Intelligent Signal Processing

Streaming
and
Compressive Sensing

Angelo Ciaramella

Introduction

- Compressive Sensing (o Compressed Sensing) technique
- client-server architecture
- Compressice Sensing for
  - compression
  - packet loss reconstruction

Compressive Sensing

- Compressive Sensing (CS)
  - is a new sensing modality, which compresses the signal being acquired at the time of sensing
  - Signals can have sparse or compressible representation either in original domain or in some transform domain
  - Relying on the sparsity of the signals, CS allows us to sample the signal at a rate much below the Nyquist sampling rate
  - the varied reconstruction algorithms of CS can faithfully reconstruct the original signal back from fewer compressive measurements

Compressive Sensing

- CS was introduced by Donoho, Candès, Romberg, and Tao in 2004

Compressive Sensing

- Emerging technique for signal processing
  - acquisition/reconstruction that violates the Nyquist-Shannon limit
    - less samples

- A signal can have sparse/compressible representation either in original domain or in some transform domains
  - Fourier transform, cosine transform, wavelet transform, etc. A few examples of signals having sparse

- Domains
  - natural images which have sparse representation in wavelet domain
  - speech signal can be represented by fewer components using Fourier transform
  - better model for medical images can be obtained using Radon transform
  - etc.

Linear inverse problems

- Many classic problems in computer can be posed as linear inverse problems

- Notation
  - Signal of interest $x \in \mathbb{R}^N$
  - Observations $y \in \mathbb{R}^M$
  - Measurement model $y = \Phi x + e$

measurement matrix

measurement noise

  - Problem definition: given y, recover x

Linear inverse problems

- Scenario 1

$$M \geq N$$

$$\hat{x} = \Phi^{-1} y$$

- Scenario 2

$$M < N$$

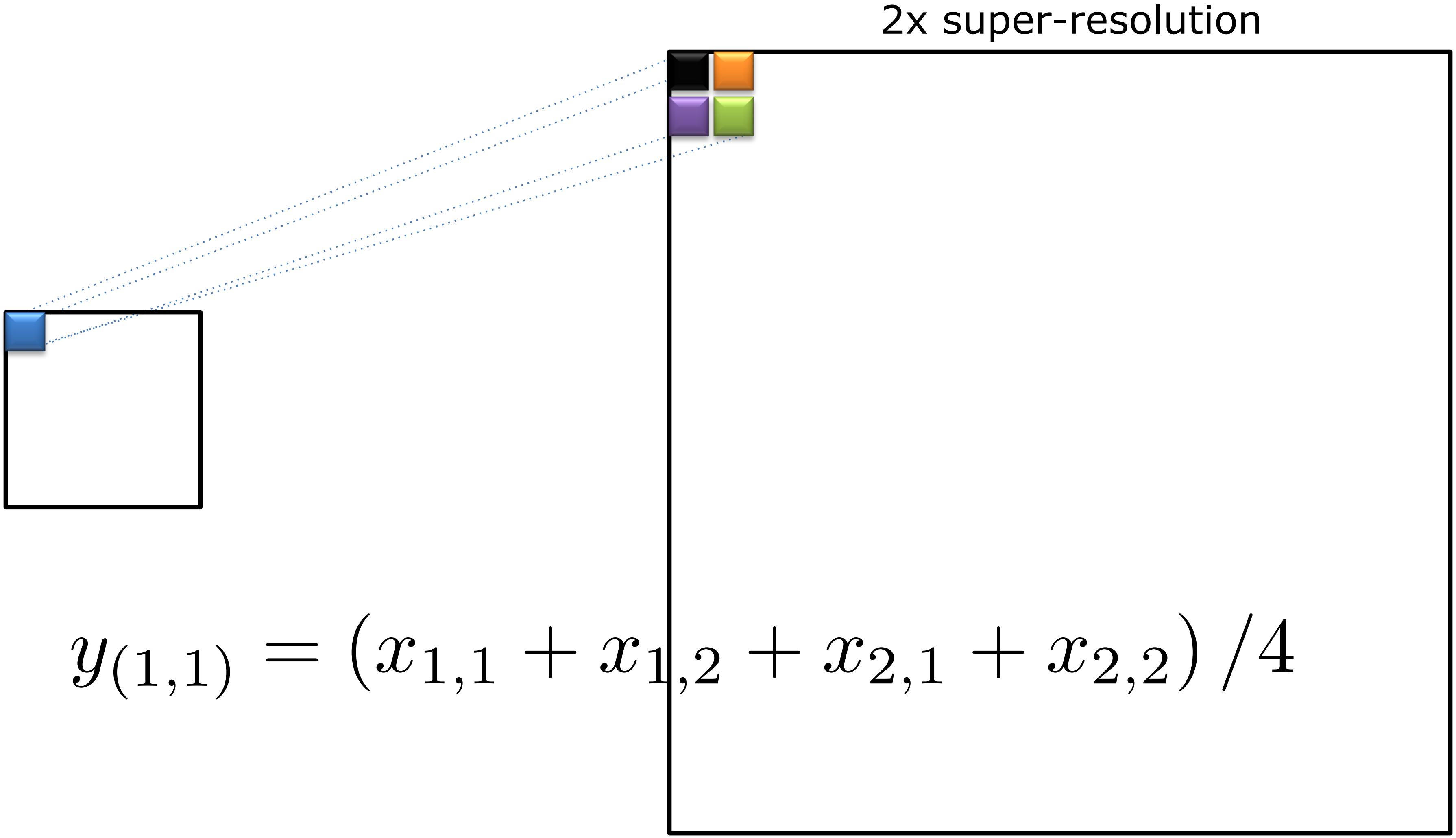
  - Measurement matrix has a (N-M) dimensional null-space
  - Solution is no longer unique
  - Under-sampling ratio M/N

Image super-resolution

Low resolution input/observation

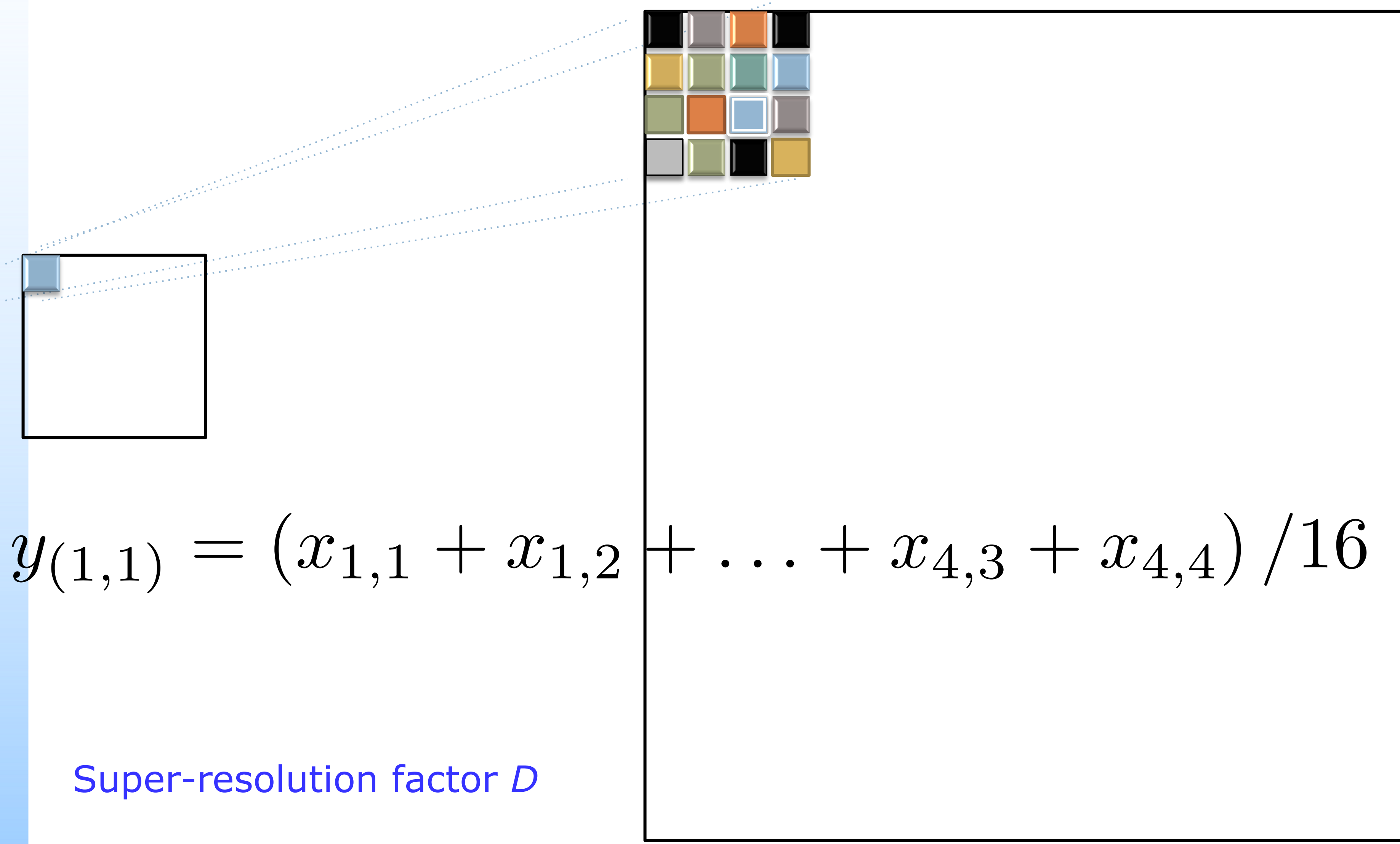
128x128 pixels

Image super-resolution



$$y_{(1,1)} = \left(x_{1,1} + x_{1,2} + x_{2,1} + x_{2,2}\right)/4$$

Image super-resolution

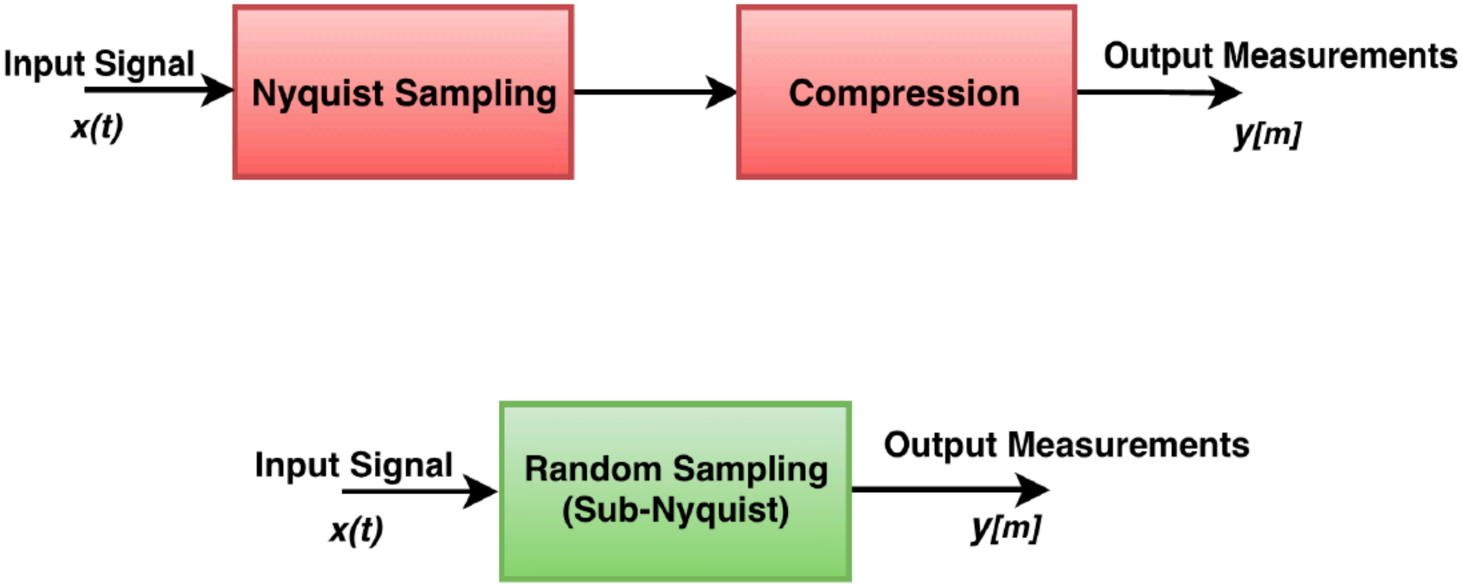


$$y_{(1,1)} = (x_{1,1} + x_{1,2} + \ldots + x_{4,3} + x_{4,4})/16$$

Super-resolution factor *D*

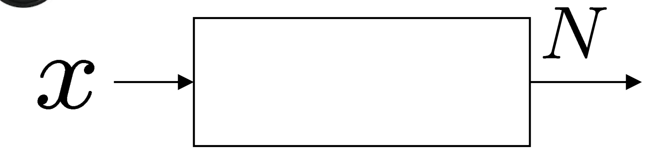
Under-sampling factor *M/N = 1/D*^{2}

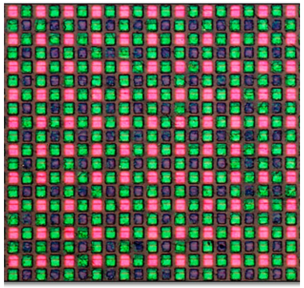
Compressive Sensing



A comparision of sampling techniques: (a) traditional sampling, (b) compressive sensing.

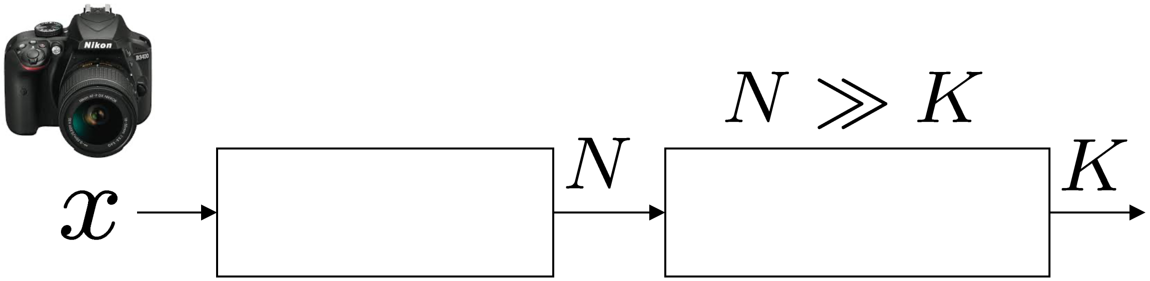
Sampling



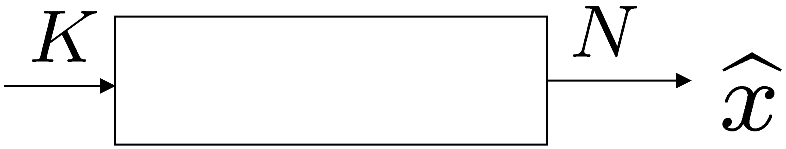


too
much
data!

Compression



JPEG
JPEG2000
...



ISP – Streaming and Compressive Sensing

Sparsity

N

pixels

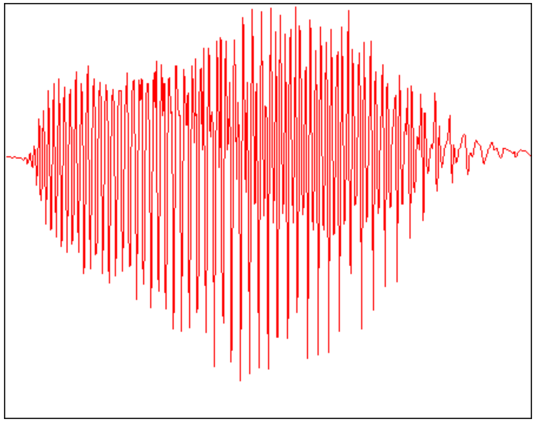
$K \ll N$

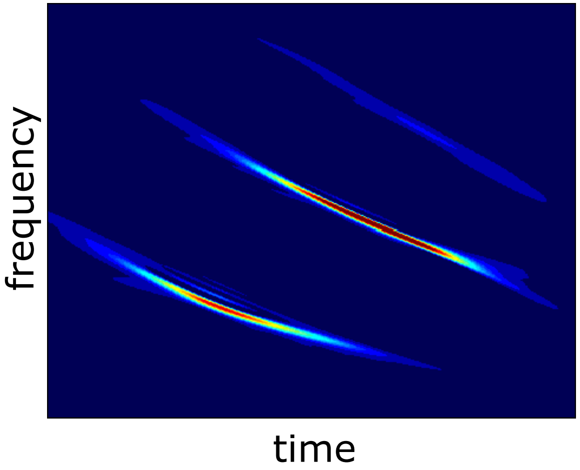
large wavelet coefficients

(blue = 0)

N

wideband signal samples



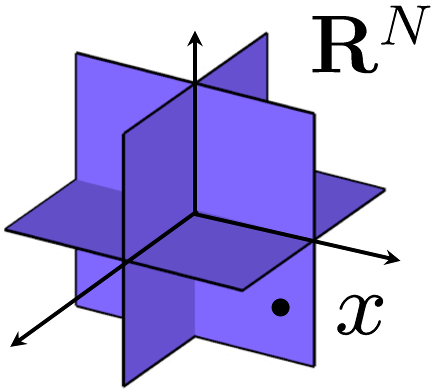


$K \ll N$

large Gabor (TF) coefficients

Sparsity

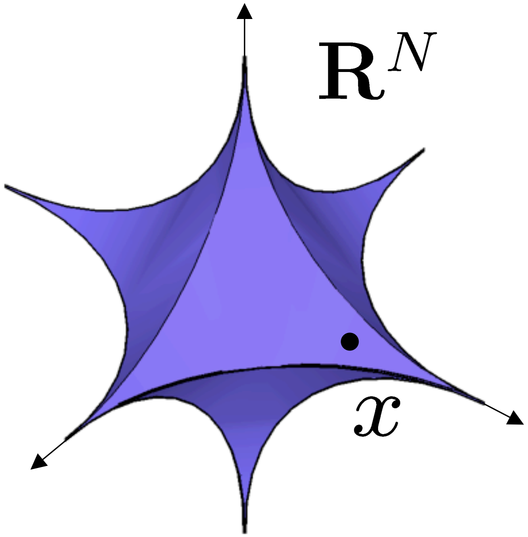
- Sparse signal
  - only K out of N coordinates nonzero
  - Model – union of *k*-dimensional subspaces

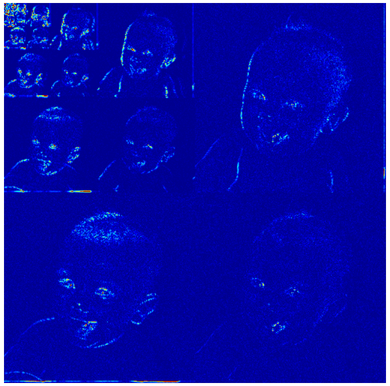


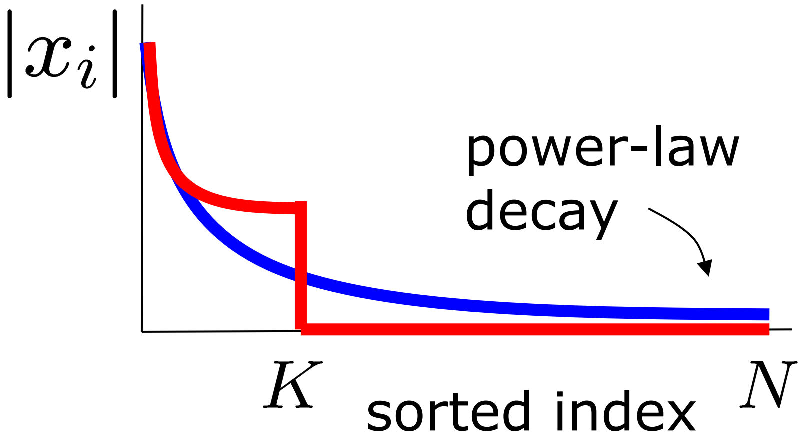
Sparsity

- Compressible signal
  - sorted coordinates decay rapidly with power-law
  - Model based on ℓ_p

$$\|x\|_p^p = \sum_i |x_i|^p \leq 1, \;\; p \leq 1$$

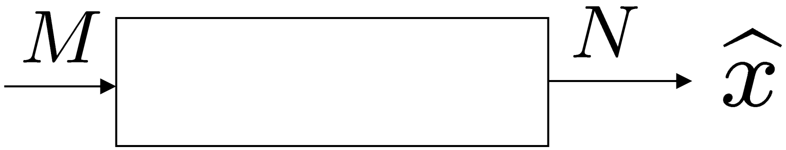




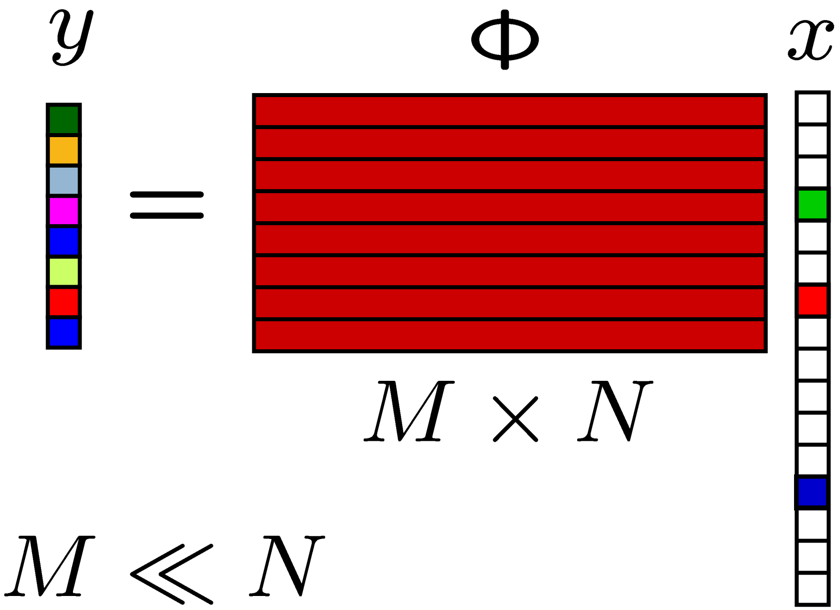


Compressive sensing

$$K \approx \underline{M} \ll N$$



Compressive Sampling



non full rank

$$y = \Phi x$$

$M \times 1$
measurements

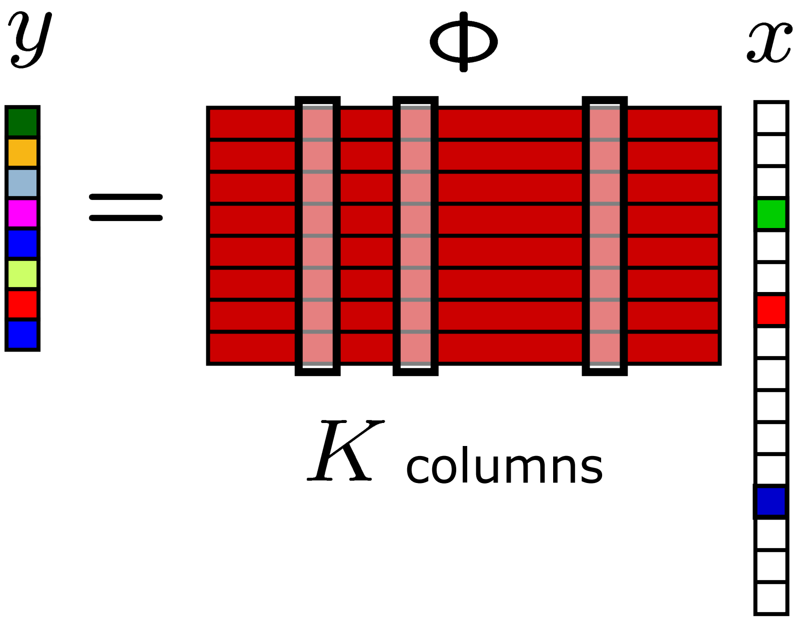
$M \times N$

$N \times 1$
sparse
signal

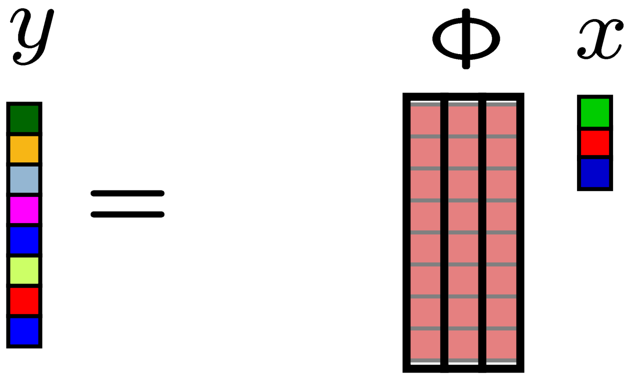
K
nonzero
entries

$$K < M \ll N$$

How can it work?



K columns

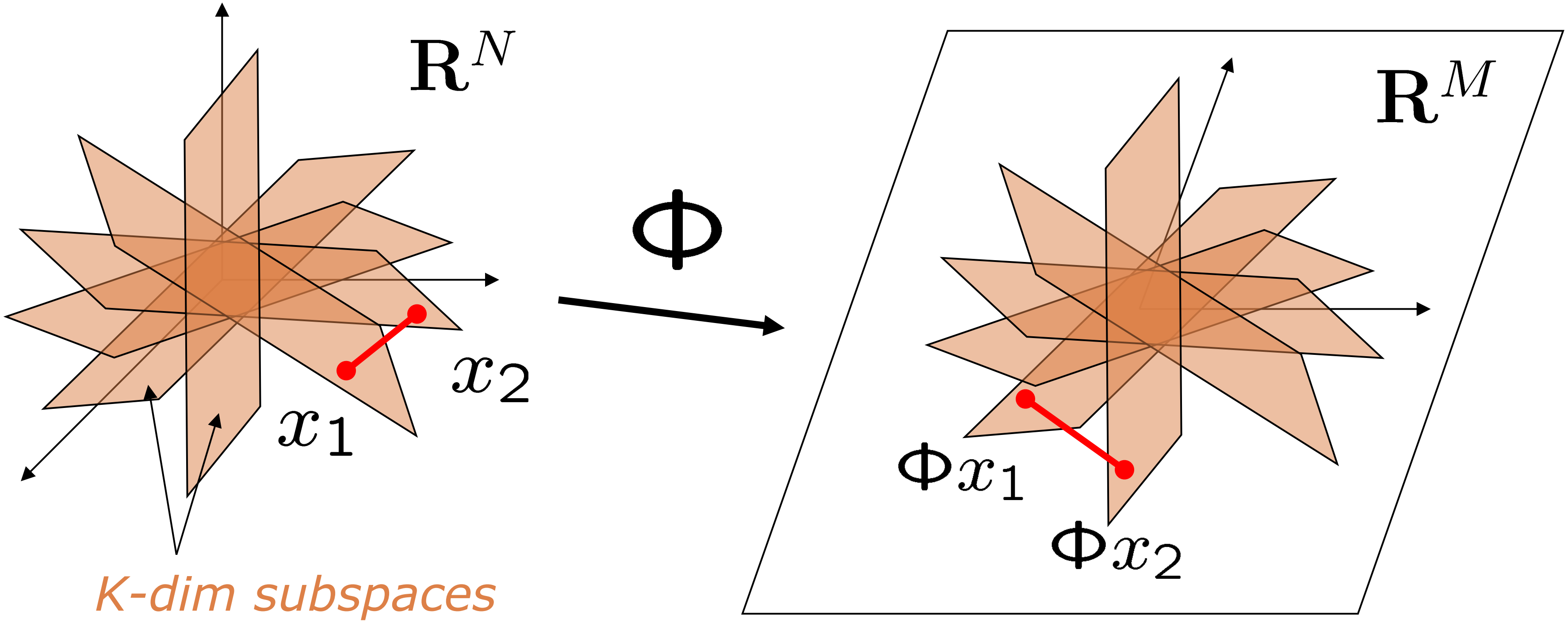


K columns

Restricted Isometry Property

- Design Φ so that each of its MxK submatrices are full rank (ideally close to orthobasis)
  - Restricted Isometry Property (RIP)

Preserve the structure of sparse/compressible signals



Restricted Isometry Property

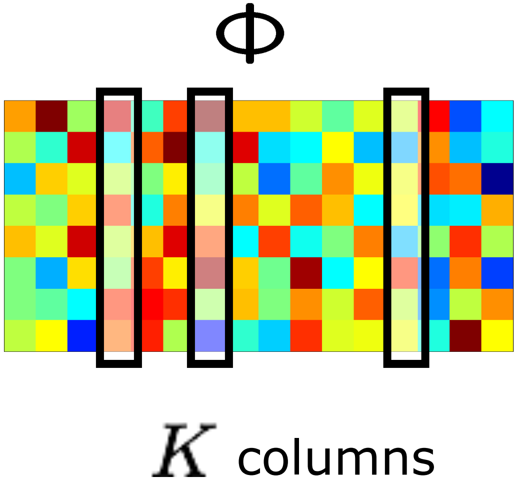
- RIP of order 2K implies
  - for all K-sparse x_1 and x_2

$$(1-\delta_{2K}) \leq \frac{\|\Phi x_1 - \Phi x_2\|_2^2}{\|x_1 - x_2\|_2^2} \leq (1+\delta_{2K})$$

  - Ensure that

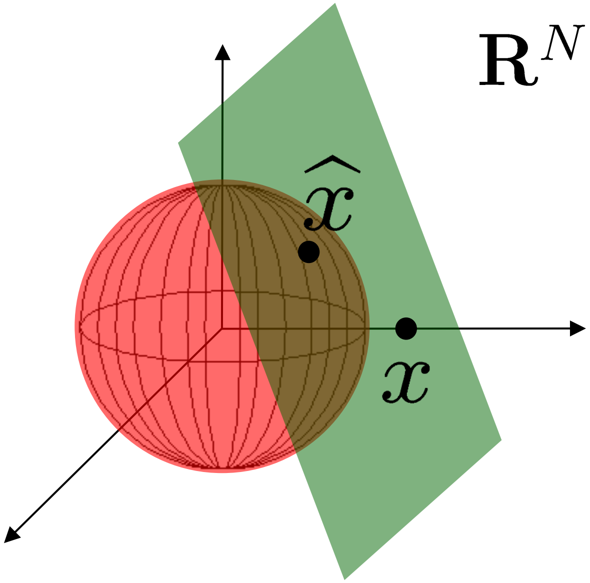
$$\|x_1 - x_2\|_2 \approx \|\Phi x_1 - \Phi x_2\|_2$$

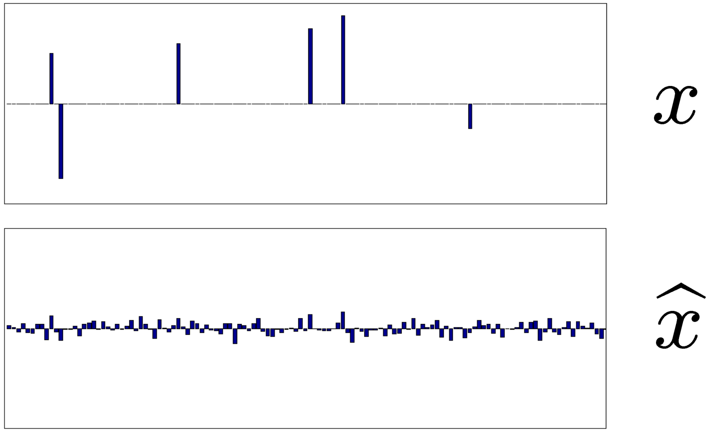
  - Draw Φ at random
    - iid Gaussian
    - iid Bernoulli



L_2 signal recovery

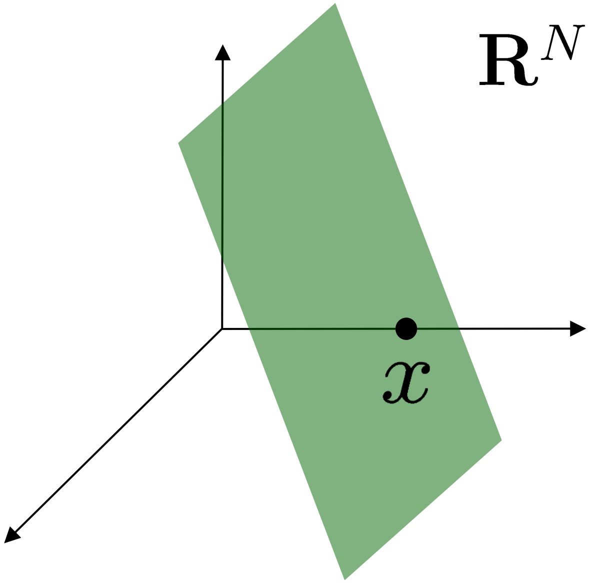
$$\widehat{x} = \arg\min_{y=\Phi x} \|x\|_2$$



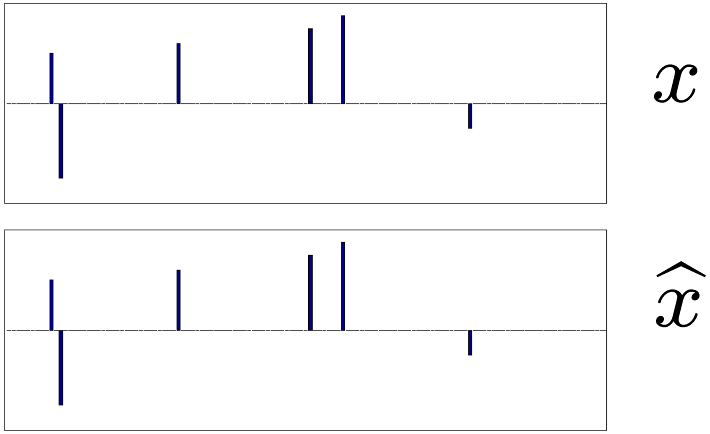


L_0 signal recovery

$$\widehat{x} = \arg\min_{y=\Phi x} \|x\|_0$$

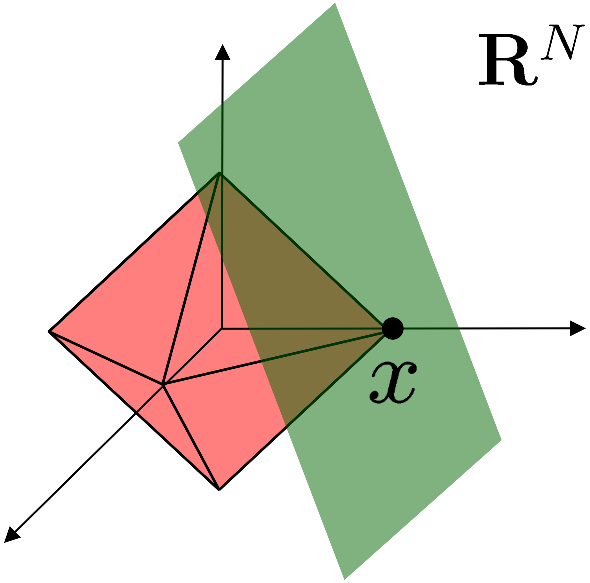


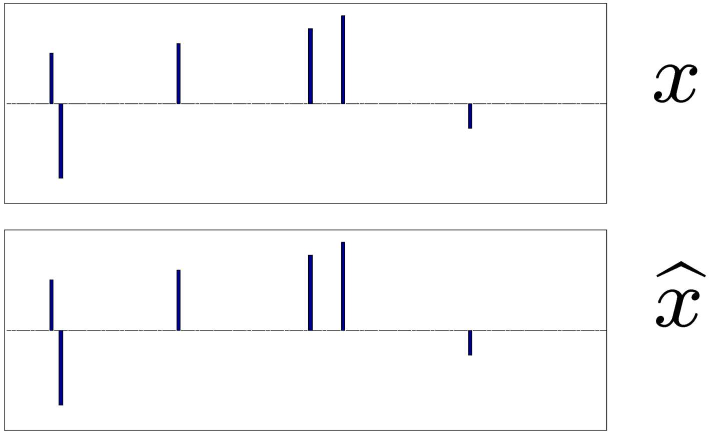
NP-Complete



L_0 signal recovery

$$\widehat{x} = \arg\min_{y=\Phi x} \|x\|_1$$





Polynomial time alg
(linear programming)

Compressive Sensing

observed signal

$$\mathbf{y} = \Phi_s \mathbf{f} \in \mathbf{R}^m$$

sensing matrix

source signal

orthonormal basis matrix

$$\mathbf{f} = \sum_{i=1}^{n} x_i \psi_i = \Psi \mathbf{x}$$

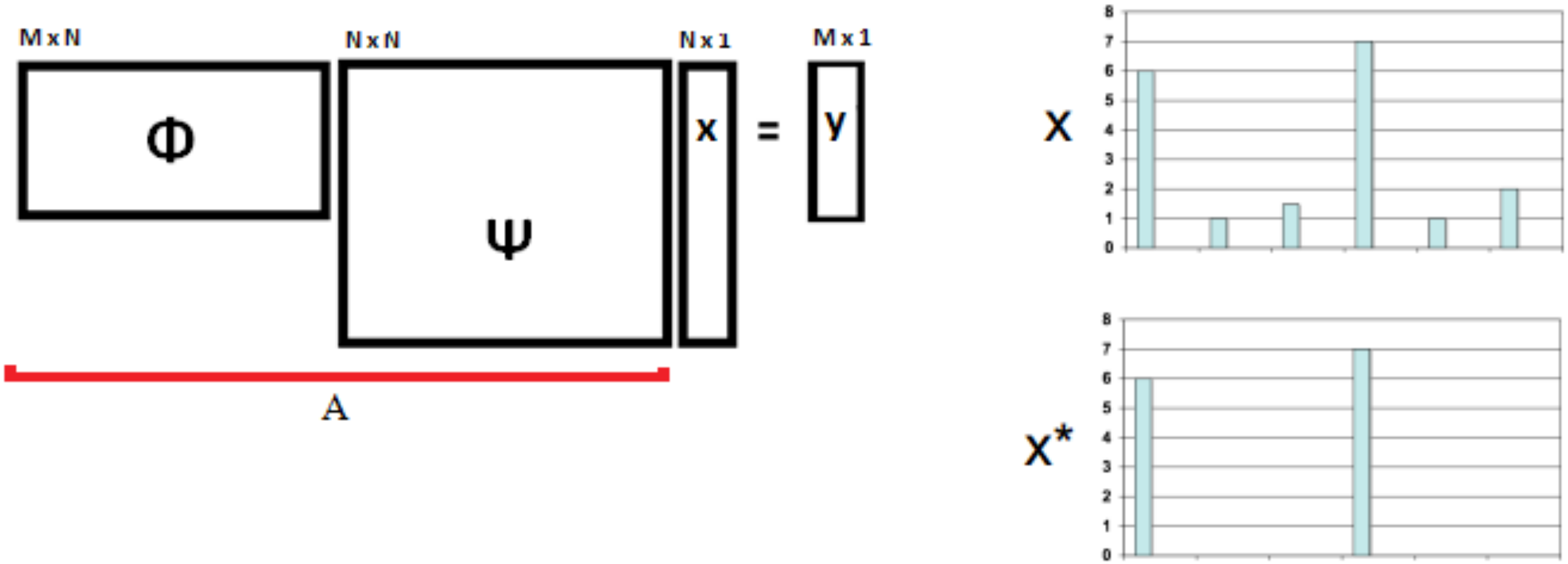
sparse representation

$$\mu(\Phi, \Psi)$$

coherence misure

In our case ϕ is the indentity matrix and ψ is a dictionay (*learned* or obtained by DCT)

Compressive Sensing



k=2

Optimization algorithm

l_0 -minimization
problem is NP-hard

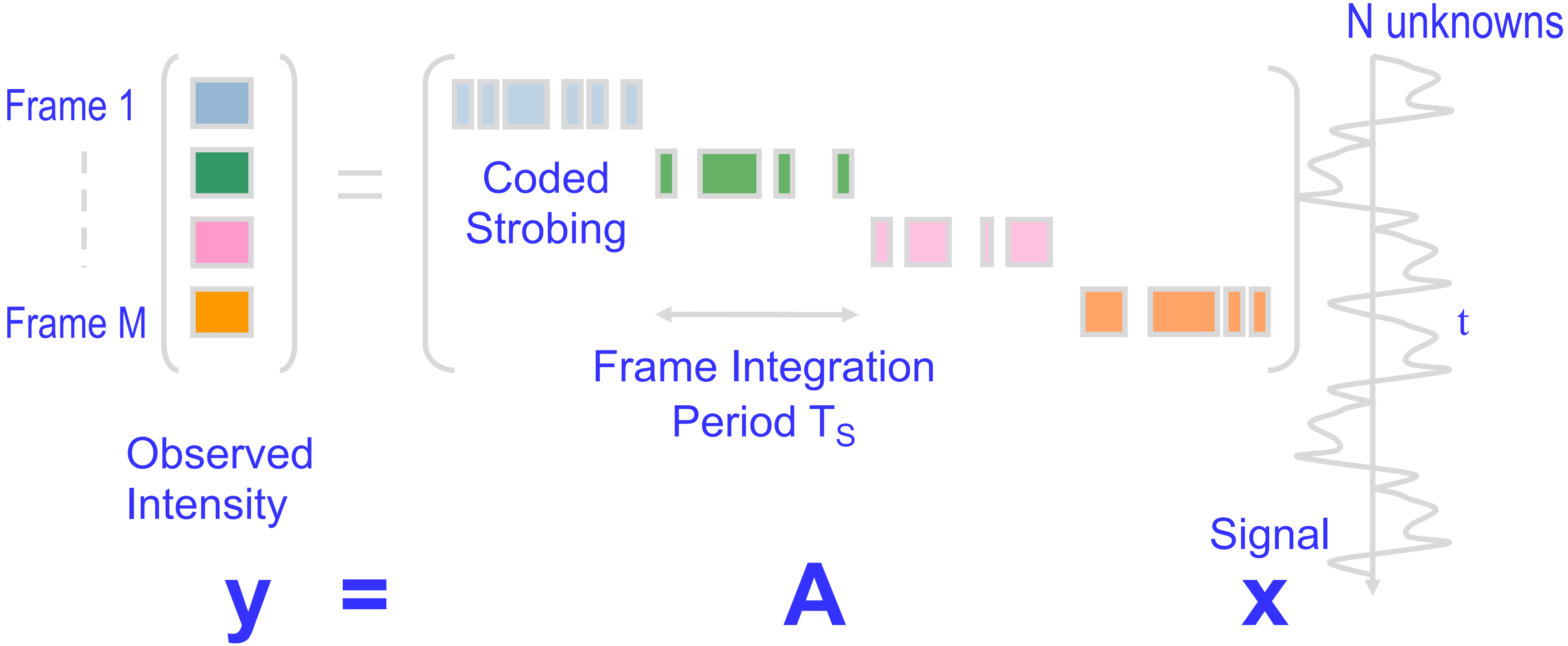
$$\mathbf{f}^* = \Psi \mathbf{x}^*$$ reconstruction

$$\min \|\mathbf{x}\|_{L_1} \quad \text{subject to} \quad y_k = \langle \phi_k, \Psi \mathbf{x} \rangle \quad \forall k \in M$$
$$\mathbf{x} \in R^n$$

Convex optimization algorithm

https://statweb.stanford.edu/~candes/l1magic/#code

ISP – Streaming and Compressive Sensing



N unknowns

Frame 1

Frame M

Coded Strobing

Frame Integration Period T_S

t

Observed Intensity

Signal

$$\mathbf{y} = \mathbf{A} \quad \mathbf{x}$$

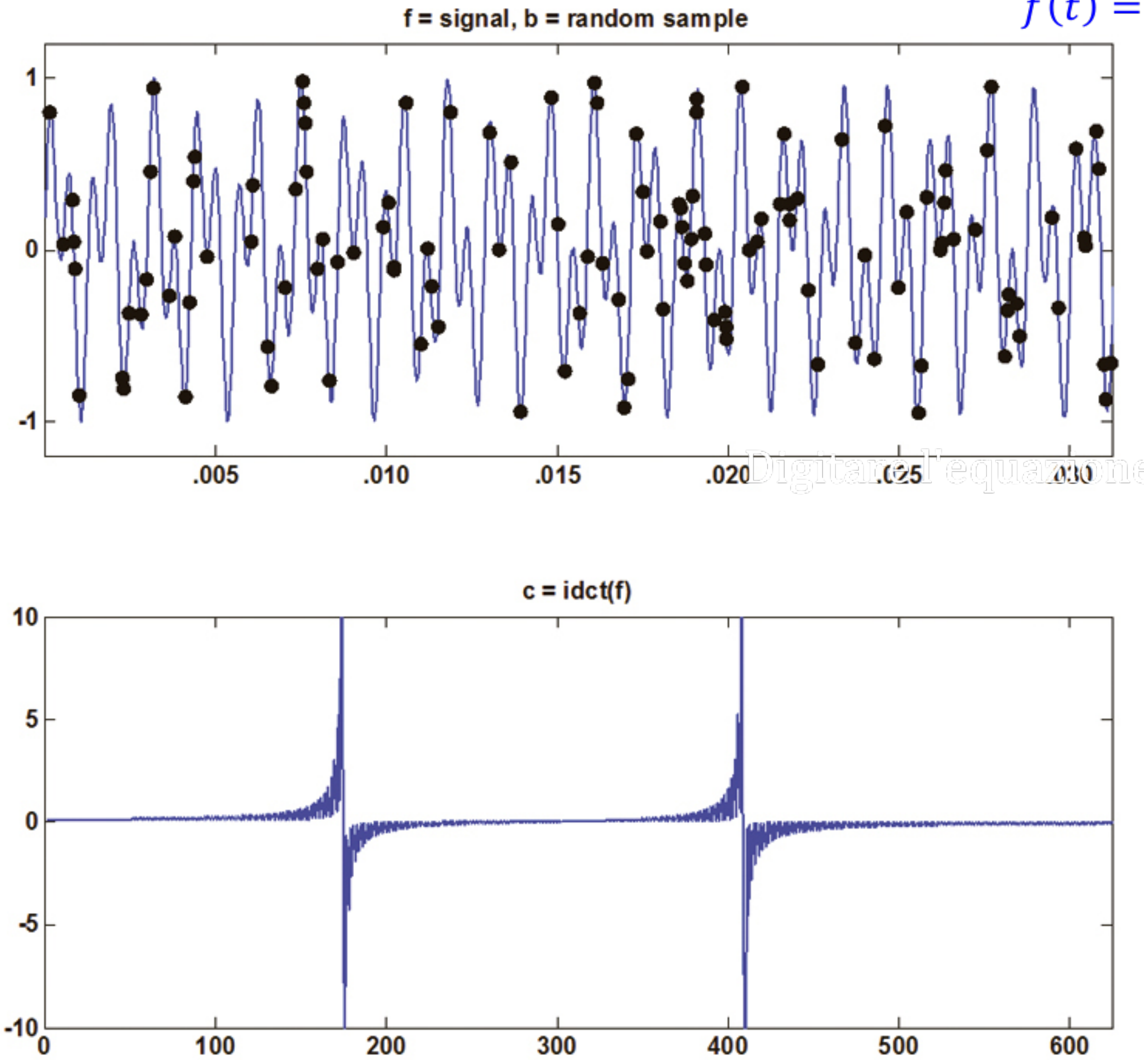
Introduction

$f(t) = \sin(1394\,\pi\,t) + \sin(3266\,\pi\,t)$

1/8 of esecond at 4 kHz

$n = 5000$

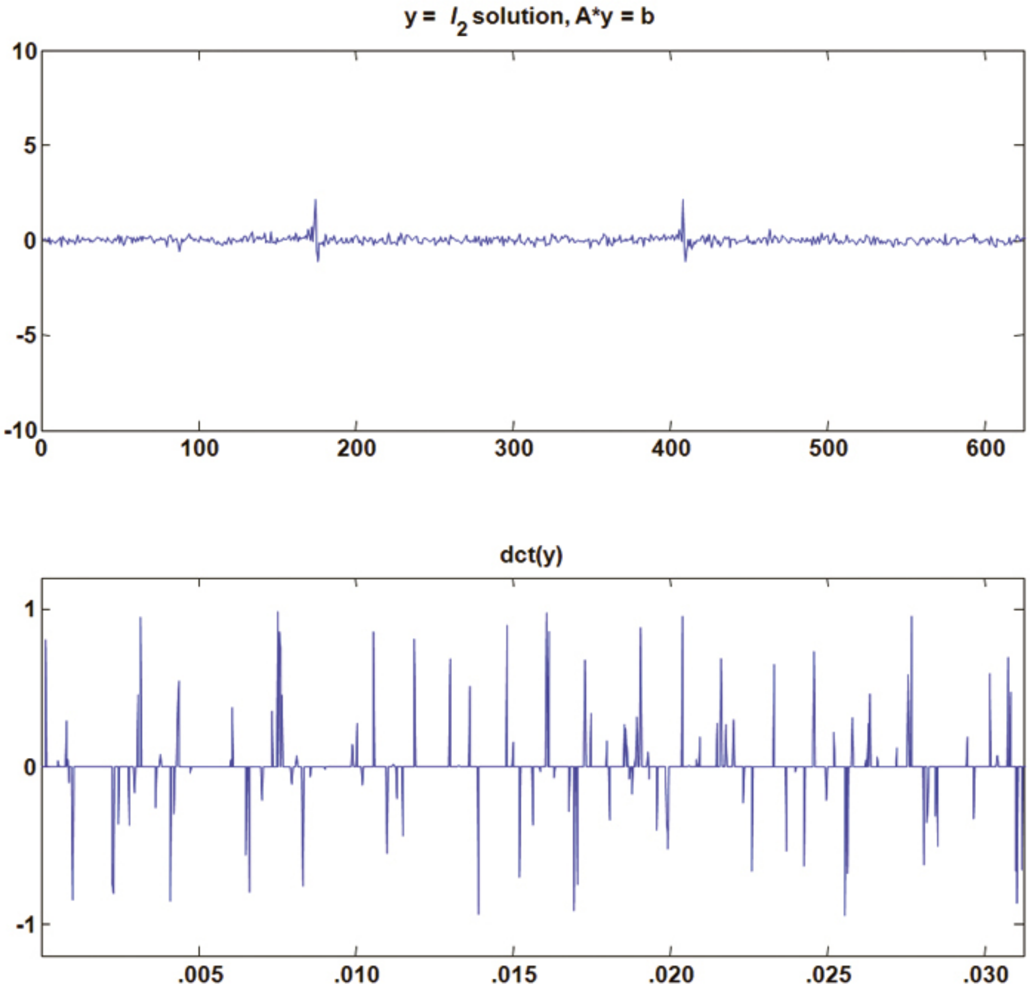
$m = 500\ random\ samples$



A DCT based dictionary is used

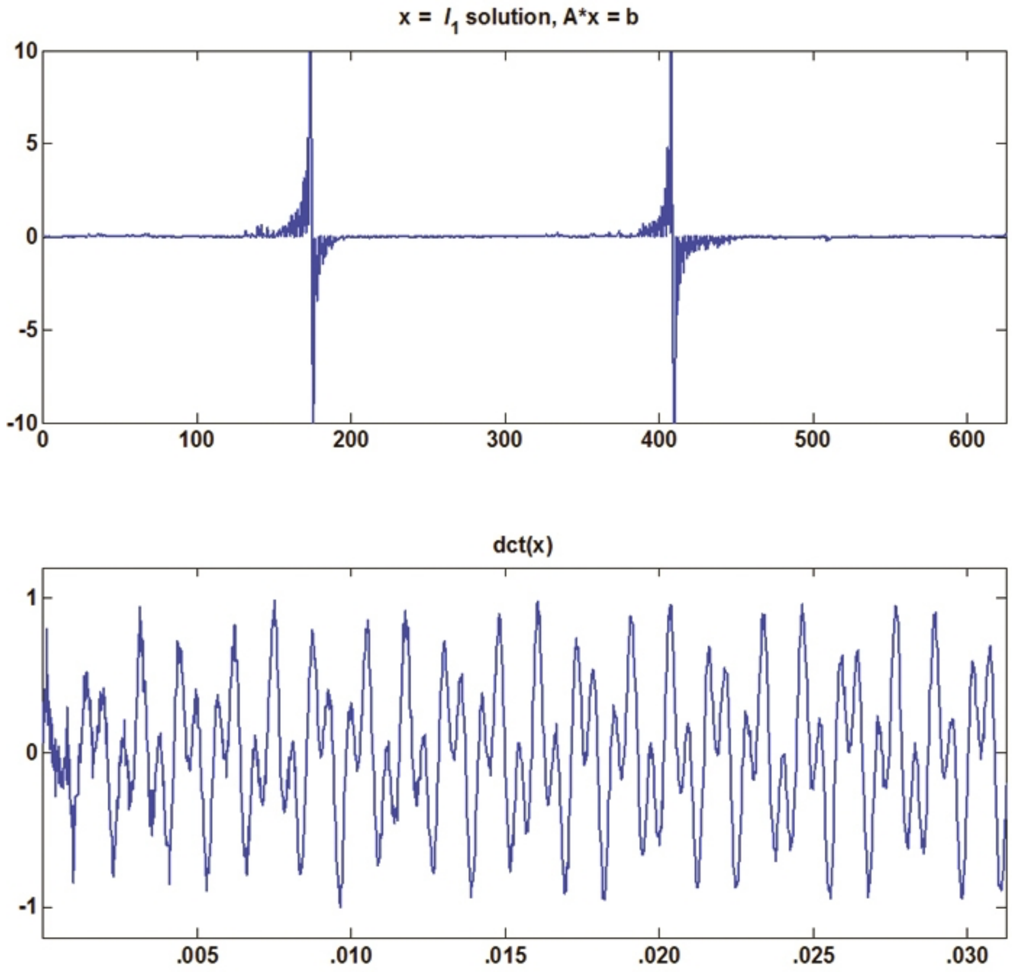
Top: Random samples of the original signal generated by the “A” key on a touch-tone phone. Bottom: The inverse discrete cosine transform of the signal.

Introduction



Results by uisng L_2 norm

Introduction



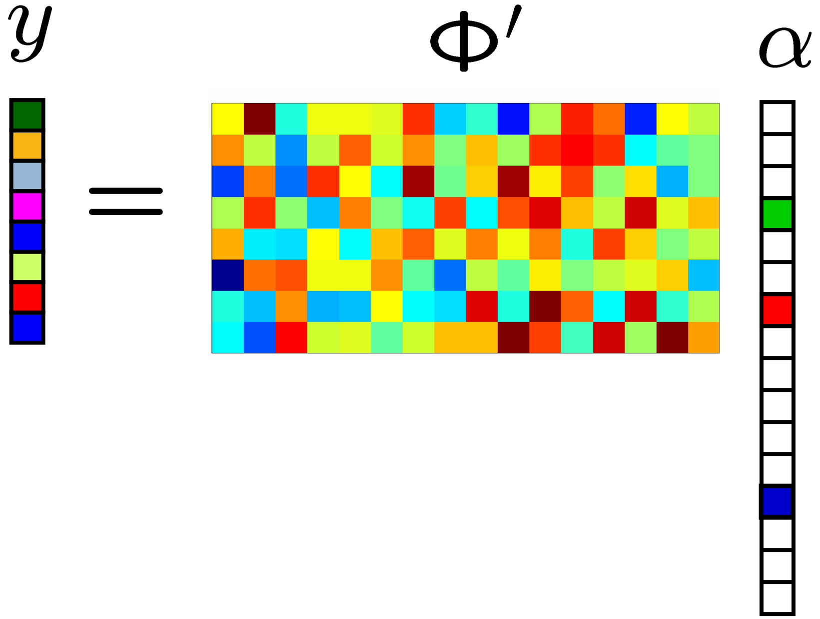
Results by uisng L_1 norm

Optimization algorithms

- Signal recovery via iterative greedy algorithm
  - (orthogonal) matching pursuit
  - iterated thresholding
  - CoSaMP

Universality

$$y = \Phi x = \Phi \Psi \alpha = \Phi' \alpha$$



$N \times 1$
sparse
coefficient
vector

K
nonzero
entries

Random measurements can be used for signals sparse in *any* basis: DCT/FFT/Wavelet/Learned Dictionary

Dictionay learning

- Goal
  - Given training data

$$x_1, x_2, \ldots, x_T \qquad x_i \in \mathbb{R}^N$$

  - learn a dictionary D

$$x_i = Ds_i \qquad D \in \mathbb{R}^{N \times Q}$$

$$s_i \in \mathbb{R}^Q$$

where s_i are sparse

Dictionay learning

- Optimization approach

$$\min_{D,S} \|X - DS\|_F$$

$$\text{s.t}$$

$$\forall i,\ \|s_i\|_0 \leq K$$

Non-convex constraint

Bilinear in *D* and *S*

Dictionay learning

- Optimization approach

$$\min_{D,S} \|X - DS\|_F$$

$$\text{s.t}$$

$$\forall i,\ \|s_i\|_0 \leq K$$

Non-convex constraint

Bilinear in *D* and *S*

- Biconvex in D and S

$$\min_{D,S} \|X - DS\|_F + \lambda \sum_k \|s_k\|_1$$

Given *D*, the optimization problem is convex in s_k
Given *S*, the optimization problem is a least squares problem

Dictionay learning

- K-SVD
  - Solve using alternate minimization techniques
    - Start with D = wavelet or DCT bases
    - Additional pruning steps to control size of the dictionary

- Sparse Modeling for Finding Representative Objects

$$\min \|\boldsymbol{Y} - \boldsymbol{Y}\boldsymbol{C}\|_F^2 \;\; \text{s.t.} \;\; \|\boldsymbol{C}\|_{1,q} \leq \tau, \;\; \mathbf{1}^\top \boldsymbol{C} = \mathbf{1}^\top$$

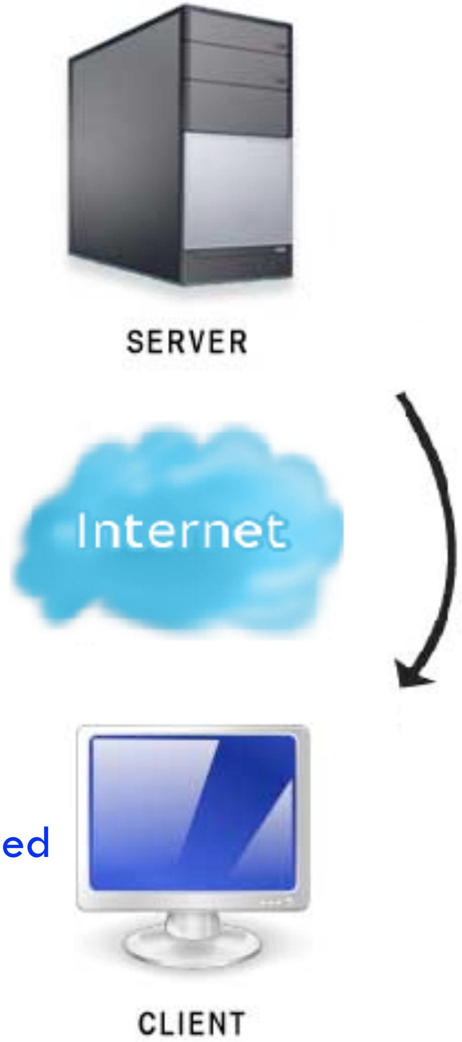
Finding Representative Objects

Deblurreing

Blurred Photos

Deblurred Result

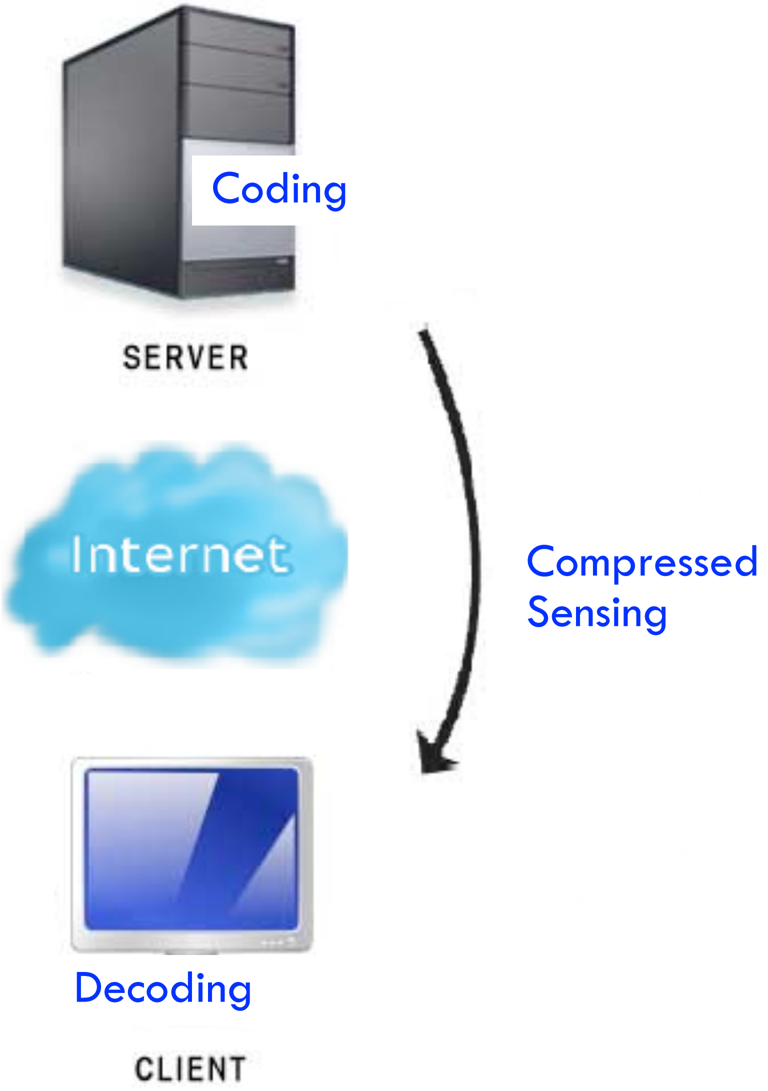
First scenario



Packet loss

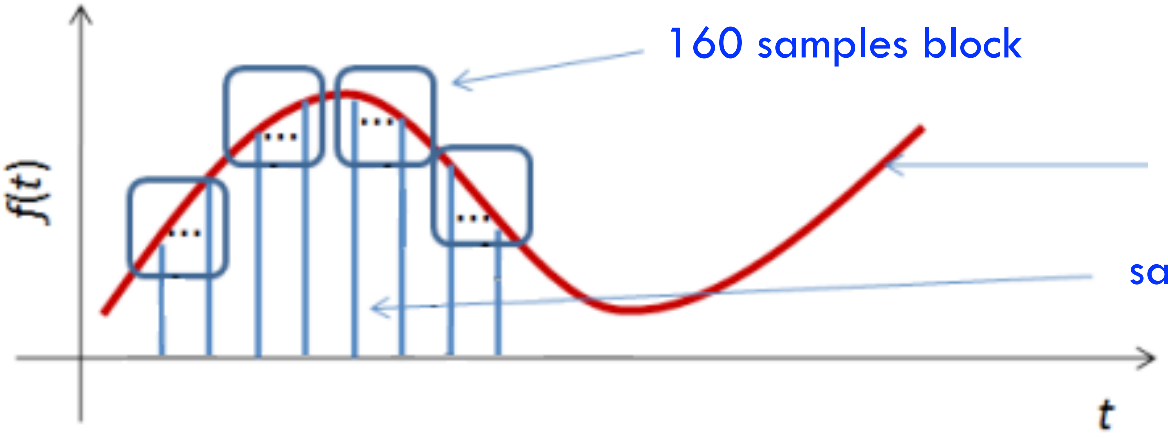
Compressed Sensing

Second Scenario



Packet loss

1 sec = 64kbit/sec ---> 1 msec = 64bit ---> 20msec = 1280bit ---> 160 byte

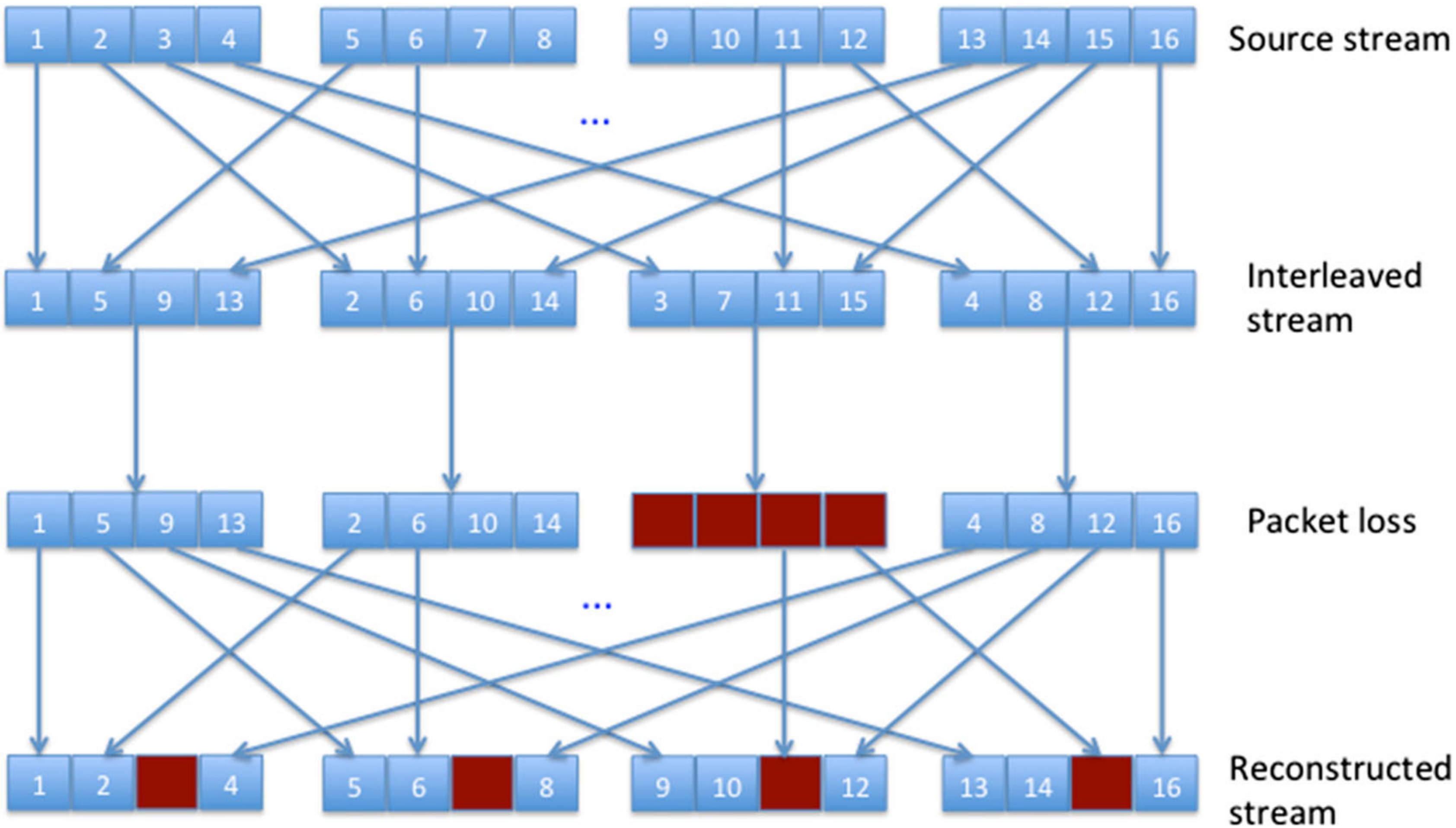


160 samples block

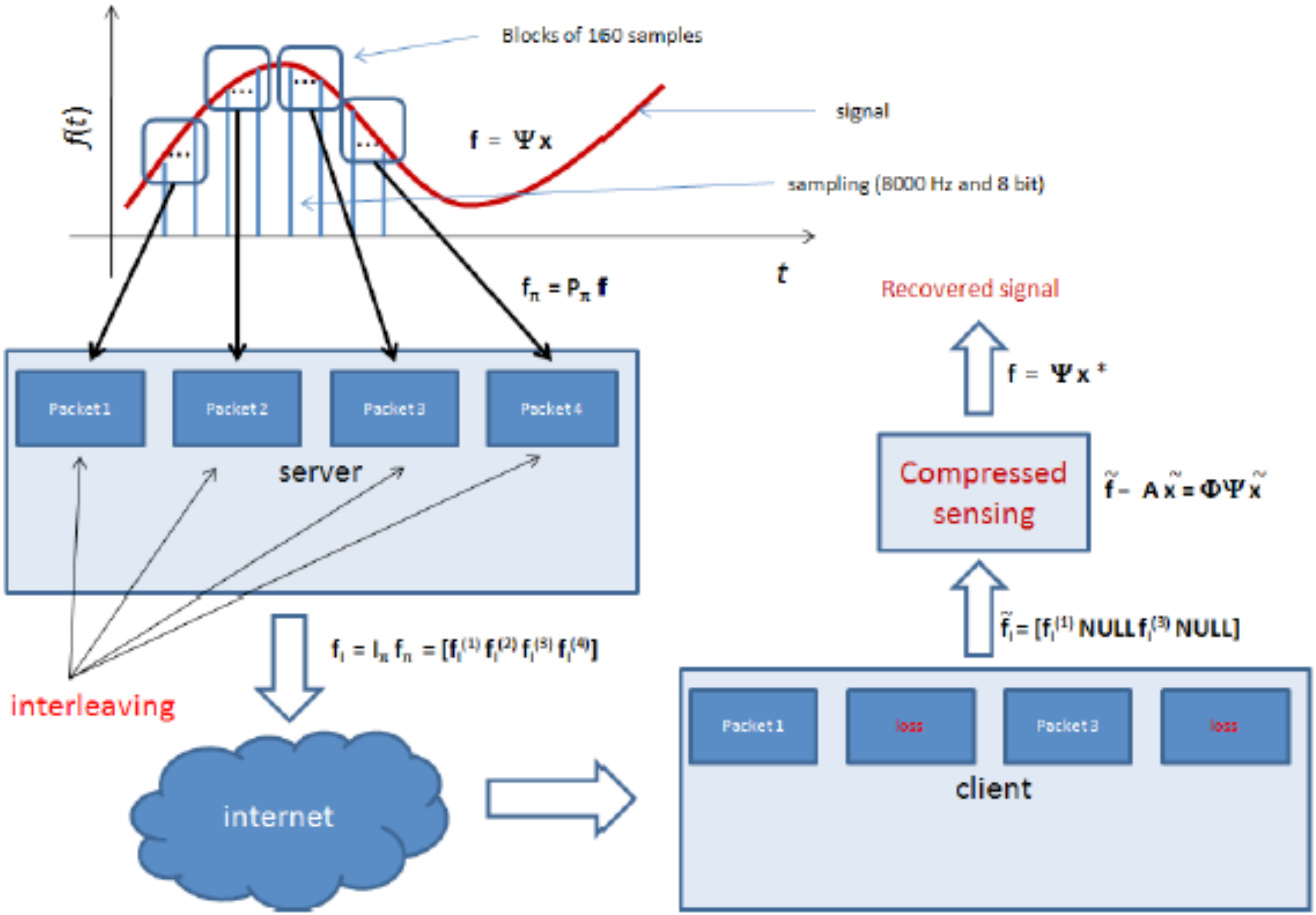
signal

sampling (8 KHz, 8 bit)

Interleaving



Reconstruction scheme



Blocks of 160 samples

$f(t)$

signal

$\mathbf{f} = \Psi \mathbf{x}$

sampling (8000 Hz and 8 bit)

t

$\mathbf{f}_\pi = \mathbf{P}_\pi \mathbf{f}$

Recovered signal

Packet1 Packet2 Packet3 Packet4

server

$\mathbf{f} = \Psi \mathbf{x}^*$

Compressed sensing

$\tilde{\mathbf{f}} - \mathbf{A}\tilde{\mathbf{x}} = \Phi\Psi\tilde{\mathbf{x}}$

interleaving

$\mathbf{f}_I = \mathbf{I}_\pi \mathbf{f}_\pi = [\mathbf{f}_I^{(1)}\ \mathbf{f}_I^{(2)}\ \mathbf{f}_I^{(3)}\ \mathbf{f}_I^{(4)}]$

$\tilde{\mathbf{f}}_I = [\mathbf{f}_I^{(1)}\ \mathrm{NULL}\ \mathbf{f}_I^{(3)}\ \mathrm{NULL}]$

internet

Packet1 loss Packet3 loss

client

Experimental results

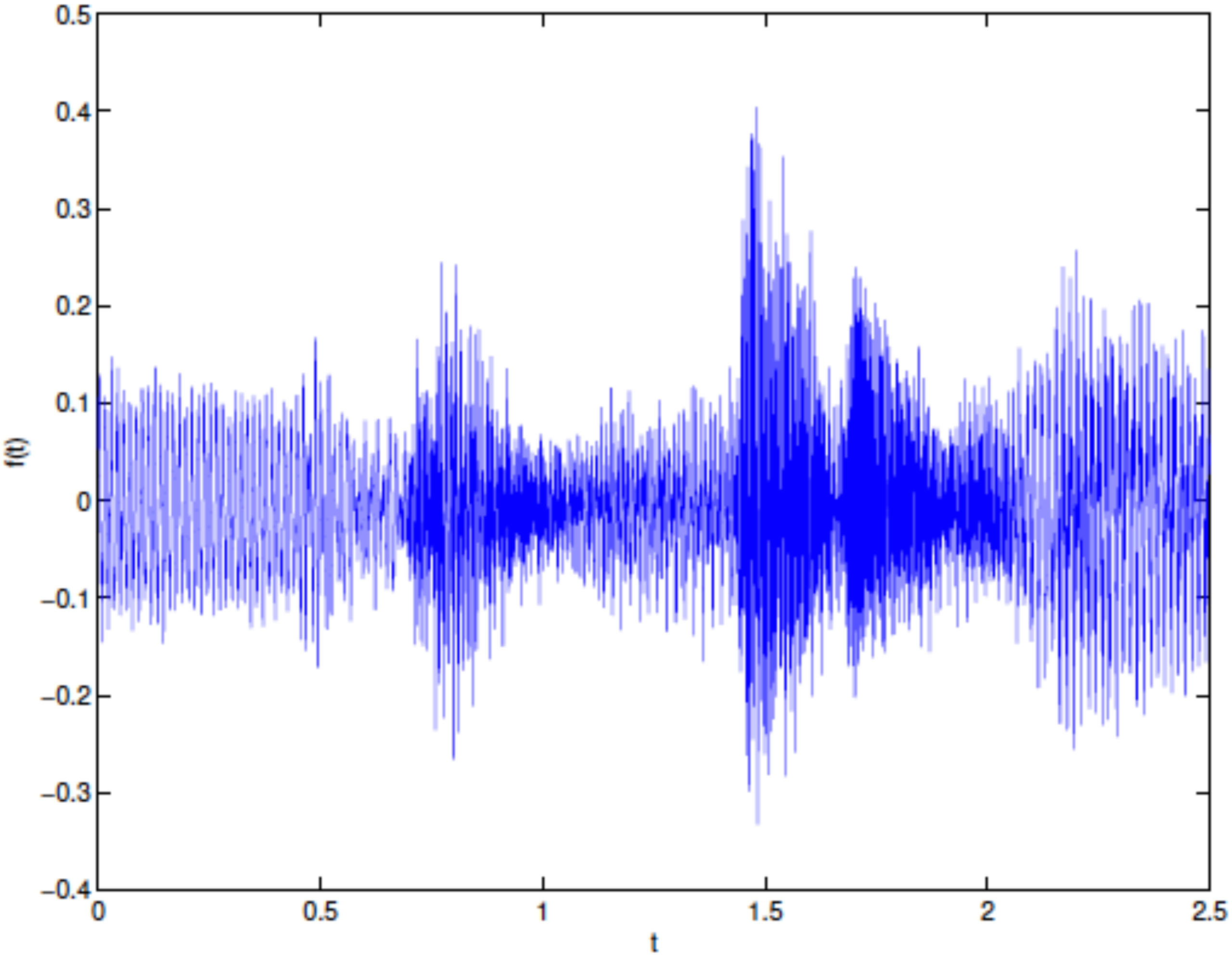


Fig. 4. Audio signal of a female speaker.

Experimental results

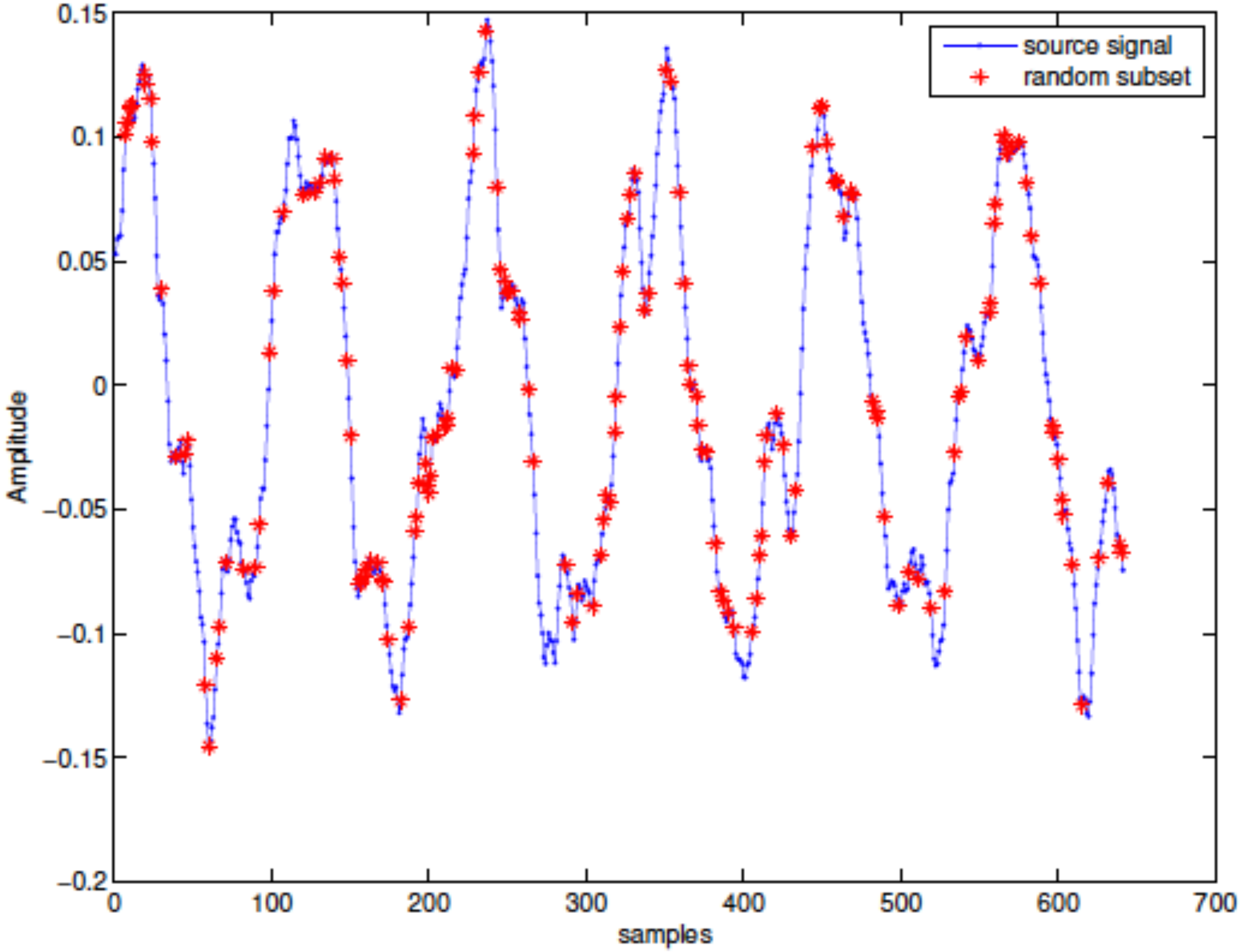


Fig. 5. Frame information after 3 packets lost.

Experimental results

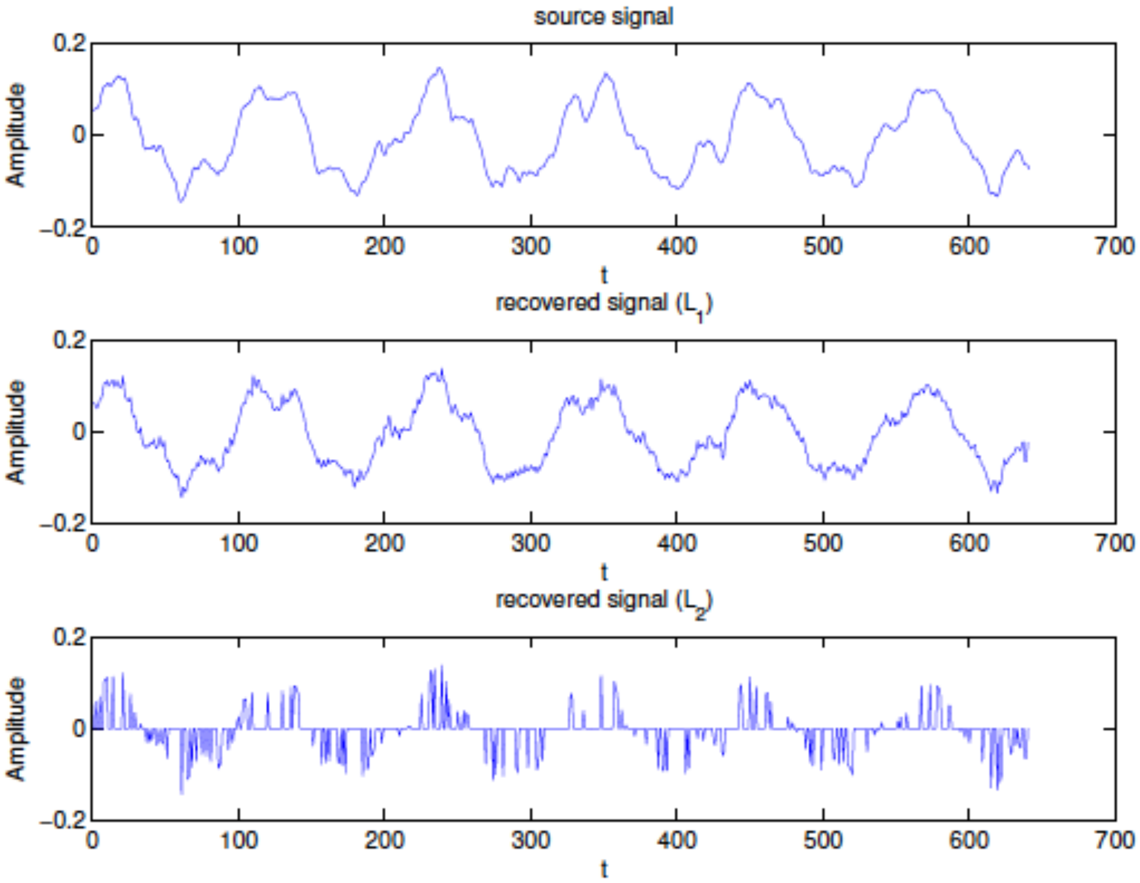


Fig. 6. Comparison between a frame of the source signal and those of the recovered signals by using L_1 and L_2 norms.

Experimental results

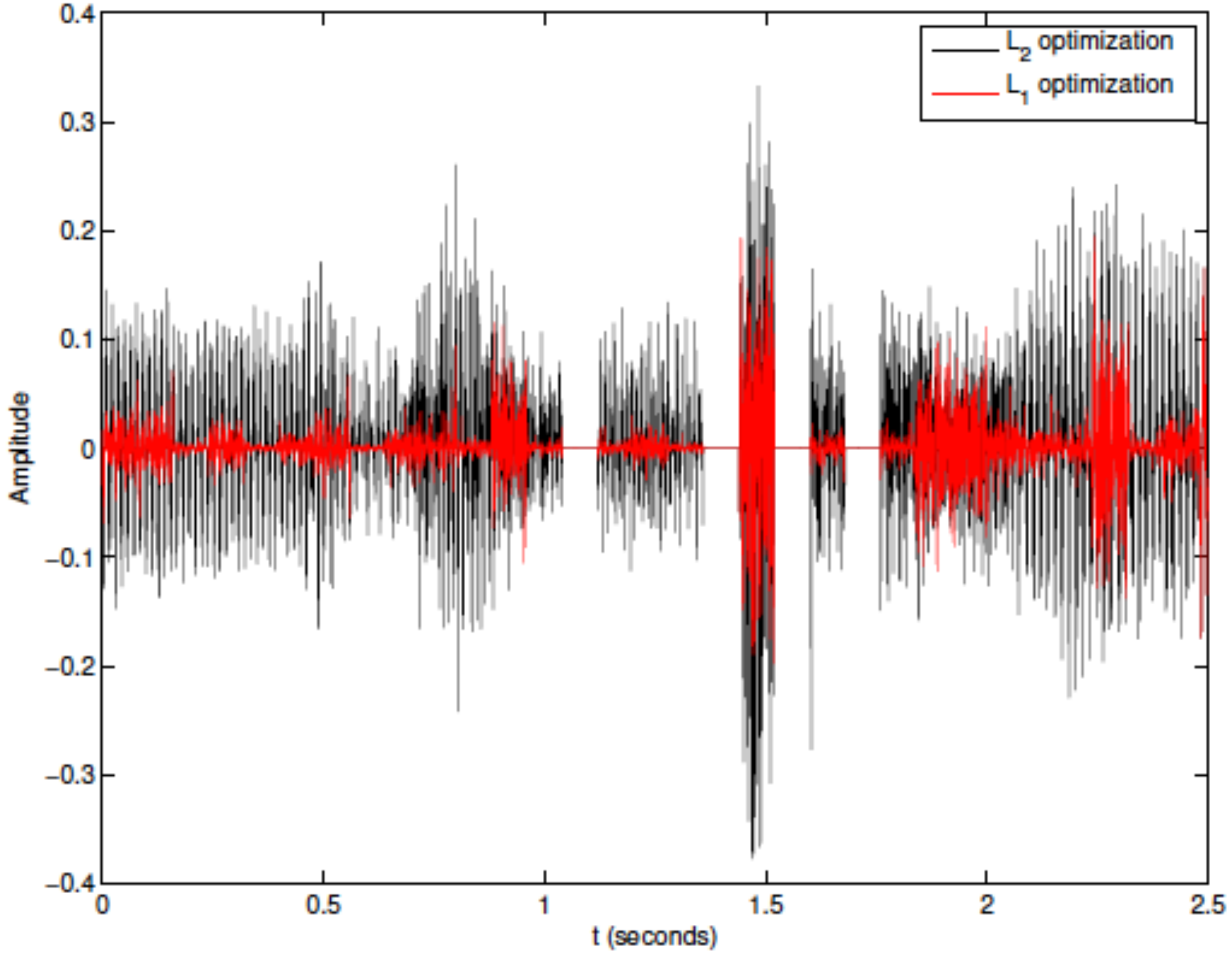


Fig. 7. Resisdua between the source signal and the recovered signals by using L_1 and L_2 norms.

Experimental results

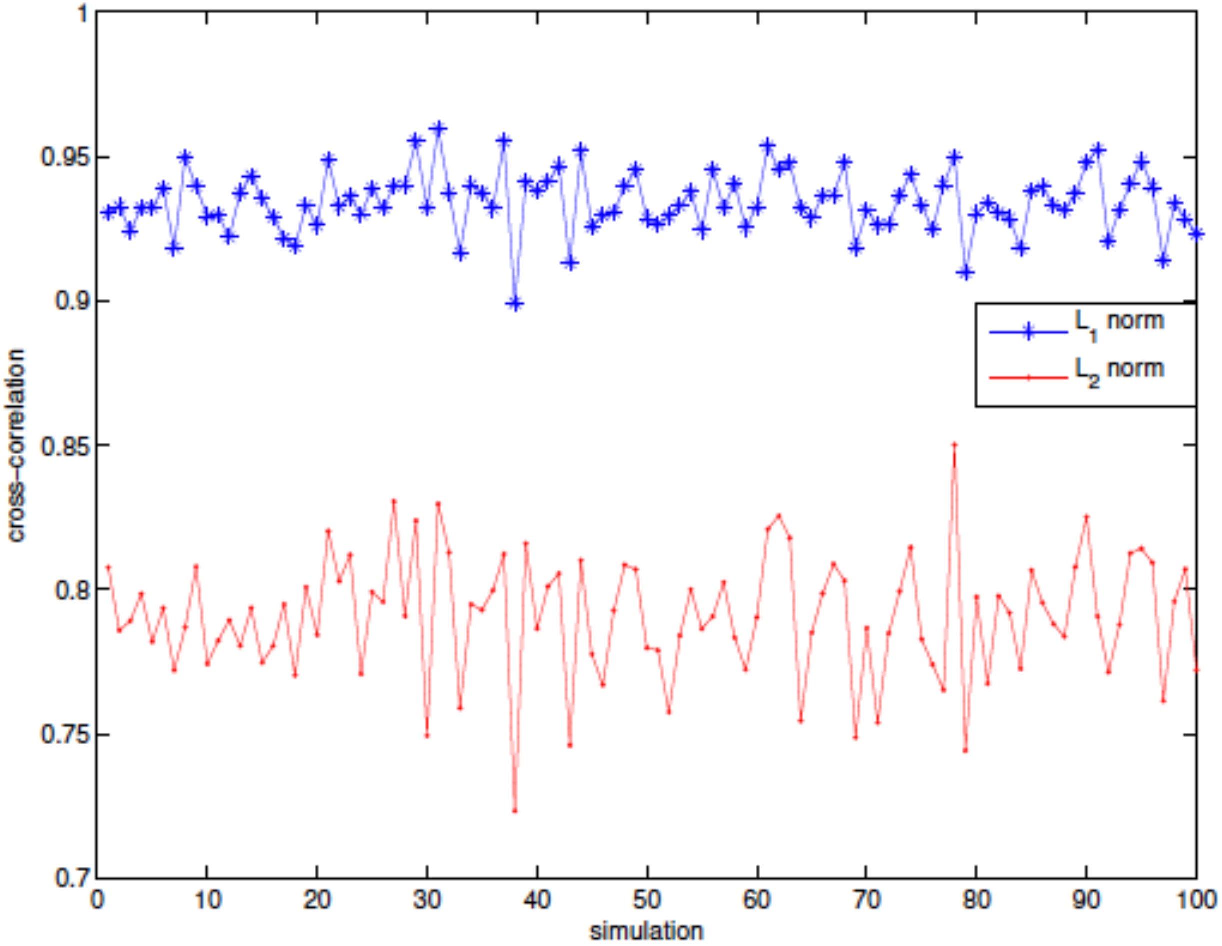


Fig. 8. Cross-correlation coefficients after 100 simulations: audio female speaker.

Experimental results

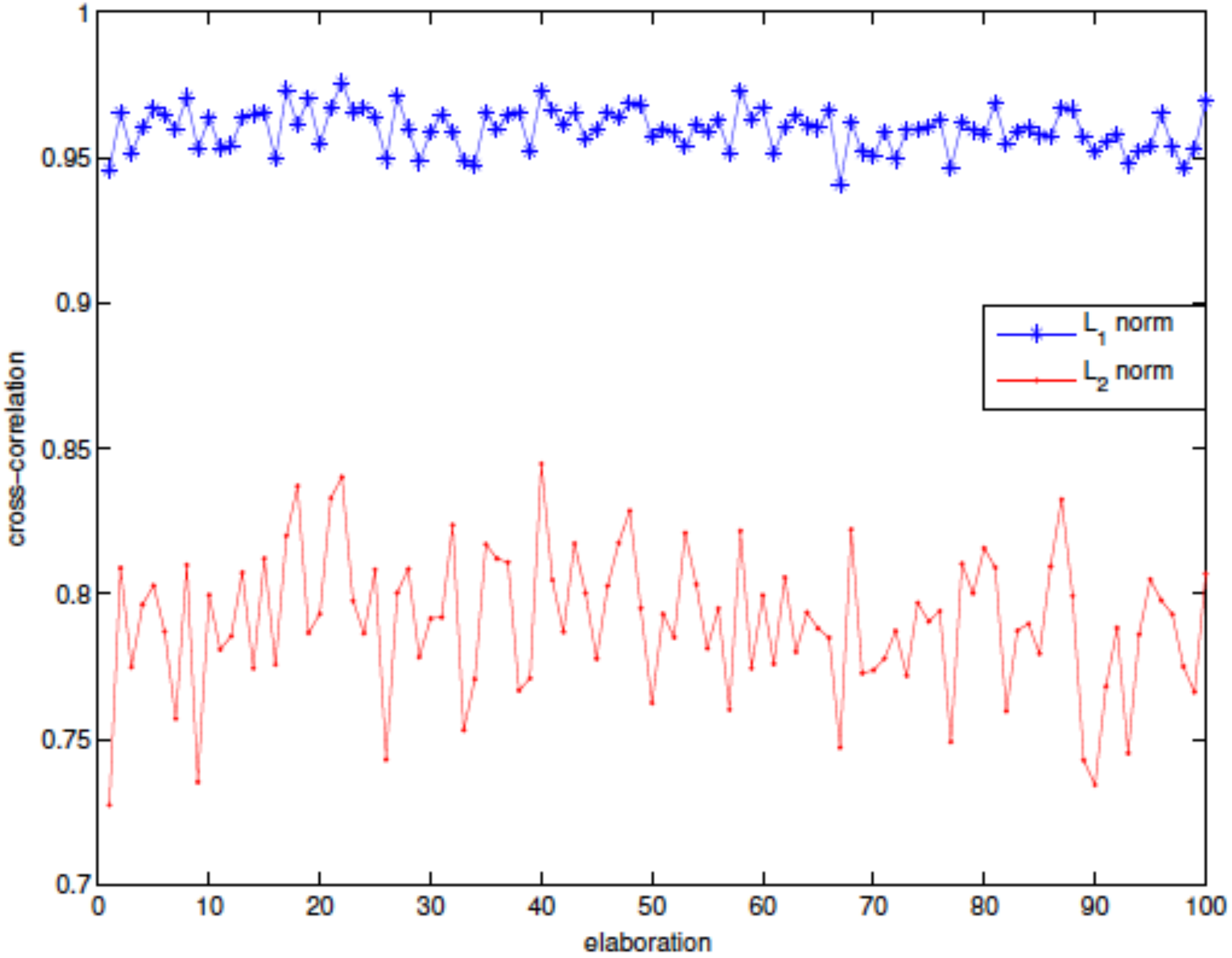


Fig. 9. Cross-correlation coefficients after 100 simulations: audio male speaker.

Experimental results

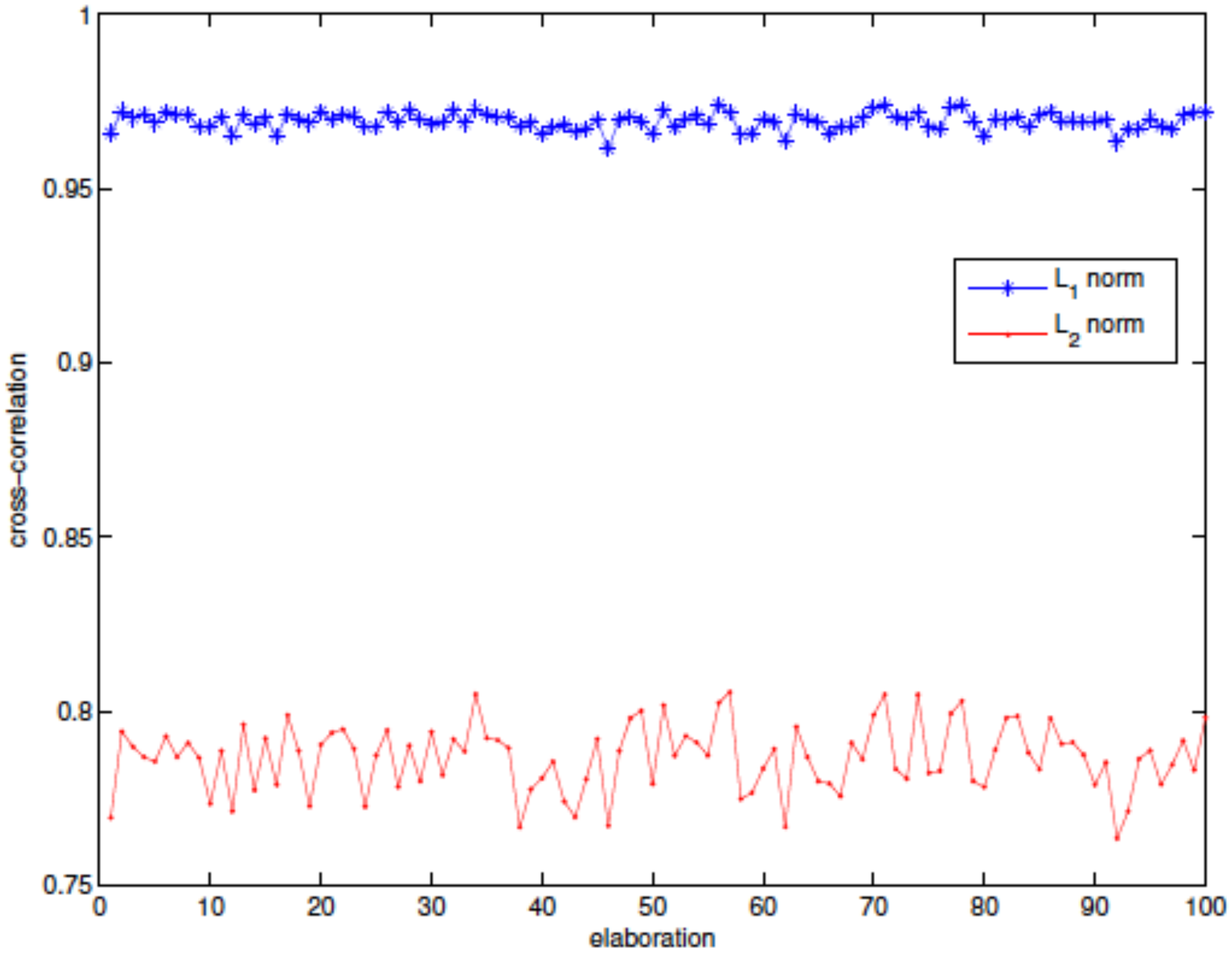


Fig. 10. Cross-correlation coefficients after 100 simulations: audio song.

Compression scheme