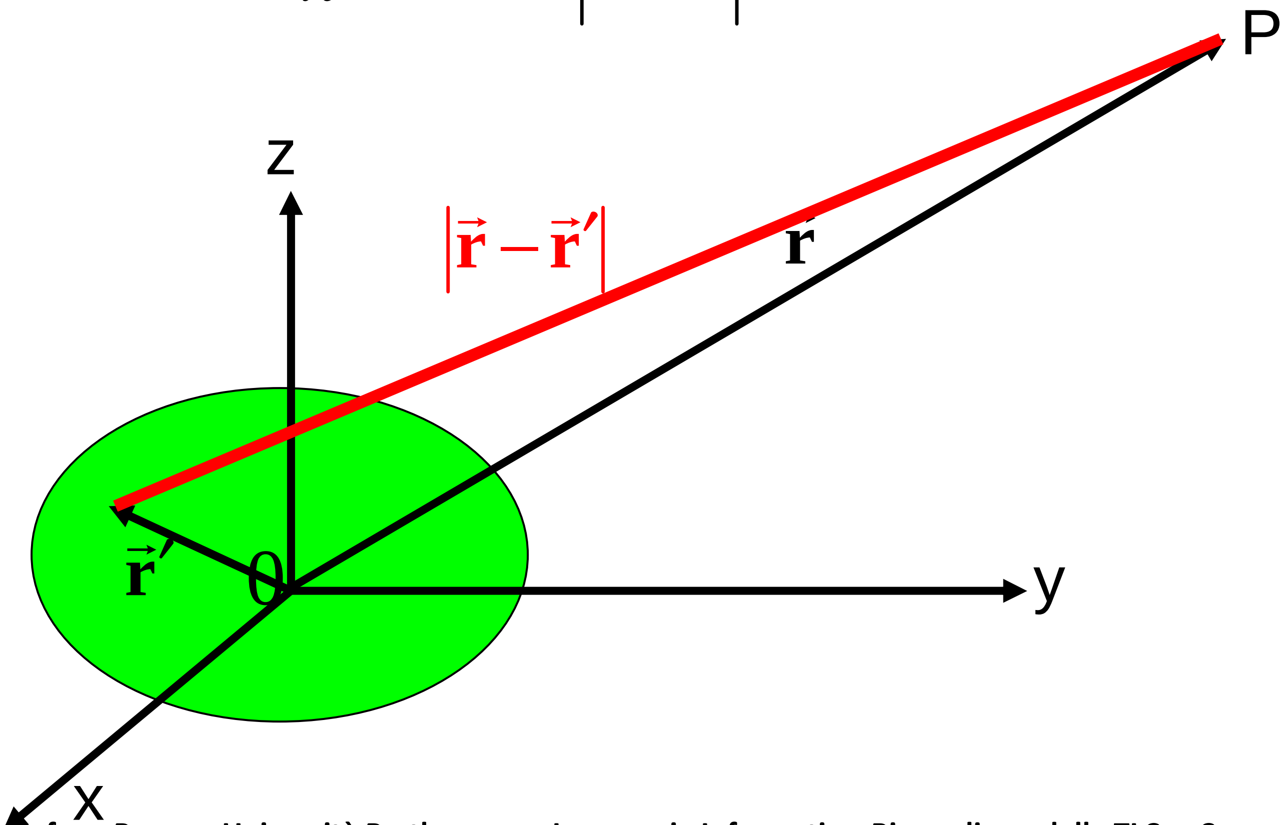
Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

Corso di Campi Elettromagnetici
a.a. 2017-2018

14 Maggio 2018

Summary of the past lecture

$$\mathbf{A}(\vec{\mathbf{r}}) = \frac{\mu}{4\pi}\int \mathbf{J}(\vec{\mathbf{r}}')\frac{e^{-jk|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}d\vec{\mathbf{r}}'$$



Summary of the past lecture

$\downarrow$ $\boldsymbol{J}(\boldsymbol{r})$

$$\nabla^2 \boldsymbol{A} + k^2 \boldsymbol{A} = -\mu \boldsymbol{J}$$ Vector Helmholtz equation

$$\downarrow \quad \mathbf{A}(\mathbf{r}) = \frac{\mu}{4\pi} \int \mathbf{J}(\mathbf{r}') \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r}'$$

$$\boldsymbol{H} = \frac{1}{\mu} \nabla \times \boldsymbol{A} \quad \text{and} \quad \boldsymbol{E} = -j\omega \boldsymbol{A} + \frac{\nabla\nabla \cdot \boldsymbol{A}}{j\omega\varepsilon\mu}$$

$\downarrow$ $\boldsymbol{E}(\boldsymbol{r})$, $\boldsymbol{H}(\boldsymbol{r})$

Summary of the past lecture

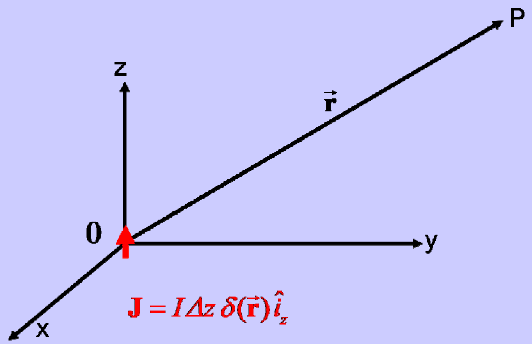
Elementary electrical dipole

$$\mathbf{J} = I\Delta z\,\delta(\vec{\mathbf{r}})\,\hat{i}_z$$

• Why are we interested in such a radiating element?

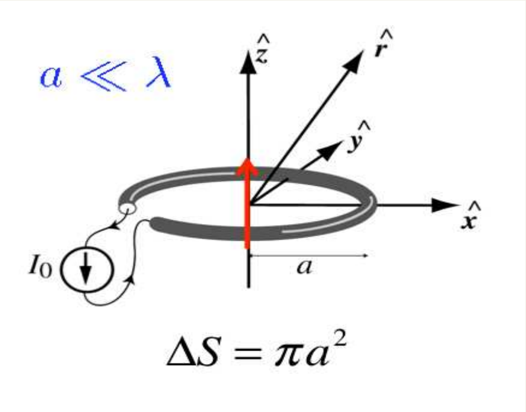
• Why is such a radiating element referred to as elementary electrical dipole?

• How can we physically approximate an elementary electrical dipole?



Small loop antenna

$$\mathbf{J} = I\delta(z)\,\delta(r-a)\hat{i}_\varphi$$



Summary of the past lecture

$\downarrow$ $\boldsymbol{J}(\boldsymbol{r})$

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$\downarrow$ $\boldsymbol{E}(\boldsymbol{r})$, $\boldsymbol{H}(\boldsymbol{r})$

Summary of the past lecture

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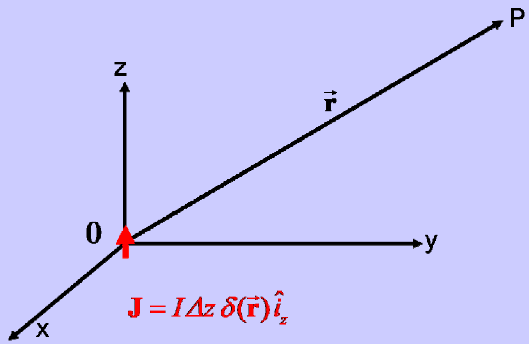
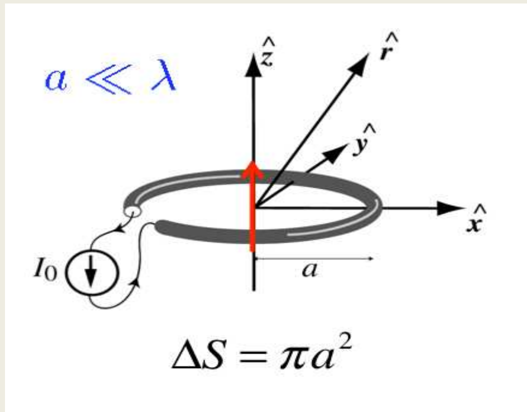


Small loop antenna

$$\mathbf{J} = I \delta(z) \, \delta(r - a) \hat{i}'_\varphi$$

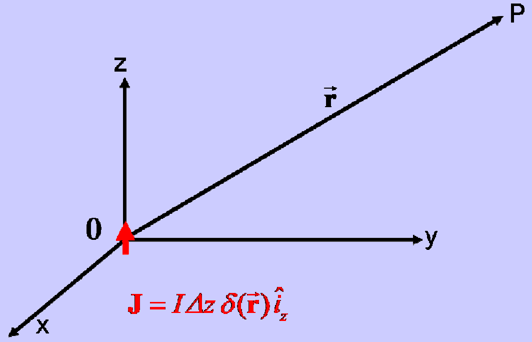


Summary of the past lecture

Elementary electrical dipole

$$\mathbf{J} = I\Delta z\, \delta(\vec{\mathbf{r}})\, \hat{i}_z$$

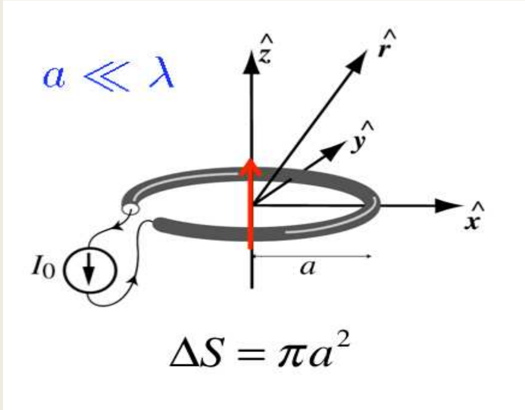
$$\begin{cases} E_r = \zeta \dfrac{I\Delta z}{2\pi}\left(\dfrac{1}{r^2} + \dfrac{1}{j\beta r^3}\right)\cos\vartheta \exp(-j\beta r) \\ E_\vartheta = \zeta \dfrac{I\Delta z}{4\pi}\left(\dfrac{j\beta}{r} + \dfrac{1}{r^2} + \dfrac{1}{j\beta r^3}\right)\sin\vartheta \exp(-j\beta r) \\ H_\varphi = \dfrac{I\Delta z}{4\pi}\left(\dfrac{j\beta}{r} + \dfrac{1}{r^2}\right)\sin\vartheta \exp(-j\beta r) \end{cases}$$



Small loop antenna

$$\mathbf{J} = I\delta(z)\,\delta(r-a)\,\hat{i}'_\varphi$$

$$\begin{cases} H_r = \dfrac{I\Delta S}{2\pi}\left(\dfrac{j\beta}{r^2} + \dfrac{1}{r^3}\right)\cos\vartheta \exp(-j\beta r) \\ H_\vartheta = \dfrac{I\Delta S}{4\pi}\left(\dfrac{(j\beta)^2}{r} + \dfrac{j\beta}{r^2} + \dfrac{1}{r^3}\right)\sin\vartheta \exp(-j\beta r) \\ E_\varphi = -\dfrac{\zeta I\Delta S}{4\pi}\left(\dfrac{(j\beta)^2}{r} + \dfrac{j\beta}{r^2}\right)\sin\vartheta \exp(-j\beta r) \end{cases}$$



Summary of the past lecture

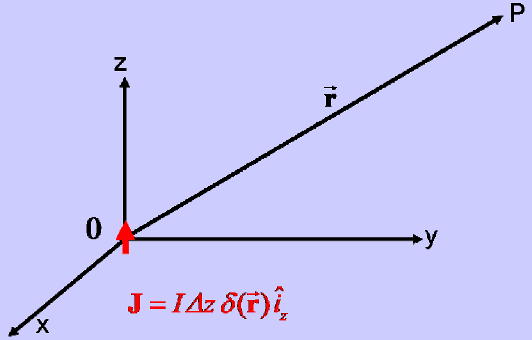
Elementary electrical dipole

$$\mathbf{J} = I\,\Delta z\,\delta(\vec{\mathbf{r}})\,\hat{i}_z$$

for $r >> \lambda$

$$\mathbf{E} = \frac{j\zeta I}{2\lambda}\frac{\exp(-j\beta r)}{r}\Delta z \sin\vartheta\;\hat{i}_\vartheta$$

$$\zeta\mathbf{H} = \hat{i}_r \times \mathbf{E}$$



Small loop antenna

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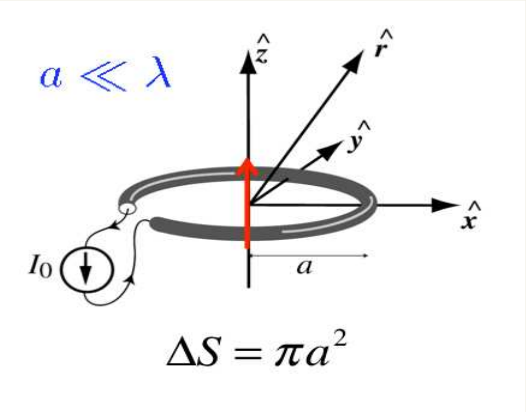

Summary of the past lectures

$$P = \frac{1}{2} \oiint_S \left[\mathbf{E} \times \mathbf{H}^* \right] \cdot \hat{i}_r \, dS$$

Elementary electrical dipole

$$P = P_1 + jP_2$$

$$P_1 = \frac{1}{2} \frac{2\pi}{3} \zeta \left(\frac{\Delta z}{\lambda} \right)^2 |I|^2$$

$$P_2 = -\frac{1}{2} \frac{2\pi}{3} \zeta \left(\frac{\Delta z}{\lambda} \right)^2 \frac{1}{(\beta r)^3} |I|^2$$

Small loop antenna

$$P = P_1 + jP_2$$

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Small loop antenna

WHY?

Magnetic Sources

James Clerk Maxwell 1831-1879

$$\begin{cases} \nabla \times \mathbf{E} = -j\omega\mu\mathbf{H} \\ \nabla \times \mathbf{H} = j\omega\varepsilon\mathbf{E} + \mathbf{J} \\ \nabla \cdot \varepsilon\mathbf{E} = \rho \\ \nabla \cdot \mu\mathbf{H} = 0 \end{cases}$$

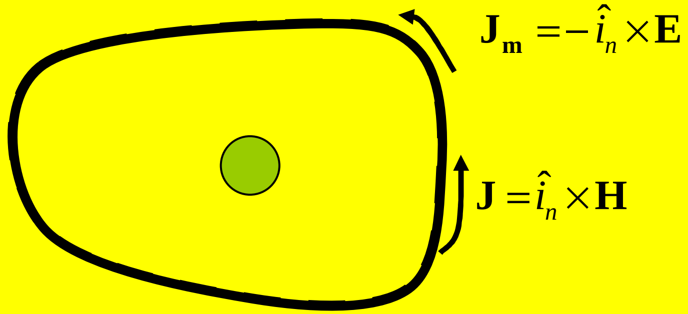
Magnetic Sources

James Clerk Maxwell 1831-1879

$$\begin{cases} \nabla \times \mathbf{E} = -j\omega\mu\mathbf{H} - \mathbf{J}_{\mathrm{m}} \\ \nabla \times \mathbf{H} = j\omega\varepsilon\mathbf{E} + \mathbf{J} \\ \nabla \cdot \varepsilon\mathbf{E} = \rho \\ \nabla \cdot \mu\mathbf{H} = \rho_{\mathrm{m}} \end{cases}$$

Magnetic Sources

Equivalence theorem



$$
\begin{cases}
\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H} - \mathbf{J}_{\mathrm{m}} \\
\nabla \times \mathbf{H} = j\omega\varepsilon\mathbf{E} + \mathbf{J} \\
\nabla \cdot \varepsilon\mathbf{E} = \rho \\
\nabla \cdot \mu\mathbf{H} = \rho_{\mathrm{m}}
\end{cases}
$$

Exploitation of the **equivalence theorem** requires to manage electrical as well as magnetic sources

Magnetic Sources

What is the relation between sources and fields in this case?

$$\begin{cases} \nabla \times \mathbf{E} = -j\omega\mu\mathbf{H} - \mathbf{J}_{\mathrm{m}} \\ \nabla \times \mathbf{H} = j\omega\varepsilon\mathbf{E} + \mathbf{J} \\ \nabla \cdot \varepsilon\mathbf{E} = \rho \\ \nabla \cdot \mu\mathbf{H} = \rho_{\mathrm{m}} \end{cases}$$

Magnetic Sources

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Let's simplify the question. What is the relation between sources and fields in this case?

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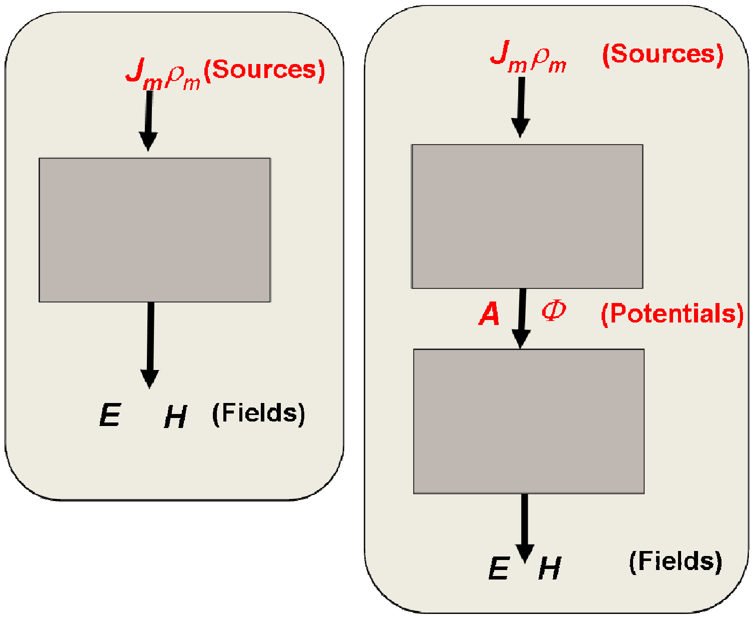

In principle, we could replace the same approach as that exploited for the electric sources

Magnetic Sources

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$$\begin{cases} \nabla \times \mathbf{E} = -j\omega\mu\mathbf{H} - \mathbf{J}_m \\ \nabla \times \mathbf{H} = j\omega\varepsilon\mathbf{E} \\ \nabla \cdot \varepsilon\mathbf{E} = 0 \\ \nabla \cdot \mu\mathbf{H} = \rho_m \end{cases}$$

In practice, we follow an easier way, provided by the **duality theorem**

Magnetic Sources

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In practice, we follow an easier way, provided by the **duality theorem**

$$\mathbf{J} \quad \rho \;\longrightarrow\; \mathbf{E}, \mathbf{H}$$

$$\Longrightarrow$$

$$\mathbf{J}_m = -\zeta\mathbf{J} \quad \rho_m = -\zeta\rho \;\longrightarrow\; \mathbf{E}' = \zeta\mathbf{H}, \; \mathbf{H}' = -\frac{1}{\zeta}\mathbf{E}$$

Duality Theorem

$$\mathbf{J}\ \rho \;\longrightarrow\; \mathbf{E},\mathbf{H}$$

$$\Longrightarrow$$

$$\mathbf{J}_{\mathrm{m}} = -\zeta\mathbf{J} \qquad \rho_{\mathrm{m}} = -\zeta\rho \;\longrightarrow\; \mathbf{E}' = \zeta\mathbf{H}\ ,\ \mathbf{H}' = -\frac{1}{\zeta}\mathbf{E}$$

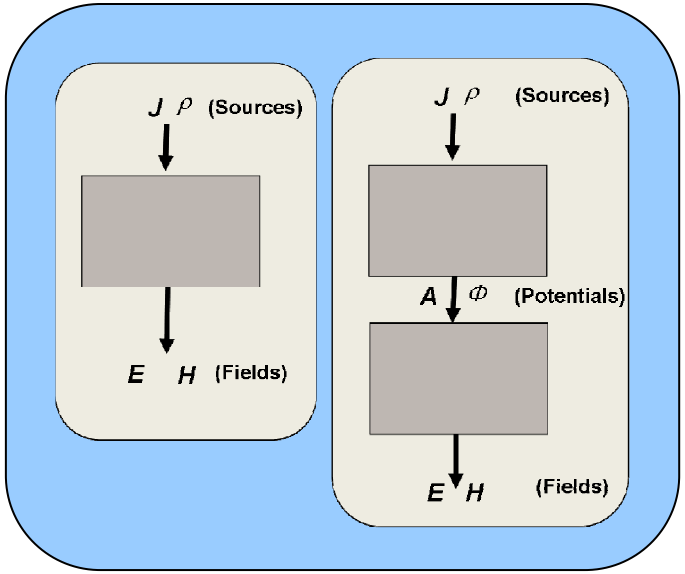
$$\mathbf{J}_{\mathrm{m}}\quad \rho_{\mathrm{m}}$$

$$\downarrow$$

$$\mathbf{J} = -\frac{1}{\zeta}\mathbf{J}_{\mathrm{m}}$$

$$\rho = -\frac{1}{\zeta}\rho_{\mathrm{m}}$$

$$\rightarrow$$



$$\rightarrow$$

$$\mathbf{E}' = \zeta\mathbf{H}$$

$$\mathbf{H}' = -\frac{1}{\zeta}\mathbf{E}$$

Elementary electrical and magnetic dipoles

Elementary electrical dipole

$$\mathbf{J} = I\Delta z\,\delta(\vec{\mathbf{r}})\,\hat{i}_z = I\Delta z\,\delta(x)\delta(y)\delta(z)\,\hat{i}_z$$

$$I\Delta z = j\omega Q\Delta z = j\omega U$$

Elementary magnetic dipole

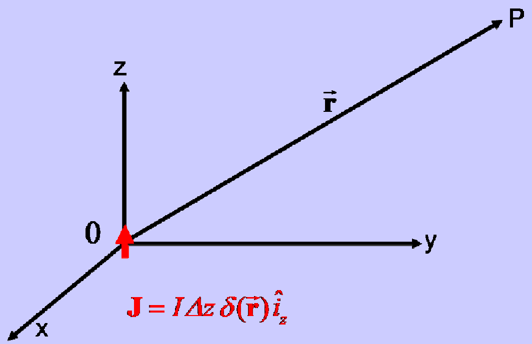
$$\mathbf{J}_m = I_m\Delta z\,\delta(\vec{\mathbf{r}})\,\hat{i}_z = I_m\Delta z\,\delta(x)\delta(y)\delta(z)\,\hat{i}_z$$

$$I_m\Delta z = j\omega U_m$$

Elementary electrical and magnetic dipoles

Elementary electrical dipole

$$\mathbf{J} = I\,\Delta z\,\delta(\vec{\mathbf{r}})\,\hat{i}_z$$



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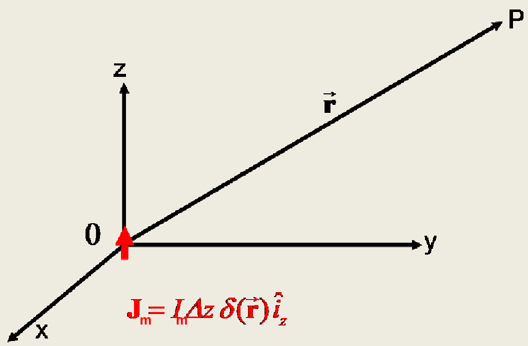
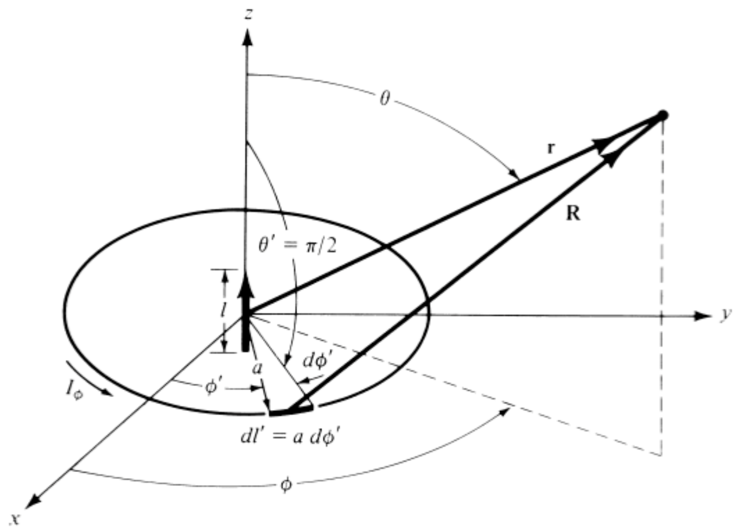


Elementary electrical and magnetic dipoles

Ampere equivalence theorem

By invoking the Duality theorem it is possible to demonstrate that the small loop antenna is equivalent to an elementary magnetic dipole, provided that:



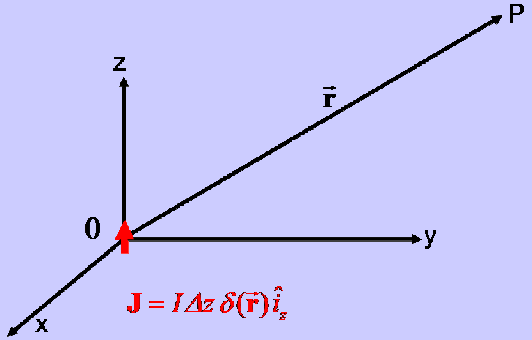
Geometry for circular loop

$$U_m = \mu I \Delta S$$

Elementary electrical and magnetic dipoles

Elementary electrical dipole

$$\mathbf{J} = I\,\Delta z\,\delta(\vec{\mathbf{r}})\,\hat{i}_z$$



- Why are we interested in such a radiating element?
- Why is such a radiating element referred to as elementary electrical dipole?
- How can we physically approximate an elementary electrical dipole?

Elementary magnetic dipole

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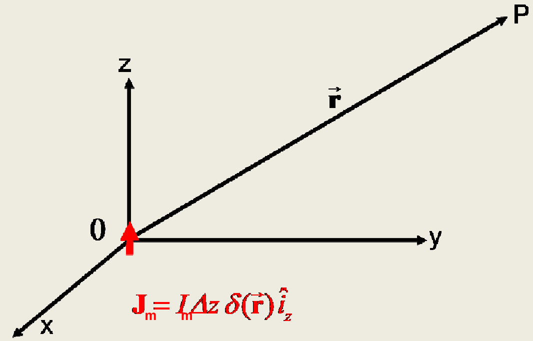

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References

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- C.A.Balanis, *Antenna Theory*, John Wiley, 1982.
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