CI&SS Lab

UNIVERSITÀ PARTHENOPE
DiST
DIPARTIMENTO DI SCIENZE E TECNOLOGIE

Intelligent Signal Processing

Signal Analysis

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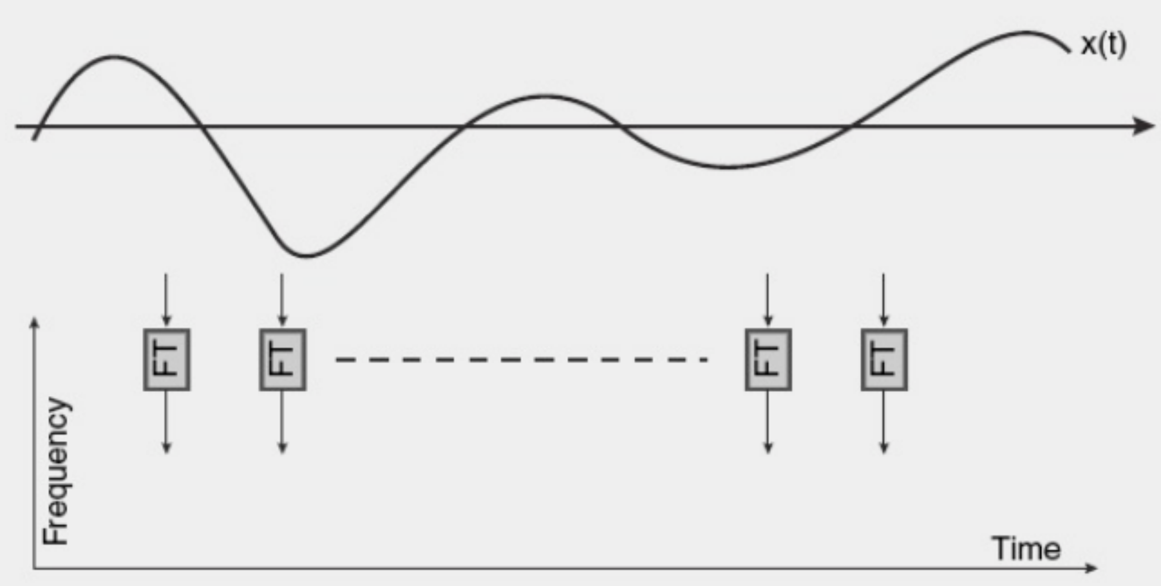
Non-stationary signals

- No trivial problem
- The autocorrelation function is no longer a function of lag only
- Time-frequency representation
  - Break the timeseries into segments
  - Estimate the spectrum for each segment
- Approaches
  - Gabor filtering/transform
  - Short Time Fourier Transform
  - Wavelet analysis
  - Spectrogram

Gabor Transform



Named after Dannis Gabor. Determine sinusoidal frequency and phase content of local sections

Gabor Transform

- The function to be transformed is first multiplied by a Gaussian function (window)
- Transformed with a Fourier transform to derive the time-frequency analysis
- The window function means that the signal near the time being analyzed will have higher weight

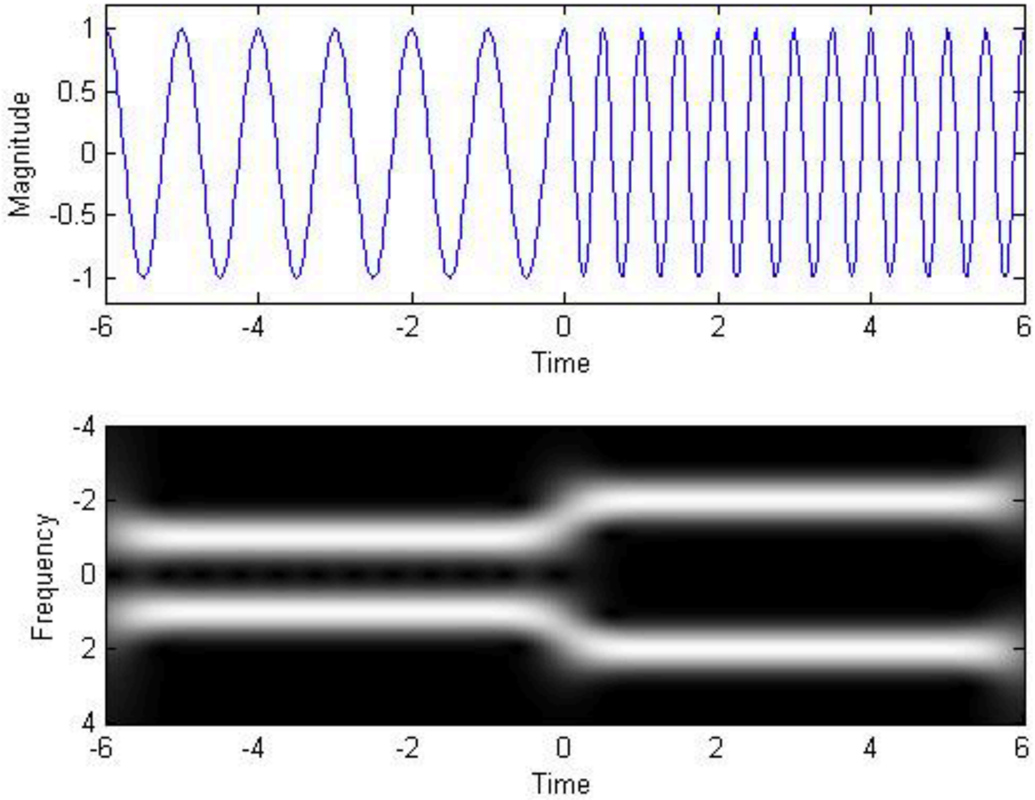
$$G_X(t,f) = \int_{-\infty}^{+\infty} e^{-\pi(\tau-t)^2}\, e^{-j2\pi f\tau} x(\tau) d\tau$$

Transform

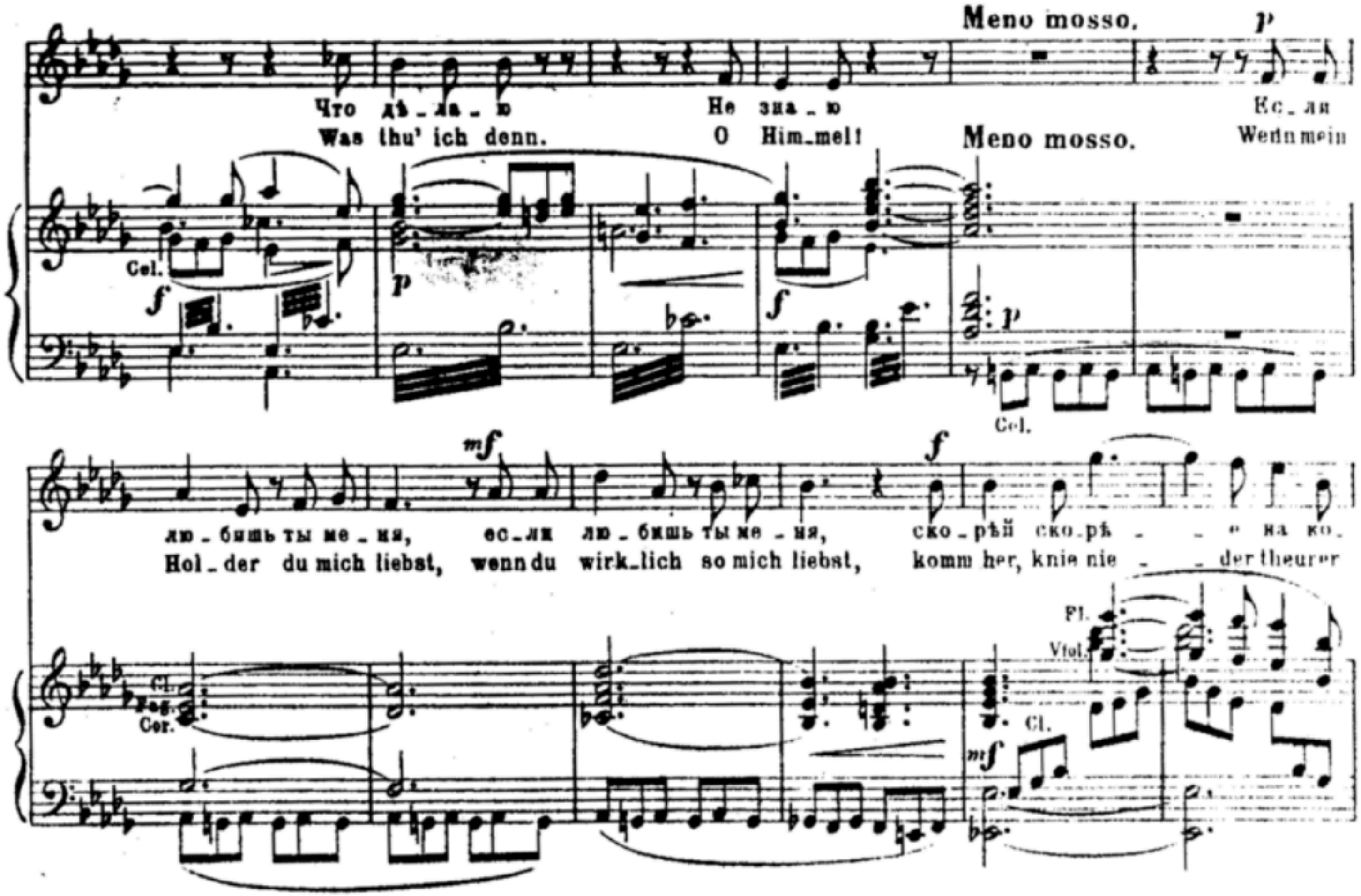
$$x(t) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} G_X(\tau,f) e^{j2\pi tf}\, df\, d\tau$$

Inverse

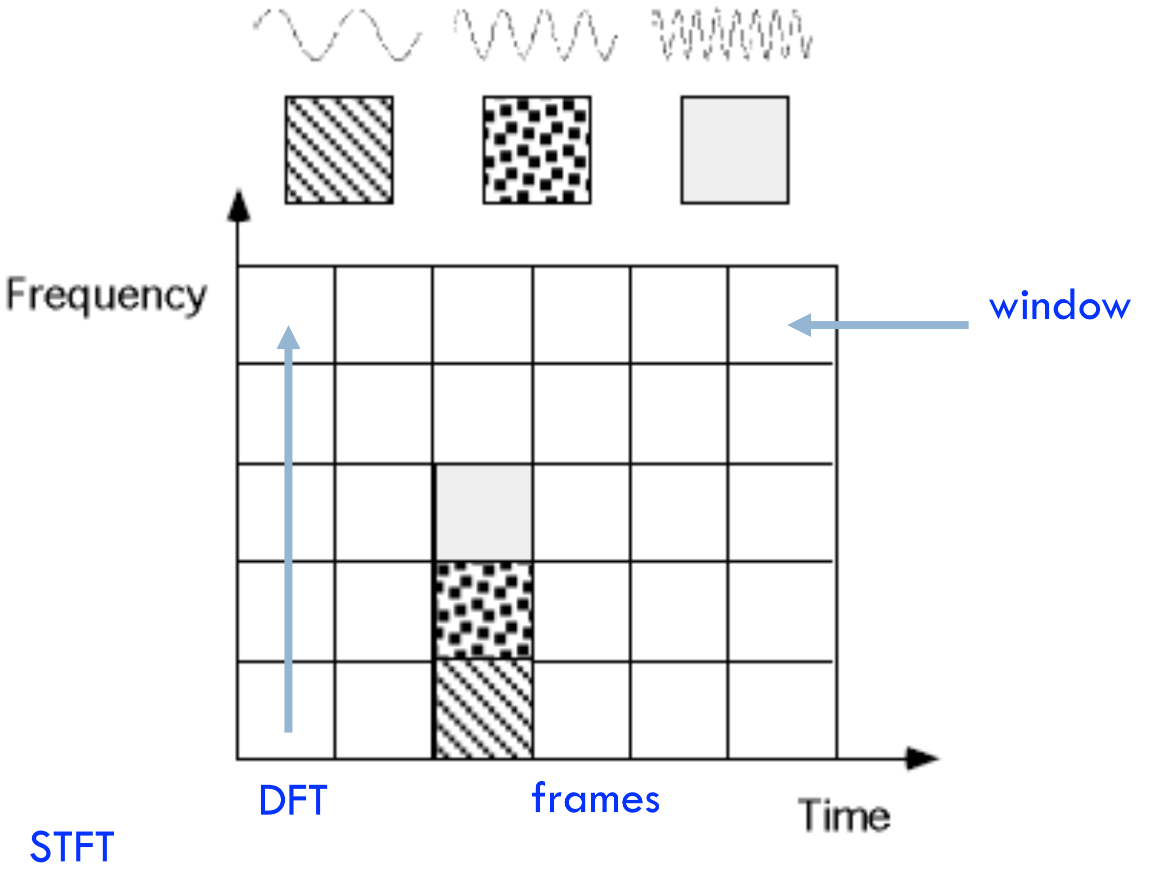
Gabor Transform Example



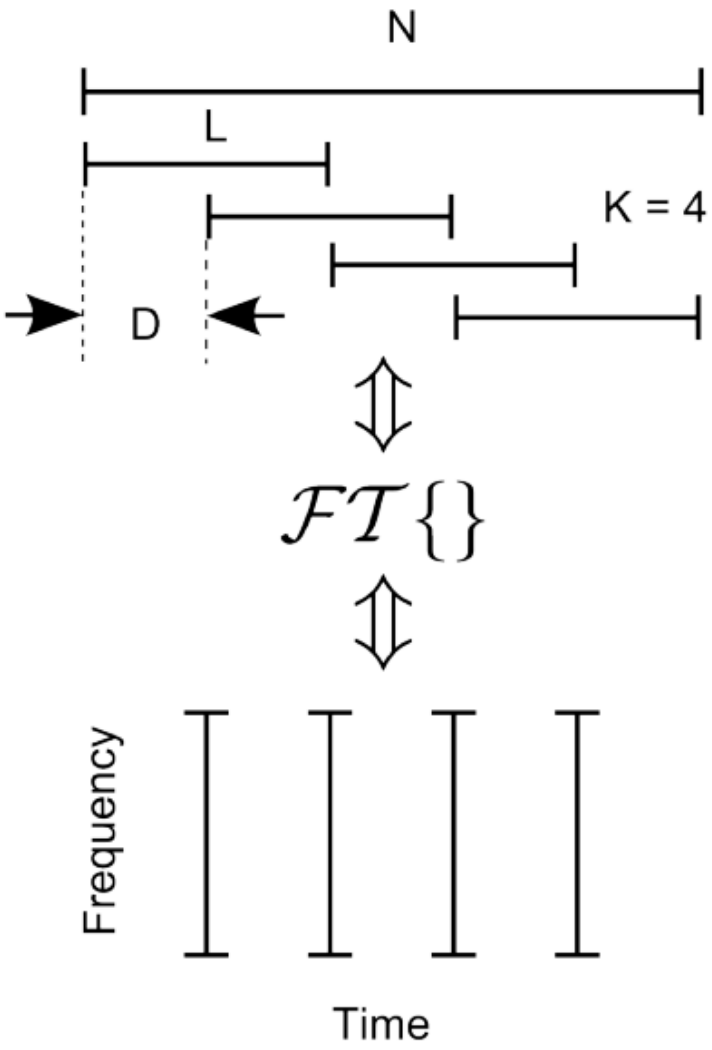
Adding the frequency axis we can detect different time-dependent components in the signal

Time->Frequency Representation

Short Time Fourier Transform

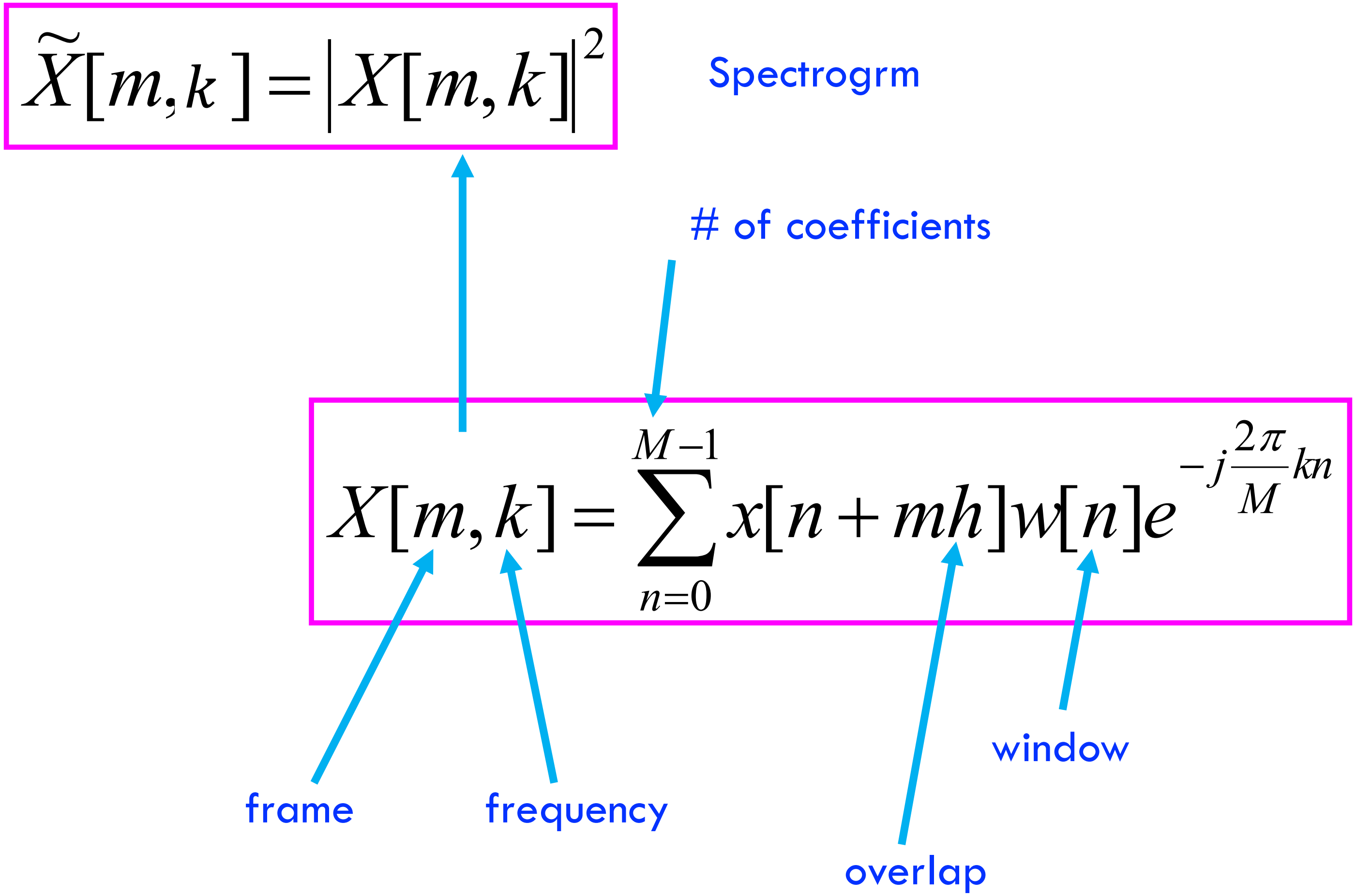
- Short Time Fourier Transform (STFT)
  - resolution in both time and frequency domains



STFT



STFT



$$\widetilde{X}[m,k] = \left|X[m,k]\right|^2$$

Spectrogrm

# of coefficients

$$X[m,k] = \sum_{n=0}^{M-1} x[n+mh]w[n]e^{-j\frac{2\pi}{M}kn}$$

frame

frequency

overlap

window

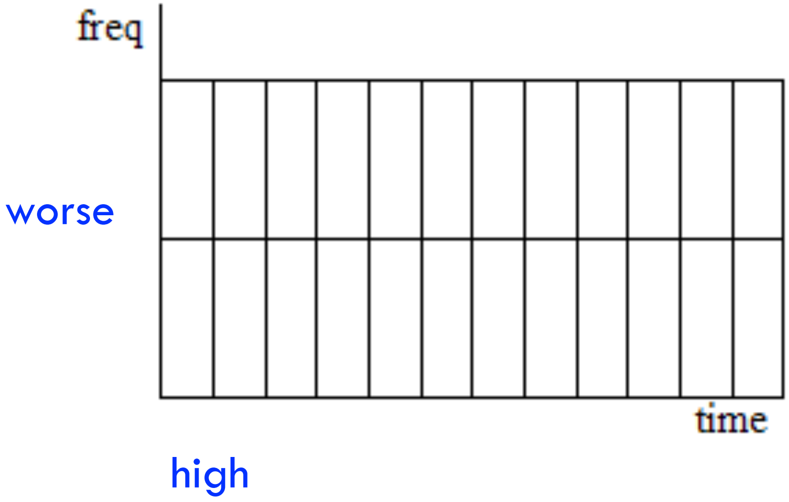
Applications

- Sound synthesis
- Graphical equalizers
- Mood and emotional detection

STFT Uncertainty

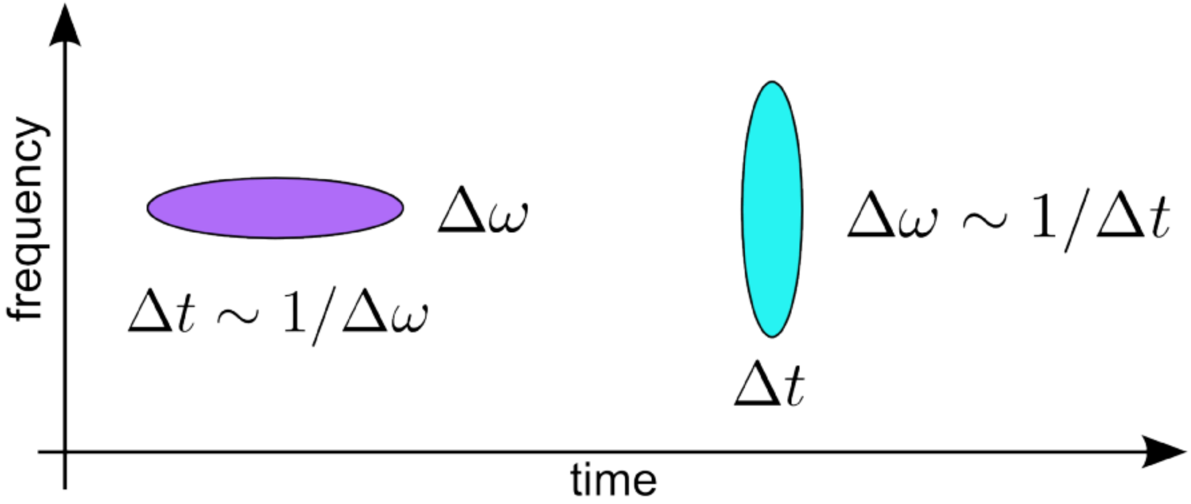
- spectrogram behaviour
  - similar to the *Heisenberg uncertainty principle*
  - higher time resolution imply worse frequencies resolution and vice versa

$$\Delta t \Delta \omega \geq \frac{1}{2}$$

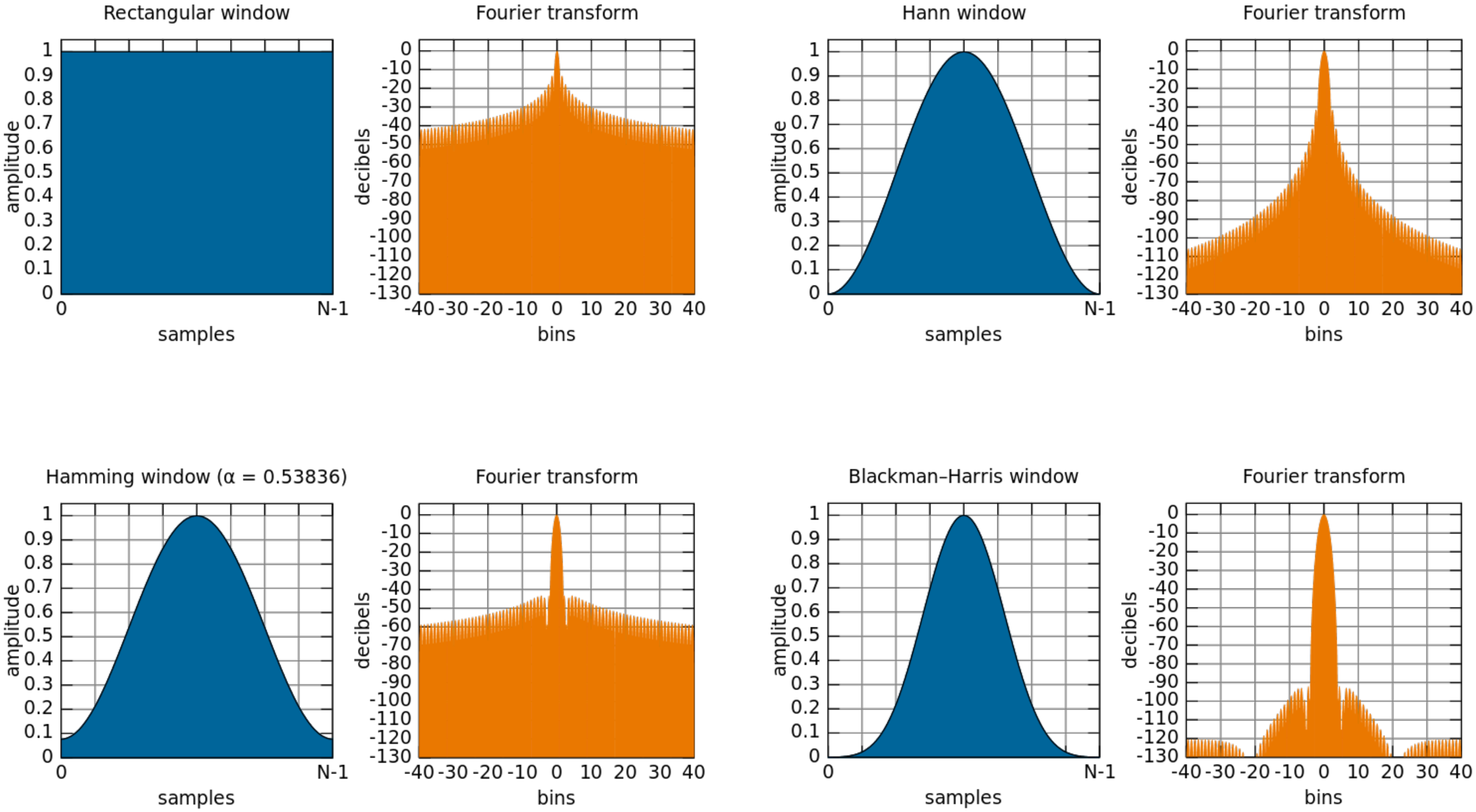


STFT Uncertainty

$$\Delta t \Delta \omega \geq \frac{1}{2}$$

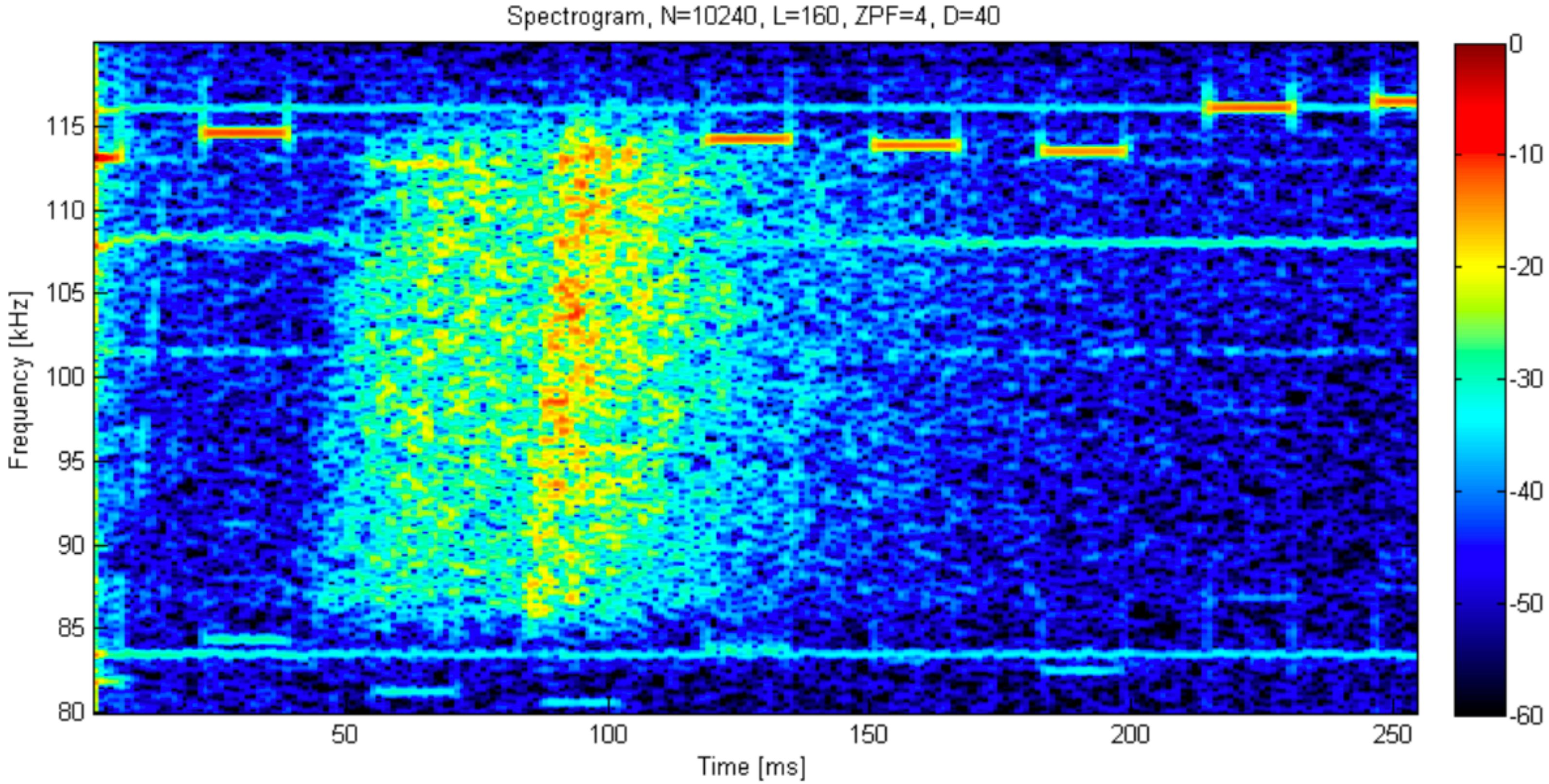


Windows



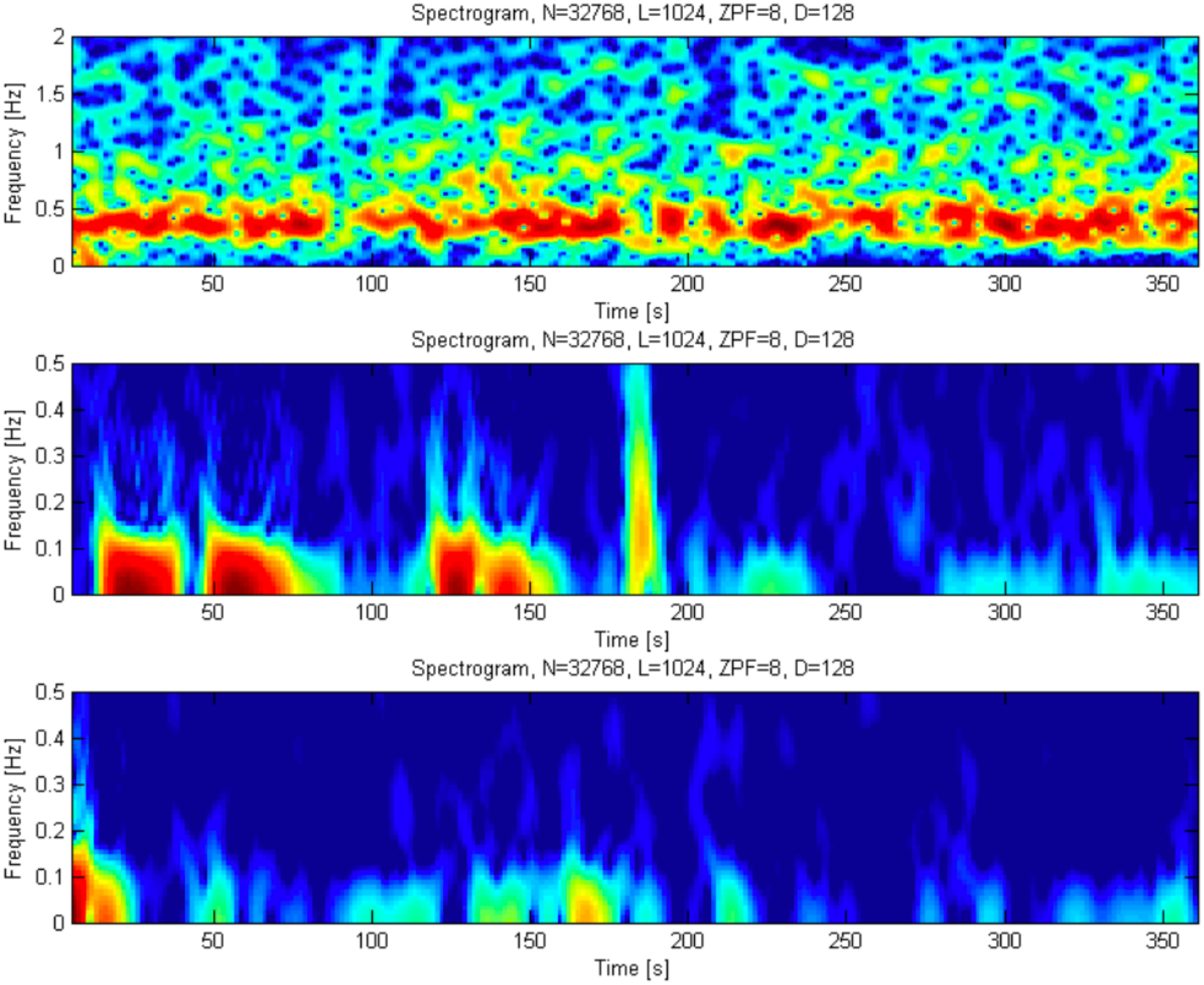
$$G_X(t,f) = \int_{-\infty}^{+\infty} w(t-\tau)\, e^{-j2\pi f\tau} x(\tau) d\tau$$

STFT of Sonar data



Single ping of sonar data

STFT of Sonar data



Navigation data

Wavelet

- Wavelet Transform
  - solves the resolution problem
  - the signal is analyzed at different frequencies ans resolutions
    - High frequencies
      - High time resolution, low frequencies resolution
    - Low frequencies
      - High frequencies resolution, low time resolution

Wavelet Analysis

$$G_X(t,f) = \int_{-\infty}^{+\infty} w(t-\tau)\, e^{-j2\pi f\tau} x(\tau)d\tau$$

Mother Wavelet

shift

$$w(t-\tau) = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} x(t)\, \psi\left(\frac{t-\tau}{s}\right) dt$$

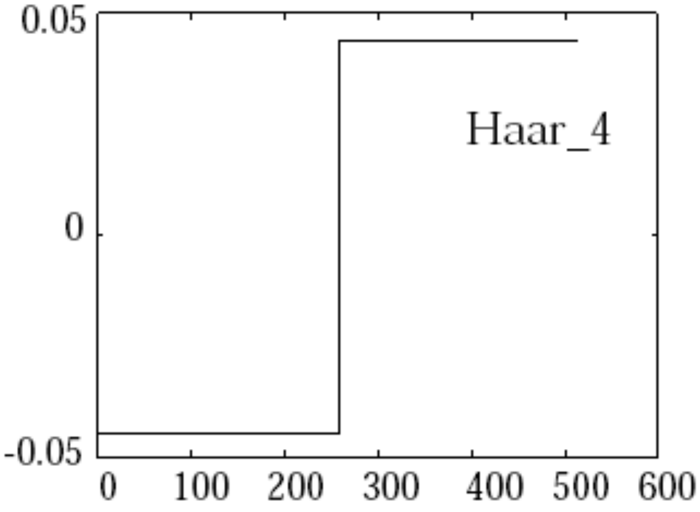
scale

The wavelet transform is simply a kind of correlation function between the mother wavelet scaled and shifted, and the input signal

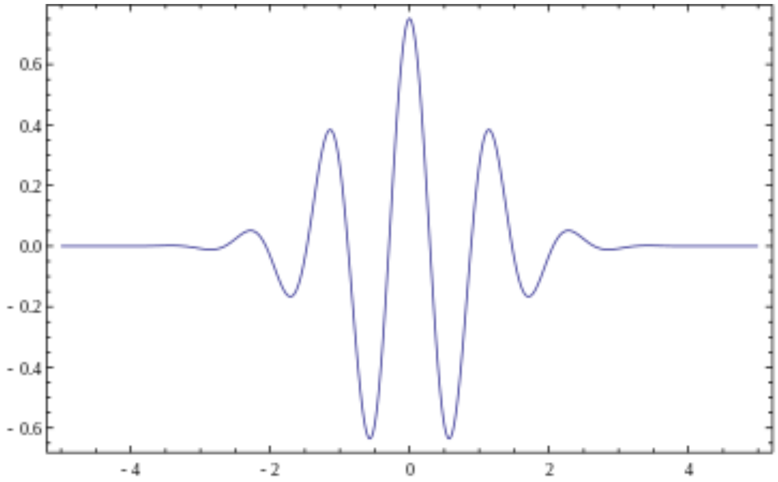
Scale factor

- s>1: dilated
- s<1: compressed

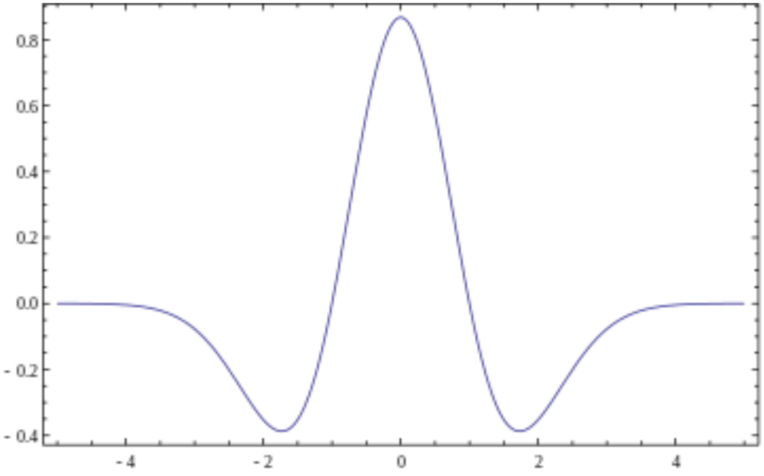
Mother Wavelet



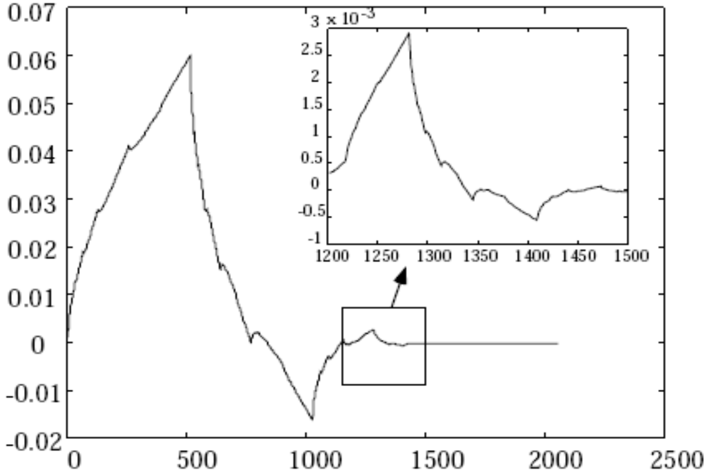
Haar



Morlet

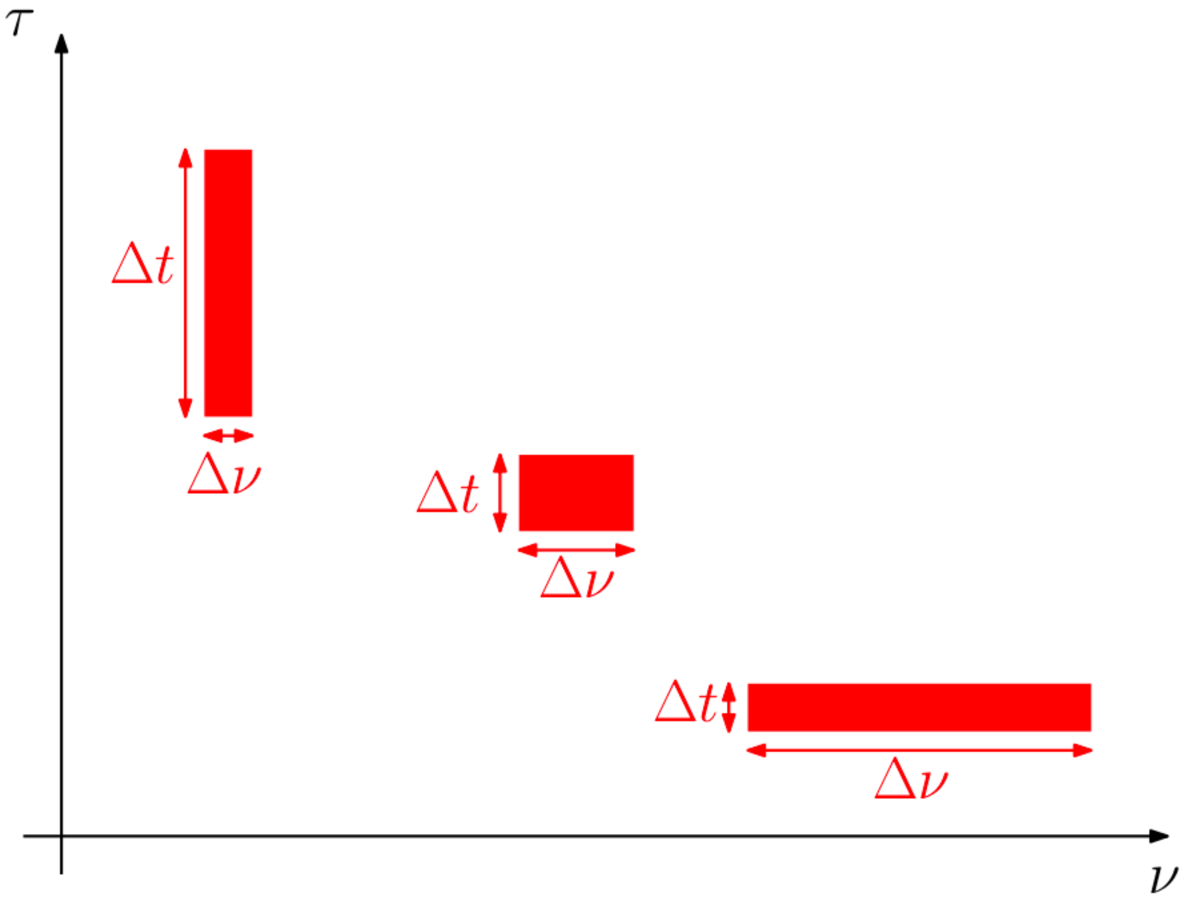


Mexican hat



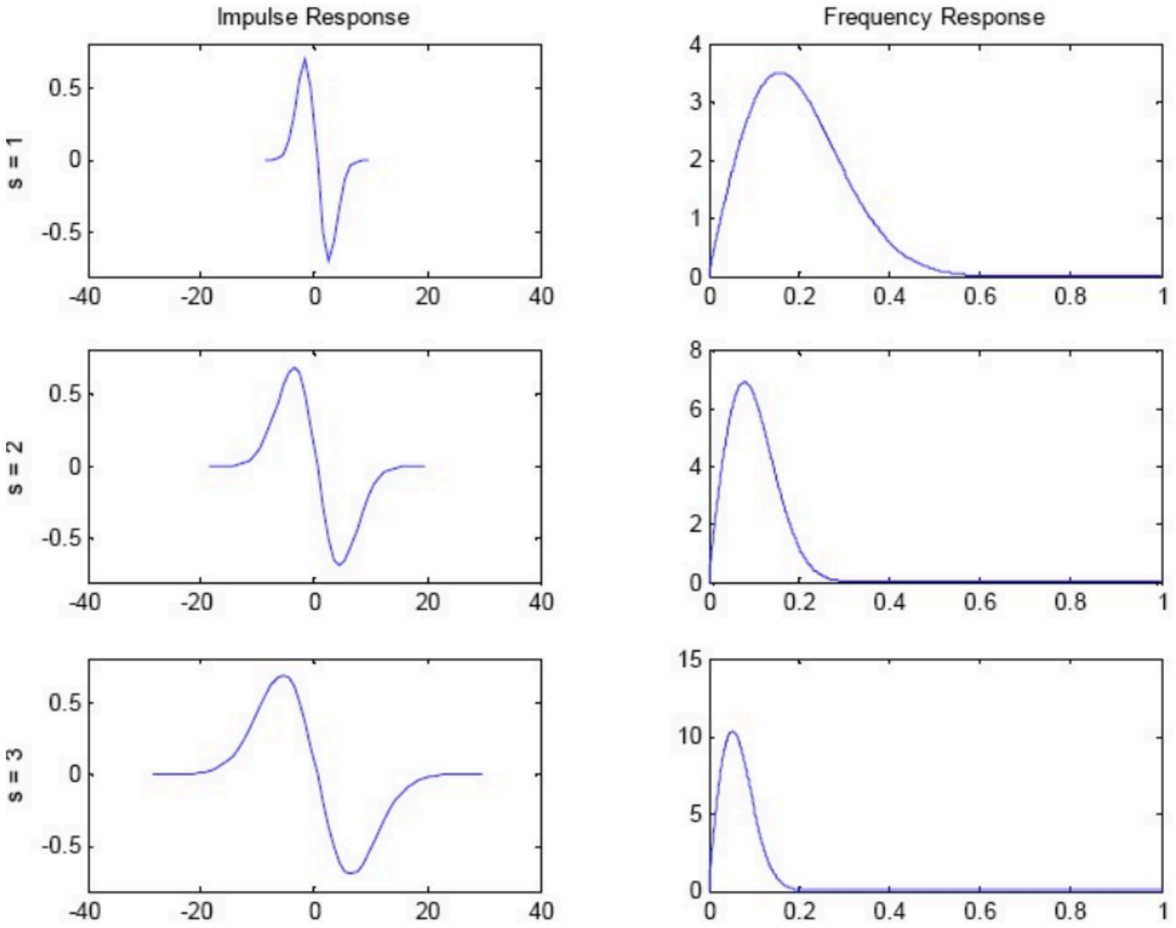
Daubechies (fractal)

Resolutions



Fast changes: low frequency resolution, high time resolution
Slow changes: high frequency resolution, low time resolution

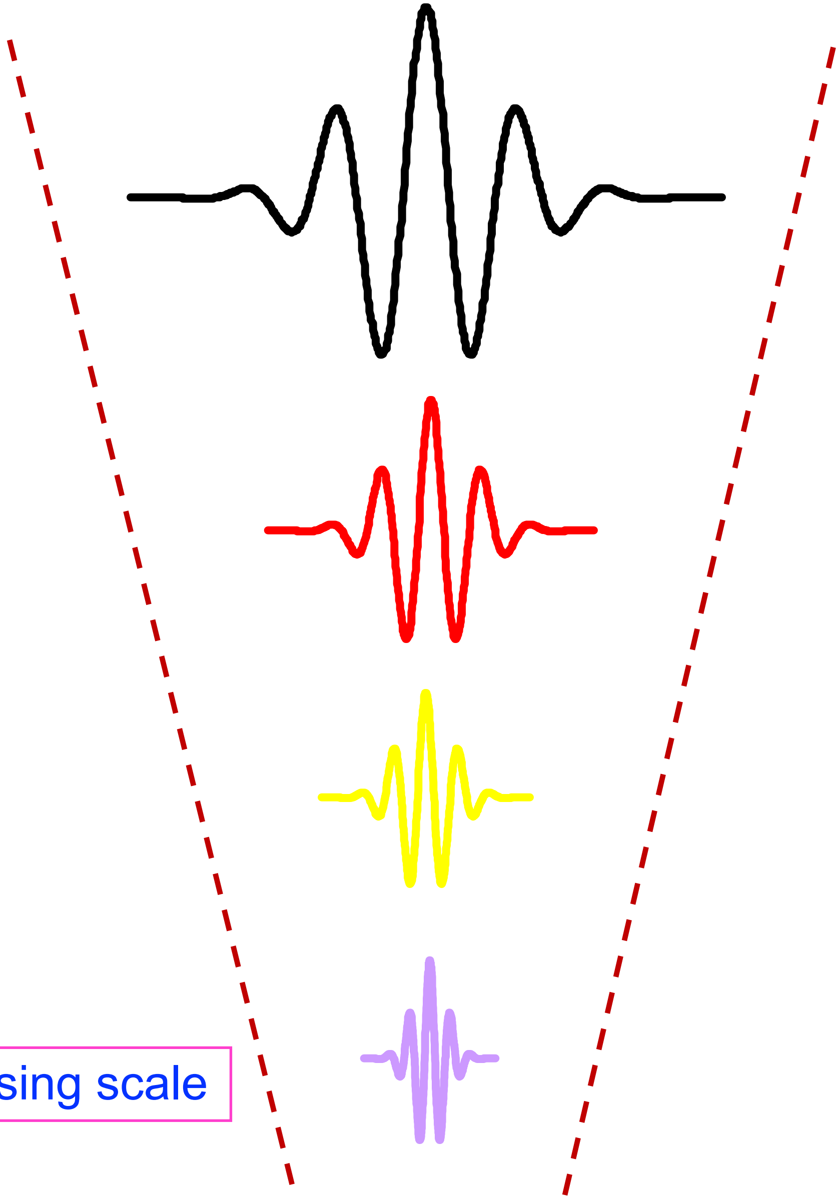
Resolutions



Low (time) scales is equivalent to study low frequency components, i.e. the rough features of the signal

High (time) scales is equivalent to study high frequency components, i.e. the details in the signal

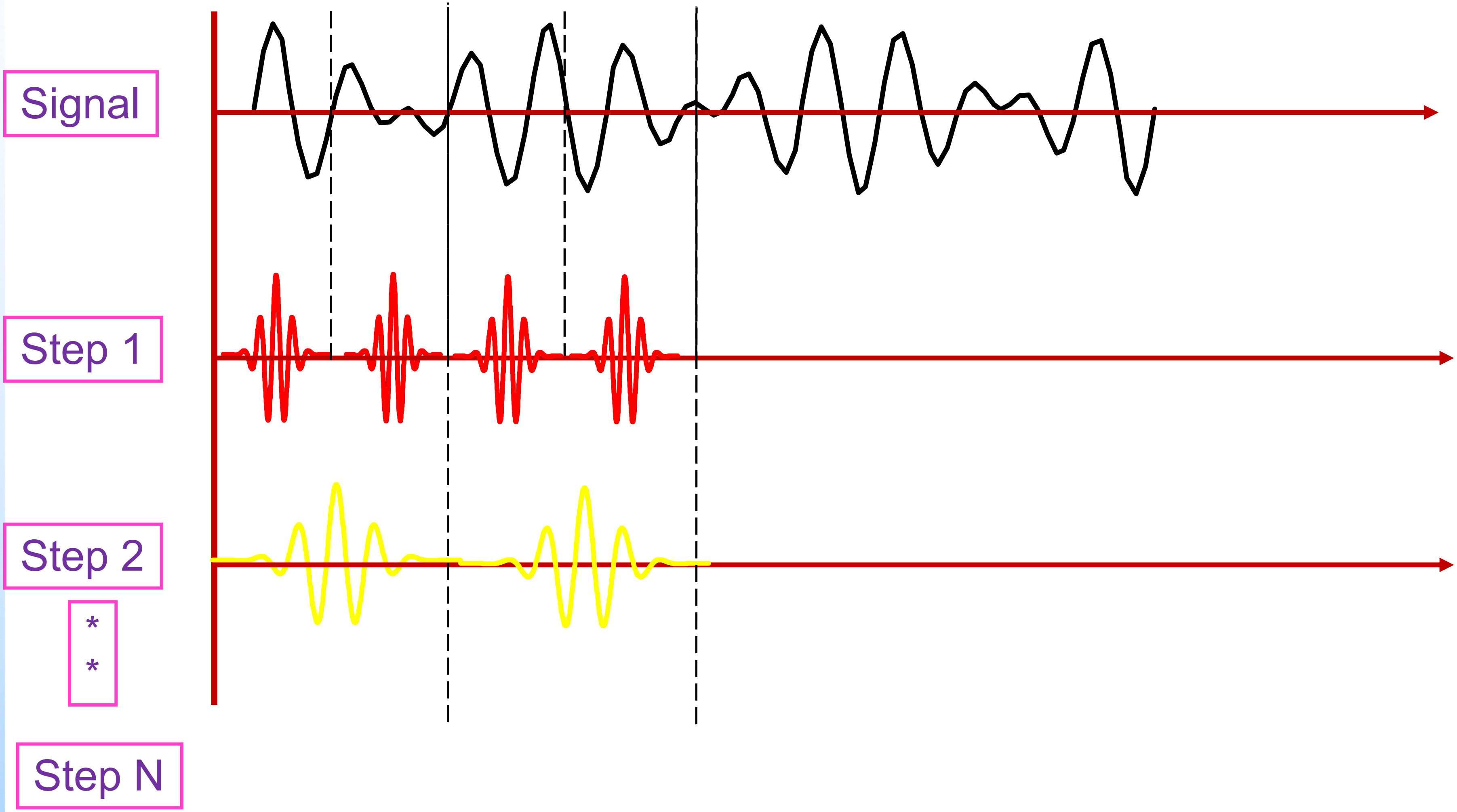
Mother Wavelet



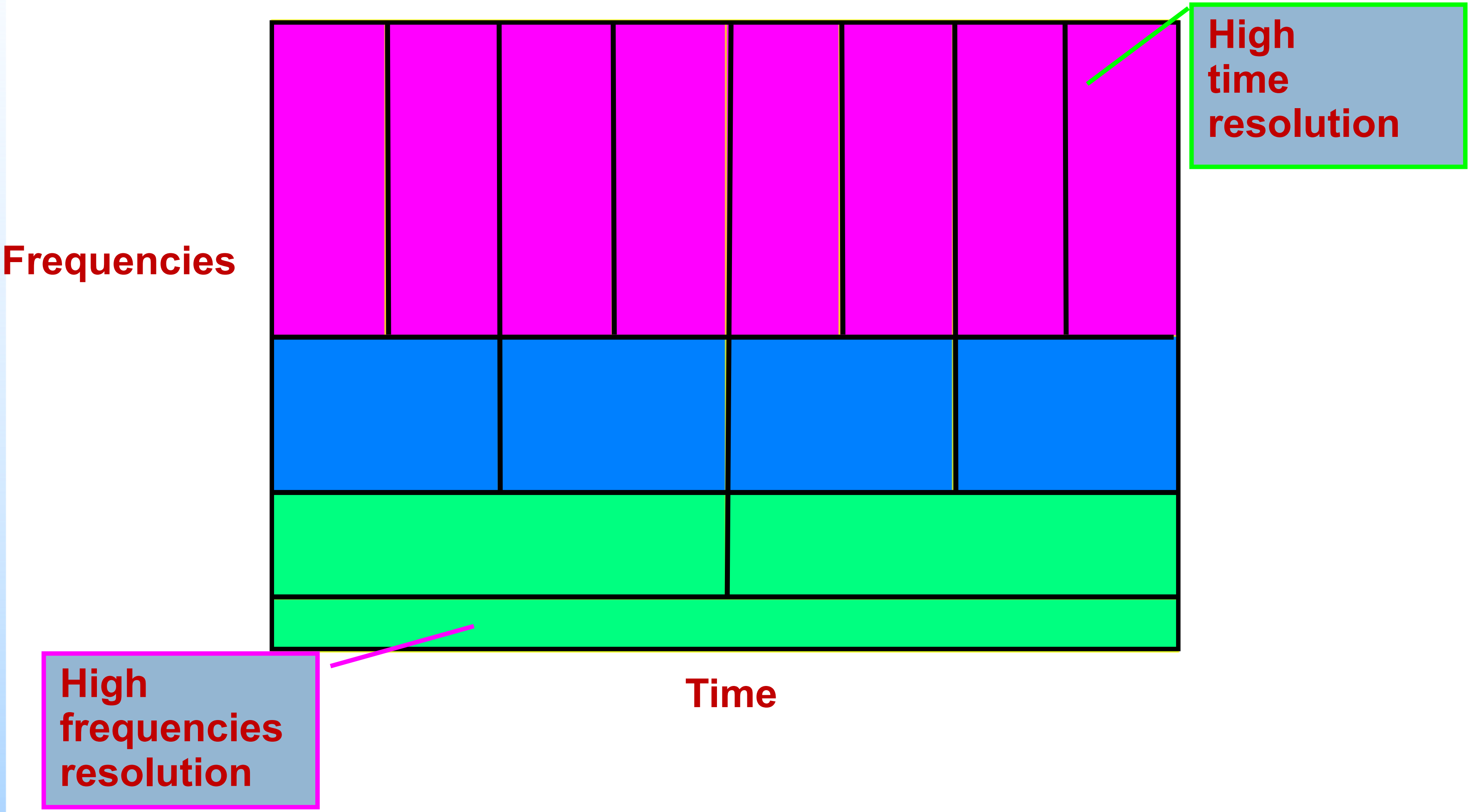
decreasing frequency

decreasing scale

Wavelet Transform

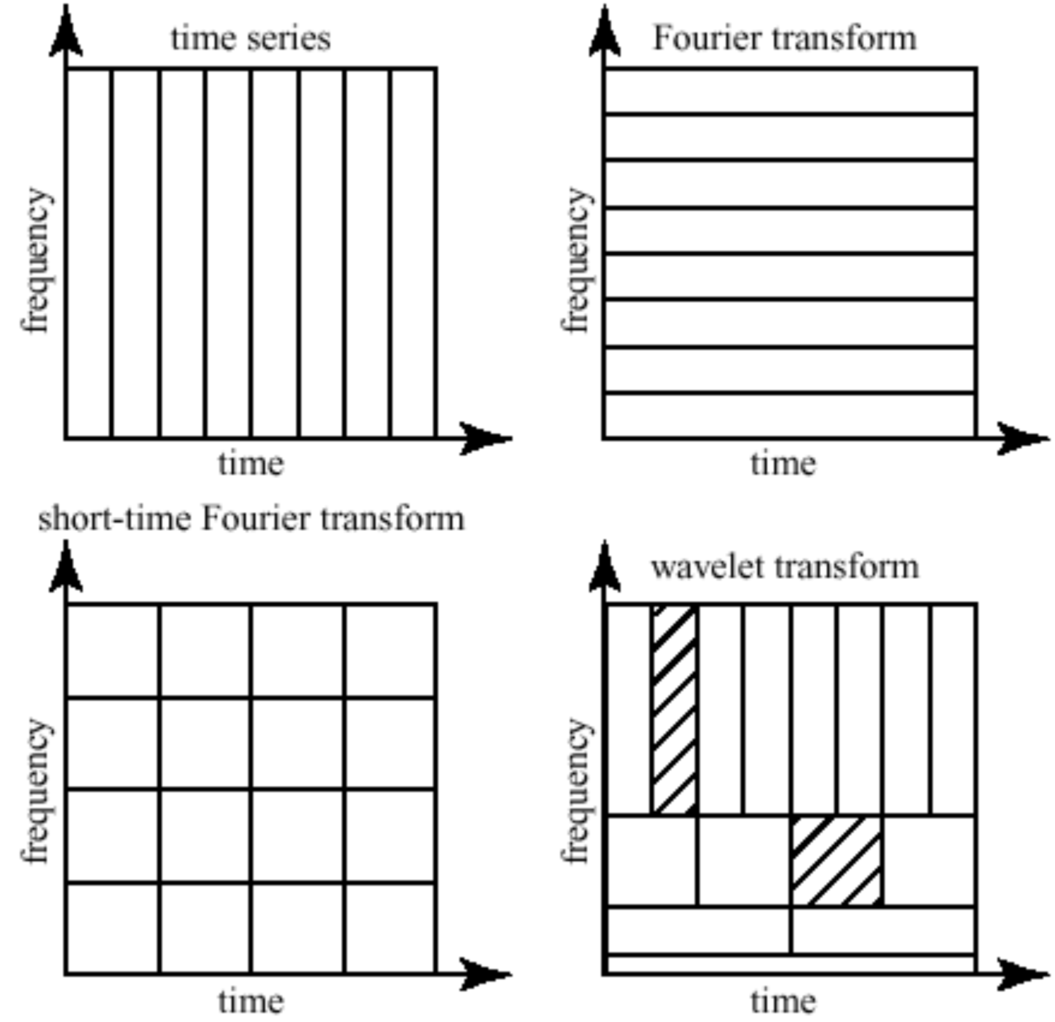


Resolution



Frequencies
High time resolution
High frequencies resolution
Time

STFT vs CWT



From http://www.cerm.unifi.it/EUcourse2001/Gunther_lecturenotes.pdf, p.10

Discrete Wavelet Transform (DWT)

- Sub-bands encoding
  - High pass filters
    - impulse response g[n]
  - Low pass filters
    - impulse response h[n]

DWT

$$x[n]\otimes h[n]=\sum_{k=-\infty}^{\infty}x[k]h[n-k]$$

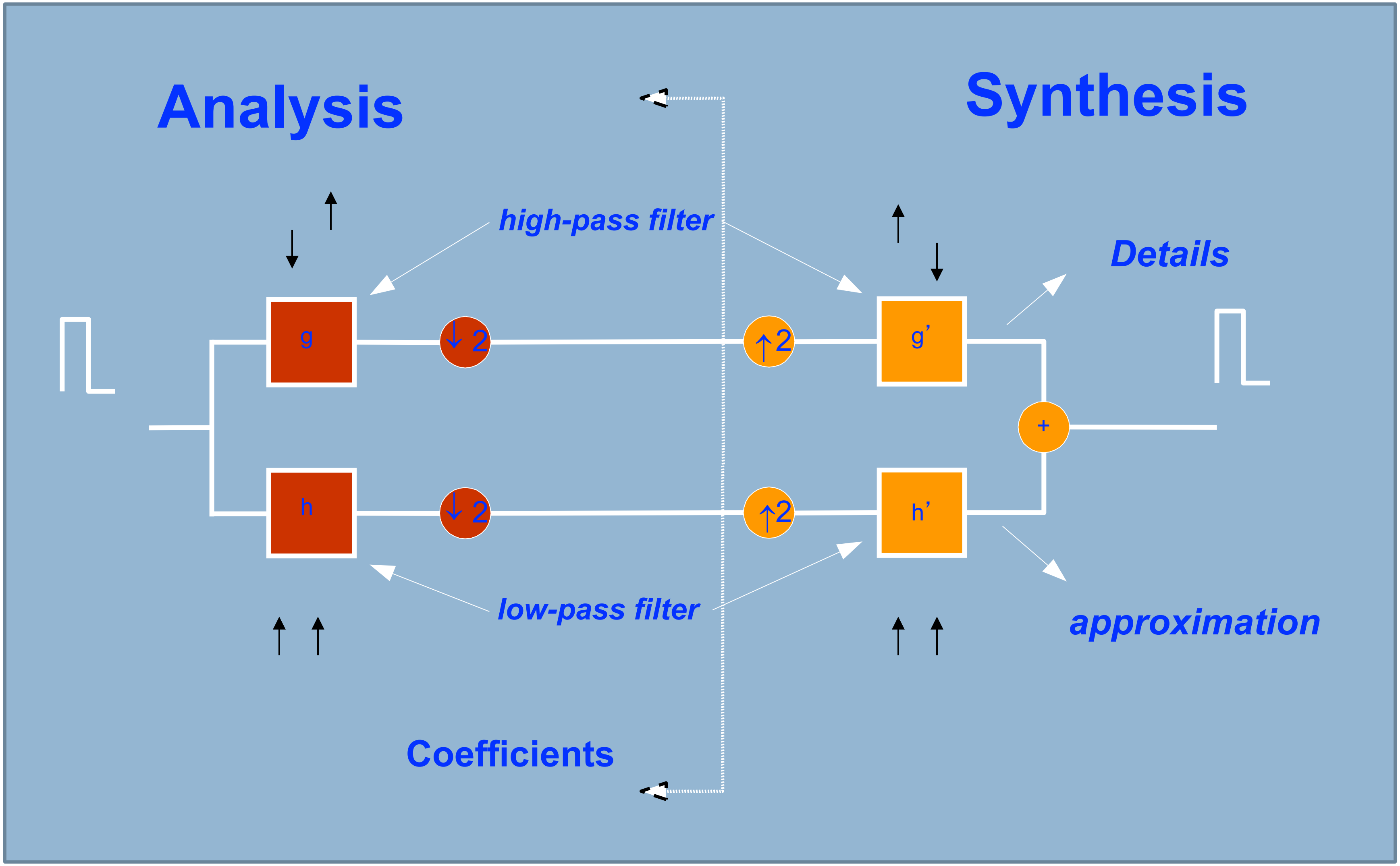
filtering

$$y_H[k]=\sum_{n=-\infty}^{\infty}x[n]g[2k-n]$$

$$y_L[k]=\sum_{n=-\infty}^{\infty}x[n]h[2k-n]$$

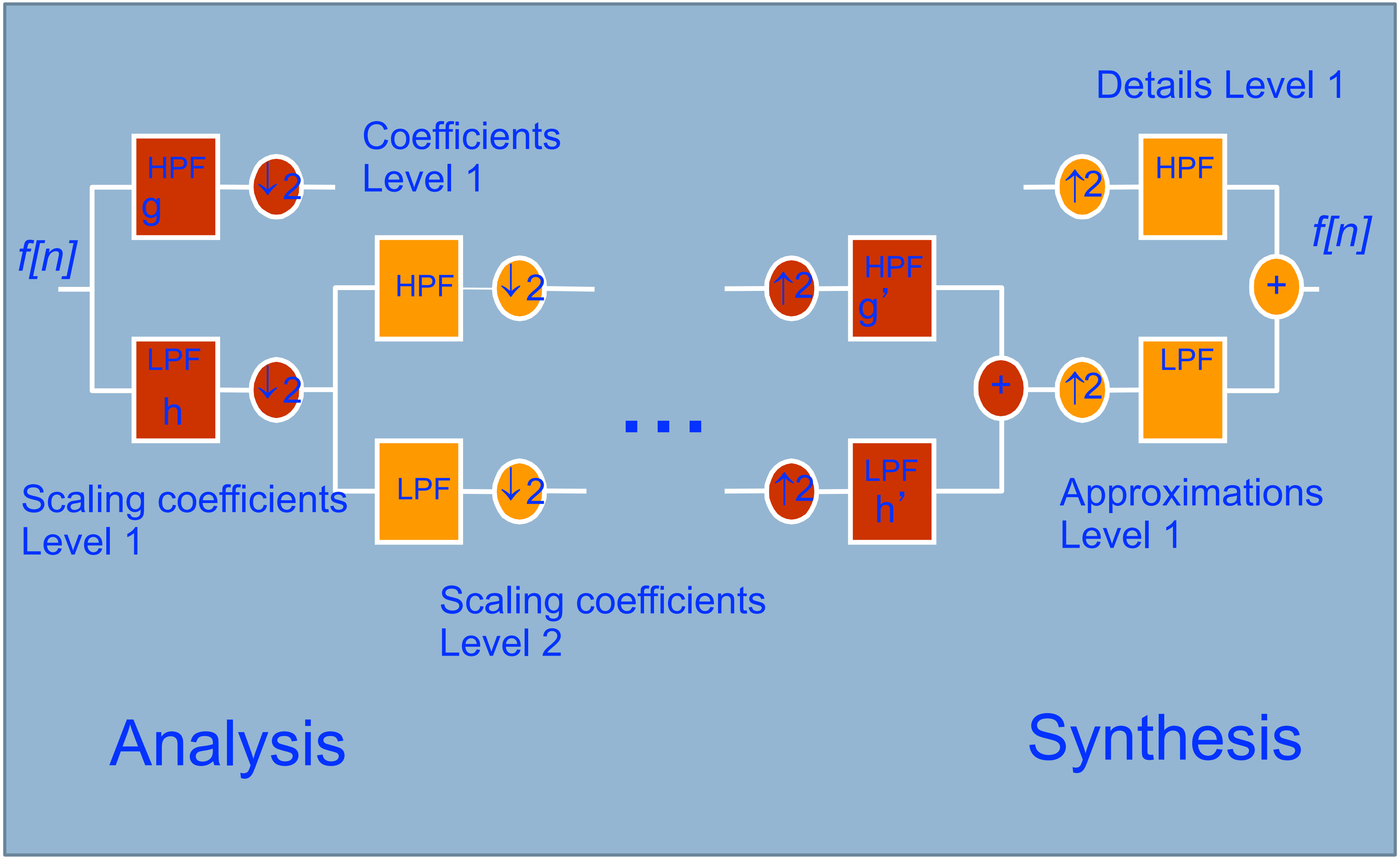
Filtering and downsampling

DWT



Analysis
Synthesis
high-pass filter
Details
g
↓2
↑2
g'
+
h
↓2
↑2
h'
low-pass filter
approximation
Coefficients

Mallat's algorithm



Coefficients Level 1

Scaling coefficients Level 1

Scaling coefficients Level 2

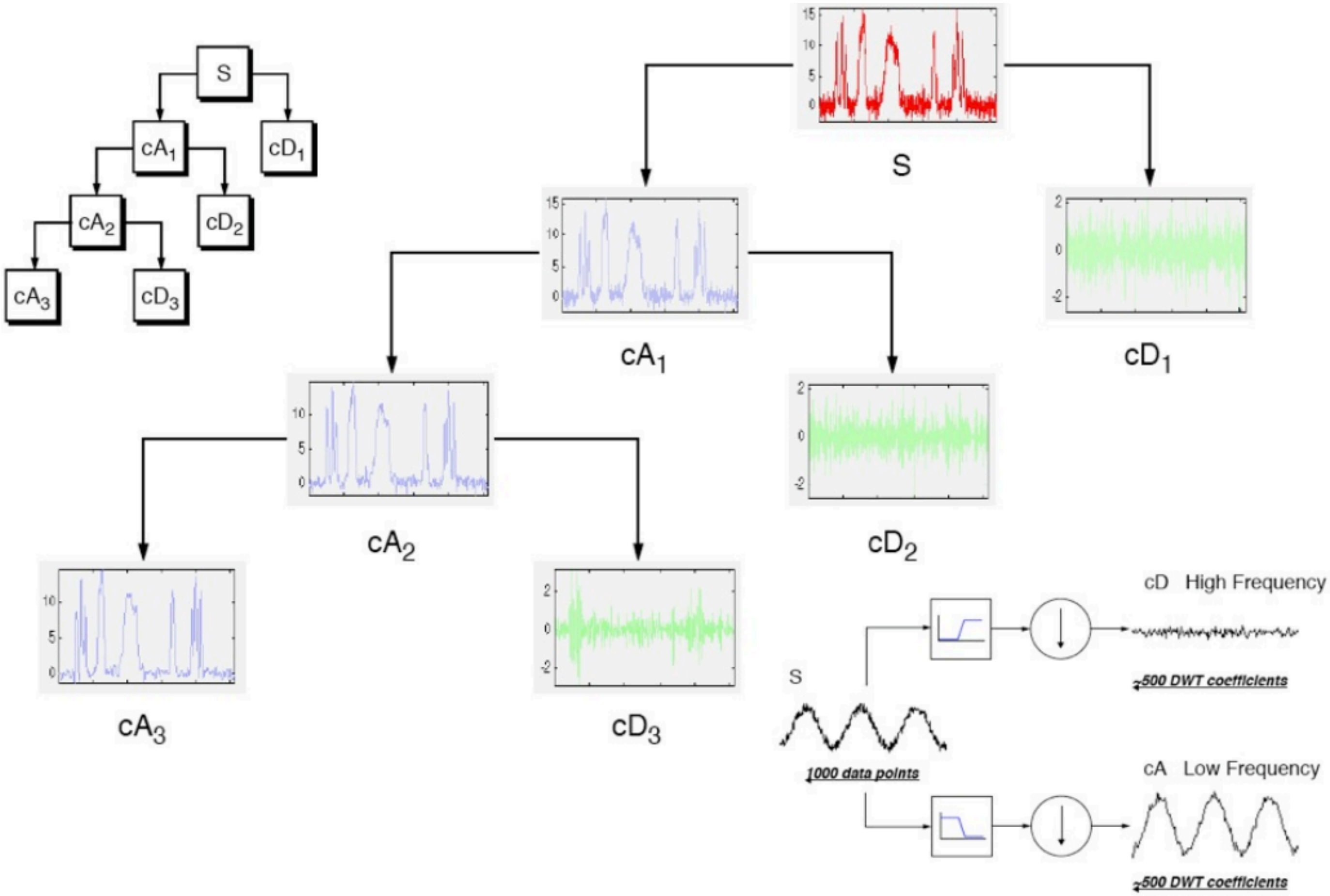
Details Level 1

Approximations Level 1

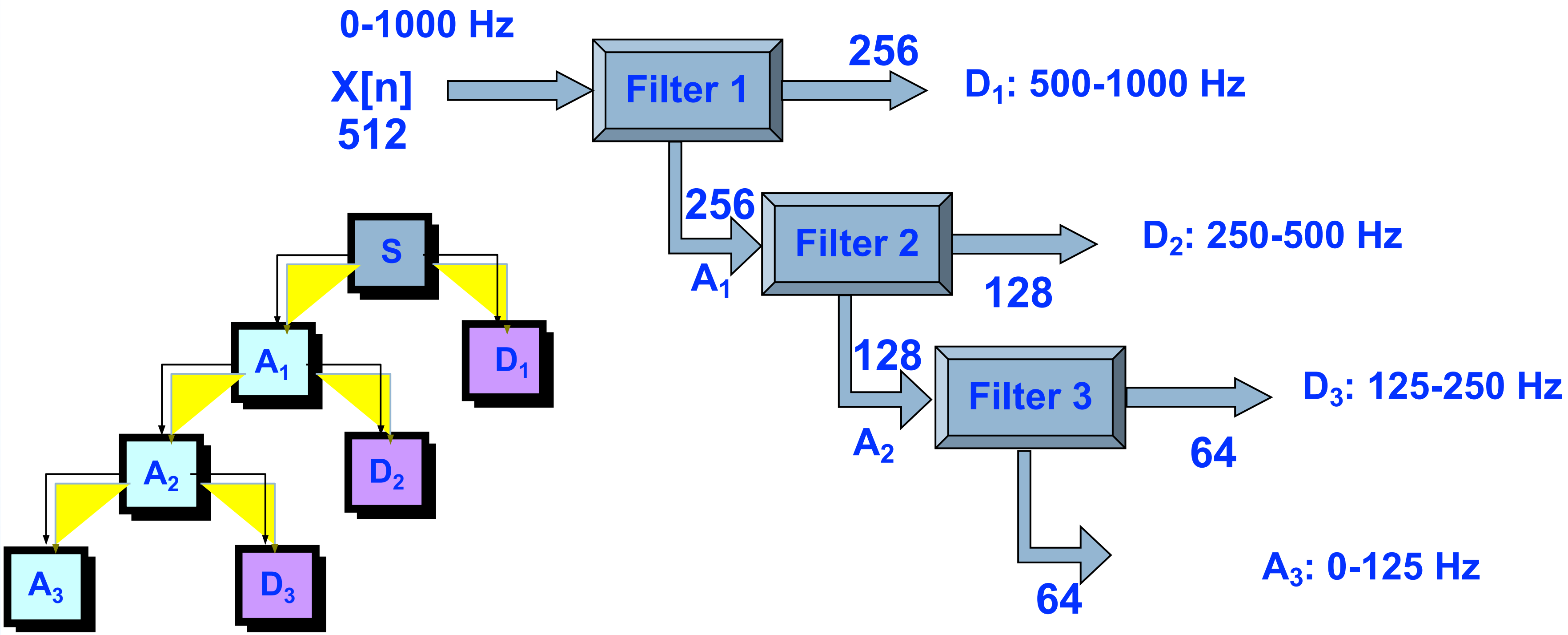
Analysis

Synthesis

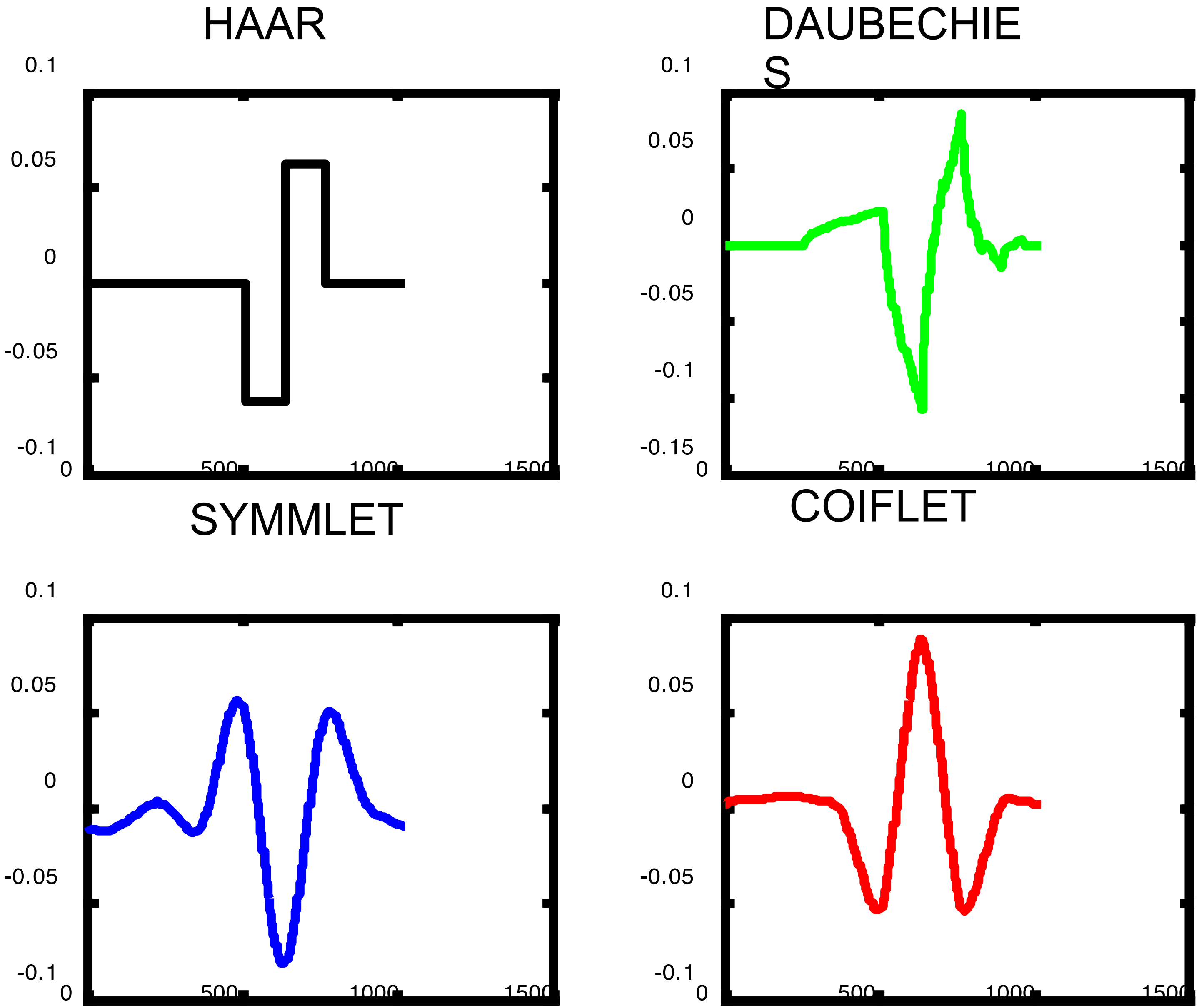
Mallat's algorithm



Mallat's algorithm



Wavelets


HAAR
0.1
0.05
0
-0.05
-0.1
0
500
1000
1500
DAUBECHIES
0.1
0.05
0
-0.05
-0.1
-0.15
0
500
1000
1500
SYMMLET
0.1
0.05
0
-0.05
-0.1
0
500
1000
1500
COIFLET
0.1
0.05
0
-0.05
-0.1
0
500
1000
1500

Example – 1D signal

Decomposition at level 5 : s = a5 + d5 + d4 + d3 + d2 + d1 .

Data (Size) xn1 (1001)

Wavelet db 4

Level 5

Display mode : Full Decomposition

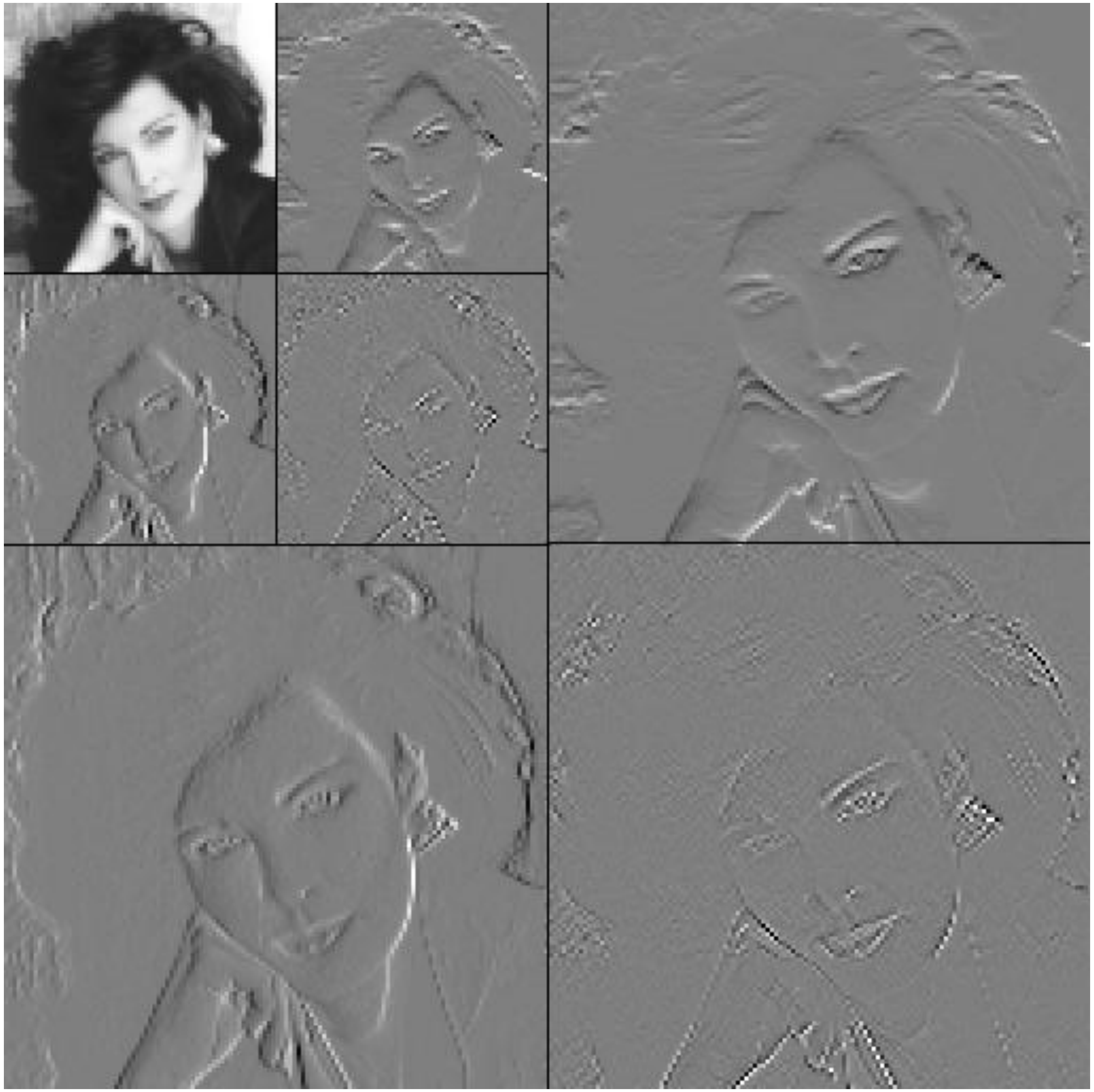
at level 5

Signal:
0.0-0.4: 2 Hz
0.4-0.7: 10 Hz
0.7-1.0: 20Hz

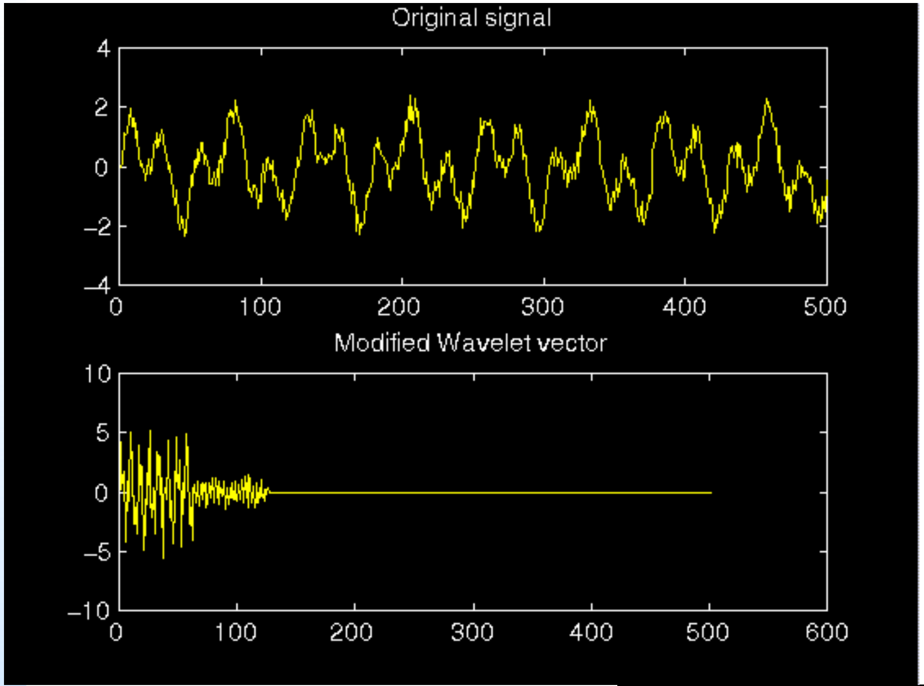
Wavelet: db4

Level: 6

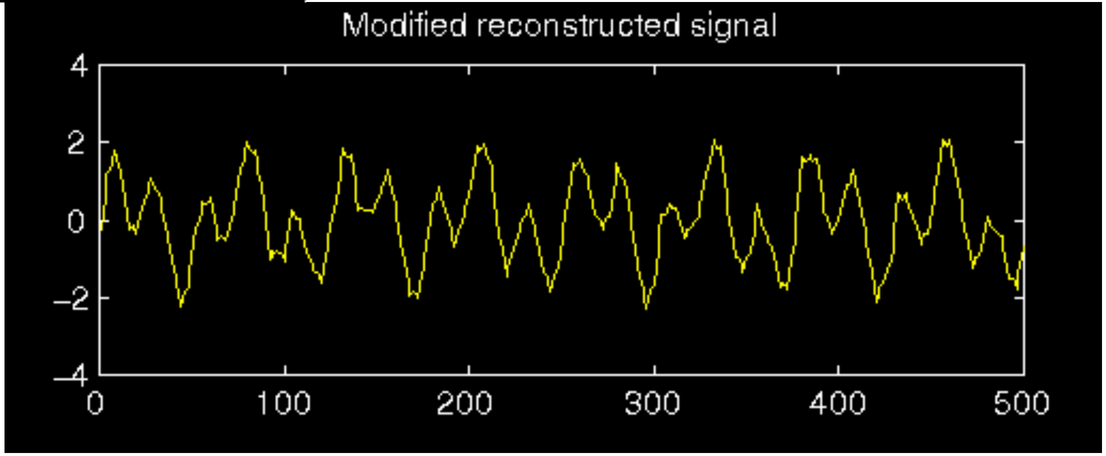
Example – 2D signal



Example – 1D signal denoising


Original signal

Modified Wavelet vector


Modified reconstructed signal

Example – compression

Comparison of Performance on Color Images

JPEG-1 at 0.27 bpp

Original

JPEG-2000 at 0.27 bpp

Los Alamos National Laboratory

Computer & Computational Sciences, CCS-3

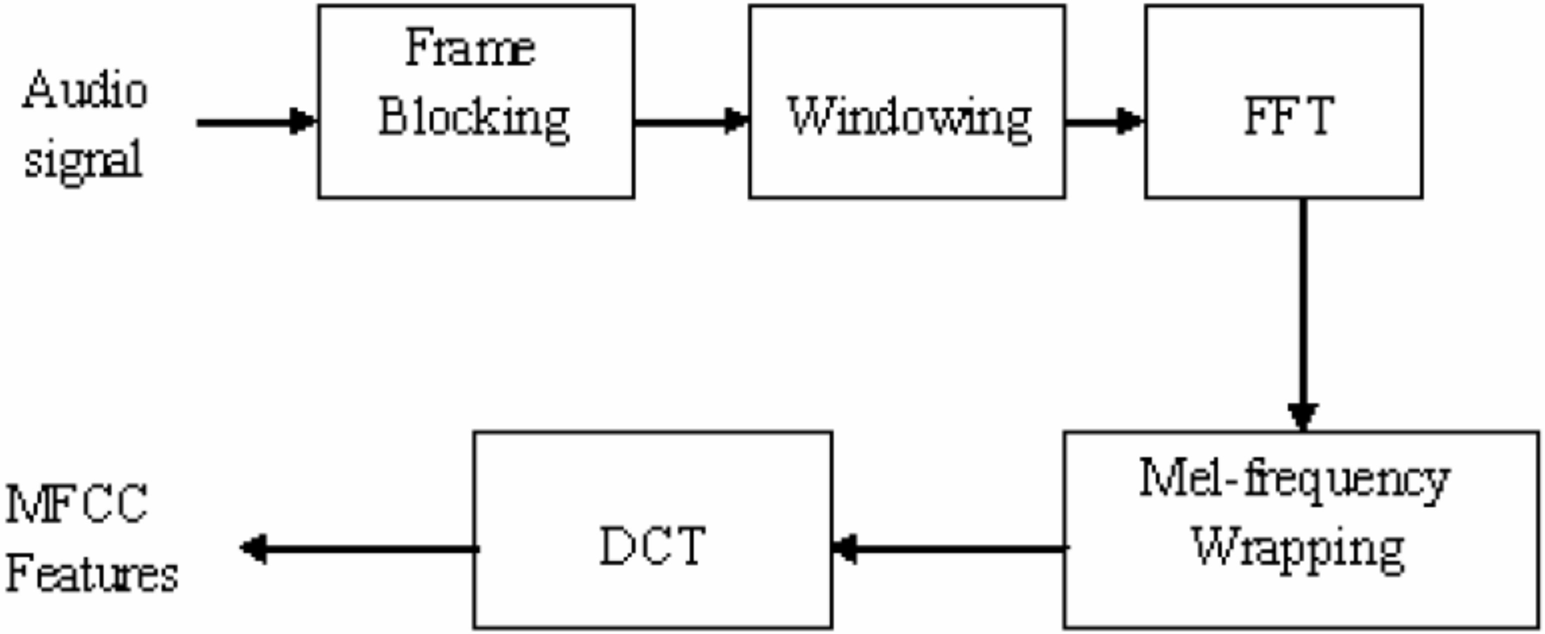
Mel Frequency Cepstral Coefficients

- Mel Frequency Cepstral Coefficients (MFCC)
  - based on perceptual techniques

- Main applications
  - Speech recognition
  - Music information retrieval
  - Musical genre classification

MFCC

Bartlett or Hamming



Block diagram

MFCC wrapping

- Melody scale (mel)
  - proposed by Stevens, Volkman and Newman in 1937
  - based on the non-linear human auditory perception
  - the human hearing system cannot differentiate very close frequencies
  - A 1000 Hz tone at 40 dB corresponds to 1000 mels

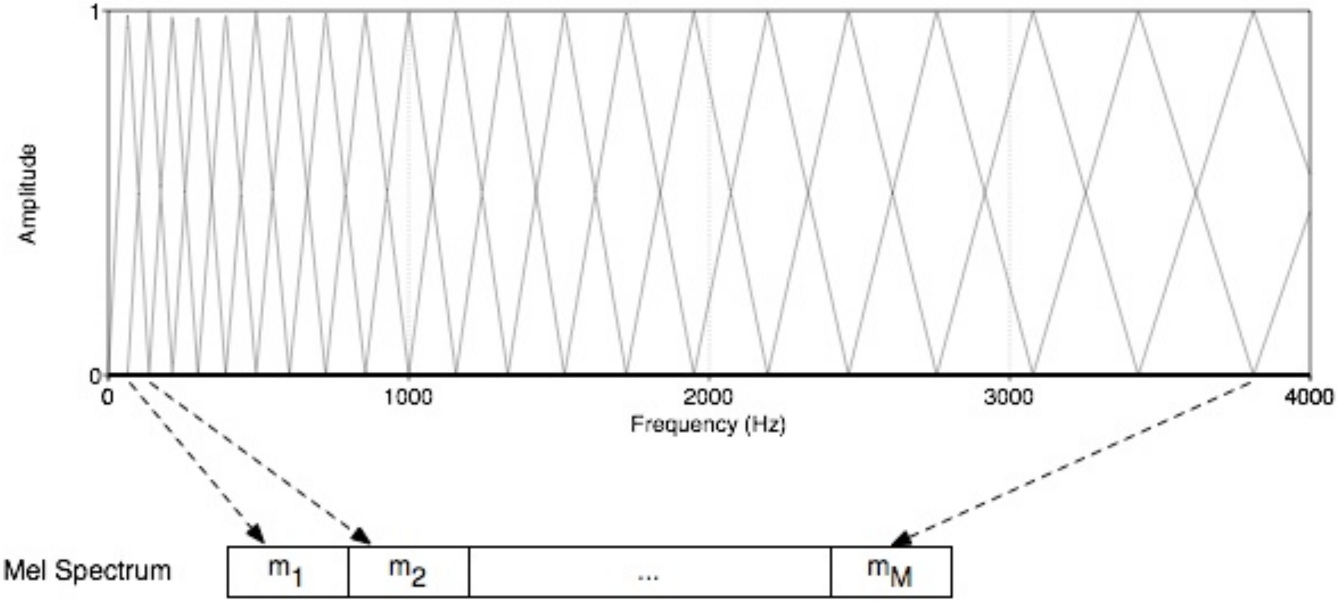
$$\boxed{Mel(f) = 2595\log_{10}\left(1+\frac{f}{700}\right)}$$

MFCC wrapping

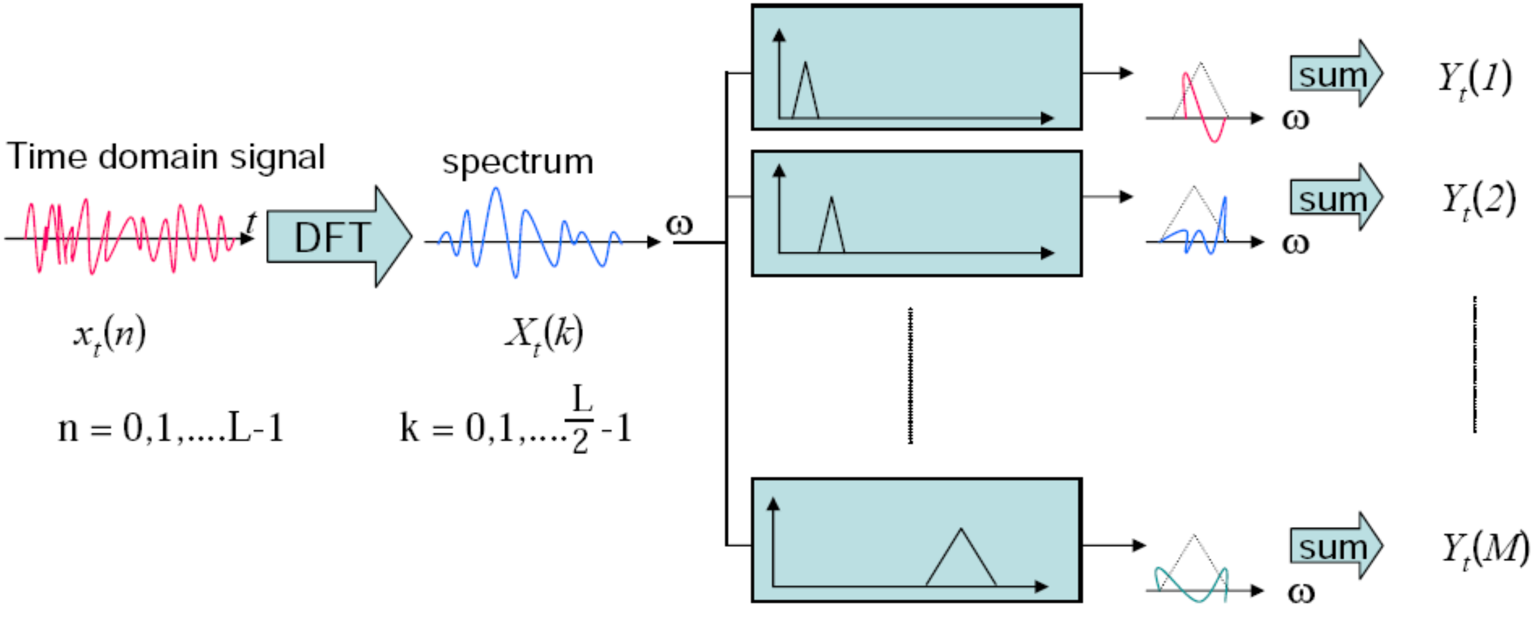
$$BW = 25 + 75\left[1 + 1.4(f_{mel}/1000)^2\right]^{0.69}$$

Band-pass filter

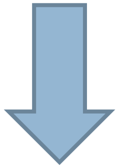
Filters Bank



MFCC process



MFCC process

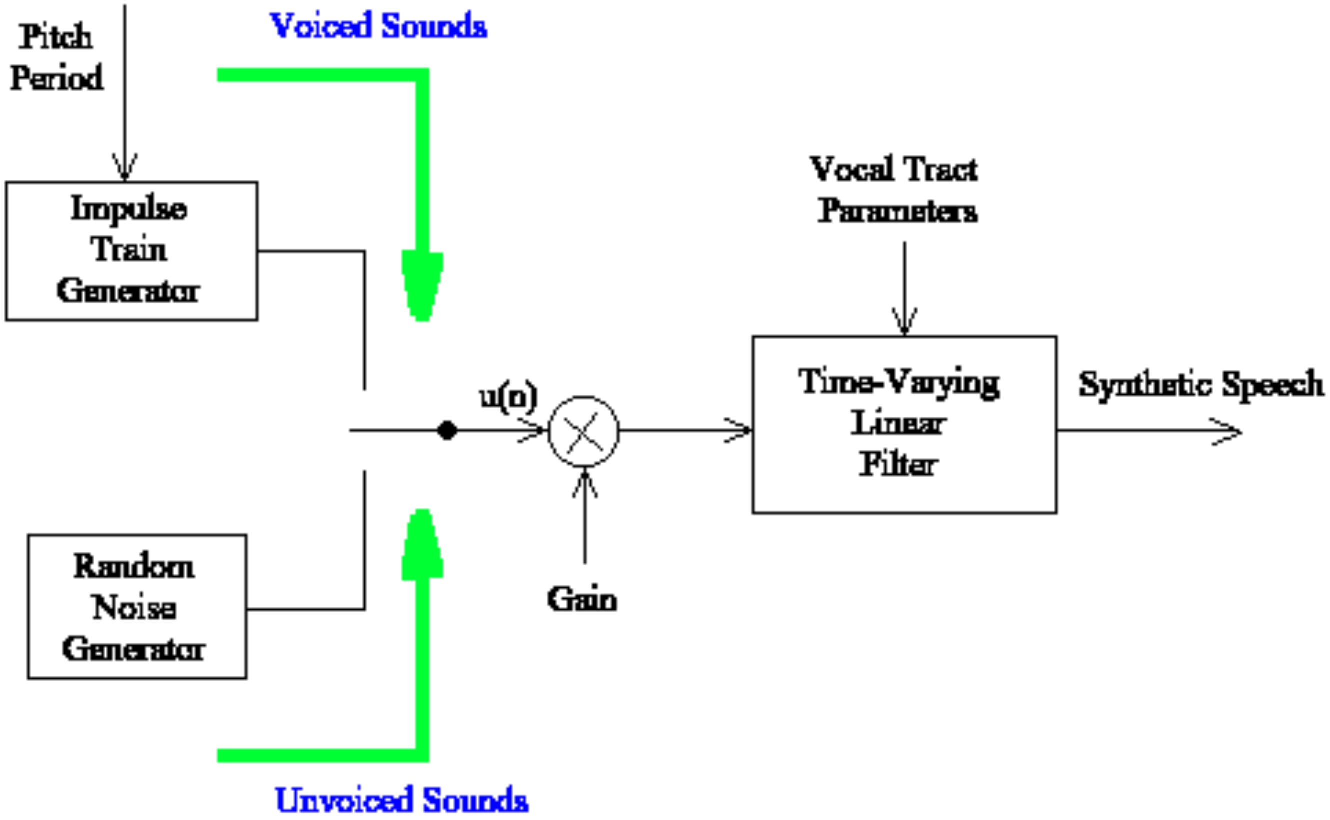


Time domain coefficients by PCA or DCT

Linear Predictive Coding

- Linear Predictive Coding (LPC)
    - Analysis and synthesis of signals
    - Feature extraction
    - Compression
    - Synthesis of the vocal tract
- Voice
    - modulation result caused by the throat and mouth (formant) on the sound emitted by the vocal cords (residue)

LPC



Speech Synthesis model based on LPC model

LPC

source signal

$$\boxed{\tilde{x}(n) = \sum_{i=1}^{M} a_i x(n-i)}$$

prediction (formant)

linear coefficients

$$\boxed{\varepsilon(n) = x(n) - \sum_{i=1}^{M} a_i x(n-i)}$$

residue

prediction error

LPC

- Mean Squared Error (MSE)

$$\boxed{E=\sum_{n}\varepsilon(n)^{2}=\sum_{n}\left(x(n)-\sum_{i=1}^{M}a_{i}x(n-i)\right)^{2}}$$

coefficients to estimate

- Optimization
  - Autocorrelation method (N^3)
    - QR decomposition
    - Gauss elimination
  - Levison-Durbin Algorithm (N^2)

Adaptive filters

- Adaptive filter
  - The parameters are estimated
    - learning algorithm
  - An error function is used
  - e.g., Linear Artificial Neural Network (Adaline)

Adaptive filters

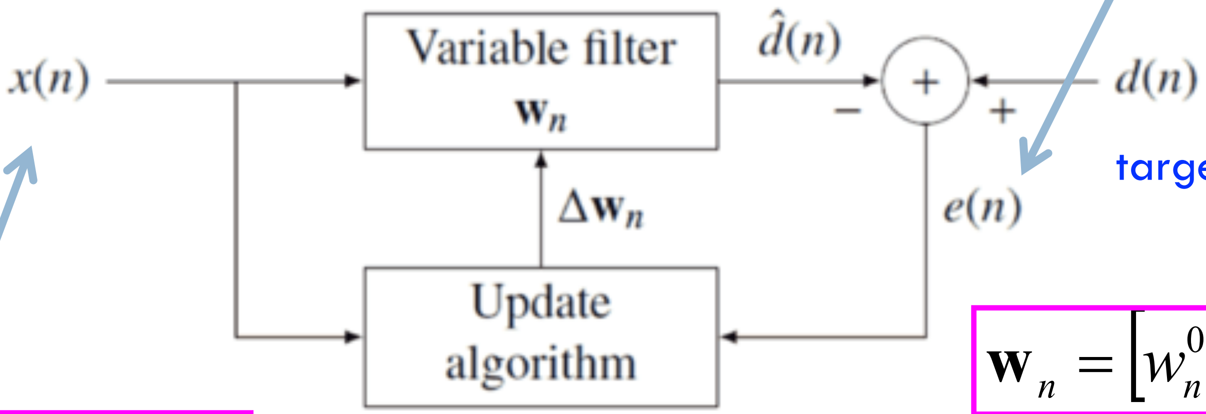
- Hospital
  - ECG (electrocardiogram) corrupted by noise at 50 Hz (electricity)
  - The current can vary between 47 Hz and 53 Hz
  - A filter for the elimination of static noise at 50 Hz could give errors
  - An adaptive filter can learn from the current shape of noise

- Helicopter
  - Pilot speaking with noise from rotating propeller
  - The noise has not a spectrum well defined
  - An adaptive filter learns the shape of the noise
  - The noise can be subtracted from the signal for only the pilot's voice

Adaptive filters

$$e(n) = d(n) - \hat{d}(n)$$

input



target signal

$$x(n) = d(n) + v(n)$$

noise

$$\mathbf{w}_n = \left[w_n^0, w_n^1, \ldots, w_n^p\right]$$

FIR structure

$$\tilde{d}(n) = \mathbf{w}_n \cdot \mathbf{x}(n)$$

$$\mathbf{w}_{n+1} = \mathbf{w}_n + \Delta\mathbf{w}_n$$

Learning algorithm (e.g., LMS and descent gradient)

$$\mathbf{x}(n) = \left[x(n), x(n-1), \ldots, x(n-p)\right]$$

source signal