

# Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

Corso di Campi Elettromagnetici  
a.a. 2017-2018

20 Maggio 2018

# Antenna Parameters

# Summary of the past lecture

## Antenna Parameters

### Parameters of the Tx Antenna

### Parameters of the Rx Antenna

# Summary of the past lecture

## Tx Antenna parameters

- Effective length
  - Radiation pattern
  - Radiation pattern lobes
  - Beamwidth
- Directivity
- Gain
- Radiation Resistance
- Equivalent circuit of the tx antenna
- Input Impedance and Input Resistance



# Summary of the past lecture

## Tx Antenna parameters

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# Summary of the past lecture

## Parameters of the Tx Antenna

### Effective length

$$r \gg D$$

$$r > \frac{2D^2}{\lambda}$$

$$r \gg \lambda$$

$$\left\{ \begin{array}{l} \mathbf{E}(\vec{\mathbf{r}}) = \mathbf{E}(r, \vartheta, \varphi) = \frac{j\zeta I}{2\lambda} \frac{e^{-j\beta r}}{r} \mathbf{l}(\vartheta, \varphi) \\ \zeta \mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}}) \end{array} \right.$$

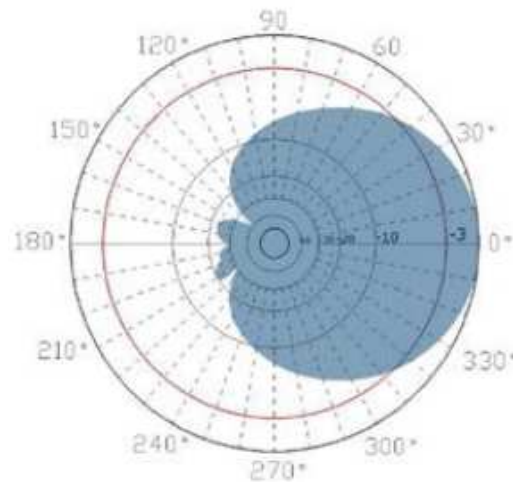
Fraunhofer region

$\mathbf{l}(\vartheta, \varphi) = l_\vartheta(\vartheta, \varphi) \hat{i}_\vartheta + l_\varphi(\vartheta, \varphi) \hat{i}_\varphi$  is the effective length of the antenna

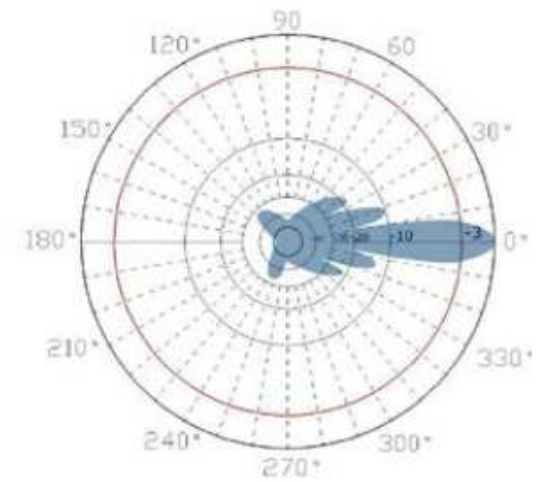
# Summary of the past lecture

## Radiation pattern

### three examples from the real life



Horizontal



Vertical

**HPBW (vertical) = 14°**

**HPBW (horizontal) = 90°**

**ZDADJ800-13-90 Patterns**

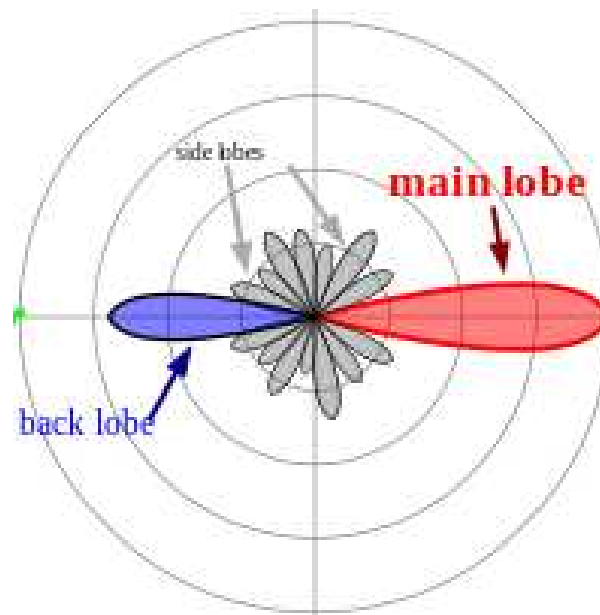


# Tx Antenna parameters

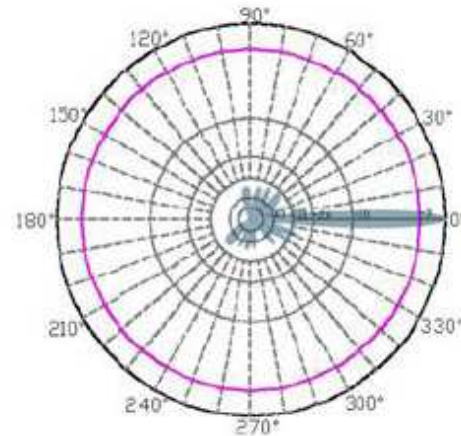
- Effective length
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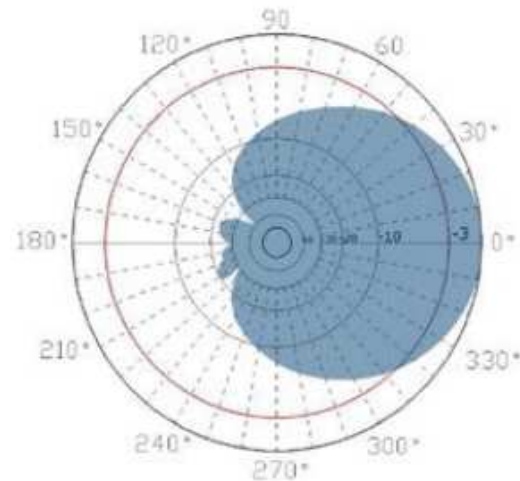
# Directivity



# Directivity

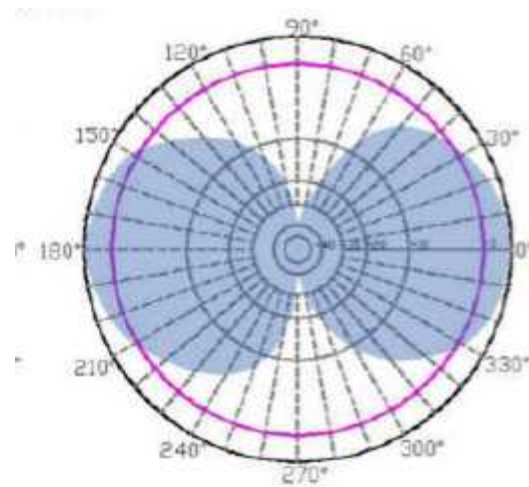


**HPBW = 6°**

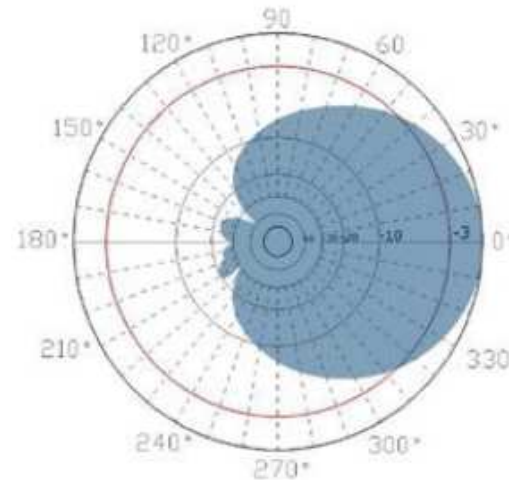


**HPBW = 90°**

# Directivity



**HPBW = 60°**



**HPBW = 90°**

# Directivity

- The directivity of an antenna is :

$$D(\vartheta, \varphi) = \lim_{r \rightarrow \infty} \frac{\frac{1}{2} \frac{|\mathbf{E}(r, \vartheta, \varphi)|^2}{\zeta}}{\frac{1}{4\pi r^2} P_{rad}}$$

....memo....

$$P = \frac{1}{2} \oiint_S [\mathbf{E} \times \mathbf{H}^*] \cdot \hat{i}_r dS = P_1 + jP_2$$

$P_1$  being the radiated power (hereafter named  $P_{rad}$  )

The Poynting vector simplifies as

$$\mathbf{S} = \frac{1}{2} \mathbf{E} \times \mathbf{H}^* = \frac{1}{2\zeta} |\mathbf{E}|^2 \hat{i}_r$$

$$r \gg D$$

$$r > \frac{2D^2}{\lambda}$$

$$r \gg \lambda$$

Fraunhofer region

.... and thus, in a spherical reference system centered in the antenna,

$$P = \int_0^{2\pi} d\varphi \int_0^{\pi} d\vartheta \frac{|\mathbf{E}|^2}{2\zeta} r^2 \sin \vartheta = P_1 = P_{rad}$$

# Directivity

- The directivity of an antenna is :

$$D(\vartheta, \varphi) = \lim_{r \rightarrow \infty} \frac{\frac{1}{2} \frac{|\mathbf{E}(r, \vartheta, \varphi)|^2}{\zeta}}{\frac{1}{4\pi r^2} P_{rad}} = \lim_{r \rightarrow \infty} \frac{\frac{1}{2} \frac{|\mathbf{E}(r, \vartheta, \varphi)|^2}{\zeta}}{\frac{1}{4\pi r^2} \int_0^{2\pi} d\varphi \int_0^{\pi} d\vartheta \frac{|\mathbf{E}(r, \vartheta, \varphi)|^2}{2\zeta} r^2 \sin \vartheta}$$

- The directivity of an isotropic source is equal to 1 (that is, 0 dB)

..... Memo ....

## Elementary electrical dipole

$$P = \frac{1}{2} \oiint_S [\mathbf{E} \times \mathbf{H}^*] \cdot \hat{i}_r dS = \frac{1}{2} \int_0^{2\pi} d\varphi \int_0^\pi d\vartheta r^2 \sin \vartheta E_\vartheta H_\varphi^* = \frac{1}{2} \frac{2\pi}{3} \zeta \left( \frac{\Delta z}{\lambda} \right)^2 \left[ 1 - j \frac{1}{(\beta r)^3} \right] |I|^2$$

$$P = P_1 + jP_2$$

$$P_1 = P_{rad} = \frac{1}{2} \frac{2\pi}{3} \zeta \left( \frac{\Delta z}{\lambda} \right)^2 |I|^2$$

$$P_2 = -\frac{1}{2} \frac{2\pi}{3} \zeta \left( \frac{\Delta z}{\lambda} \right)^2 \frac{1}{(\beta r)^3} |I|^2$$

far field

$$\mathbf{E}(\vec{r}) = \frac{j\zeta I e^{-j\beta r}}{2\lambda r} \Delta z \sin \vartheta \hat{i}_\vartheta$$

$$D(\vartheta, \varphi) = \frac{3}{2} \sin^2 \vartheta$$

$$D_{\max} = 10 \log_{10} 1.5 = 1.76 \text{ dB}$$



..... Memo ....

## Small loop antenna

$$P = \frac{1}{2} \oiint_S [\mathbf{E} \times \mathbf{H}^*] \cdot \hat{i}_r dS = -\frac{1}{2} \int_0^{2\pi} d\varphi \int_0^\pi d\vartheta r^2 \sin \vartheta E_\varphi H_\vartheta^* = \frac{1}{2} \frac{2\pi}{3} \zeta \left( \frac{\beta \Delta S}{\lambda} \right)^2 \left[ 1 + j \frac{1}{(\beta r)^3} \right] |I|^2$$

$$P = P_1 + jP_2$$

$$P_1 = P_{rad} = \frac{1}{2} \frac{2\pi}{3} \zeta \left( \frac{\beta \Delta S}{\lambda} \right)^2 |I|^2$$

$$P_2 = \frac{1}{2} \frac{2\pi}{3} \zeta \left( \frac{\beta \Delta S}{\lambda} \right)^2 \frac{1}{(\beta r)^3} |I|^2$$

far field

$$\mathbf{E}(\vec{r}) = \frac{\zeta \beta \Delta S I}{2\lambda} \frac{e^{-j\beta r}}{r} \sin \vartheta \hat{i}_\varphi$$

$$D(\vartheta, \varphi) = \frac{3}{2} \sin^2 \vartheta$$

$$D_{max} = 10 \log_{10} 1.5 = 1.76 \text{ dB}$$

# Tx Antenna parameters

- Effective length
  - Radiation pattern
  - Radiation pattern lobes
  - Beamwidth
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- Input Impedance and Input Resistance



# Gain

## Directivity

$$D(\vartheta, \varphi) = \lim_{r \rightarrow \infty} \frac{\frac{1}{2} \frac{|\mathbf{E}|^2}{\zeta}}{\frac{1}{4\pi r^2} P_{rad}}$$

## Gain

$$G(\vartheta, \varphi) = \lim_{r \rightarrow \infty} \frac{\frac{1}{2} \frac{|\mathbf{E}|^2}{\zeta}}{\frac{1}{4\pi r^2} P_{in}}$$

**$P_{rad}$** : radiated power

**$P_{in}$** : input power

# Gain

- If one replace  $P_{\text{rad}}$  with the input real power to the antenna  $P_{\text{in}}$  one finds the definition of the *Gain*.
- For a lossless antenna,  $P_{\text{in}}=P_{\text{rad}}$  and  $G=D$ . If losses are present  $P_{\text{in}}>P_{\text{rad}}$  and  $G<D$ .
- *Note that both  $D$  and  $G$  are dimensionless.*

# Gain

## three examples from the real life

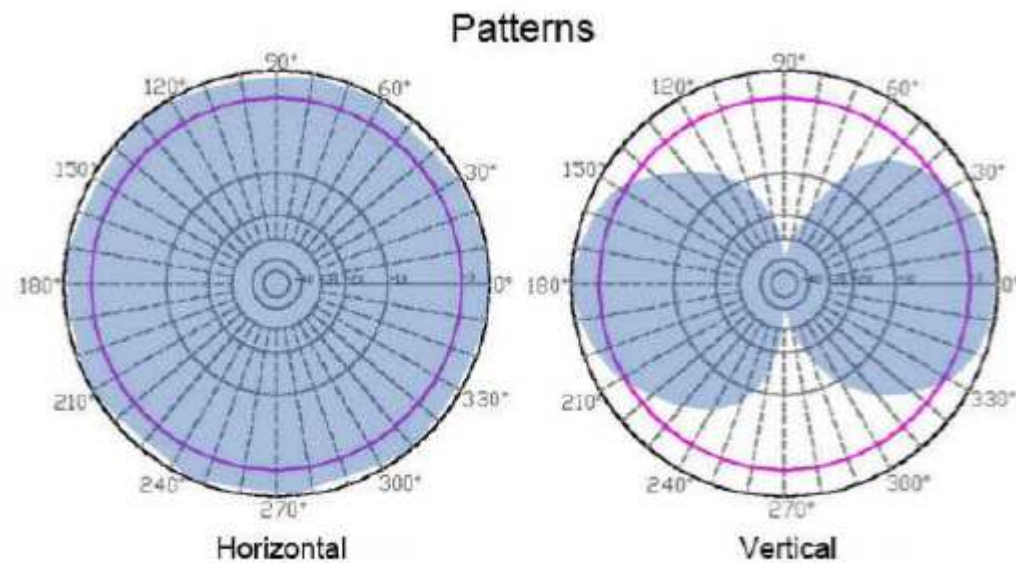


# Gain

## three examples from the real life



(maximum) Gain = 3 dB



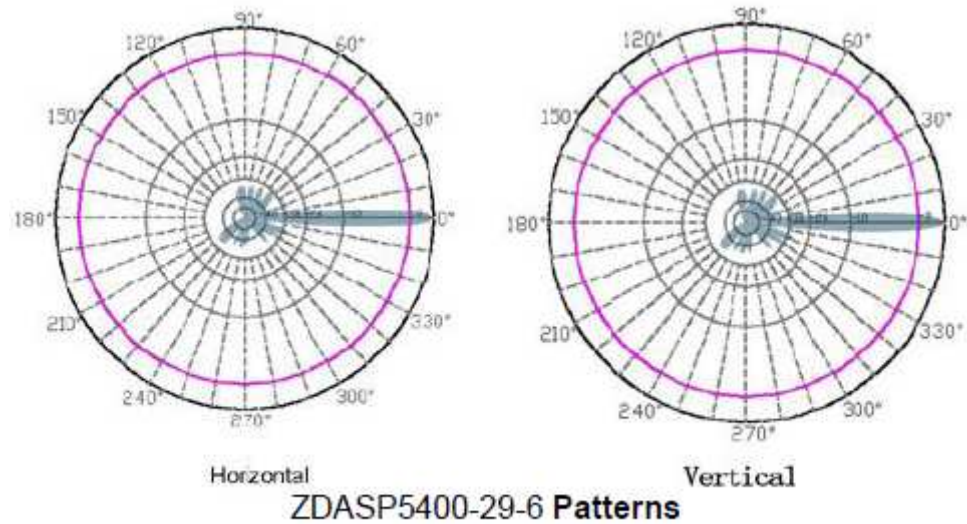
ZDAEW1900-3 Patterns

# Gain

## three examples from the real life



**(maximum) Gain = 29 dB**

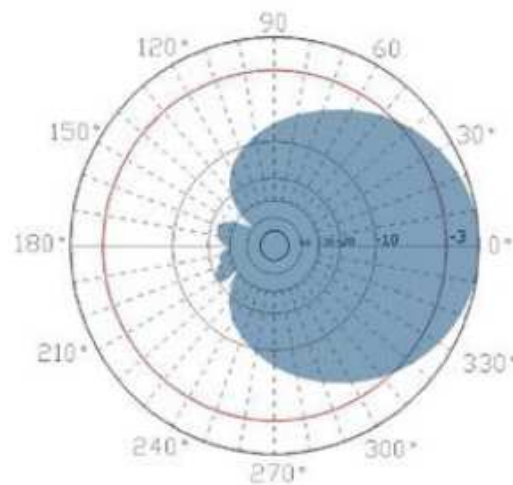


# Gain

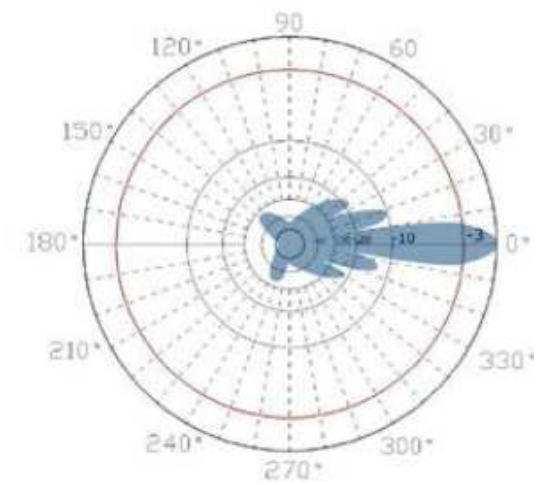
## three examples from the real life



(maximum) Gain = 13 dB



Horizontal



Vertical

### ZDADJ800-13-90 Patterns



# Tx Antenna parameters

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- **Radiation Resistance**
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# Radiation resistance

- Associated to the far-field radiated power one can define the radiation Resistance  $R_{rad}$ :

$$P_{rad} = \int_0^{2\pi} d\varphi \int_0^{\pi} d\vartheta \frac{|\mathbf{E}|^2}{2\zeta} r^2 \sin \vartheta = \frac{1}{2} R_{rad} |I|^2$$

..... Memo ....

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$$P = \frac{1}{2} \oiint_S [\mathbf{E} \times \mathbf{H}^*] \cdot \hat{i}_r dS = \frac{1}{2} \int_0^{2\pi} d\varphi \int_0^\pi d\vartheta r^2 \sin \vartheta E_\vartheta H_\varphi^* = \frac{1}{2} \frac{2\pi}{3} \zeta \left( \frac{\Delta z}{\lambda} \right)^2 \left[ 1 - j \frac{1}{(\beta r)^3} \right] |I|^2$$

$$P = P_1 + jP_2$$

$$P_1 = P_{rad} = \frac{1}{2} \frac{2\pi}{3} \zeta \left( \frac{\Delta z}{\lambda} \right)^2 |I|^2$$

$$P_2 = -\frac{1}{2} \frac{2\pi}{3} \zeta \left( \frac{\Delta z}{\lambda} \right)^2 \frac{1}{(\beta r)^3} |I|^2$$

$$R_{rad} = \frac{2\pi}{3} \zeta \left( \frac{\Delta z}{\lambda} \right)^2$$

..... Memo ....

## Small loop antenna

$$P = \frac{1}{2} \oiint_S [\mathbf{E} \times \mathbf{H}^*] \cdot \hat{i}_r dS = -\frac{1}{2} \int_0^{2\pi} d\varphi \int_0^\pi d\vartheta r^2 \sin \vartheta E_\varphi H_\vartheta^* = \frac{1}{2} \frac{2\pi}{3} \zeta \left( \frac{\beta \Delta S}{\lambda} \right)^2 \left[ 1 + j \frac{1}{(\beta r)^3} \right] |I|^2$$

$$P = P_1 + jP_2$$

$$P_1 = P_{rad} = \frac{1}{2} \frac{2\pi}{3} \zeta \left( \frac{\beta \Delta S}{\lambda} \right)^2 |I|^2$$

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$$R_{rad} = \frac{2\pi}{3} \zeta \left( \frac{\beta \Delta S}{\lambda} \right)^2$$

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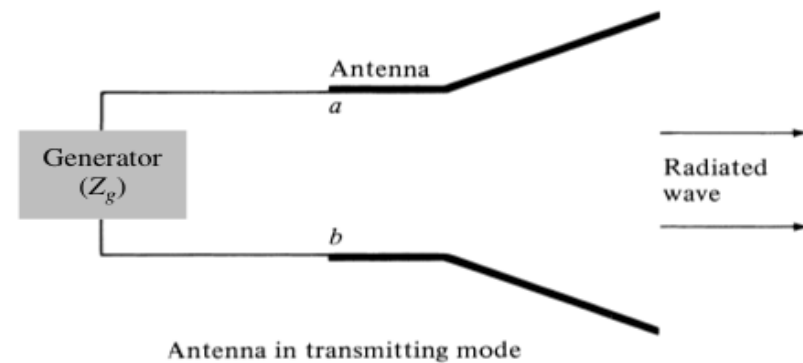


# Input impedance

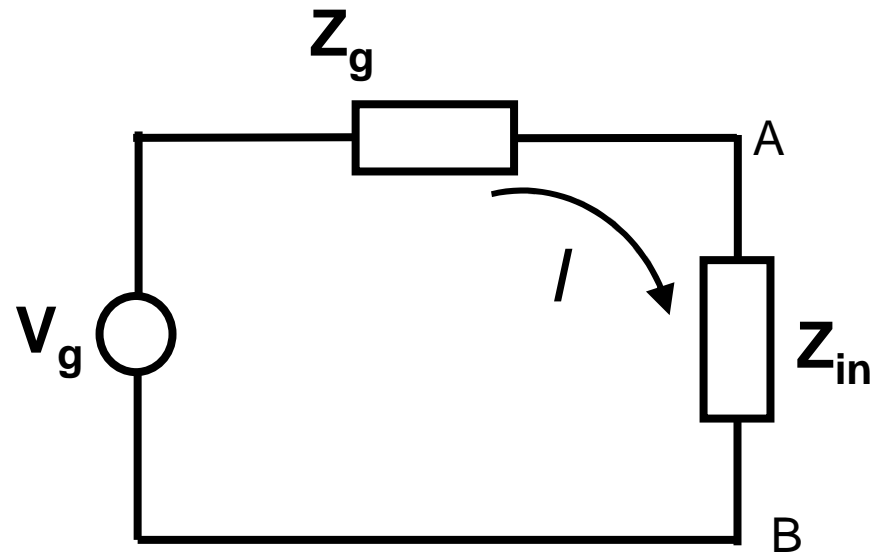
- *Input impedance* is defined as “the impedance presented by an antenna at its terminals or the ratio of the voltage to current at a pair of terminals or the ratio of the appropriate components of the electric to magnetic fields at a point.”

# Input impedance

- We are primarily interested in the input impedance at a pair of terminals which are the input terminals of the antenna. In Figure these terminals are denoted as  $a - b$ .

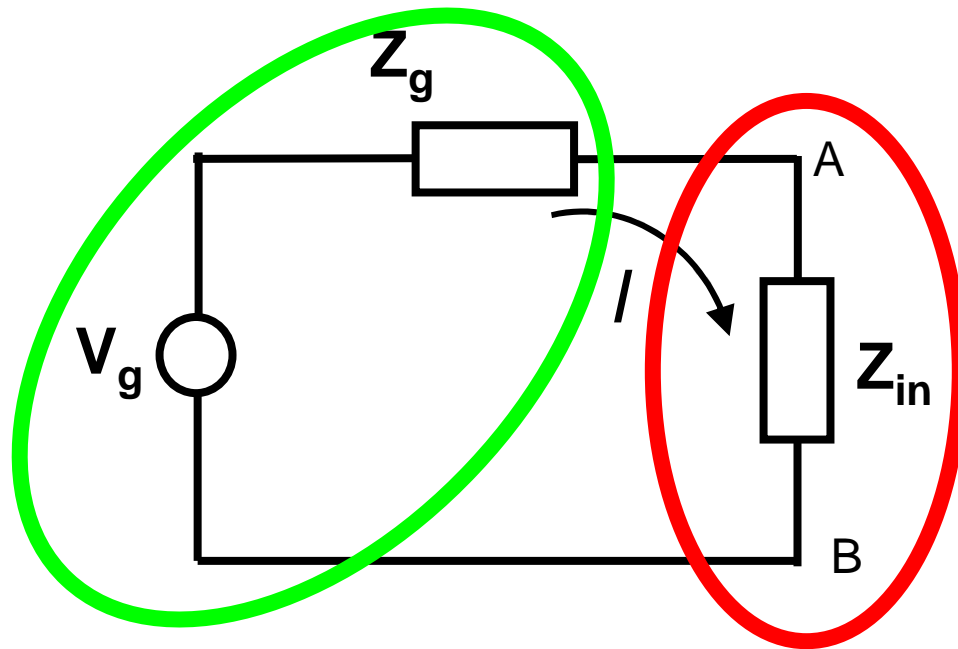


# Equivalent circuit of the Tx antenna





# Equivalent circuit of the Tx antenna



$$Z_{in} = R_{in} + jX_{in}$$

$$P_{in} = \frac{1}{2} R_{in} |I|^2$$

Tx circuit

Tx Antenna

Step

$$P_{rad} = \frac{1}{2} R_{rad} |I|^2$$

$$P_{rad} \leq P_{in} \quad \Rightarrow \quad R_{rad} \leq R_{in} \quad \Rightarrow \quad R_{in} = R_{rad} + R_{\Omega}$$

## Directivity

$$D(\vartheta, \phi) = \lim_{r \rightarrow \infty} \frac{\frac{1}{2} \frac{|\mathbf{E}|^2}{\zeta}}{\frac{1}{4\pi r^2} P_{rad}}$$

## Gain

$$G(\vartheta, \phi) = \lim_{r \rightarrow \infty} \frac{\frac{1}{2} \frac{|\mathbf{E}|^2}{\zeta}}{\frac{1}{4\pi r^2} P_{in}}$$

$$P_{rad} = \frac{1}{2} R_{rad} |I|^2$$

$$P_{in} = \frac{1}{2} R_{in} |I|^2 = \frac{1}{2} (R_{rad} + R_{\Omega}) |I|^2$$

**Radiation Efficiency**

$$\eta = \frac{P_{rad}}{P_{in}} = \frac{R_{rad}}{R_{rad} + R_{\Omega}} = \frac{G}{D}$$

# Input impedance

- The input impedance of an antenna is generally a function of frequency.
- The input impedance of the antenna depends on many factors including its geometry, its method of excitation, and its proximity to surrounding objects.
- Because of their complex geometries, only a limited number of practical antennas have been investigated analytically. For many others, the input impedance has been determined experimentally.

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All these parameters are related each other

# Bandwidth

- The *bandwidth* of an antenna is defined as “the range of frequencies within which the performance of the antenna, with respect to some characteristic, conforms to a specified standard.”
- The bandwidth can be considered to be the range of frequencies, where the antenna characteristics are within an acceptable value of those at the center frequency.

# Bandwidth

- For *broadband antennas*, the bandwidth is usually expressed as the ratio of the upper-to-lower frequencies of acceptable operation. For example, a 10:1 bandwidth indicates that the upper frequency is 10 times greater than the lower.
- For *narrowband antennas*, the bandwidth is expressed as a percentage of the frequency difference (upper minus lower) over the center frequency of the bandwidth. For example, a 5% bandwidth indicates that the frequency difference of acceptable operation is 5% of the center frequency of the bandwidth.

# Bandwidth

**Log.-periodic Antenna, Frequency  
400-4000 MHz for Amateur radio**



**UHA 9125 D, Half – Wave Dipole**

