \\ \zeta \mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}}) \end{cases}$$

$$\text{I)} \quad \mathbf{S} = \frac{1}{2\zeta} |\mathbf{E}|^2 \hat{i}_r$$

$$\text{II)} \quad \mathbf{E}(\vec{\mathbf{r}}) = \mathbf{E}(r, \vartheta, \varphi) = \frac{j\zeta I}{2\lambda} \frac{e^{-j\beta r}}{r} \mathbf{l}(\vartheta, \varphi)$$

where

$I$  is the input current at the antenna input terminals

$\mathbf{l}(\vartheta, \varphi) = l_\vartheta(\vartheta, \varphi) \hat{i}_\vartheta + l_\varphi(\vartheta, \varphi) \hat{i}_\varphi$  is said **effective length** of the antenna

# Field regions

$$\mathbf{A}(\vec{\mathbf{r}}) = \frac{\mu}{4\pi} \int \mathbf{J}(\vec{\mathbf{r}}') \frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} d\vec{\mathbf{r}}' \quad |\vec{\mathbf{r}}-\vec{\mathbf{r}}'| = r - r' \cos \gamma + \frac{(r')^2}{2r} \sin^2 \gamma + \dots$$

$$\frac{1}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} = \frac{1}{r - r' \cos \gamma + \frac{(r')^2}{2r} \sin^2 \gamma + \dots} \approx \frac{1}{r} \quad \text{if } D \ll r$$

$$e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} = e^{-j\beta r} e^{j\beta r' \cos \gamma} e^{-j\beta \frac{(r')^2}{2r} \sin^2 \gamma} e^{-j\beta \frac{(r')^3}{2r^2} \cos \gamma \sin^2 \gamma} \dots$$

$$\approx e^{-j\beta r} e^{j\beta \vec{\mathbf{r}}' \cdot \hat{\mathbf{i}}_r} e^{-j\beta \frac{(r')^2}{2r} \sin^2 \gamma} \quad \text{if } r > 0.62 \sqrt{\frac{D^3}{\lambda}}$$

## Fresnel region

# Fresnel region

- *Radiating near-field (Fresnel) region* is defined as “that region of the field of an antenna between the reactive near-field region and the far-field region.”
- The inner boundary is taken to be the distance  $R \geq 0.62 \sqrt{D^3/\lambda}$  and the outer boundary the distance  $R < 2D^2/\lambda$  where  $D$  is the largest dimension of the antenna. ( $D$  is meant to be greater than  $\lambda$ )

## Field regions

The space surrounding an antenna is usually subdivided into three regions: (a) reactive near-field, (b) radiating near-field (*Fresnel*) and (c) far-field (*Fraunhofer*) regions as shown in Figure.

