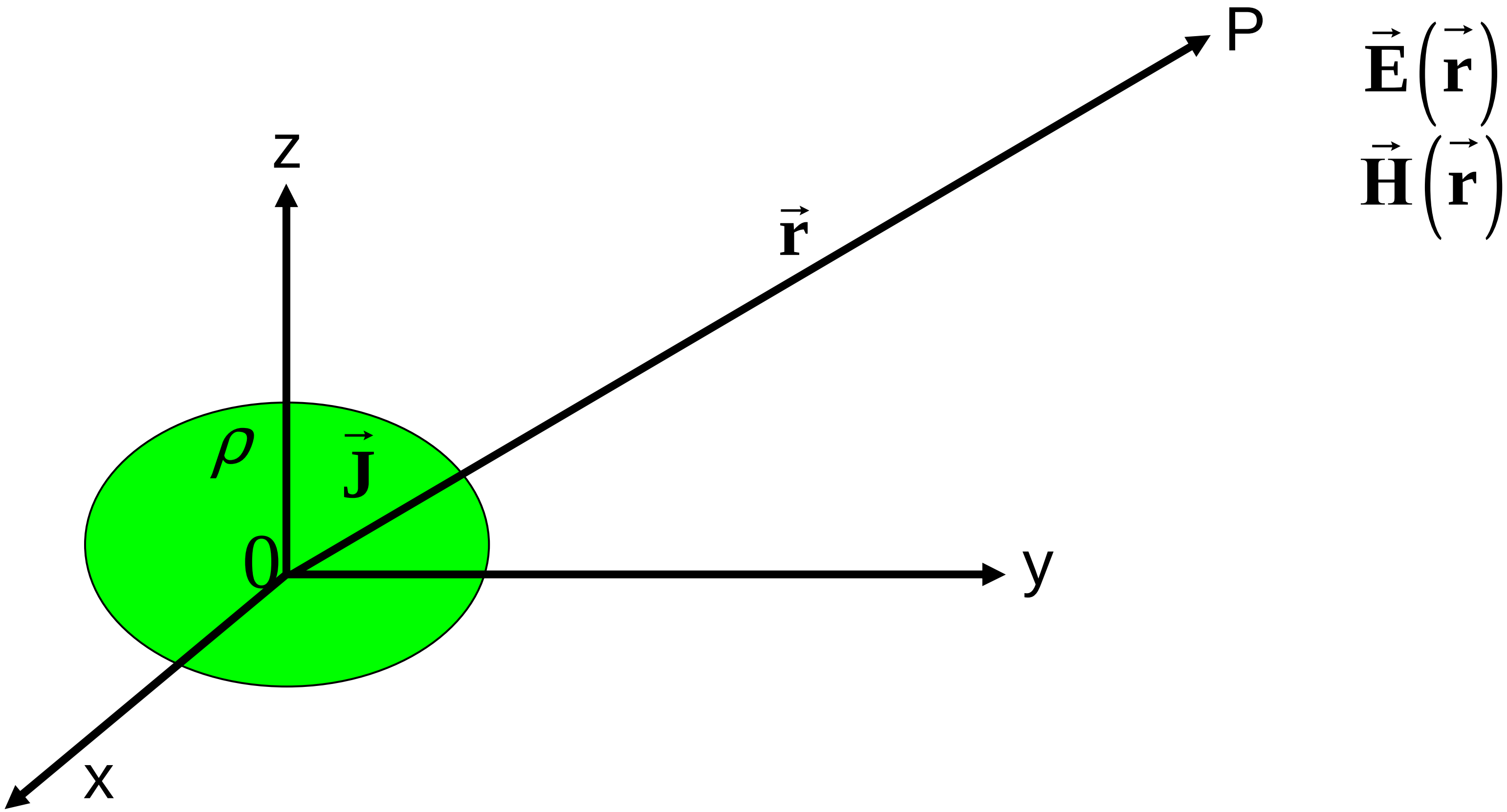
Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

Corso di Campi Elettromagnetici
a.a. 2017-2018

11 Maggio 2018

Summary of the past lecture
Radiation problem

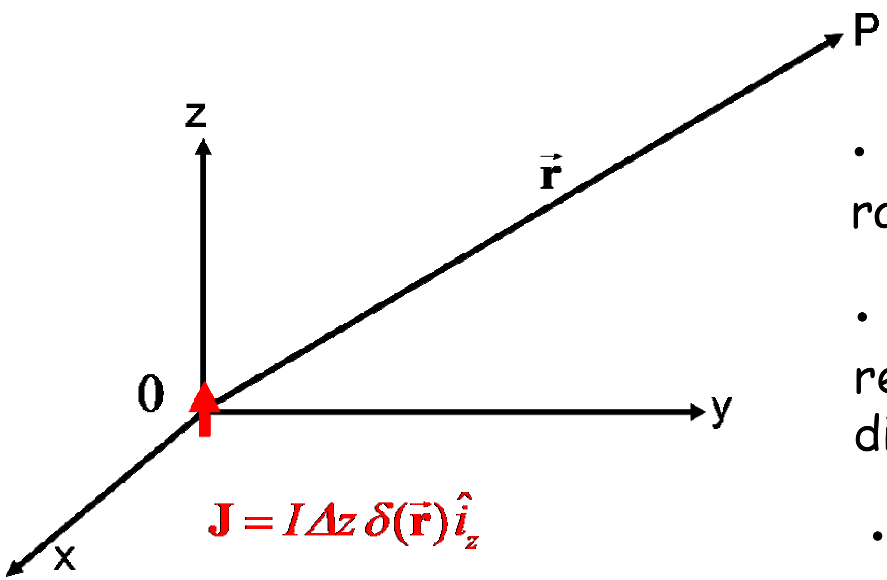


Summary of the past lecture

Elementary electrical dipole

- A δ-source radiating element is also known as elementary electrical dipole.

$$\mathbf{J} = I\Delta z\,\delta(\vec{\mathbf{r}})\,\hat{i}_z = I\Delta z\,\delta(x)\delta(y)\delta(z)\,\hat{i}_z$$



- Why are we interested in such a radiating element?
- Why is such a radiating element referred to as elementary electrical dipole?
- How can we physically approximate an elementary electrical dipole?

Summary of the past lecture

Elementary electrical dipole

The E.M. field radiated by the elementary electrical dipole

$$
\begin{cases}
E_r = \zeta \dfrac{I\Delta z}{2\pi}\left(\dfrac{1}{r^2} + \dfrac{1}{j\beta r^3}\right)\cos\vartheta \exp(-j\beta r) \\
E_\vartheta = \zeta \dfrac{I\Delta z}{4\pi}\left(\dfrac{j\beta}{r} + \dfrac{1}{r^2} + \dfrac{1}{j\beta r^3}\right)\sin\vartheta \exp(-j\beta r) \\
H_\varphi = \dfrac{I\Delta z}{4\pi}\left(\dfrac{j\beta}{r} + \dfrac{1}{r^2}\right)\sin\vartheta \exp(-j\beta r)
\end{cases}
$$

… for $\omega = 0$ simplifies as

$$
\begin{cases}
E_r = \dfrac{Q\Delta z}{2\pi}\dfrac{1}{\varepsilon r^3}\cos\vartheta \\
E_\vartheta = \dfrac{Q\Delta z}{4\pi}\dfrac{1}{\varepsilon r^3}\sin\vartheta \\
H_\varphi = 0
\end{cases}
$$

Summary of the past lecture

Elementary electrical dipole

…the E.M. field radiated by the elementary electrical dipole..

$$\begin{cases} E_r = \zeta \dfrac{I\Delta z}{2\pi}\left(\dfrac{1}{r^2}+\dfrac{1}{j\beta r^3}\right)\cos\vartheta \exp(-j\beta r) \\ E_\vartheta = \zeta \dfrac{I\Delta z}{4\pi}\left(\dfrac{j\beta}{r}+\dfrac{1}{r^2}+\dfrac{1}{j\beta r^3}\right)\sin\vartheta \exp(-j\beta r) \\ H_\varphi = \dfrac{I\Delta z}{4\pi}\left(\dfrac{j\beta}{r}+\dfrac{1}{r^2}\right)\sin\vartheta \exp(-j\beta r) \end{cases}$$

… for $r \gg \lambda$ simplifies as

$$\begin{cases} E_r = 0 \\ E_\vartheta = j\zeta \dfrac{I\Delta z}{2\lambda r}\sin\vartheta \exp(-j\beta r) \\ H_\varphi = j\dfrac{I\Delta z}{2\lambda r}\sin\vartheta \exp(-j\beta r) = \dfrac{E_\vartheta}{\zeta} \end{cases}$$

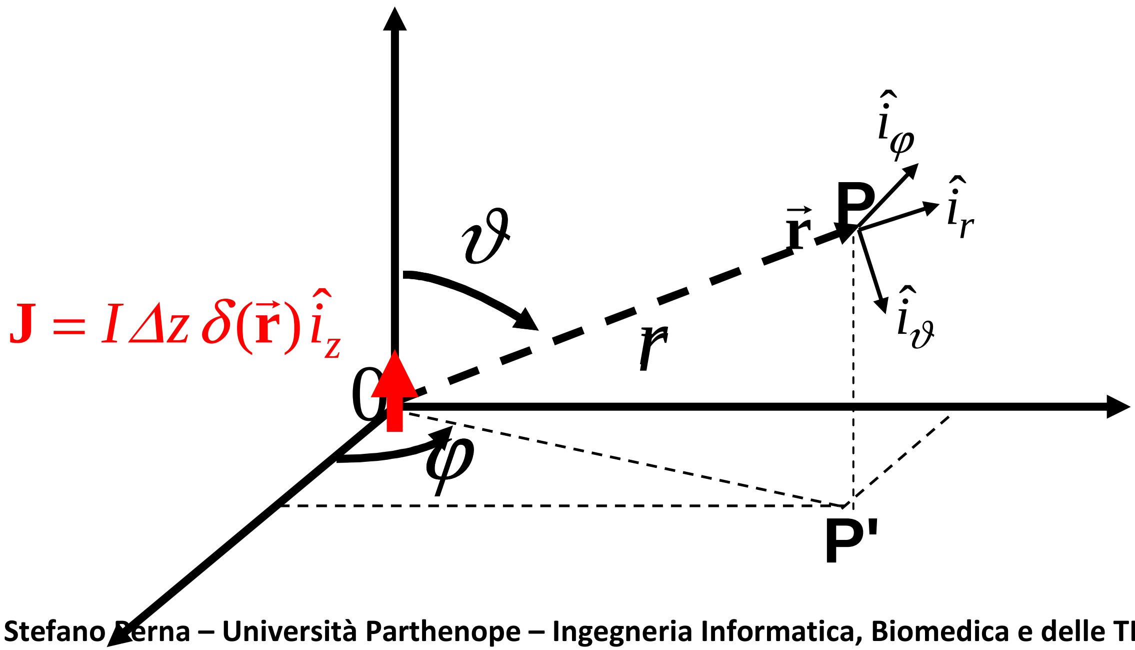
Elementary electrical dipole

for $r >> \lambda$

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathrm{E}_{\vartheta}(r,\vartheta)\hat{i}_{\vartheta}$$

$$\vec{\mathbf{H}} = \vec{\mathbf{H}}(\vec{\mathbf{r}}) = \mathrm{H}_{\varphi}(r,\vartheta)\hat{i}_{\varphi}$$

$$\begin{cases} E_{\vartheta} = j\zeta \dfrac{I\Delta z}{2\lambda r}\sin\vartheta \exp(-j\beta r) \\ H_{\varphi} = j\dfrac{I\Delta z}{2\lambda r}\sin\vartheta \exp(-j\beta r) = \dfrac{E_{\vartheta}}{\zeta} \end{cases}$$



- the e.m. field propagates along $\hat{i}_r$
- the e.m. field lies on the plane orthogonal to the propagation direction

Elementary electrical dipole

In the far-field region, we have

$$\begin{cases} E_r = 0 \\ E_\vartheta = j\zeta \dfrac{I\Delta z}{2\lambda r}\sin\vartheta \exp(-j\beta r) \\ H_\varphi = j\dfrac{I\Delta z}{2\lambda r}\sin\vartheta \exp(-j\beta r) = \dfrac{E_\vartheta}{\zeta} \end{cases} \Rightarrow \quad \zeta\mathbf{H} = \hat{i}_r \times \mathbf{E}$$

$$\hat{i}_\varphi = \hat{i}_r \times \hat{i}_\vartheta$$

$$\hat{i}_\vartheta = \hat{i}_\varphi \times \hat{i}_r$$

$$\hat{i}_r = \hat{i}_\vartheta \times \hat{i}_\varphi$$

and the Poynting vector:

$$\mathbf{S} = \frac{1}{2}\mathbf{E}\times\mathbf{H}^* = \frac{1}{2}\frac{\left|E_\vartheta\right|^2}{\zeta}\hat{i}_r = \frac{1}{2}\frac{\left|\mathbf{E}\right|^2}{\zeta}\hat{i}_r$$

Elementary electrical dipole

- In order to further characterize its behavior one can evaluate the Poynting vector and the associated power for the overall e.m. field (over a sphere centered in the origin):

$$P = \frac{1}{2} \oiint_S \left[\mathbf{E} \times \mathbf{H}^* \right] \cdot \hat{i}_r dS = \frac{1}{2} \int_0^{2\pi} d\varphi \int_0^{\pi} d\vartheta r^2 \sin\vartheta E_\vartheta H_\varphi^*$$

I) $dS = r^2 \sin\vartheta d\vartheta d\varphi$

II) $\left[\mathbf{E} \times \mathbf{H}^* \right] \cdot \hat{i}_r = \left[\left(E_\vartheta \hat{i}_\vartheta + E_r \hat{i}_r \right) \times \left(H_\varphi^* \hat{i}_\varphi \right) \right] \cdot \hat{i}_r = E_\vartheta H_\varphi^*$

$$\begin{cases} E_r = \zeta \dfrac{I \Delta z}{2\pi} \left(\dfrac{1}{r^2} + \dfrac{1}{j\beta r^3} \right) \cos\vartheta \exp(-j\beta r) \\ E_\vartheta = \zeta \dfrac{I \Delta z}{4\pi} \left(\dfrac{j\beta}{r} + \dfrac{1}{r^2} + \dfrac{1}{j\beta r^3} \right) \sin\vartheta \exp(-j\beta r) \\ H_\varphi = \dfrac{I \Delta z}{4\pi} \left(\dfrac{j\beta}{r} + \dfrac{1}{r^2} \right) \sin\vartheta \exp(-j\beta r) \end{cases}$$

$$P=\frac{1}{2}\oiint_S\left[\mathbf{E}\times\mathbf{H}^*\right]\cdot\hat{i}_r dS=\frac{1}{2}\int_0^{2\pi}d\varphi\int_0^{\pi}d\vartheta r^2\sin\vartheta E_\vartheta H_\varphi^* =\frac{1}{2}\frac{2\pi}{3}\zeta\left(\frac{\Delta z}{\lambda}\right)^2\left[1-j\frac{1}{(\beta r)^3}\right]|I|^2$$

$$\begin{cases}E_r=\zeta\frac{I\Delta z}{2\pi}\left(\frac{1}{r^2}+\frac{1}{j\beta r^3}\right)\cos\vartheta\exp(-j\beta r)\\ E_\vartheta=\zeta\frac{I\Delta z}{4\pi}\left(\frac{j\beta}{r}+\frac{1}{r^2}+\frac{1}{j\beta r^3}\right)\sin\vartheta\exp(-j\beta r)\\ H_\varphi=\frac{I\Delta z}{4\pi}\left(\frac{j\beta}{r}+\frac{1}{r^2}\right)\sin\vartheta\exp(-j\beta r)\end{cases}$$

$$P=P_1+jP_2$$

$$P_1=\frac{1}{2}\frac{2\pi}{3}\zeta\left(\frac{\Delta z}{\lambda}\right)^2|I|^2$$

$$P_2=-\frac{1}{2}\frac{2\pi}{3}\zeta\left(\frac{\Delta z}{\lambda}\right)^2\frac{1}{(\beta r)^3}|I|^2$$

$$\left(\frac{j\beta}{r}+\frac{1}{r^2}+\frac{1}{j\beta r^3}\right)\left(\frac{-j\beta}{r}+\frac{1}{r^2}\right)=\left(\frac{\beta}{r}\right)^2-j\frac{1}{\beta r^5}$$

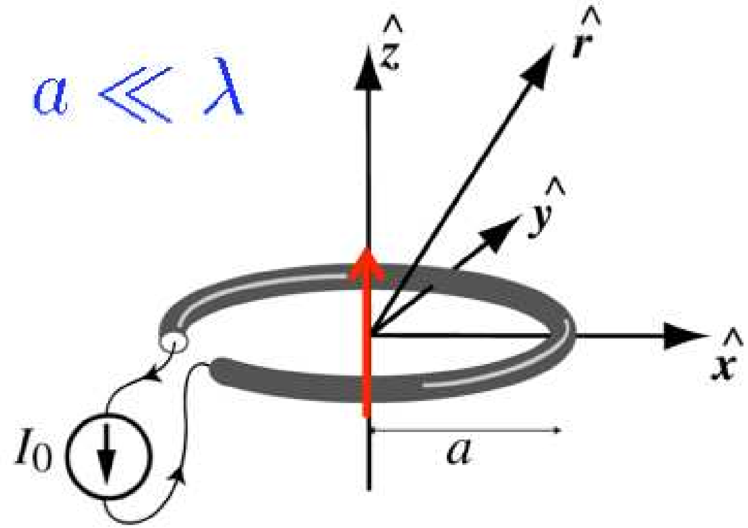
- *Note that in the far-field case only the first active power term exists and it does not depend on r.*

Elementary electrical dipole

- Note that the real part of the power, in lossless medium, is independent of *r*, therefore if one consider two different spherical surfaces one gets the same result. Only the so-called radiative terms contribute.
- The reactive part depends on *r*. Its sign is negative showing that there is an excess of stored **electric** energy in the neighbor of the electrical dipole (see Poynting's theorem)

Small loop antenna

- A simple and inexpensive antenna type is the loop antenna.



$$\Delta S = \pi a^2$$

Small loop antenna

Electrically small antennas are those whose overall length (circumference) is usually less than about one-tenth of a wavelength (C < λ/10).

Small loop antenna

$$\mathbf{J} = I\,\delta(z)\,\delta(r-a)\,\hat{i}_\varphi'$$

$\downarrow$ $\boldsymbol{J}$

$$\nabla^2 \boldsymbol{A} + k^2 \boldsymbol{A} = -\mu \boldsymbol{J}$$ Vector Helmholtz equation

$\downarrow$

$$\mathbf{A}(\mathbf{r}) = \frac{\mu}{4\pi}\int \mathbf{J}(\mathbf{r}')\frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}d\mathbf{r}'$$

$$\boldsymbol{H} = \frac{1}{\mu}\nabla\times\boldsymbol{A} \quad \text{and} \quad \boldsymbol{E} = -j\omega\boldsymbol{A} + \frac{\nabla\nabla\cdot\boldsymbol{A}}{j\omega\varepsilon\mu}$$

$\downarrow$

$\boldsymbol{E}(\boldsymbol{r})$

$\boldsymbol{H}(\boldsymbol{r})$

Small loop antenna

$$\mathbf{J} = I\,\delta(z)\,\delta(r-a)\,\hat{i}'_{\varphi}$$

$$\mathbf{A}(\vec{\mathbf{r}}) = \frac{\mu}{4\pi}\int \mathbf{J}(\vec{\mathbf{r}}')\frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}d\,\vec{\mathbf{r}}' \quad = \frac{\mu}{4\pi}\int_0^{2\pi} d\varphi'\; I\;\frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}a\;\hat{i}'_{\varphi}$$

$$\approx \frac{j\beta I\mu\,\Delta S}{4\pi}\frac{e^{-j\beta r}}{r}\left[1+\frac{1}{j\beta r}\right]\sin\vartheta\;\hat{i}_{\varphi}$$

.. by assuming that the current I in the small loop is constant and that the radius of the loop a<<λ

Small loop antenna

$$\mathbf{J} = I\,\delta(z)\,\delta(r-a)\,\hat{i}'_{\varphi}$$

$\boldsymbol{J}$ ↓

$\nabla^2 \boldsymbol{A} + k^2 \boldsymbol{A} = -\mu \boldsymbol{J}$ Vector Helmholtz equation

↓ $\mathbf{A} \approx \dfrac{j\beta I \mu \Delta S}{4\pi} \dfrac{e^{-j\beta r}}{r} \left[1 + \dfrac{1}{j\beta r}\right] \sin\vartheta\, \hat{\boldsymbol{i}}_{\varphi}$

$\boldsymbol{H} = \dfrac{1}{\mu} \nabla \times \boldsymbol{A}$ and $\boldsymbol{E} = -j\omega \boldsymbol{A} + \dfrac{\nabla\nabla \cdot \boldsymbol{A}}{j\omega\varepsilon\mu}$

↓

$\boldsymbol{E}$ $\boldsymbol{H}$

Small loop antenna

The E.M. field radiated by the small loop antenna

$$\begin{cases} H_r = \dfrac{I\Delta S}{2\pi}\left(\dfrac{j\beta}{r^2}+\dfrac{1}{r^3}\right)\cos\vartheta\exp(-j\beta r) \\ H_\vartheta = \dfrac{I\Delta S}{4\pi}\left(\dfrac{(j\beta)^2}{r}+\dfrac{j\beta}{r^2}+\dfrac{1}{r^3}\right)\sin\vartheta\exp(-j\beta r) \\ E_\varphi = -\dfrac{\zeta I\Delta S}{4\pi}\left(\dfrac{(j\beta)^2}{r}+\dfrac{j\beta}{r^2}\right)\sin\vartheta\exp(-j\beta r) \end{cases}$$

… for r>>λ simplifies as

$$\begin{cases} H_r = 0 \\ H_\vartheta = \dfrac{-E_\varphi}{\zeta} \\ E_\varphi = \dfrac{\zeta\beta\Delta sI}{2\lambda r}\sin\vartheta\exp(-j\beta r) \end{cases}$$

Small loop antenna

In the far-field region, we have

$$\begin{cases} H_r = 0 \\ H_\vartheta = \dfrac{-E_\varphi}{\zeta} \\ E_\varphi = \dfrac{\zeta\beta\Delta sI}{2\lambda r}\sin\vartheta \exp(-j\beta r) \end{cases} \Rightarrow \zeta\mathbf{H} = \hat{i}_r \times \mathbf{E}$$

$$\hat{i}_\varphi = \hat{i}_r \times \hat{i}_\vartheta$$

$$\hat{i}_\vartheta = \hat{i}_\varphi \times \hat{i}_r$$

$$\hat{i}_r = \hat{i}_\vartheta \times \hat{i}_\varphi$$

and the Poynting vector is:

$$\mathbf{S} = \frac{1}{2}\mathbf{E}\times\mathbf{H}^* = \frac{1}{2}\frac{\left|E_\varphi\right|^2}{\zeta}\hat{i}_r = \frac{1}{2}\frac{\left|\mathbf{E}\right|^2}{\zeta}\hat{i}_r$$

Small loop antenna

- Similarly to the elementary electrical dipole, to further characterize the small loop anntenna behavior one can evaluate the Poynting vector and the associated power over a sphere centered in the origin:

$$P = \frac{1}{2}\oiint_S \left[\mathbf{E}\times\mathbf{H}^*\right]\cdot\hat{i}_r dS = -\frac{1}{2}\int_0^{2\pi} d\varphi \int_0^{\pi} d\vartheta r^2 \sin\vartheta E_\varphi H_\vartheta^*$$

$$\hat{i}_\varphi = \hat{i}_r \times \hat{i}_\vartheta$$

$$\hat{i}_\vartheta = \hat{i}_\varphi \times \hat{i}_r$$

$$\hat{i}_r = \hat{i}_\vartheta \times \hat{i}_\varphi$$

I) $dS = r^2 \sin\vartheta d\vartheta d\varphi$

II) $\left[\mathbf{E}\times\mathbf{H}^*\right]\cdot\hat{i}_r = \left[\left(E_\varphi \hat{i}_\varphi\right)\times\left(H_\vartheta^* \hat{i}_\vartheta + H_r^* \hat{i}_r\right)\right]\cdot\hat{i}_r = -E_\varphi H_\vartheta^*$

Small loop antenna

$$P = \frac{1}{2}\oiint_S \left[\mathbf{E}\times\mathbf{H}^*\right]\cdot\hat{i}_r dS = -\frac{1}{2}\int_0^{2\pi} d\varphi \int_0^{\pi} d\vartheta r^2 \sin\vartheta E_\varphi H_\vartheta^* = \frac{1}{2}\frac{2\pi}{3}\zeta\left(\frac{\beta \Delta S}{\lambda}\right)^2\left[1 + j\frac{1}{(\beta r)^3}\right]|I|^2$$

$$\begin{cases} H_r = \dfrac{I\Delta S}{2\pi}\left(\dfrac{j\beta}{r^2}+\dfrac{1}{r^3}\right)\cos\vartheta \exp(-j\beta r) \\ H_\vartheta = \dfrac{I\Delta S}{4\pi}\left(\dfrac{(j\beta)^2}{r}+\dfrac{j\beta}{r^2}+\dfrac{1}{r^3}\right)\sin\vartheta \exp(-j\beta r) \\ E_\varphi = -\dfrac{\zeta I\Delta S}{4\pi}\left(\dfrac{(j\beta)^2}{r}+\dfrac{j\beta}{r^2}\right)\sin\vartheta \exp(-j\beta r) \end{cases}$$

$$P = P_1 + jP_2$$

$$P_1 = \frac{1}{2}\frac{2\pi}{3}\zeta\left(\frac{\beta \Delta S}{\lambda}\right)^2 |I|^2$$

$$P_2 = \frac{1}{2}\frac{2\pi}{3}\zeta\left(\frac{\beta \Delta S}{\lambda}\right)^2 \frac{1}{(\beta r)^3}|I|^2$$

Elementary electrical dipole vs. Small loop antenna

Elementary electrical dipole

$$P = P_1 + jP_2$$

$$P_1 = \frac{1}{2}\frac{2\pi}{3}\zeta\left(\frac{\Delta z}{\lambda}\right)^2 |I|^2$$

$$P_2 = -\frac{1}{2}\frac{2\pi}{3}\zeta\left(\frac{\Delta z}{\lambda}\right)^2 \frac{1}{(\beta r)^3}|I|^2$$

Small loop antenna

$$P = P_1 + jP_2$$

$$P_1 = \frac{1}{2}\frac{2\pi}{3}\zeta\left(\frac{\beta \Delta S}{\lambda}\right)^2 |I|^2$$

$$P_2 = \frac{1}{2}\frac{2\pi}{3}\zeta\left(\frac{\beta \Delta S}{\lambda}\right)^2 \frac{1}{(\beta r)^3}|I|^2$$

Small loop antenna

The reactive part depends on r. Its sign is positive showing that there is an excess of stored **magnetic** energy in the neighbor of the magnetic dipole (see Poynting's theorem)

$$P_r = P_1 + jP_2$$

$$P_1 = \frac{1}{2}\frac{2\pi}{3}\zeta\left(\frac{\beta \Delta S}{\lambda}\right)^2 |I|^2$$

$$P_2 = \frac{1}{2}\frac{2\pi}{3}\zeta\left(\frac{\beta \Delta S}{\lambda}\right)^2 \frac{1}{(\beta r)^3}|I|^2$$

Small loop antenna

WHY?