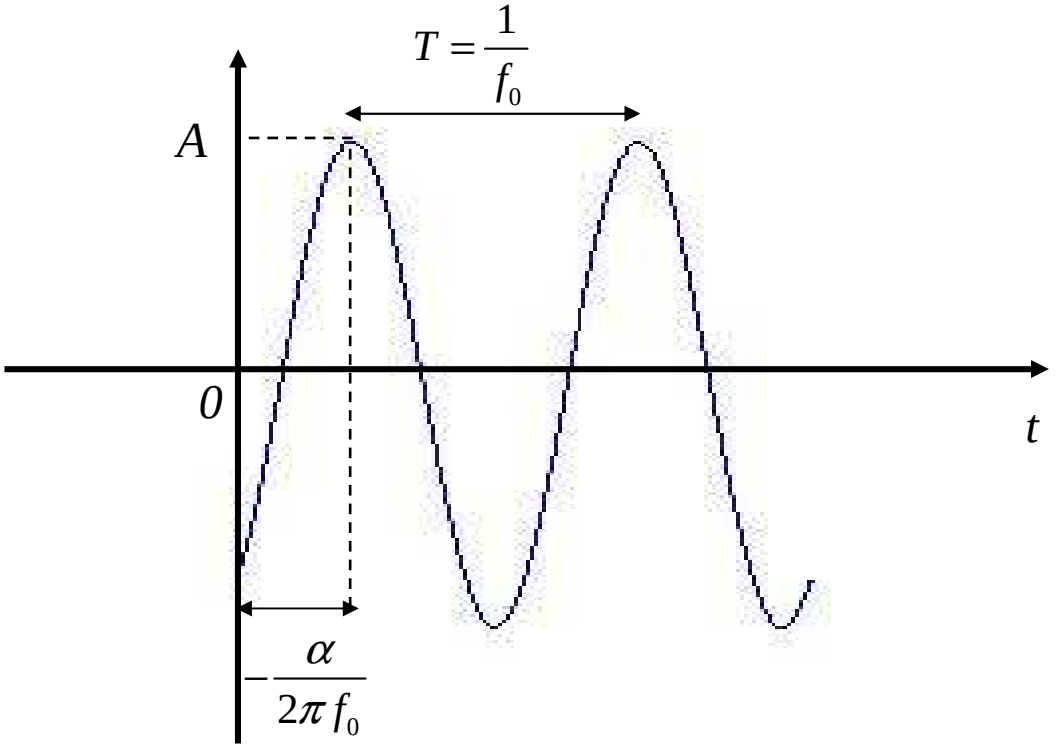
# Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

## Corso di Campi Elettromagnetici
## a.a. 2017-2018

# 15 Marzo 2018

# Sinusoidal signals and phasors

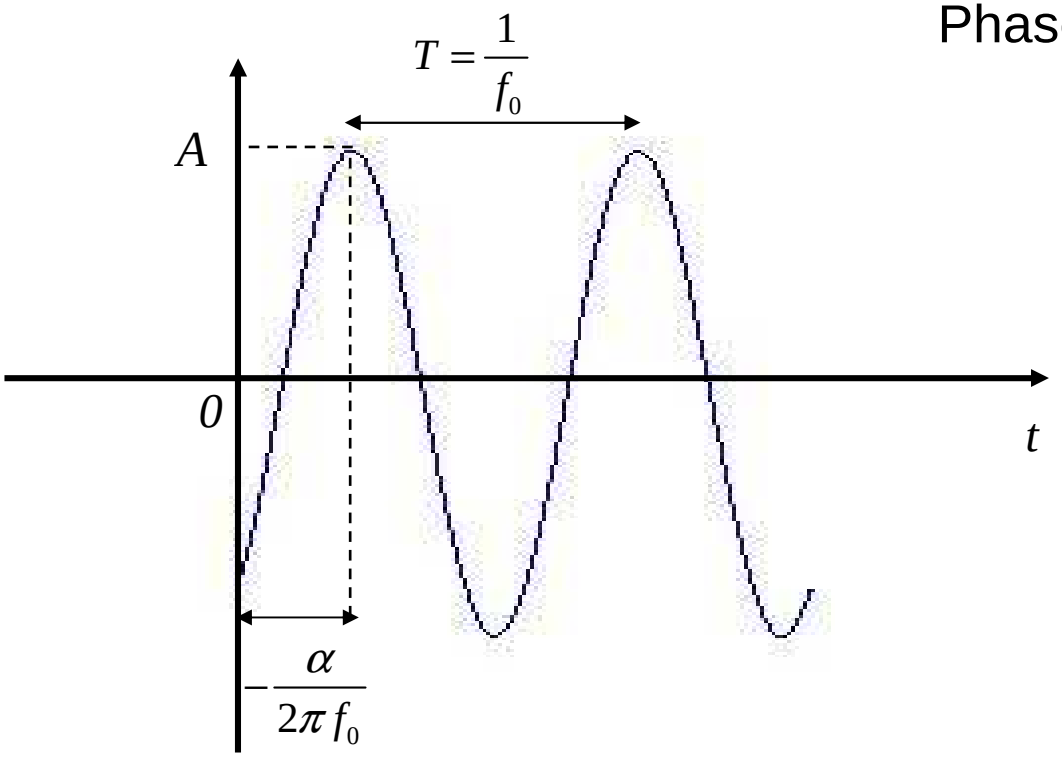
$$v(t) = A\cos\left(2\pi f_0 t + \alpha\right)$$



# Sinusoidal signals and phasors

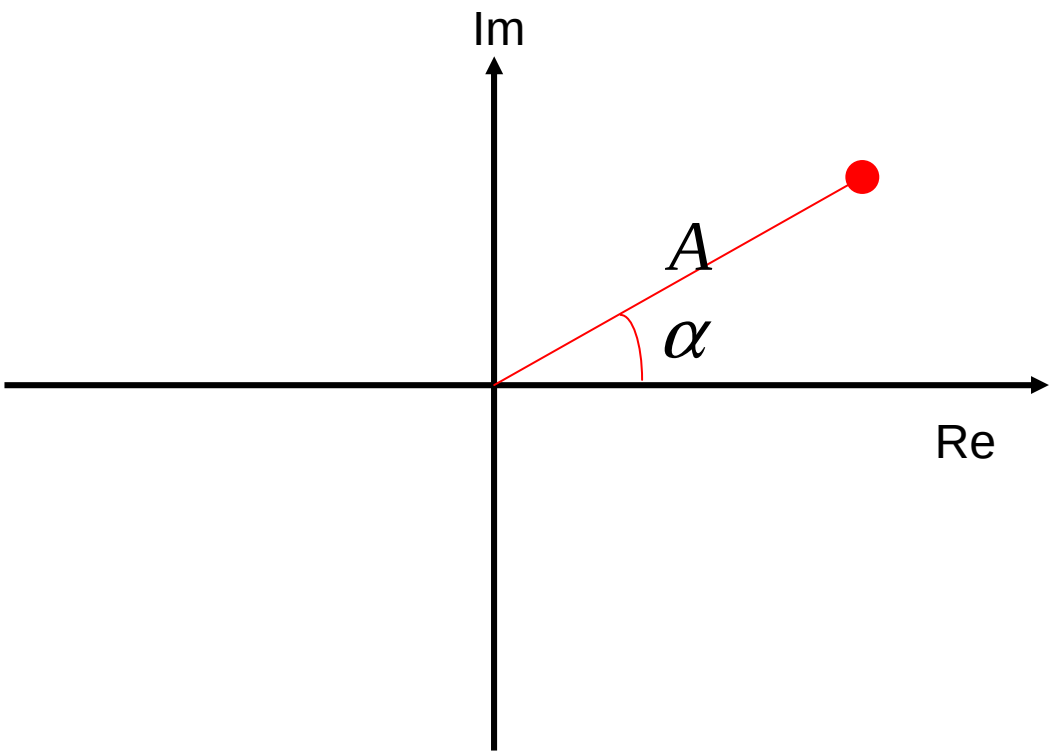
$$v(t) = A\cos\left(2\pi f_0 t + \alpha\right) \longrightarrow V = Ae^{j\alpha}$$

Phasor (complex number)



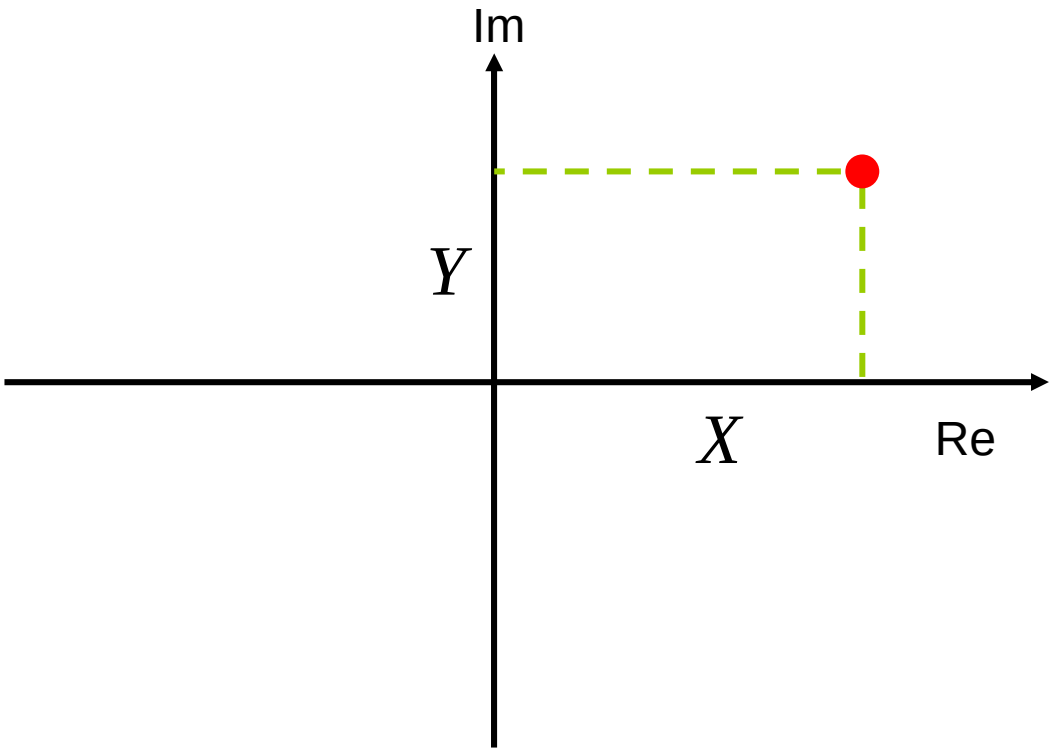
# … memo: complex numbers

$$Ae^{j\alpha}$$



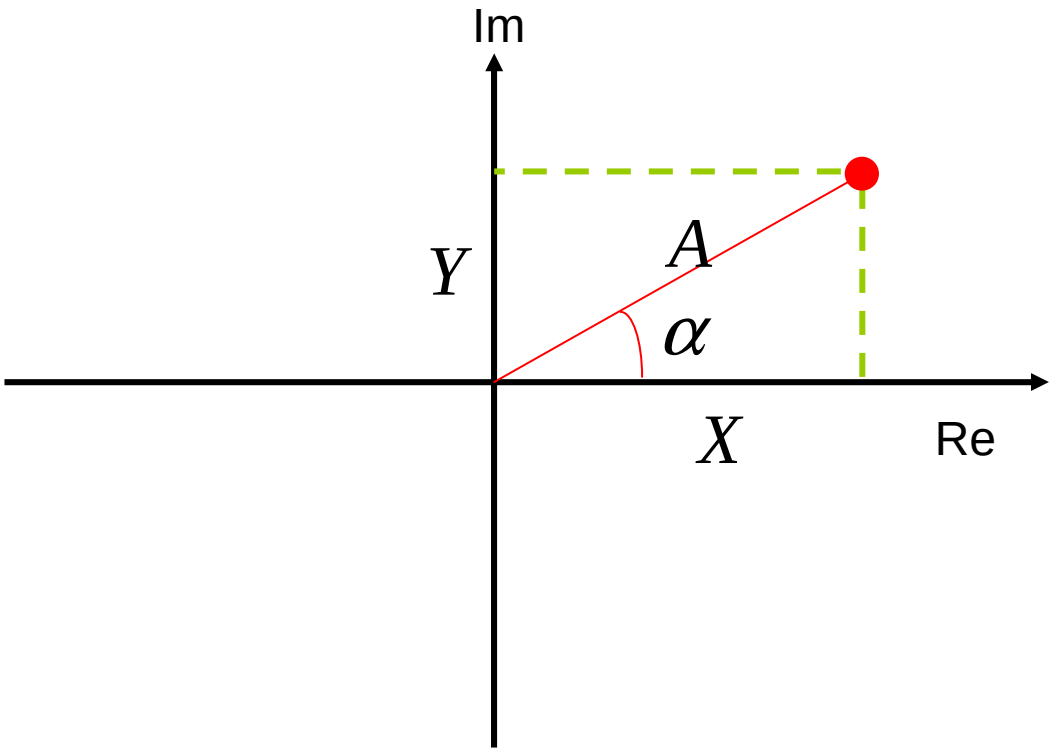
# … memo: complex numbers

$$Ae^{j\alpha} = X + jY$$



# … memo: complex numbers

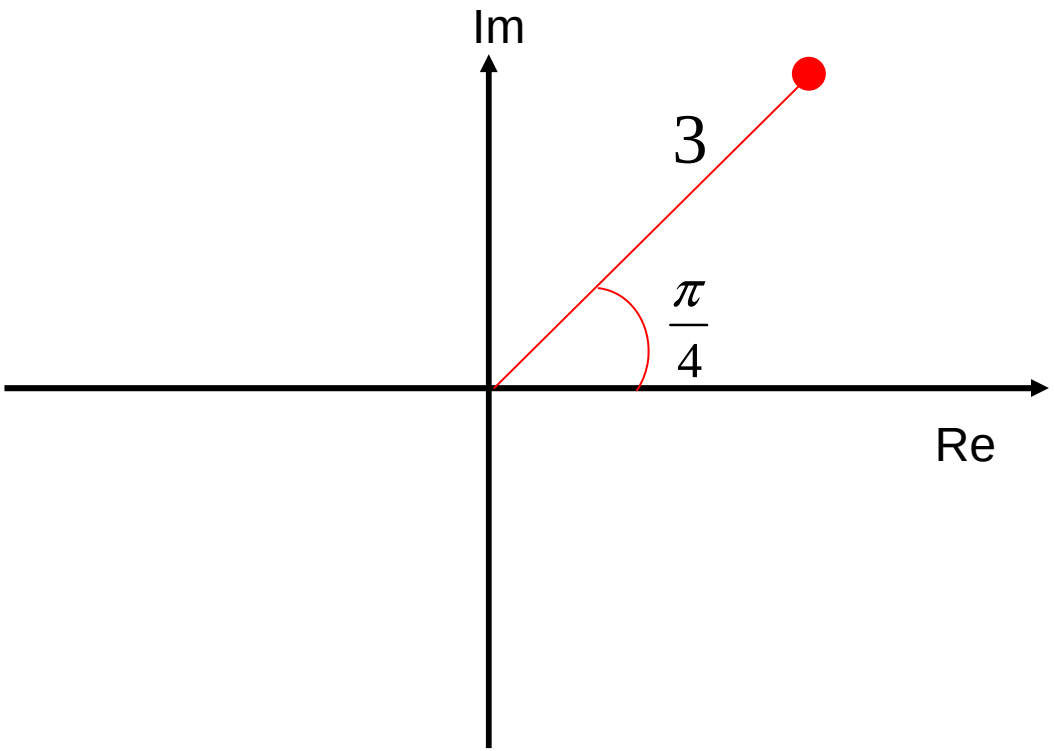
$$Ae^{j\alpha} = X + jY$$



# … memo: complex numbers

## some examples

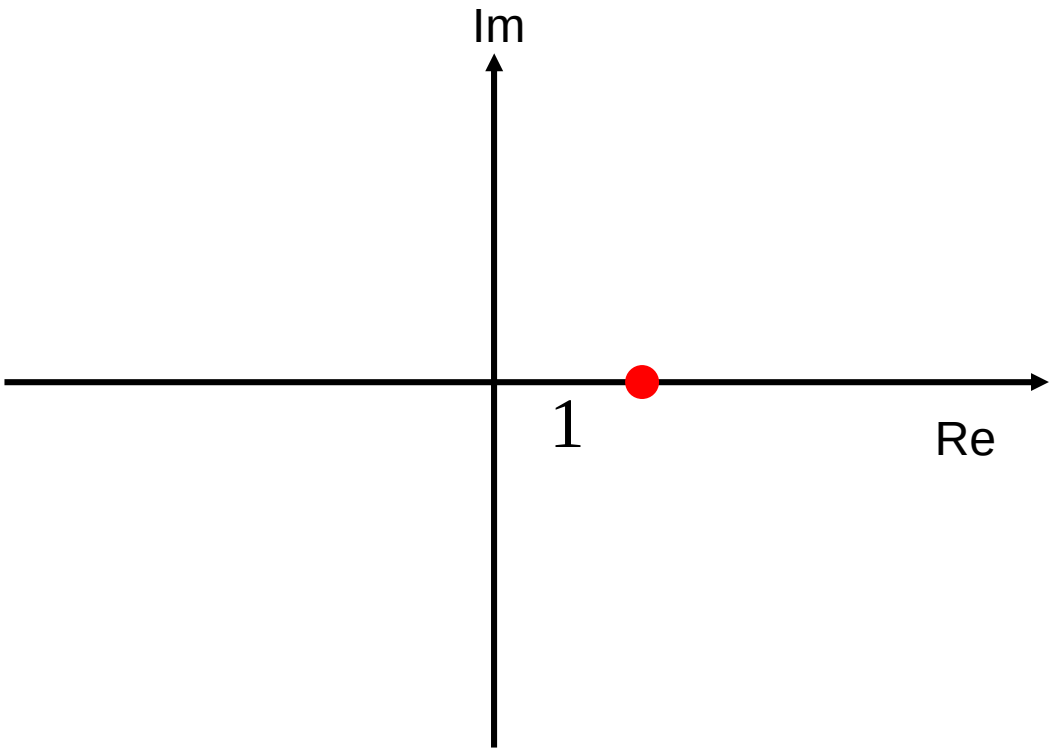
$$3e^{j\frac{\pi}{4}}$$



# … memo: complex numbers

## some examples

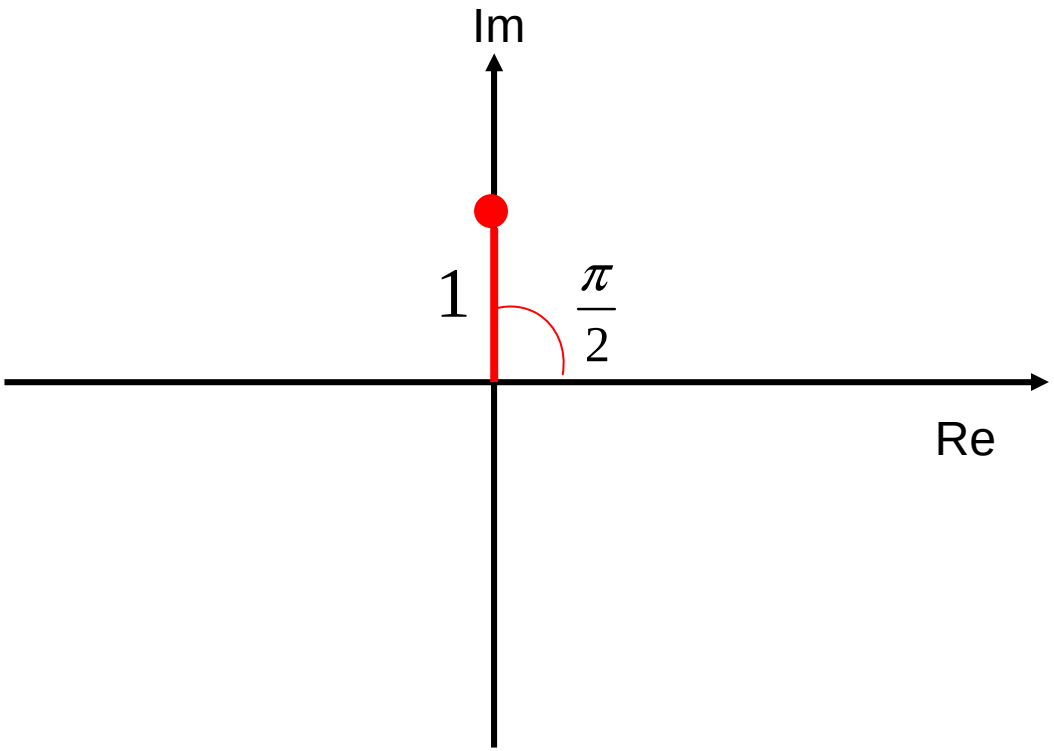
1



# … memo: complex numbers

## some examples

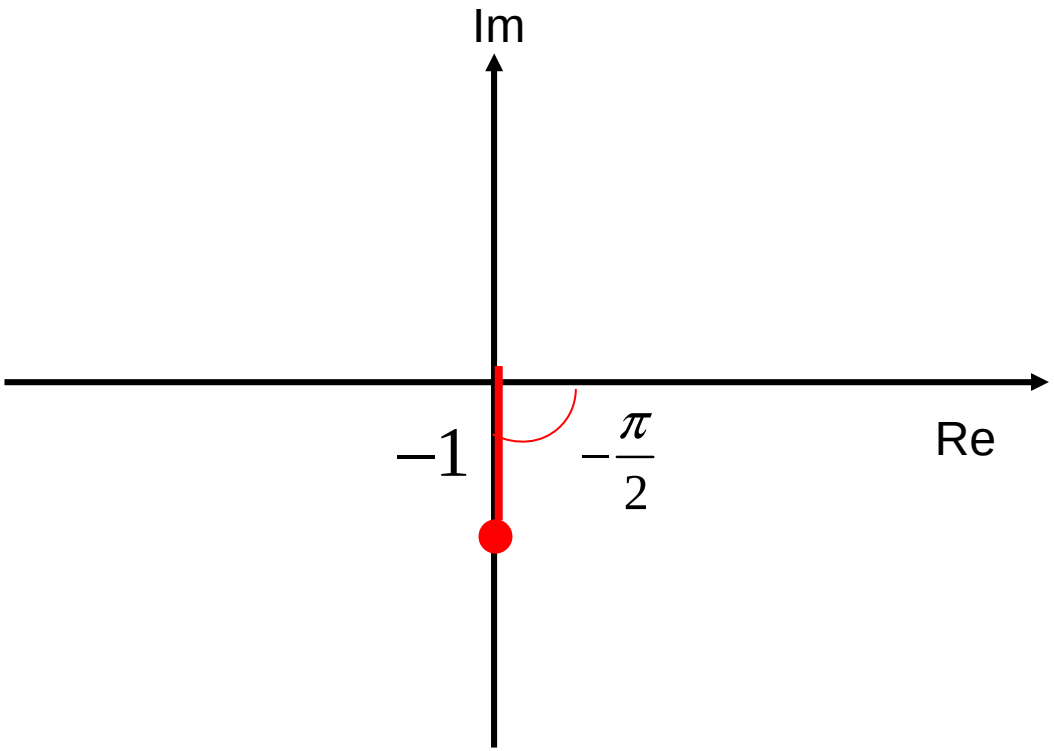
$$j = e^{j\frac{\pi}{2}} = e^{j\left(\frac{\pi}{2}+2\pi\right)}$$



# … memo: complex numbers

## some examples

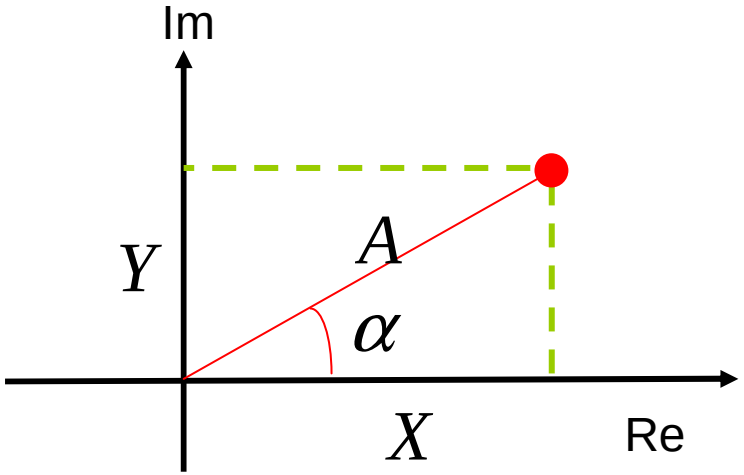
$$-j = e^{-j\frac{\pi}{2}}$$



# … memo: complex numbers

## Conversion formulas

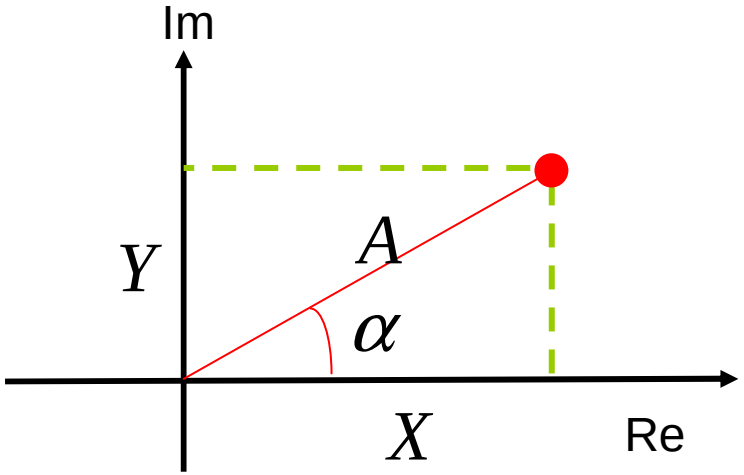
$$Ae^{j\alpha} = X + jY$$



# … memo: complex numbers

## Conversion formulas

$$Ae^{j\alpha} = X + jY$$

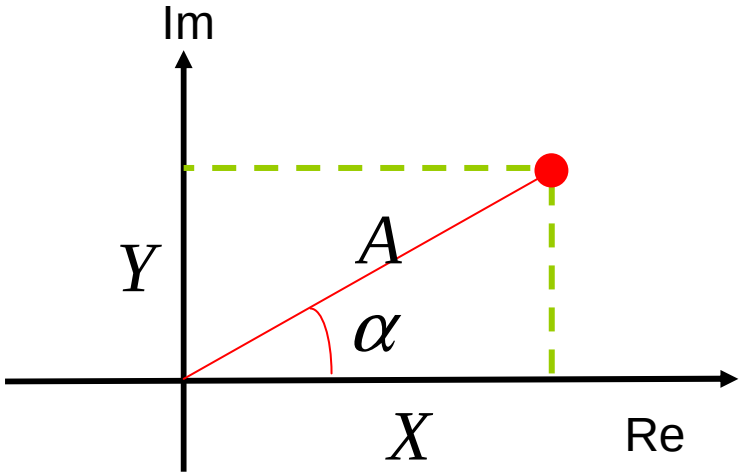



$$Ae^{j\alpha} \longrightarrow X + jY$$

# … memo: complex numbers

## Conversion formulas

$$Ae^{j\alpha} = X + jY$$




$$Ae^{j\alpha} \longrightarrow X + jY$$

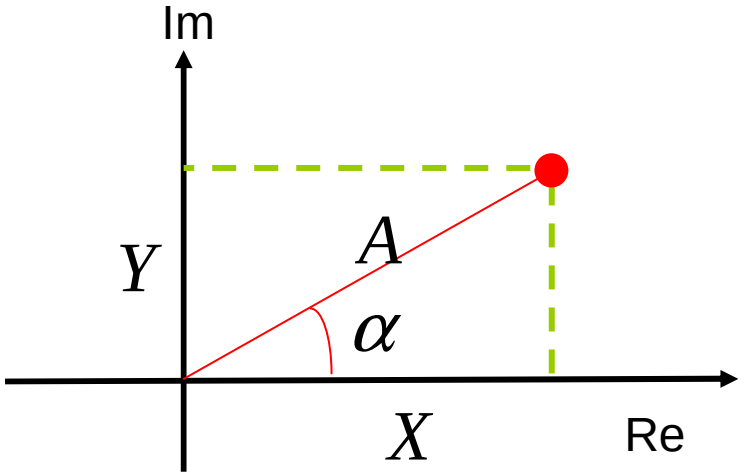
$$X = A\cos(\alpha)$$

$$Y = A\sin(\alpha)$$

# … memo: complex numbers

## Conversion formulas

$$Ae^{j\alpha} = X + jY$$




$$X + jY \longrightarrow Ae^{j\alpha}$$

$$Ae^{j\alpha} \longrightarrow X + jY$$

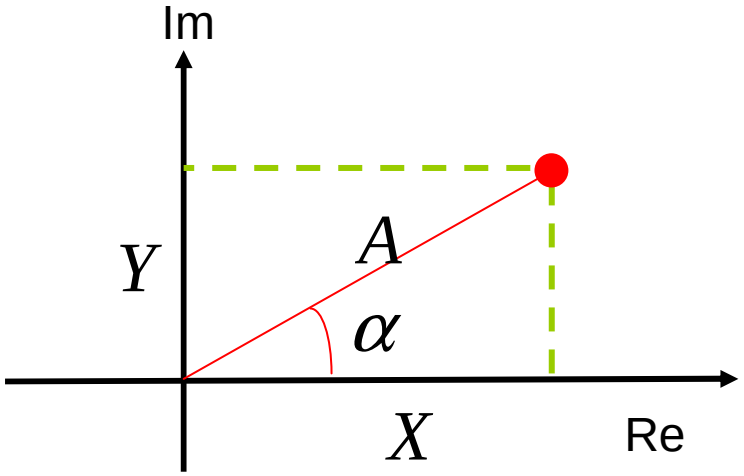
$$X = A\cos(\alpha)$$

$$Y = A\sin(\alpha)$$

# … memo: complex numbers

## Conversion formulas

$$Ae^{j\alpha} = X + jY$$




$$Ae^{j\alpha} \longrightarrow X + jY$$

$$X = A\cos(\alpha)$$

$$Y = A\sin(\alpha)$$

$$X + jY \longrightarrow Ae^{j\alpha}$$

$$A = \sqrt{X^2 + Y^2}$$

$$\begin{cases} \alpha = \text{acos}\left(\dfrac{X}{A}\right) & se\ Y \geq 0 \\ \alpha = -\text{acos}\left(\dfrac{X}{A}\right) & se\ Y < 0 \end{cases}$$

# … memo: complex numbers

## Conversion formulas: some examples

$$Ae^{j\alpha} \longrightarrow X + jY$$

$$X = A\cos(\alpha)$$

$$Y = A\sin(\alpha)$$

$$2e^{j\frac{3}{4}\pi} = -\sqrt{2} + j\sqrt{2}$$

# … memo: complex numbers

## Conversion formulas: some examples

$$X + jY \longrightarrow Ae^{j\alpha}$$

$$A = \sqrt{X^2 + Y^2}$$

$$\begin{cases} \alpha = \text{acos}\left(\dfrac{X}{A}\right) \ se\ Y \geq 0 \\ \alpha = -\text{acos}\left(\dfrac{X}{A}\right) \ se\ Y < 0 \end{cases}$$

$$\sqrt{3} + j = 2e^{j\frac{\pi}{6}}$$

$$\sqrt{3} - j = 2e^{-j\frac{\pi}{6}}$$

# … memo: complex numbers

## Summation

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 + Z_2 = \left(X_1 + X_2\right) + j\left(Y_1 + Y_2\right)$$

# … memo: complex numbers

## Product

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 \cdot Z_2 = \left( X_1 \cdot X_2 - Y_1 \cdot Y_2 \right) + j\left( X_1 \cdot Y_2 + Y_1 \cdot X_2 \right)$$

$$Z_1 \cdot Z_2 = A_1 \cdot A_2 e^{j(\alpha_1 + \alpha_2)}$$

# … memo: complex numbers

## Product: some examples

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 \cdot Z_2 = A_1 \cdot A_2 e^{j(\alpha_1+\alpha_2)}$$

$$z_1 = 3e^{j\frac{\pi}{6}}$$

$$z_2 = 4e^{j\frac{\pi}{4}}$$

$$z_1 \cdot z_2 = 12e^{j\left(\frac{\pi}{6}+\frac{\pi}{4}\right)} = 12e^{j\left(\frac{5}{12}\pi\right)}$$

# … memo: complex numbers

## Product: some examples

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 \cdot Z_2 = A_1 \cdot A_2 e^{j(\alpha_1+\alpha_2)}$$

$$z_1 = \sqrt{3} + j \quad = 2e^{j\frac{\pi}{6}}$$

$$z_2 = -\sqrt{2} + j\sqrt{2} \quad = 2e^{j\frac{3}{4}\pi}$$

$$z_1 \cdot z_2 = \quad 4e^{j\left(\frac{\pi}{6}+\frac{3\pi}{4}\right)} = 4e^{j\left(\frac{11}{12}\pi\right)}$$

# … memo: complex numbers

## Product: some examples

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

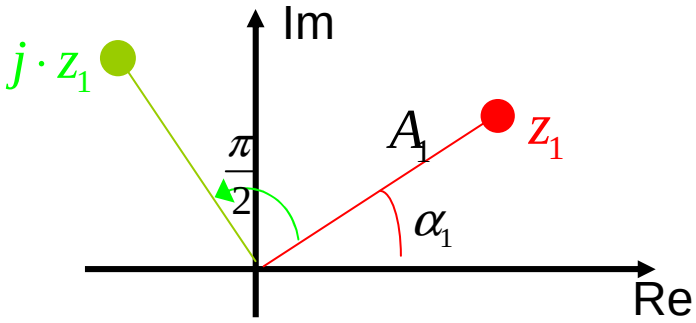
$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 \cdot Z_2 = A_1 \cdot A_2 e^{j(\alpha_1+\alpha_2)}$$

$$z_1 = A_1 e^{j\alpha_1}$$

$$z_2 = j = e^{j\frac{\pi}{2}}$$

$$j \cdot z_1 = A_1 e^{j\left(\alpha_1+\frac{\pi}{2}\right)}$$



# … memo: complex numbers

## Product: some examples

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

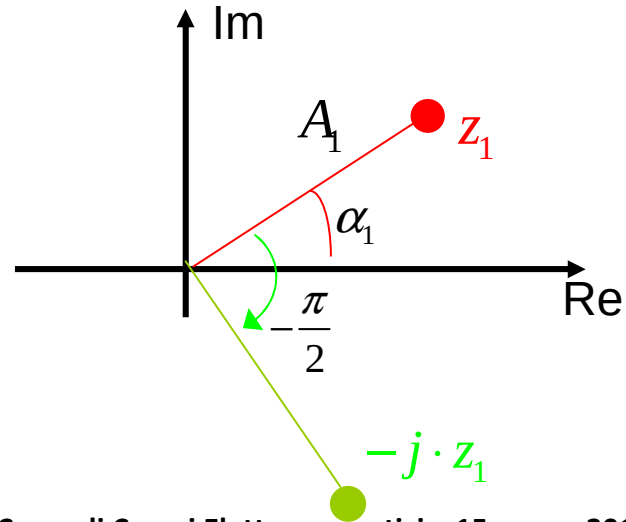
$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 \cdot Z_2 = A_1 \cdot A_2 e^{j(\alpha_1+\alpha_2)}$$

$$z_1 = A_1 e^{j\alpha_1}$$

$$z_2 = -j = e^{-j\frac{\pi}{2}}$$

$$-j \cdot z_1 = A_1 e^{j\left(\alpha_1 - \frac{\pi}{2}\right)}$$



# … memo: complex numbers

## Product: some examples

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

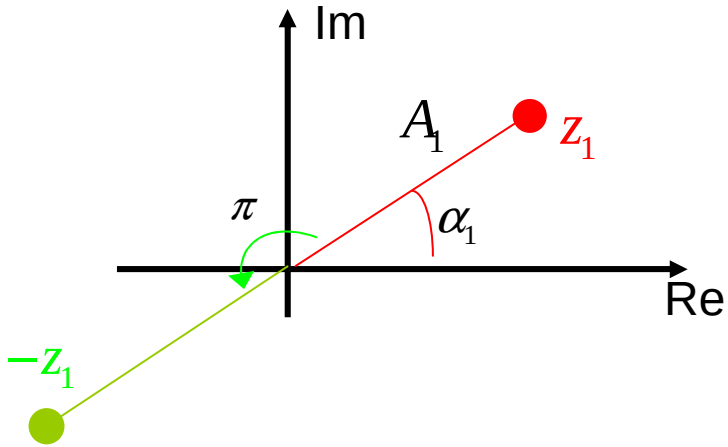
$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 \cdot Z_2 = A_1 \cdot A_2 e^{j(\alpha_1+\alpha_2)}$$

$$z_1 = A_1 e^{j\alpha_1}$$

$$z_2 = -1 = e^{j\pi}$$

$$-z_1 = A_1 e^{j(\alpha_1+\pi)}$$



# Sinusoidal signals and phasors

$$v(t) = A\cos\left(\omega_0 t + \alpha\right) \longrightarrow V = Ae^{j\alpha}$$

$$V = Ae^{j\alpha} \longrightarrow v(t) = \mathrm{Re}\left\{Ve^{j\omega_0 t}\right\} = \mathrm{Re}\left\{Ae^{j\alpha}e^{j\omega_0 t}\right\}$$

# Sinusoidal signals and phasors

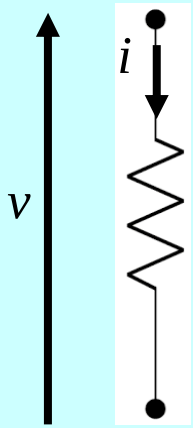
$$v(t) = A\cos\left(\omega_0 t + \alpha\right) \longrightarrow V = Ae^{j\alpha}$$

$$k \cdot v(t) \longrightarrow k \cdot V = k \cdot Ae^{j\alpha}$$

# Sinusoidal signals and phasors

$$v(t) = A\cos(\omega_0 t + \alpha) \longrightarrow V = Ae^{j\alpha}$$

$$k \cdot v(t) \longrightarrow k \cdot V = k \cdot Ae^{j\alpha}$$




$$v(t) = R \cdot i(t)$$

$$i(t) \rightarrow I$$

$$v(t) \rightarrow V$$

$$V = R \cdot I$$

$$V = Z \cdot I$$

$$Z = R$$

# Sinusoidal signals and phasors

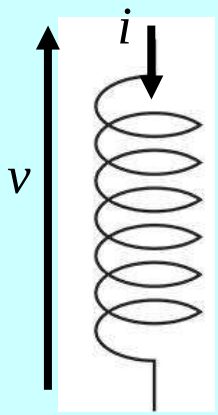
$$v(t) = A\cos\left(\omega_0 t + \alpha\right) \longrightarrow V = Ae^{j\alpha}$$

$$\frac{\partial v(t)}{\partial t} \longrightarrow j\omega_0 V = j\omega_0 Ae^{j\alpha}$$

# Sinusoidal signals and phasors

$$v(t) = A\cos(\omega_0 t + \alpha) \longrightarrow V = Ae^{j\alpha}$$

$$\frac{\partial v(t)}{\partial t} \longrightarrow j\omega_0 V = j\omega_0 Ae^{j\alpha}$$




$$v(t) = L\frac{\partial i(t)}{\partial t}$$

$$i(t) \rightarrow I$$

$$v(t) \rightarrow V$$

$$V = j\omega_0 LI$$

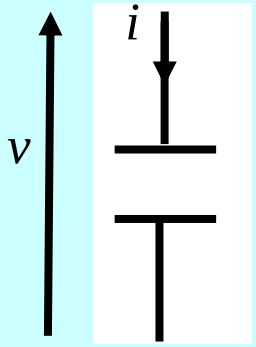
$$V = Z \cdot I$$

$$Z = j\omega_0 L$$

# Sinusoidal signals and phasors

$$v(t) = A\cos(\omega_0 t + \alpha) \longrightarrow V = Ae^{j\alpha}$$

$$\frac{\partial v(t)}{\partial t} \longrightarrow j\omega_0 V = j\omega_0 Ae^{j\alpha}$$




$$i(t) = C\frac{\partial v(t)}{\partial t}$$

$$i(t) \rightarrow I$$

$$v(t) \rightarrow V$$

$$I = j\omega_0 CV$$

$$V = Z \cdot I$$

$$Z = -j\frac{1}{\omega_0 C}$$

# Sinusoidal signals and phasors

$$v(t) = A\cos\left(\omega_0 t + \alpha\right) \longrightarrow V = Ae^{j\alpha}$$

$$i(t) = B\cos\left(\omega_0 t + \beta\right) \longrightarrow I = Be^{j\beta}$$

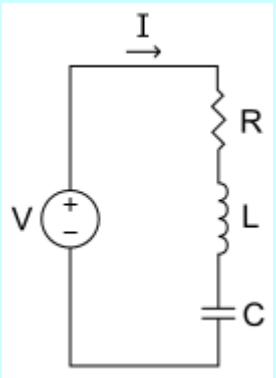
$$\left\langle v(t)\cdot i(t)\right\rangle = \frac{1}{T}\int_0^T v(t)\cdot i(t)\,dt = \frac{1}{2}\mathrm{Re}\left\{V\cdot I^*\right\}$$

# Sinusoidal signals and phasors

$$v(t) = A\cos(\omega_0 t + \alpha) \longrightarrow V = Ae^{j\alpha}$$

$$i(t) = B\cos(\omega_0 t + \beta) \longrightarrow I = Be^{j\beta}$$

$$\langle v(t)\cdot i(t)\rangle = \frac{1}{T}\int_0^T v(t)\cdot i(t)\,dt = \frac{1}{2}\mathrm{Re}\{V\cdot I^*\}$$




$$P = \frac{1}{2}V\cdot I^* = P_1 + jP_2$$

$$P = \frac{1}{2}V\cdot I^* = \frac{1}{2}(Z_R + Z_L + Z_C)\,I\cdot I^* = \frac{1}{2}\left(R + j\omega_0 L - \frac{j}{\omega_0 C}\right)|I|^2$$

$$P_1 = \frac{1}{2}R|I|^2 \;;\quad P_2 = \frac{1}{2}\left(\omega_0 L - \frac{1}{\omega_0 C}\right)|I|^2$$