Intelligent Signal Processing

Filtering

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Introduction

- Filter
  - device that increases or reduces the energy connected to certain regions of the spectrum sound
  - these operations are typically performed by equalizers
    - bank of bandpass filters
  - the critical bands of the auditory membrane are bandpass filters

Convolution

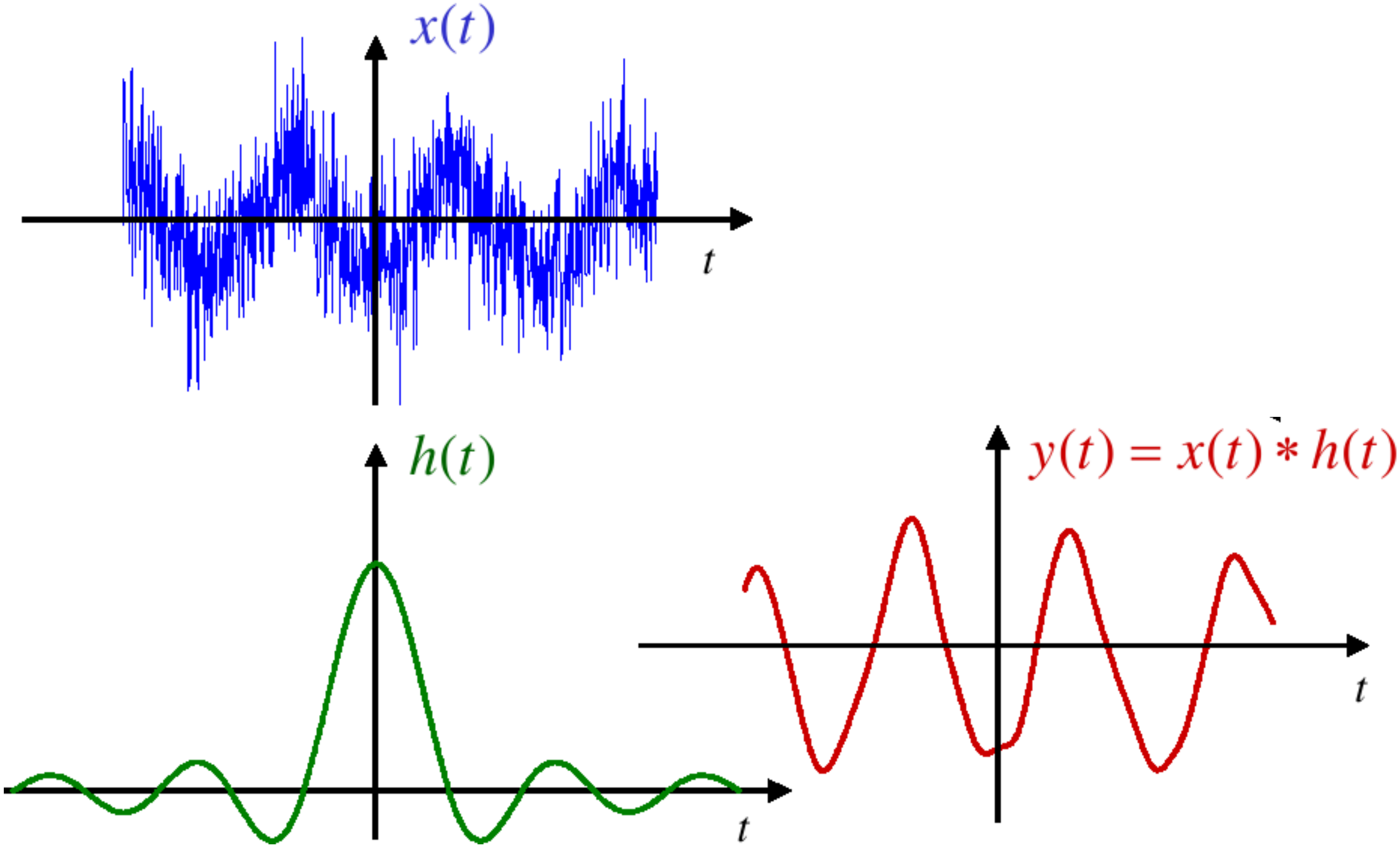
- LTI are characterized by the impulse response

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

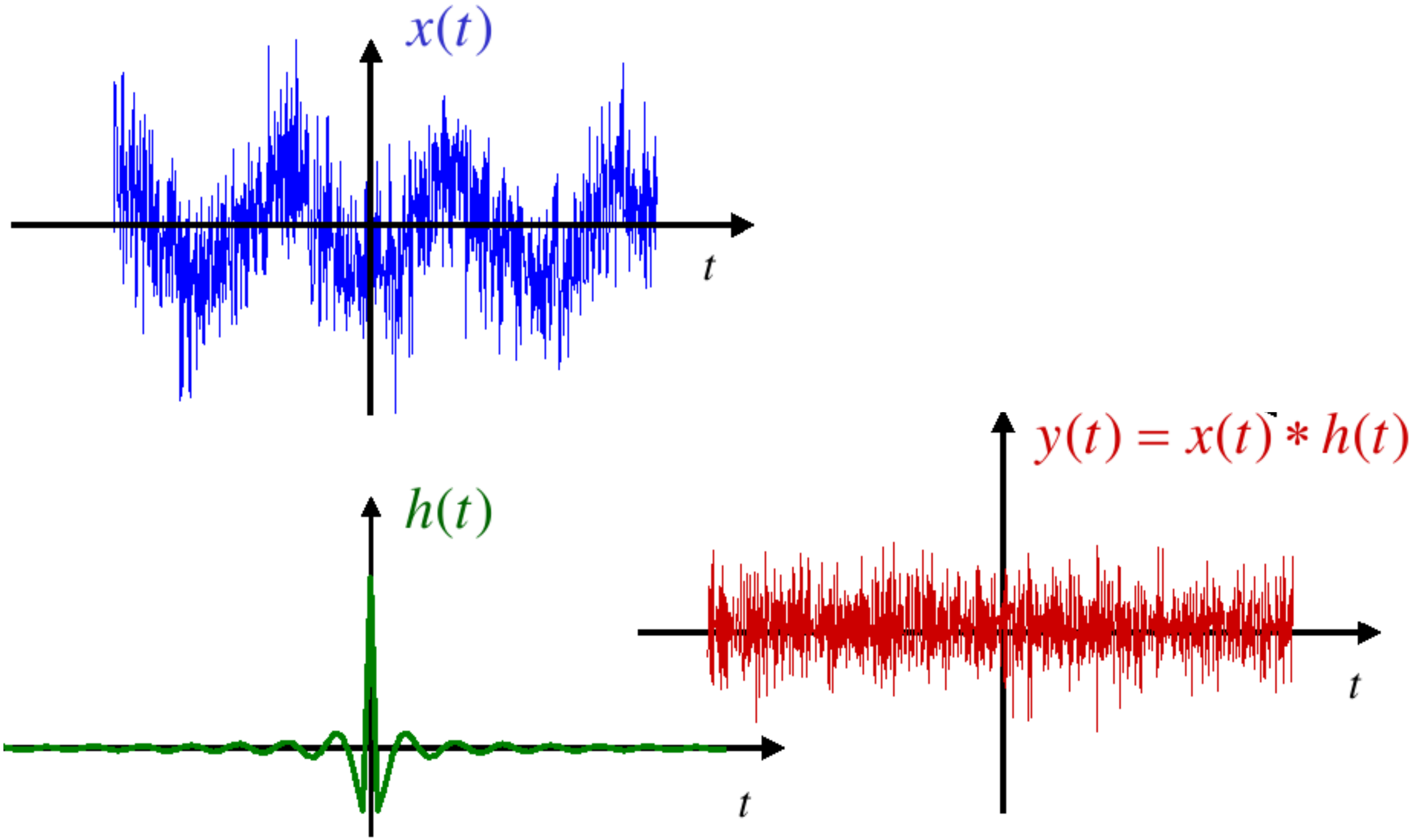
convolution

$$y(n) = x(n) * h(n)$$

Low-pass filter



High-pass filter



Transfer function

- The Fourier Transform of the impulse response corresponds to the Frequency Response of the system

Transfer function

$$y(n) = x(n) * h(n) \quad \Rightarrow \quad Y(z) = X(z)H(z)$$

Difference equations

- difference equations

$$\sum_{k=0}^{N} a_k y(n-k) = \sum_{r=0}^{M} b_r x(n-r)$$

- applying the z-transform

$$\Im\left[\sum_{k=0}^{N} a_k y(n-k)\right] = \Im\left[\sum_{r=0}^{M} b_r x(n-r)\right]$$

linearity

$$\sum_{k=0}^{N} a_k \Im[y(n-k)] = \sum_{r=0}^{M} b_r \Im[x(n-r)]$$

delay property

$$\Im[y(n-k)] = z^{-k}Y(z)$$

$$\Im[x(n-r)] = z^{-r}Y(z)$$

Difference equations

- obtaining

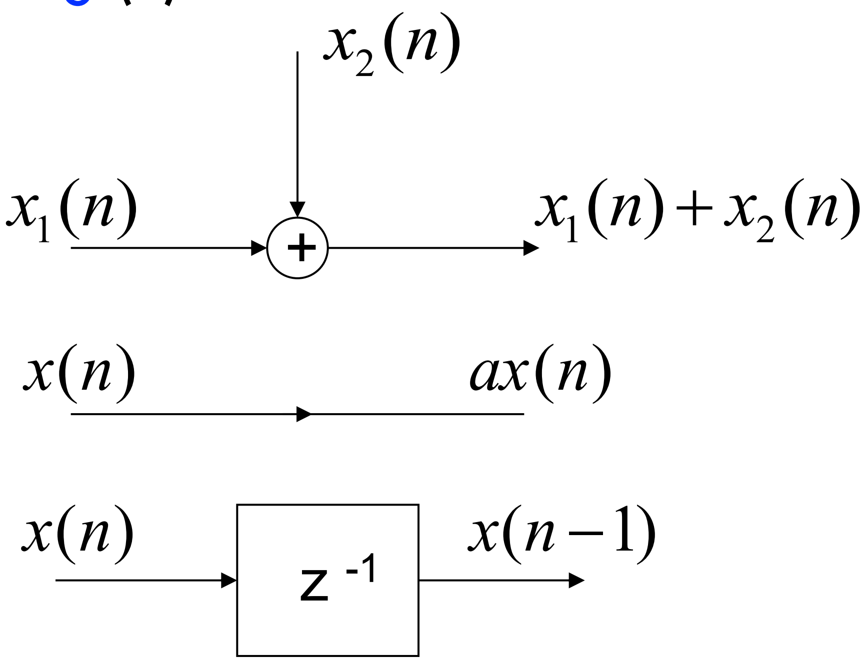
$$\sum_{k=0}^{N} a_k z^{-k} Y(z) = \sum_{r=0}^{M} b_r z^{-r} X(z) \qquad \equiv \qquad \left(\sum_{k=0}^{N} a_k z^{-k}\right) Y(z) = \left(\sum_{r=0}^{M} b_r z^{-r}\right) X(z)$$

$$\Downarrow$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{r=0}^{M} b_r z^{-r}}{\sum_{k=0}^{N} a_k z^{-k}}$$

Filter representation

- Basic operations required to realize a digital filter
  - adder (a)
  - multiplier by a constant (b)
  - delaying (c)



a)

b)

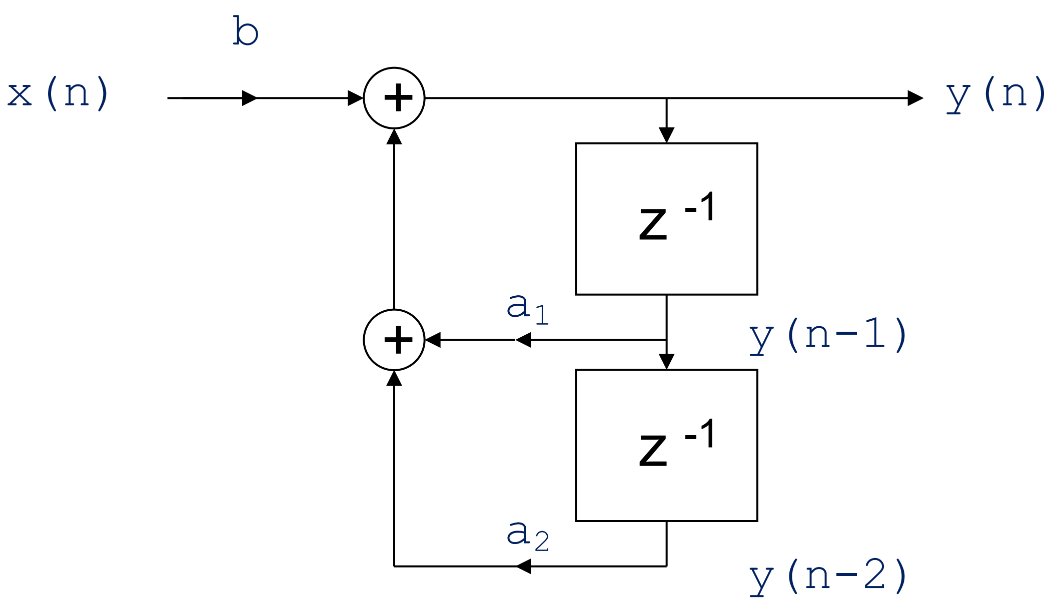
c)

Example

- Equation

$$y(n) = a_1 y(n-1) + a_2 y(n-2) + bx(n)$$

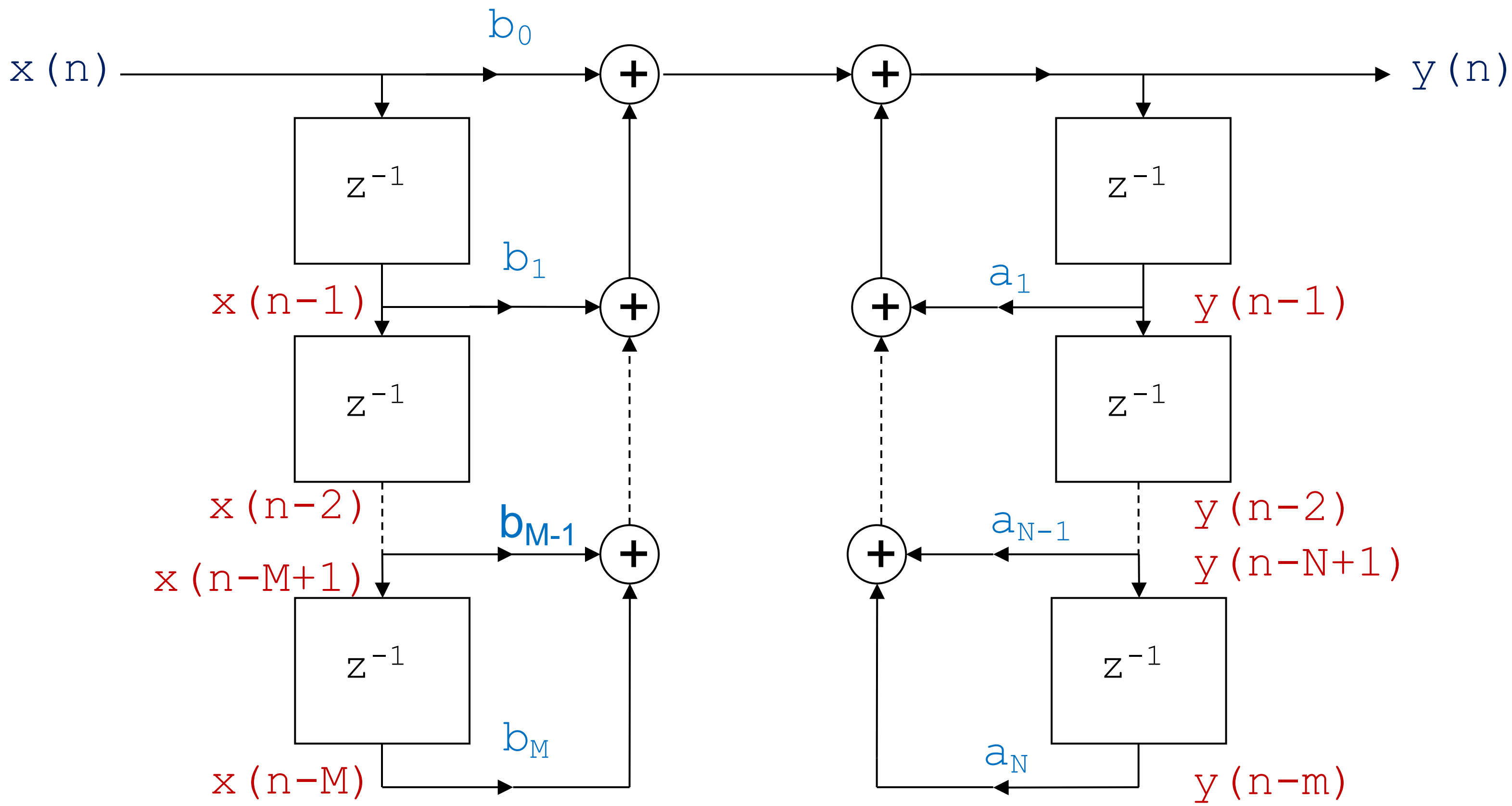
representation



transfer function

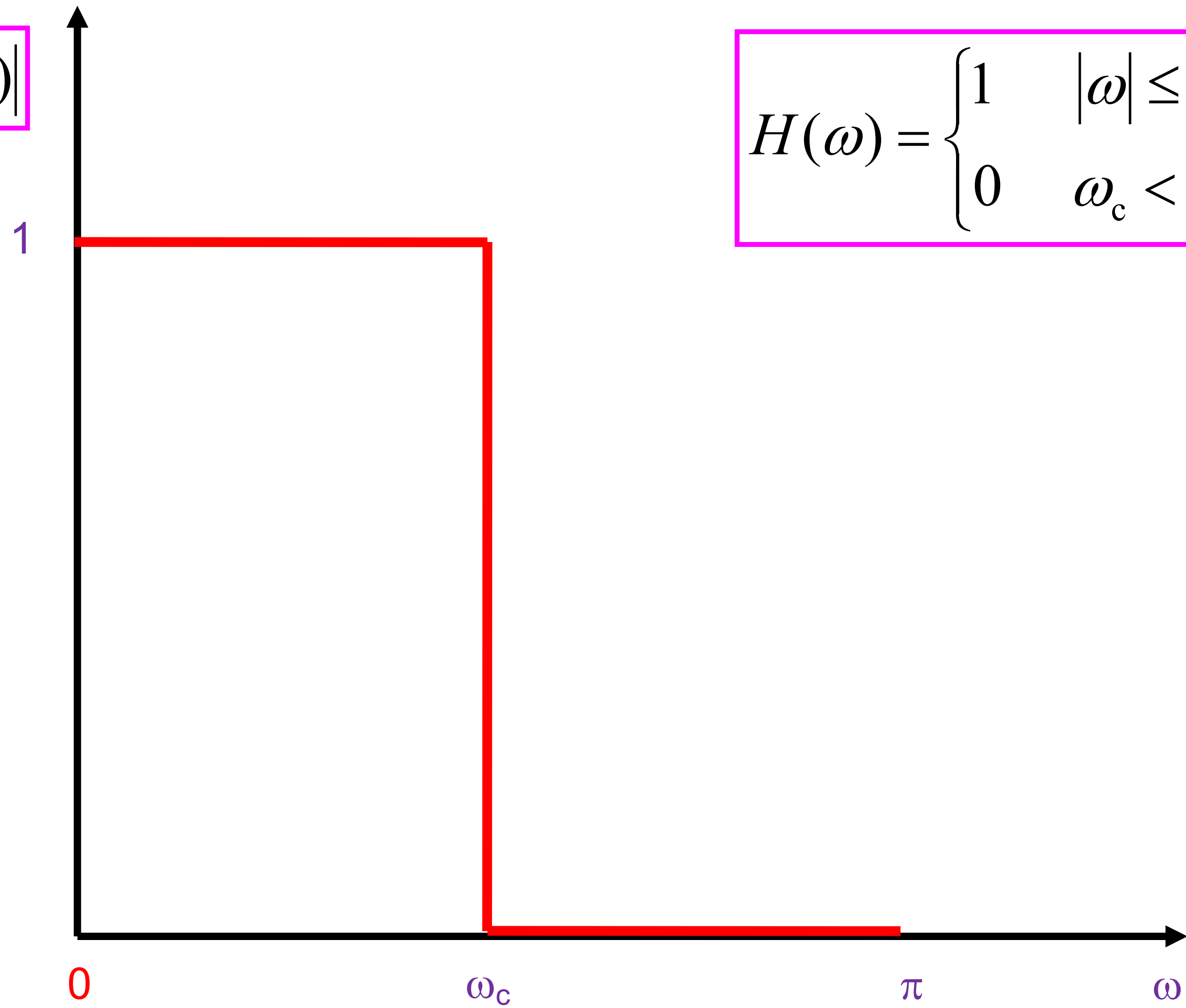
$$H(z) = \frac{b}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

General representation



Ideal low-pass filter

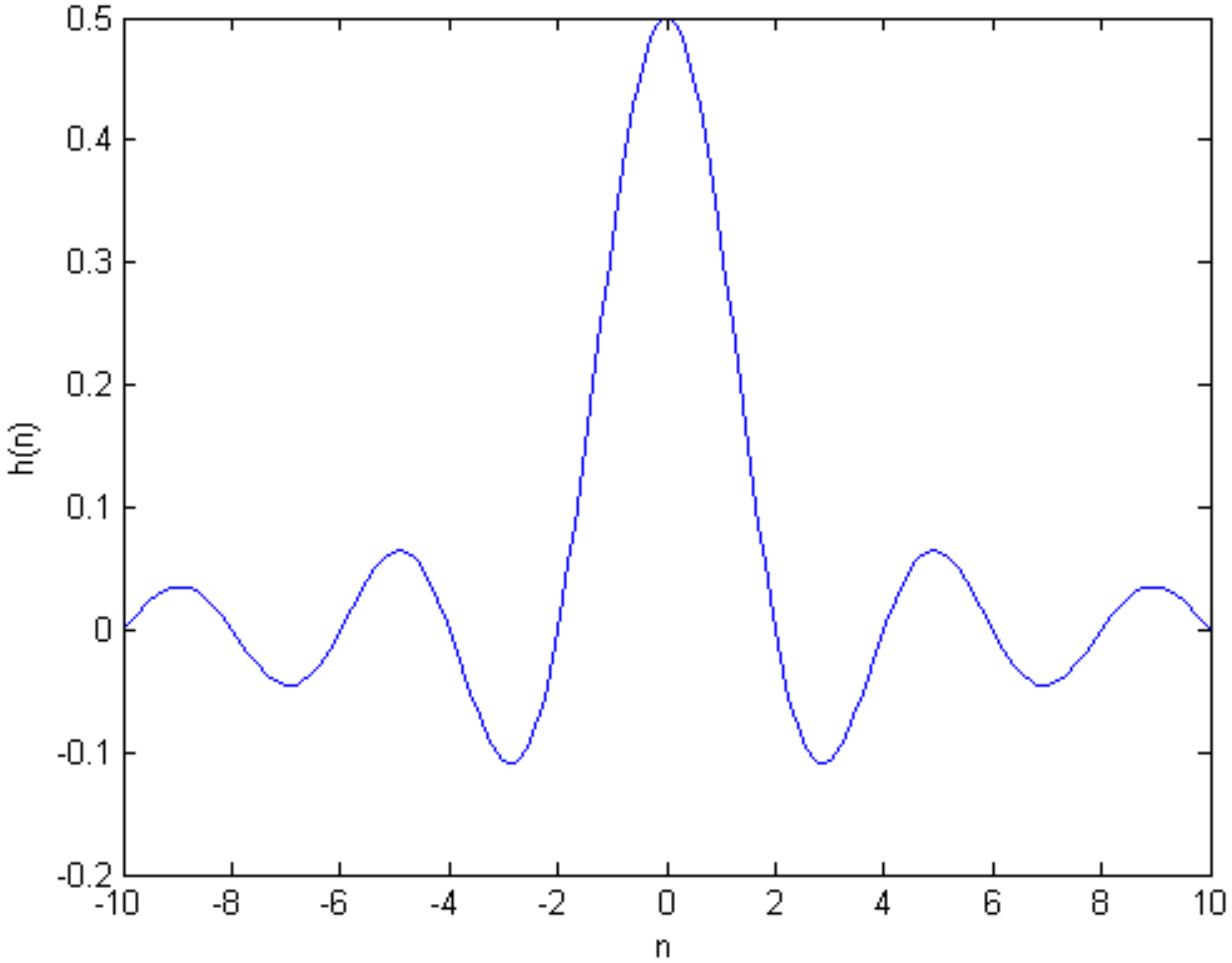
$$|H(\omega)|$$



$$H(\omega) = \begin{cases} 1 & |\omega| \le \omega_c \\ 0 & \omega_c < |\omega| < \pi \end{cases}$$

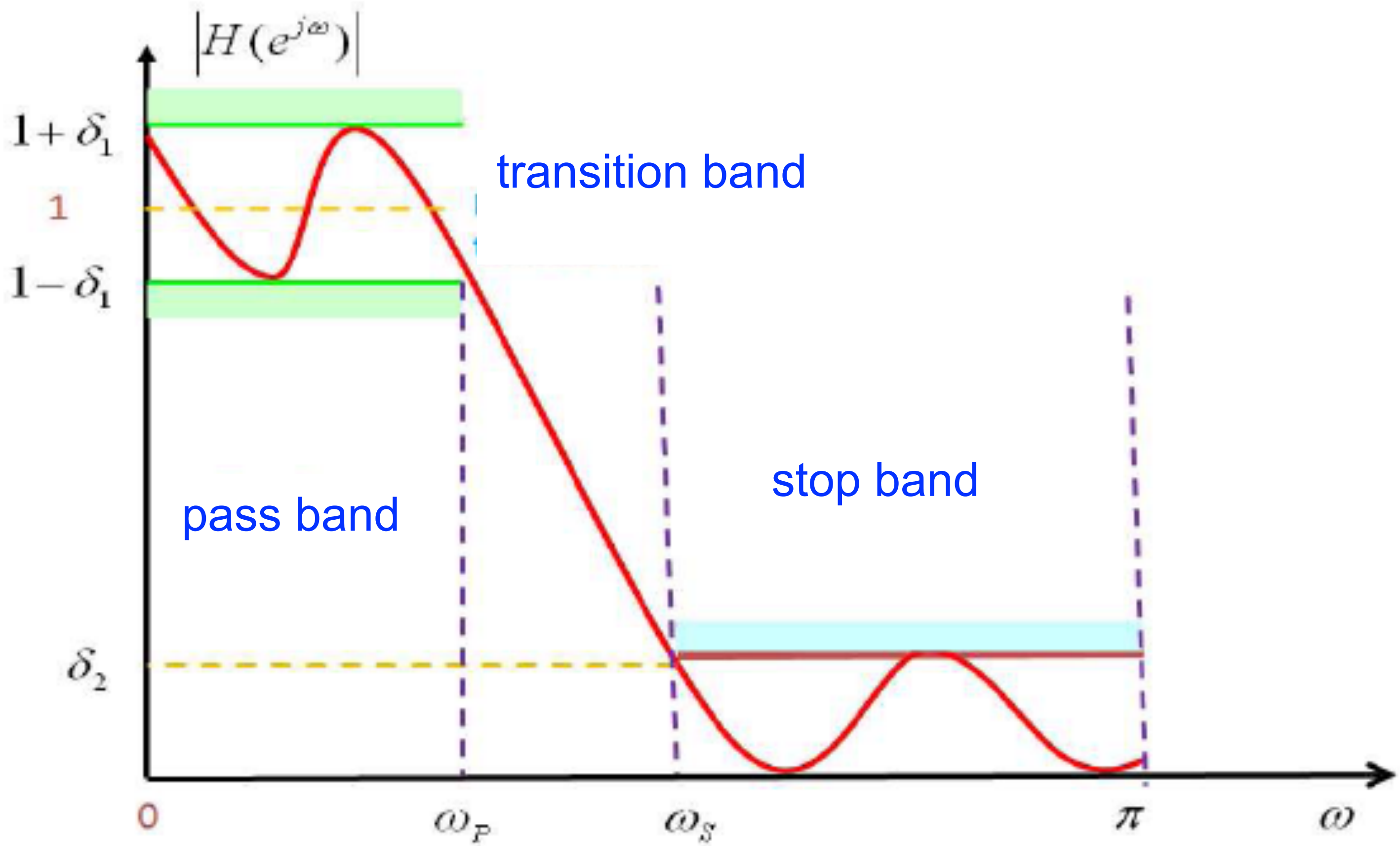
Ideal low-pass filter

h(n)



Impulse response corrisponding to low-pass filter

Real low-pass filter

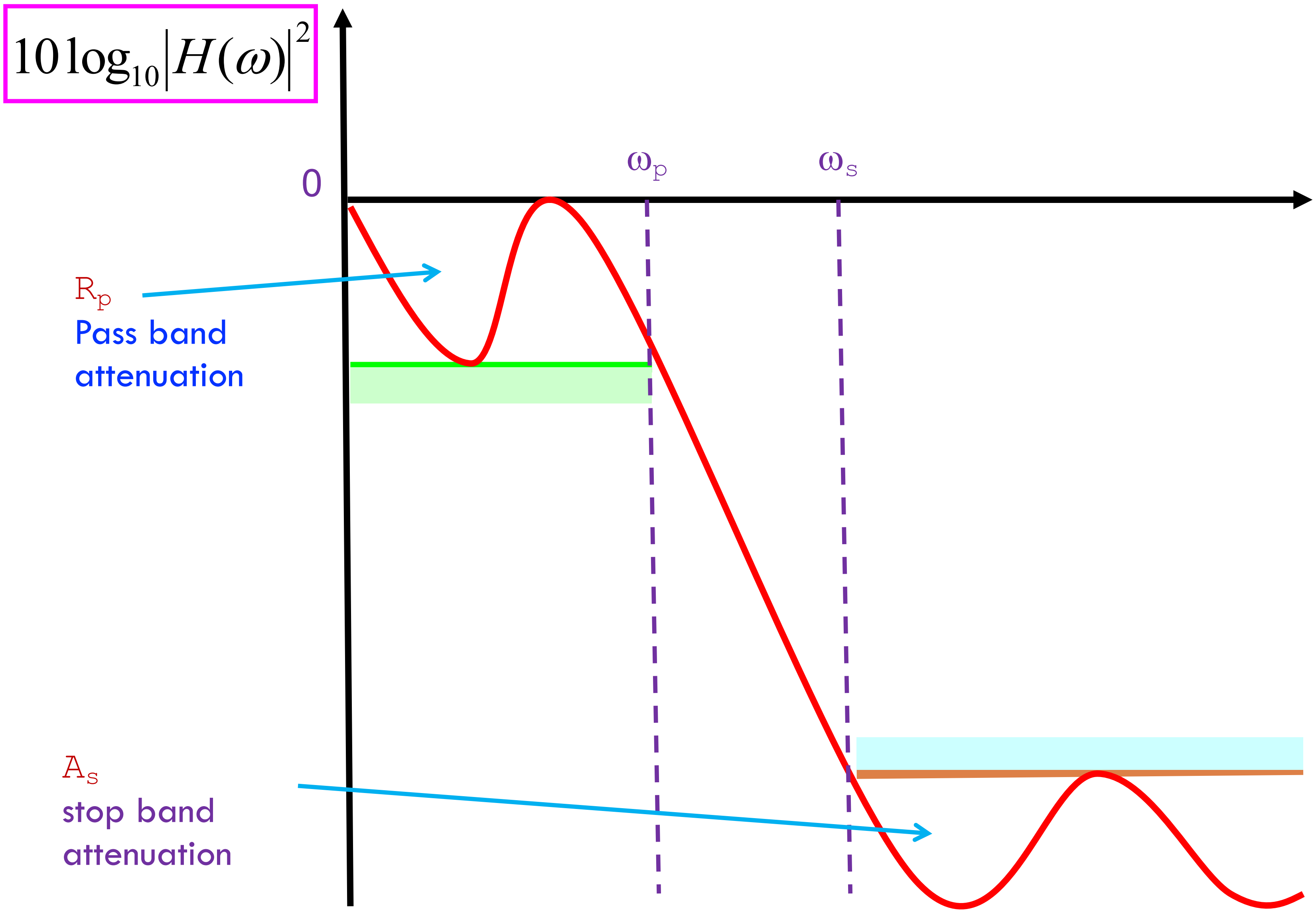


$$1-\delta_1 \le |H(\omega)| \le 1+\delta_1 \qquad |\omega| \le \omega_p$$

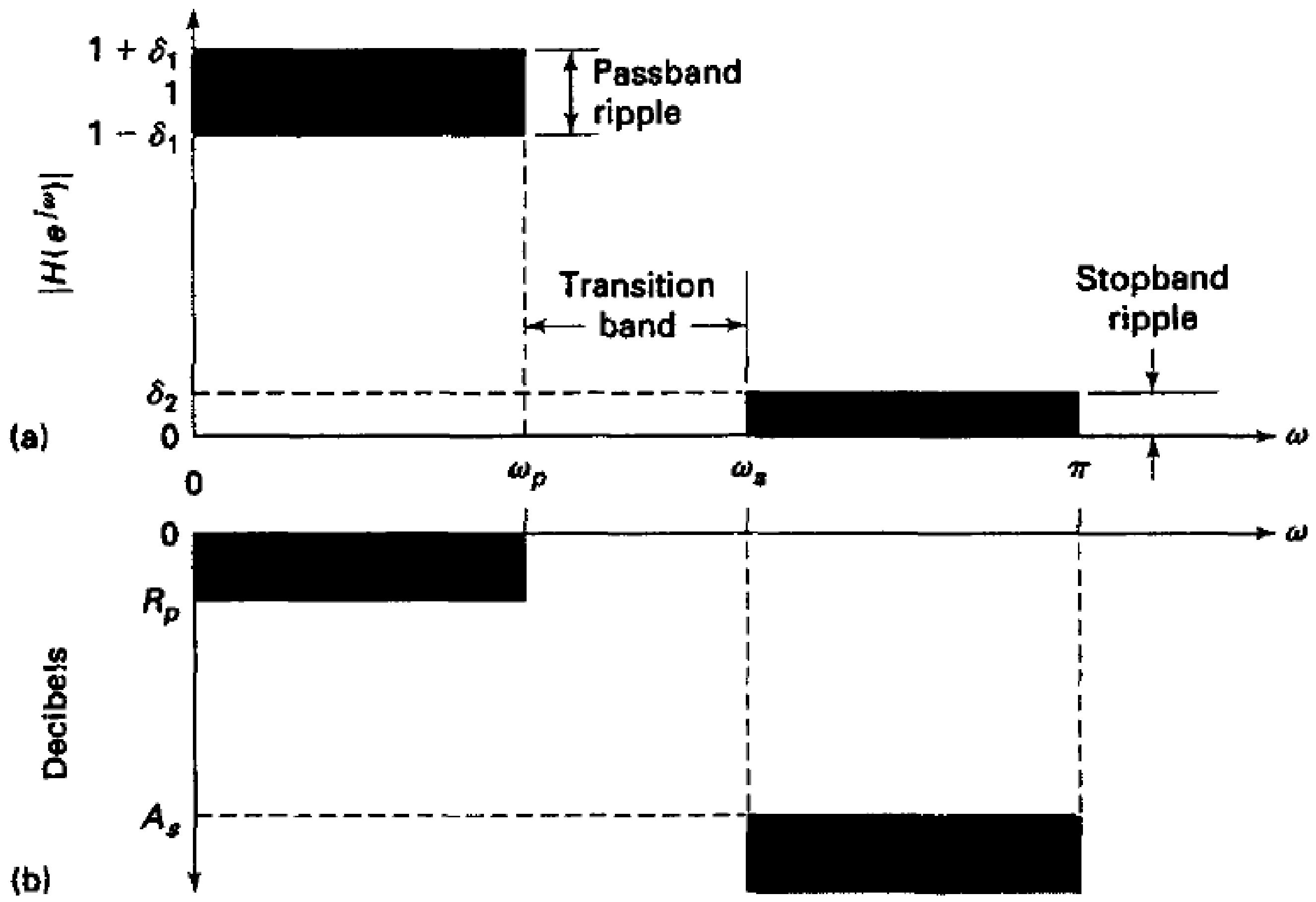
$$|H(\omega)| \le \delta_2 \qquad \omega_s \le |\omega| \le \pi$$

tollerance limits

Decibel parameters



Comparison



Comparison between paramaters with or without decibels

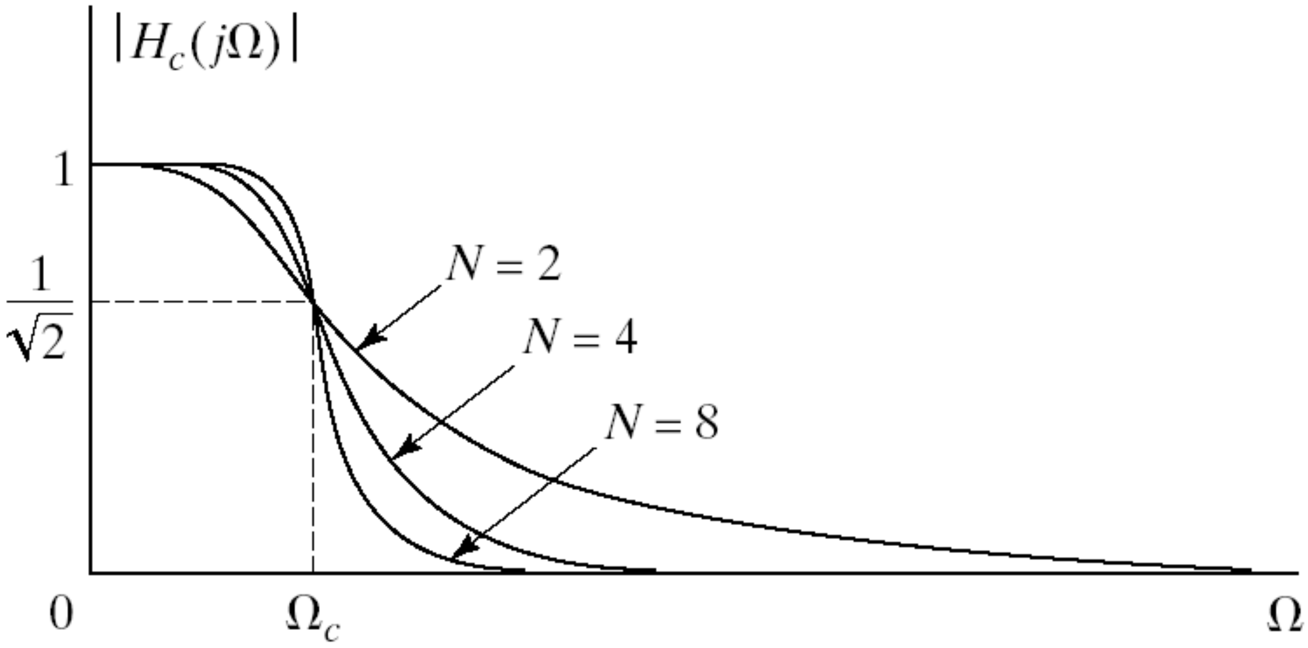
IIR and FIR

- Finite Impulse Response (FIR)
  - Polinomial Transfer function
  - Stable and linear phase
- Infinite Impulse Response (IIR)
  - Rational function
  - Non-linear phase and no stable
  - Better frequency cut

IIR

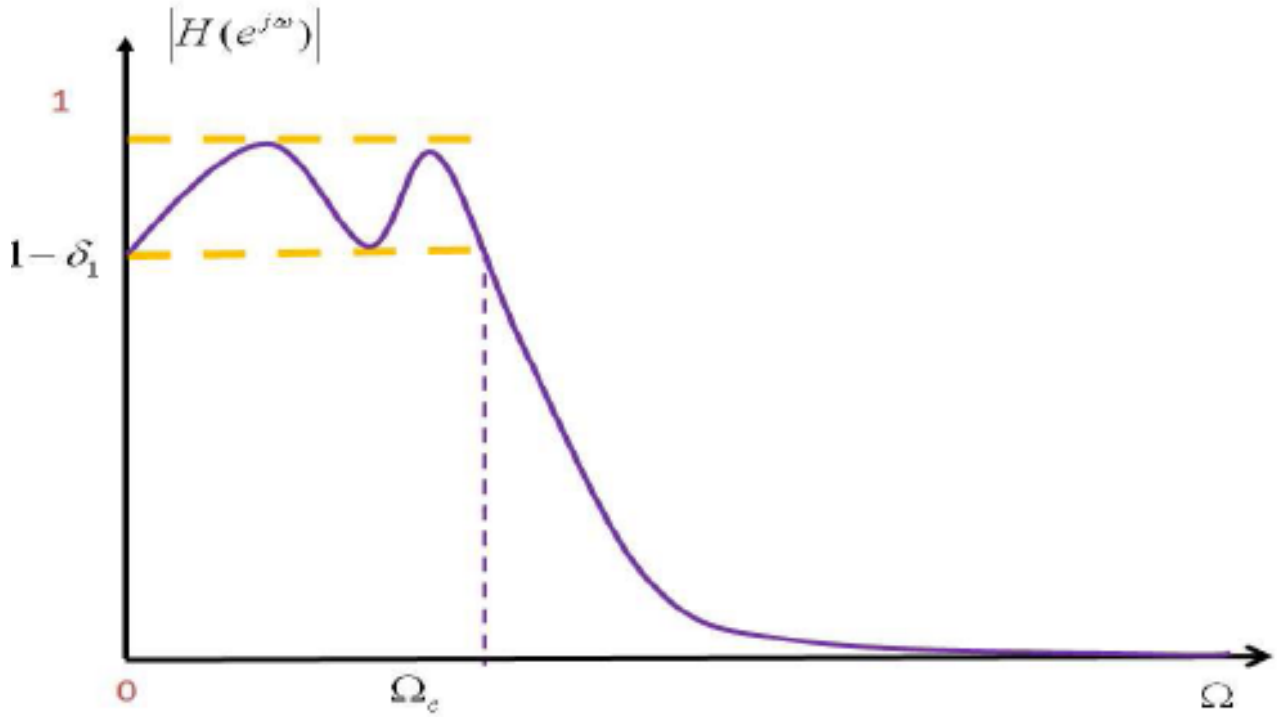
- To develop a muneric IIR filter
  - Transformation of an analogic filter in a numeric filter
- Known analogic filters
  - Butterworth
  - Chebyshev
  - Elliptic

Butterworth



Butterworth analogic filter

Chebychev



Chebychev analogic filter

Ideal low-pass filter

$$H_d(\omega) = \begin{cases} 1 & |\omega| \le \omega_c \\ 0 & \omega_c < |\omega| < \pi \end{cases}$$

frequencies

$$h_d(n) = \frac{\sin \omega_c n}{\pi n}$$

time

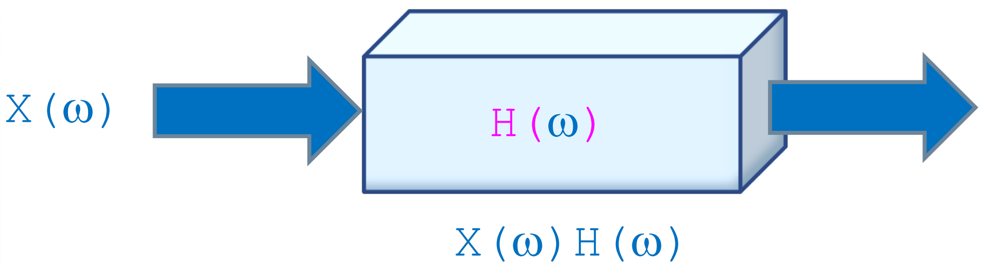
$$h(n) = h_d(n) w(n) \quad \text{dove} \quad w(n) = \begin{cases} 1 & 0 \le n \le M-1 \\ 0 & altrove \end{cases}$$

finite duaration

window

Filtering

Frequencies domain



$$Y(\omega) = X(\omega)H(\omega)$$

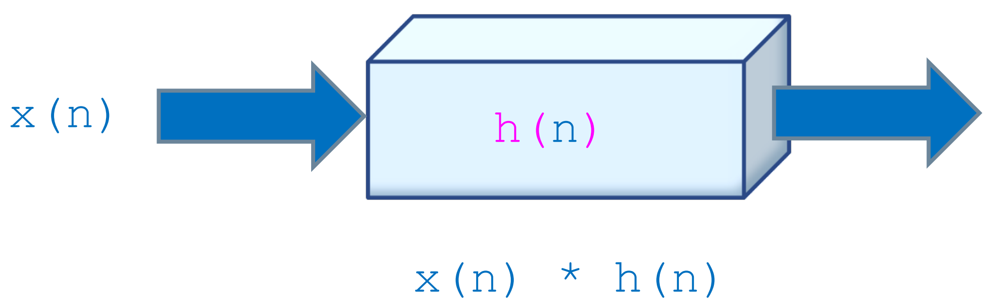
Convolution

$$H(\omega) = H_d(\omega) * W(\omega)$$

Ideal filter ($H_d(\omega)$)

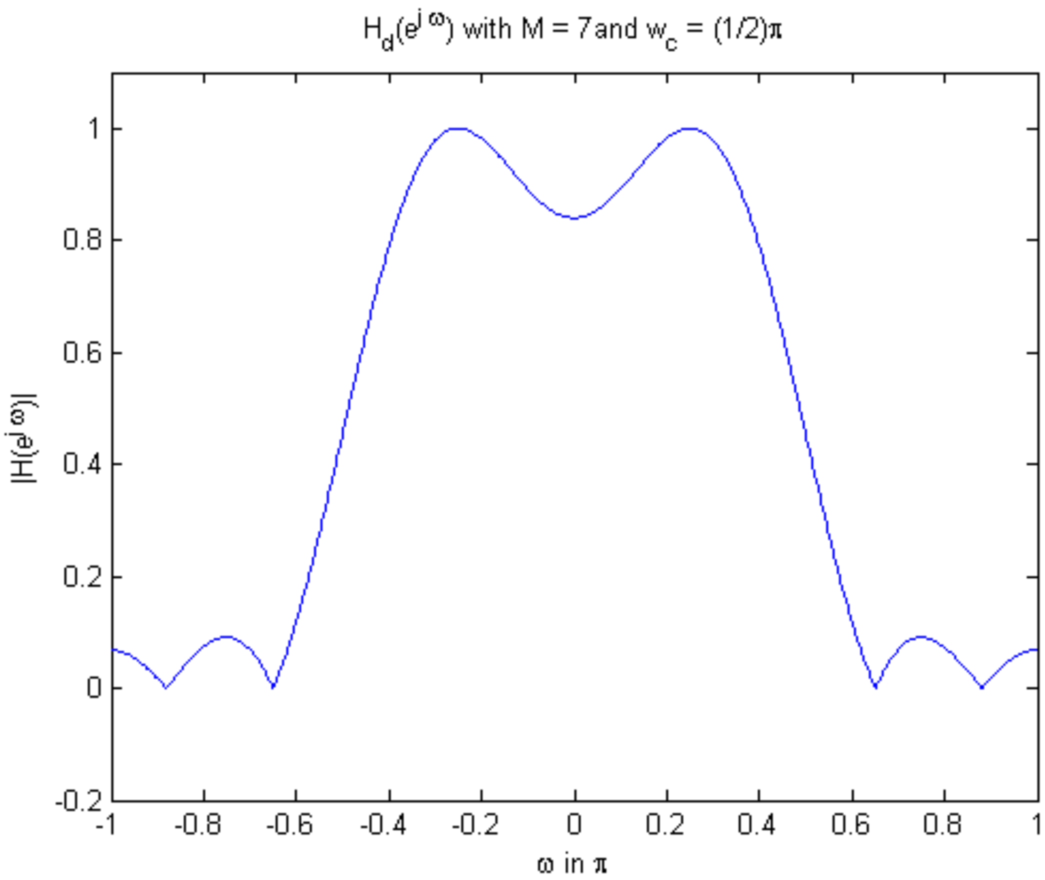
window ($W(\omega)$)

Time domain



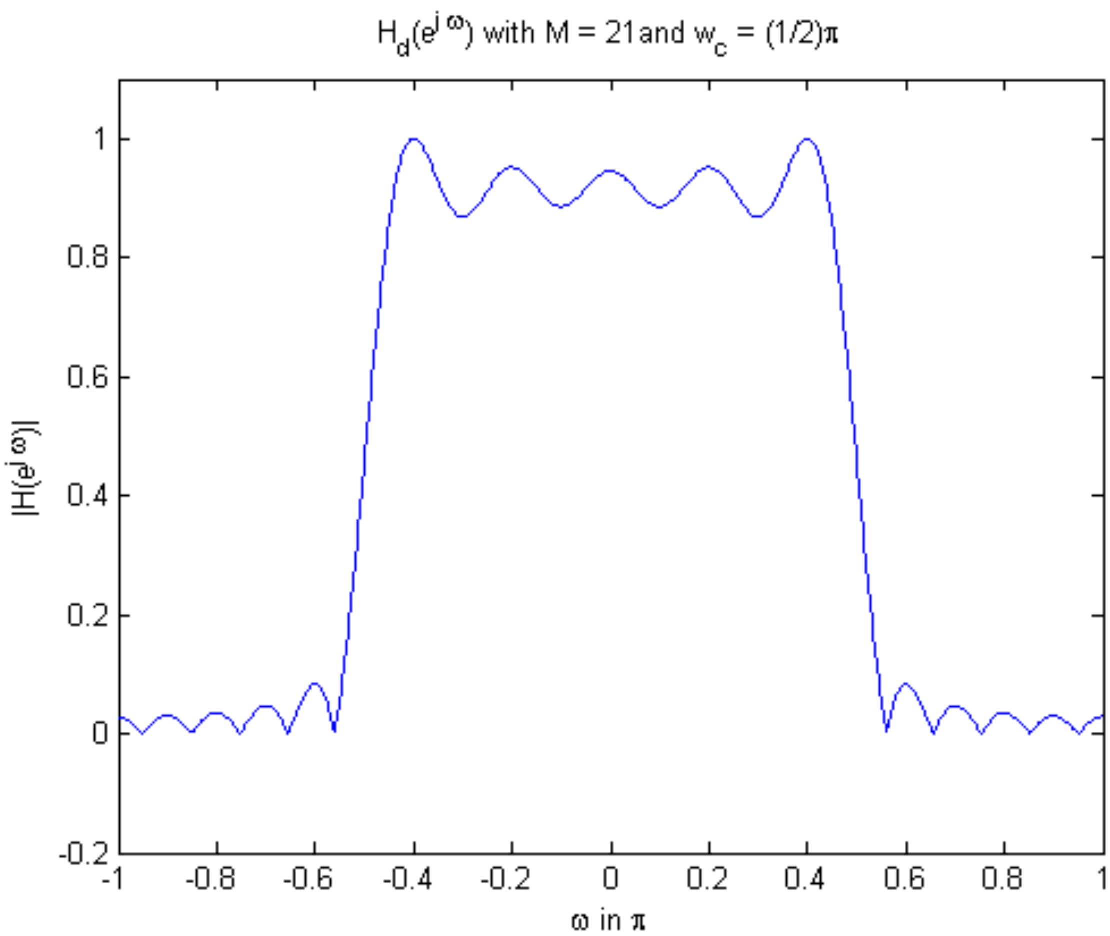
$$y(n) = x(n) * h(n)$$

Gibbs phenomenon



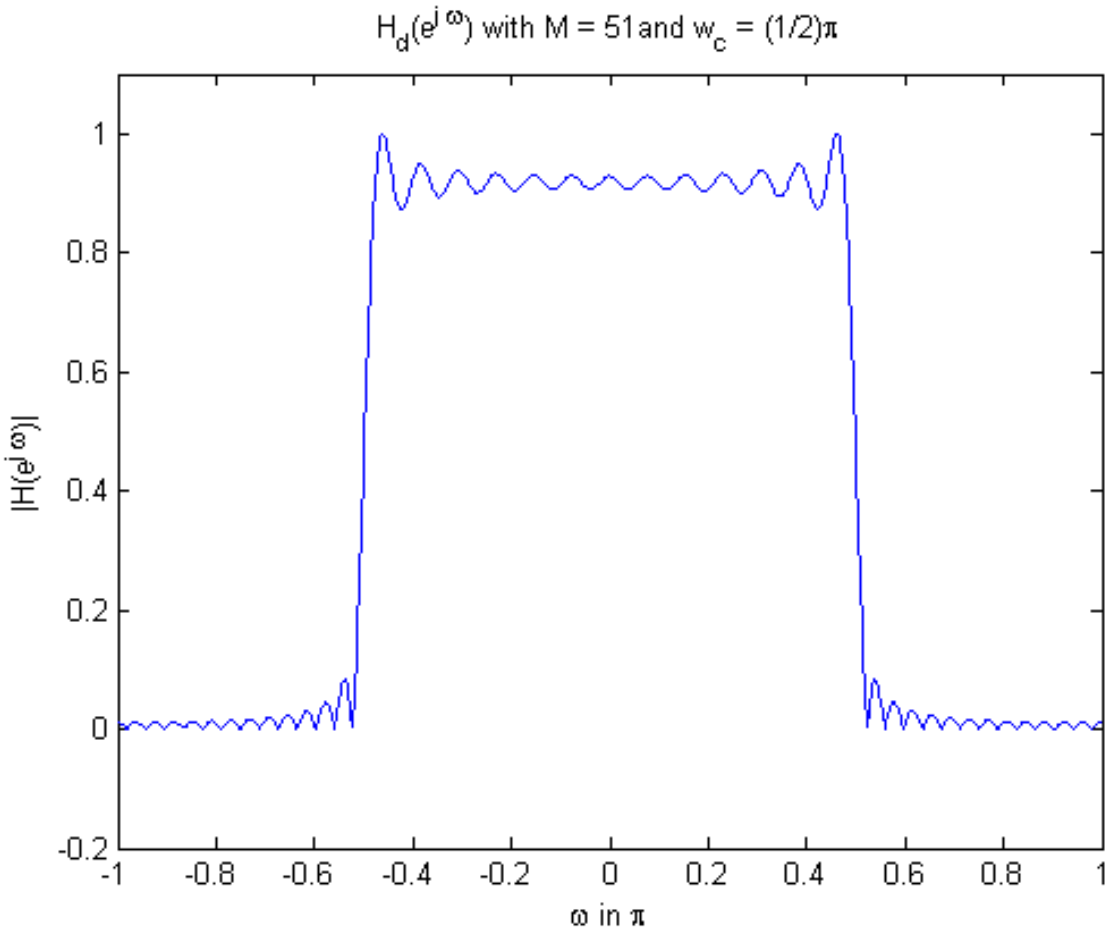
Rettangular window with $M = 7$

Gibbs phenomenon



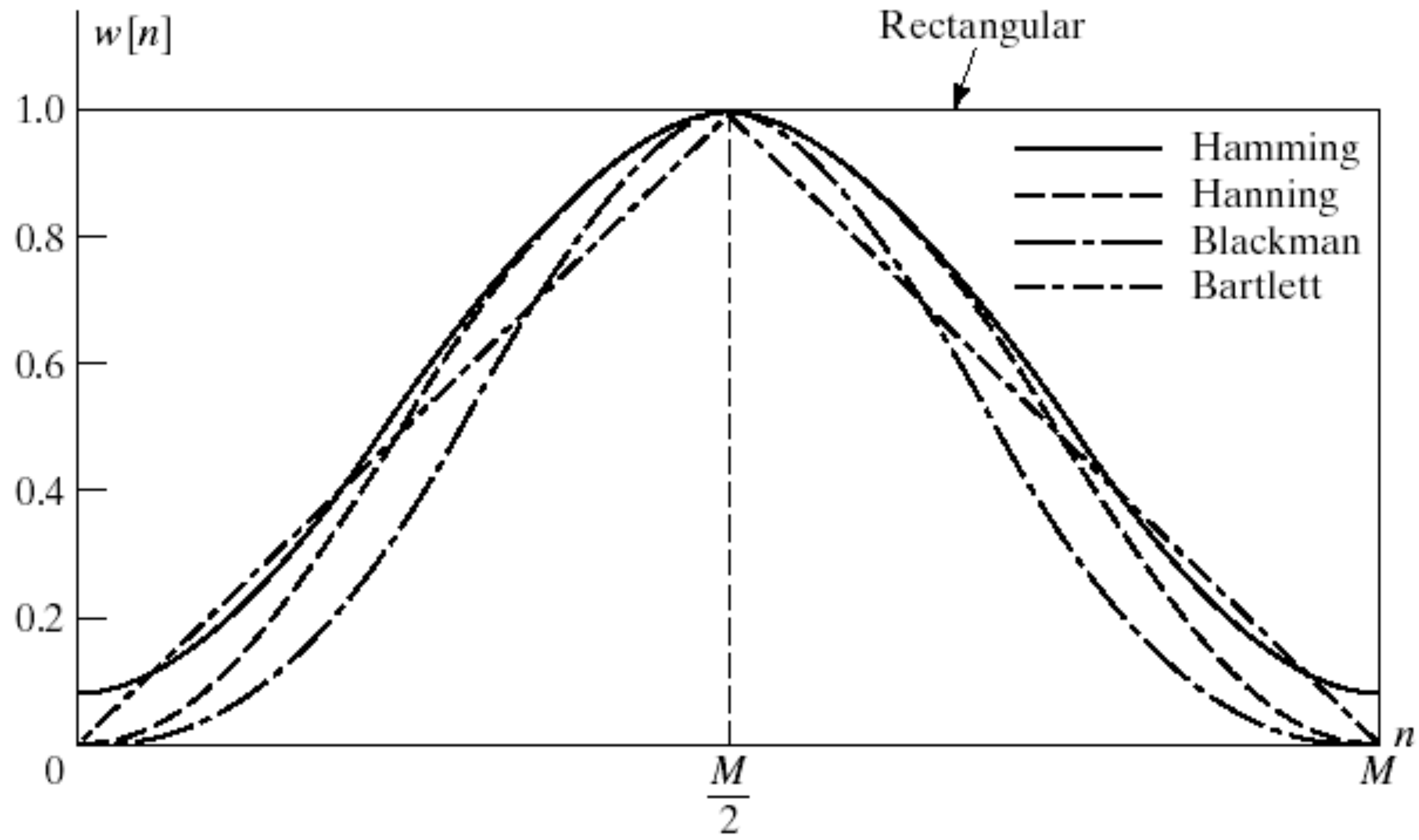
Rettangular window with M = 21

Gibbs phenomenon



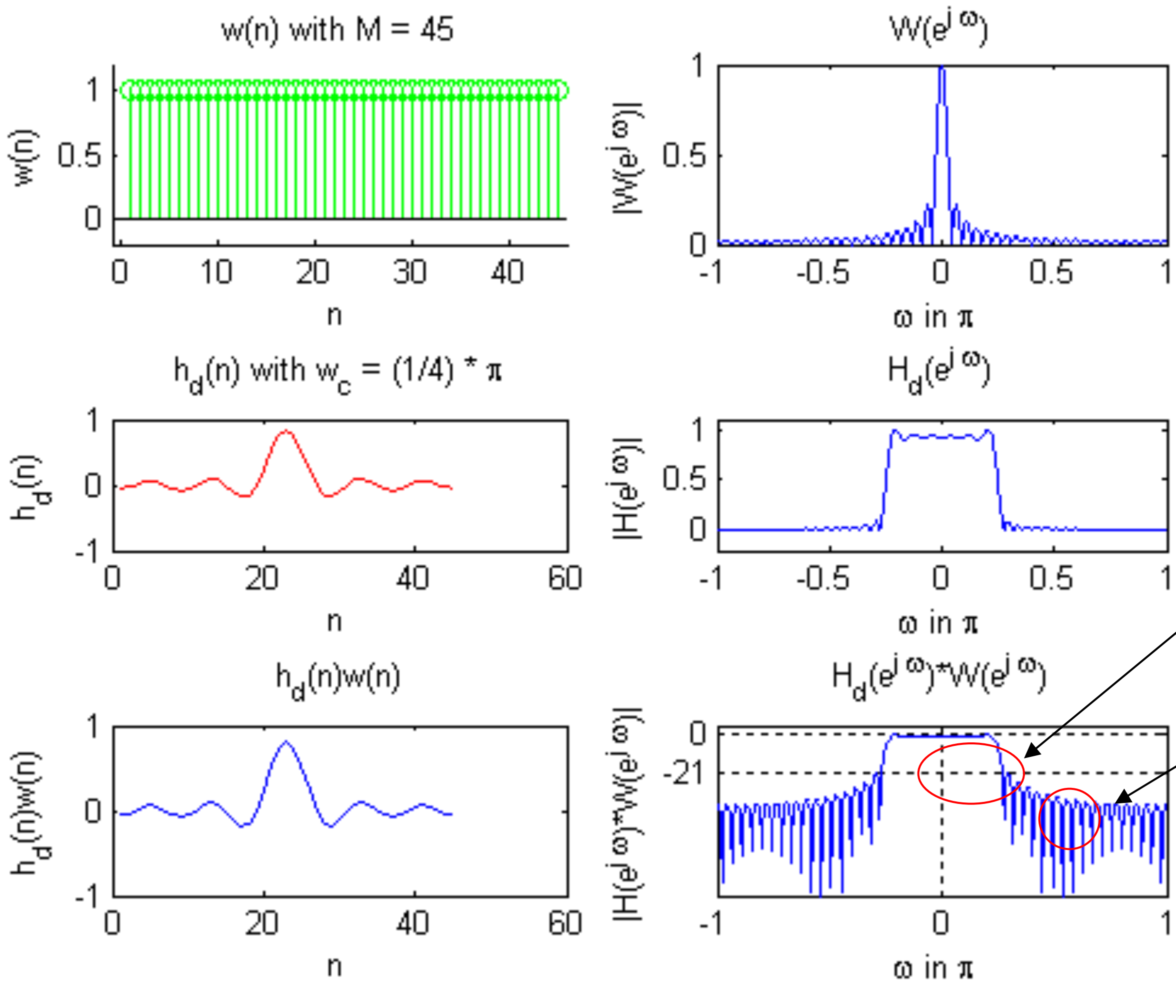
Rettangular window with M = 51

Gibbs phenomenon



To decrease the height of the side lobes different windows are used

Rectangular window



bandwidth

$$\omega_s - \omega_p = \frac{1.8\pi}{M}$$

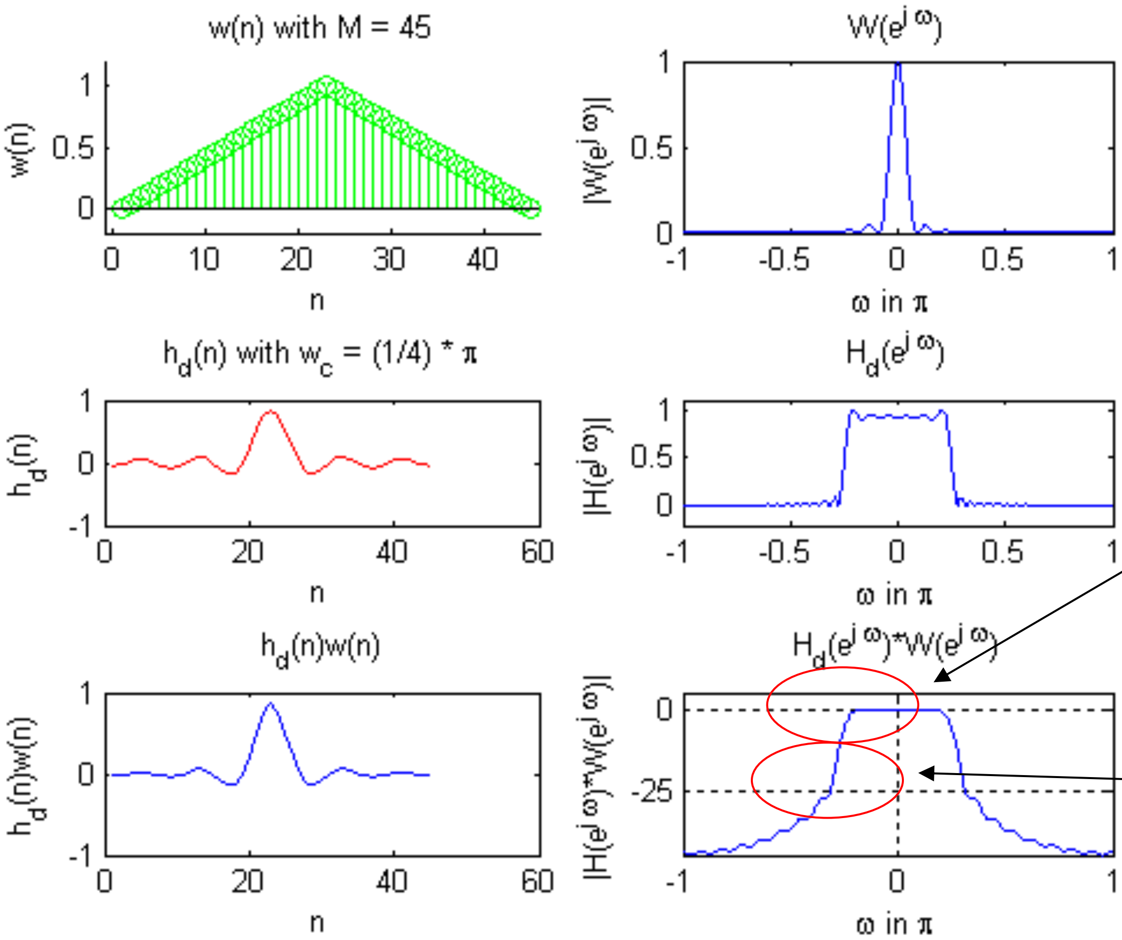
attenuation of 21 dB

M = 45

Bartlett window

$$w(n)=\begin{cases}2n/M & 0\le n\le M/2\\ 2-2n/M & M/2\le n\le M\\ 0 & else\end{cases}$$

Bartlett window



bandwidth

$$\omega_s - \omega_p = \frac{6.1\pi}{M}$$

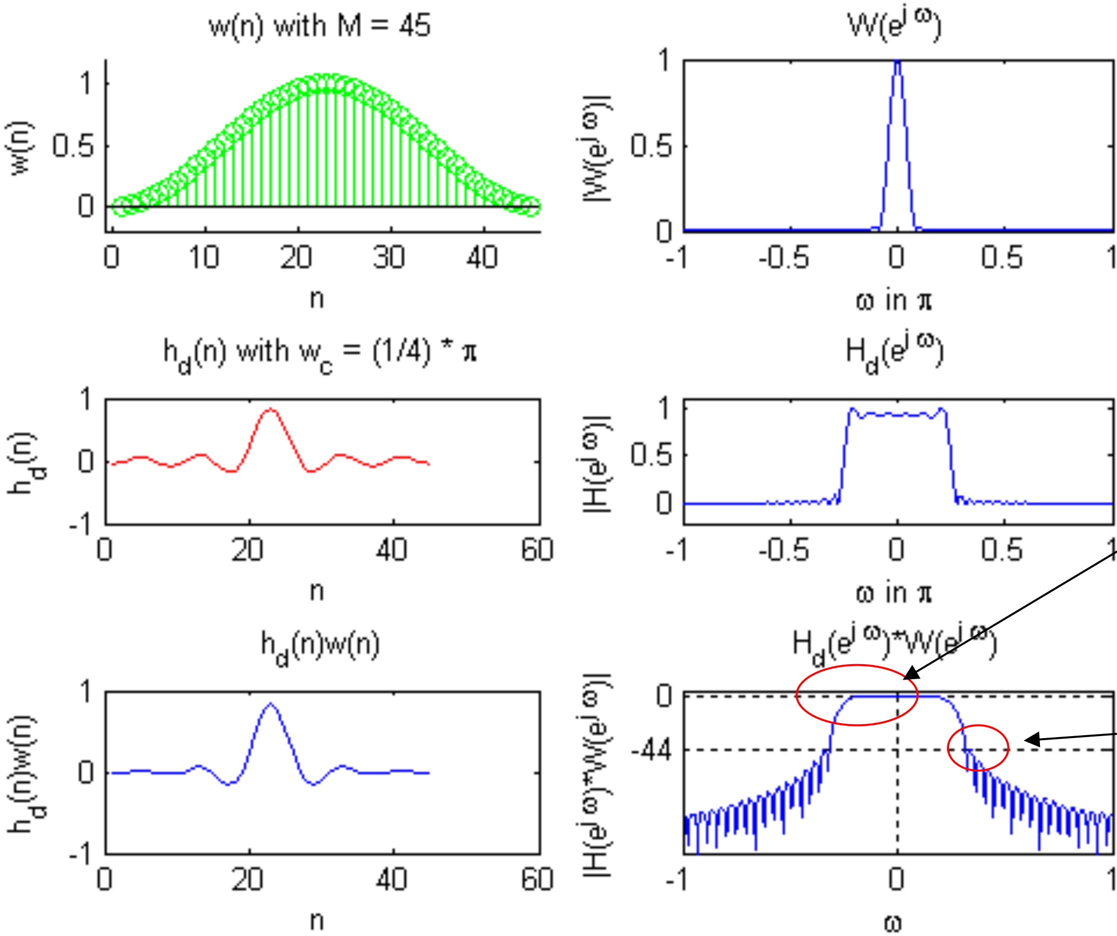
attenuation of 26 dB

M = 45

Hanning window

$$w(n) = \begin{cases} \frac{1}{2}\left[1-\cos\left(\frac{2\pi n}{M}\right)\right] & 0 \le n \le M \\ 0 & else \end{cases}$$

Hanning window



bandwidth

$$\omega_s - \omega_p = \frac{6.2\pi}{M}$$

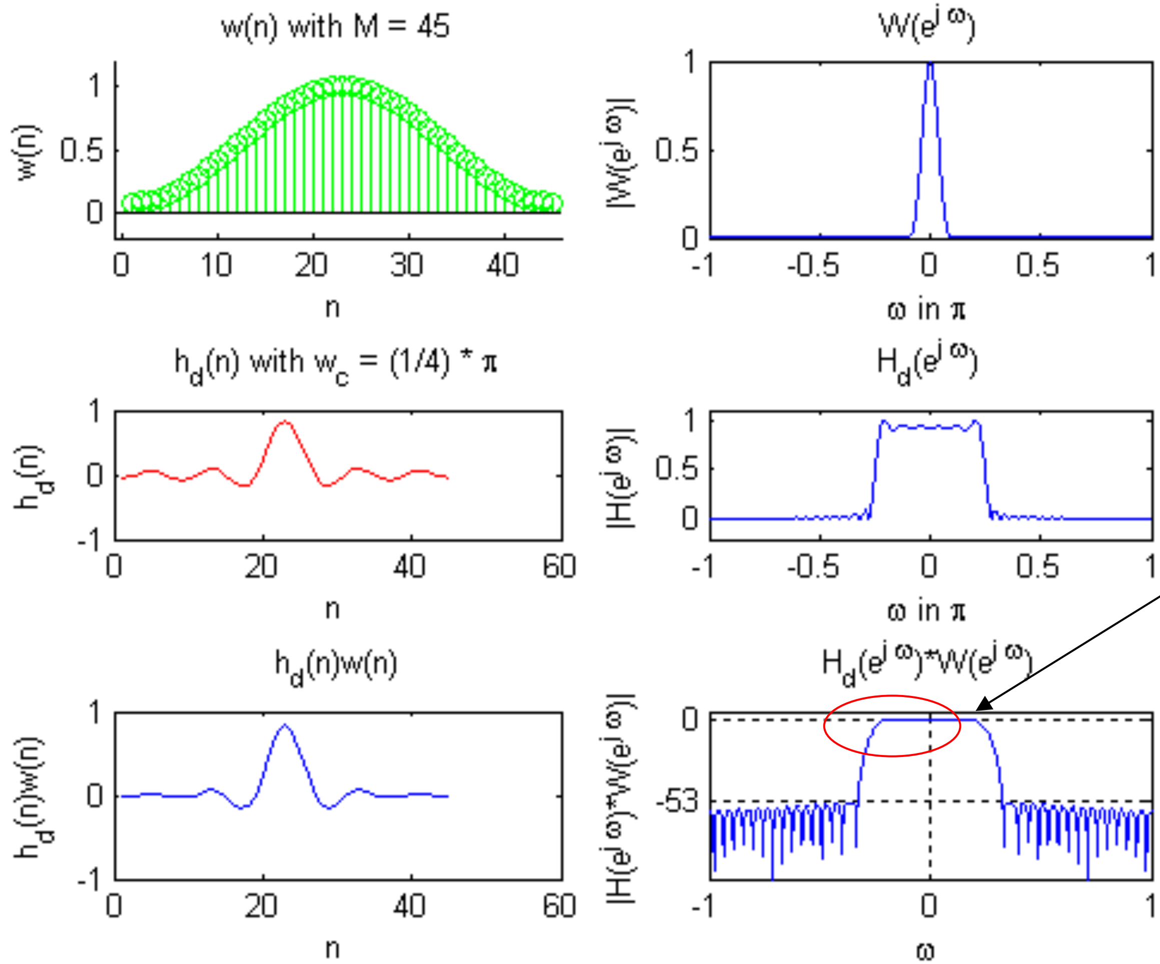
attenuation of
44 dB

M = 45

Hamming window

$$w(n) = \begin{cases} 0.54 - 0.46\cos\left(\dfrac{2\pi n}{M}\right) & 0 \le n \le M \\ 0 & else \end{cases}$$

Hamming window



bandwidth

$$\omega_s - \omega_p = \frac{6.6\pi}{M}$$

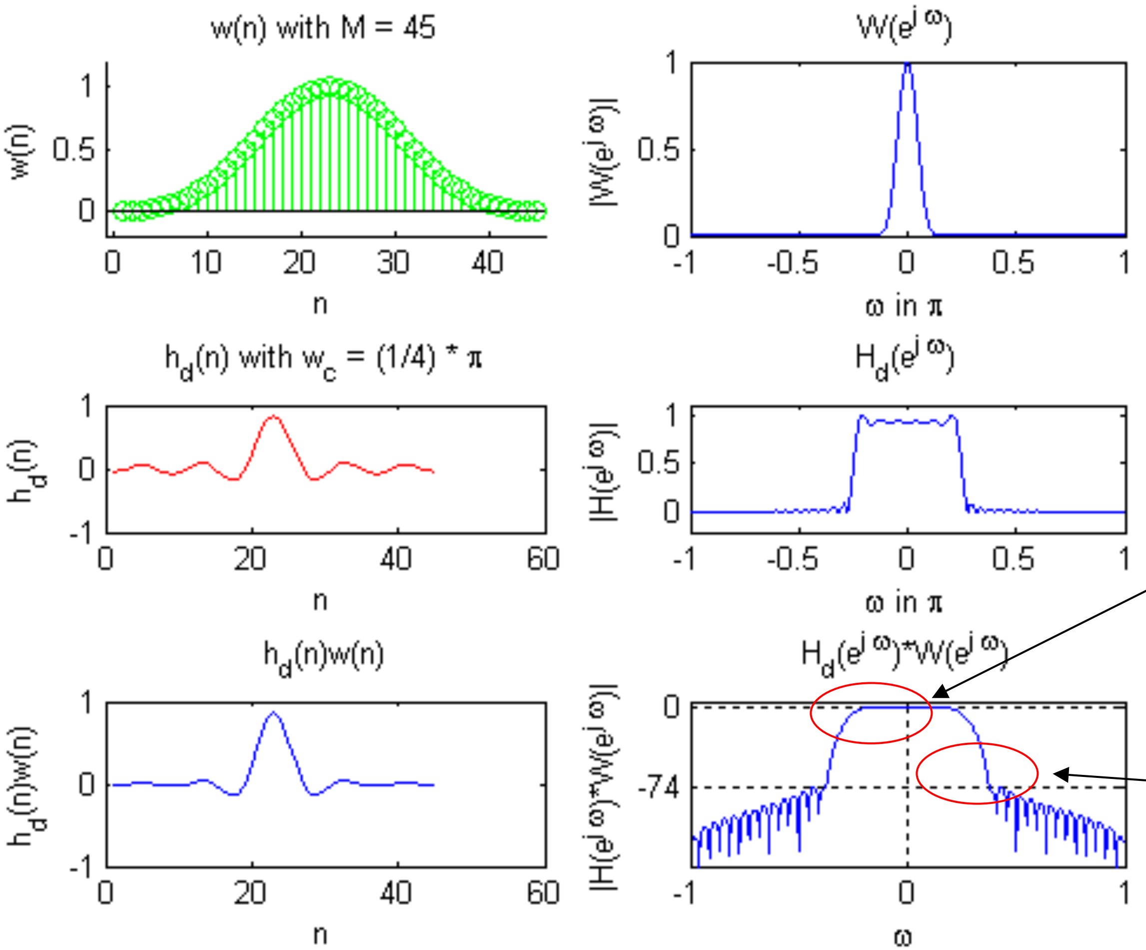
attenuation of 53 dB

M = 45

Blackman window

$$w(n) = \begin{cases} 0.42 - 0.5\cos\left(\dfrac{2\pi n}{M}\right) + 0.08\cos\left(\dfrac{4\pi n}{M}\right) & 0 \le n \le M \\ 0 & else \end{cases}$$

Blackman window



bandwidth

$$\omega_s - \omega_p = \frac{11\pi}{M}$$

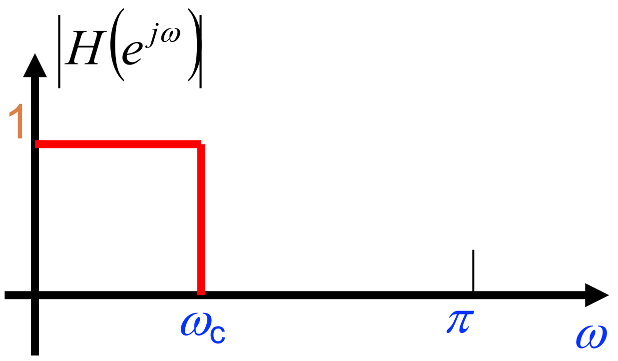
attenuation of 74 dB

M = 45

Summarazing

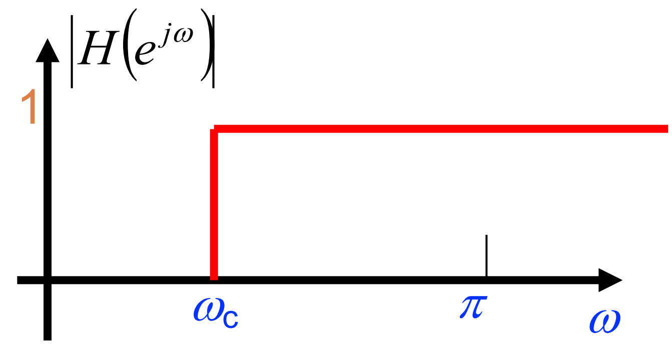
Finestra	*Altezza masima dei lobi laterali (dB)*	*Larghezza del lobo principale*	*Attenuazione minima in banda oscura (dB)*
Rettangolare	-13	$4\pi/N$	-21
Bartlett	-25	$8\pi/N$	-25
Hanning	-31	$8\pi/N$	-44
Hamming	-41	$8\pi/N$	-53
Blackman	-57	$12\pi/N$	-74

Types of filters



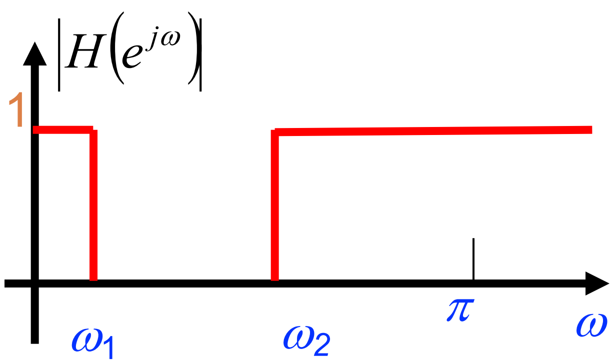
low-passa filter

a)



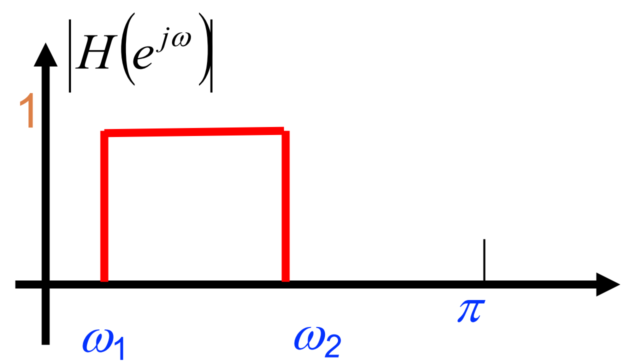
b)

high-passa filter



stop-band filter

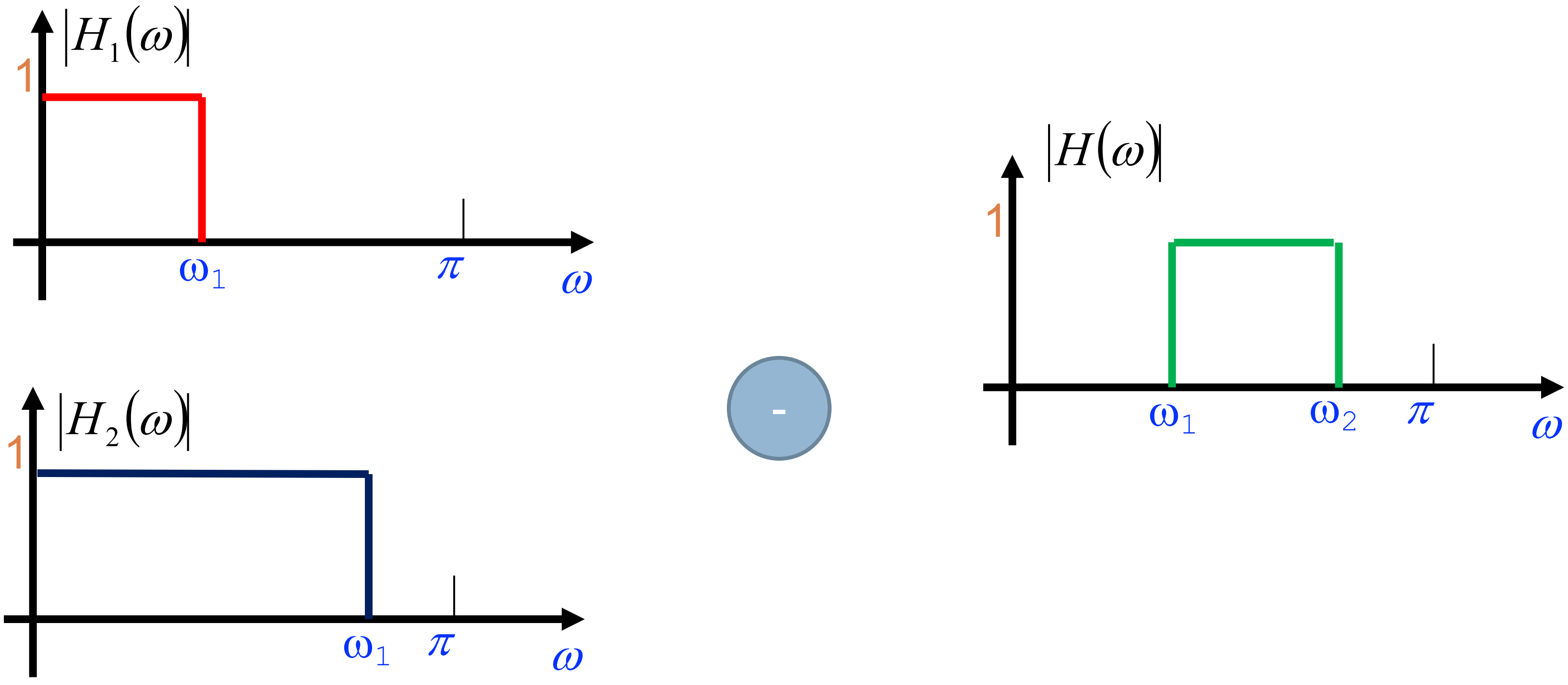
c)



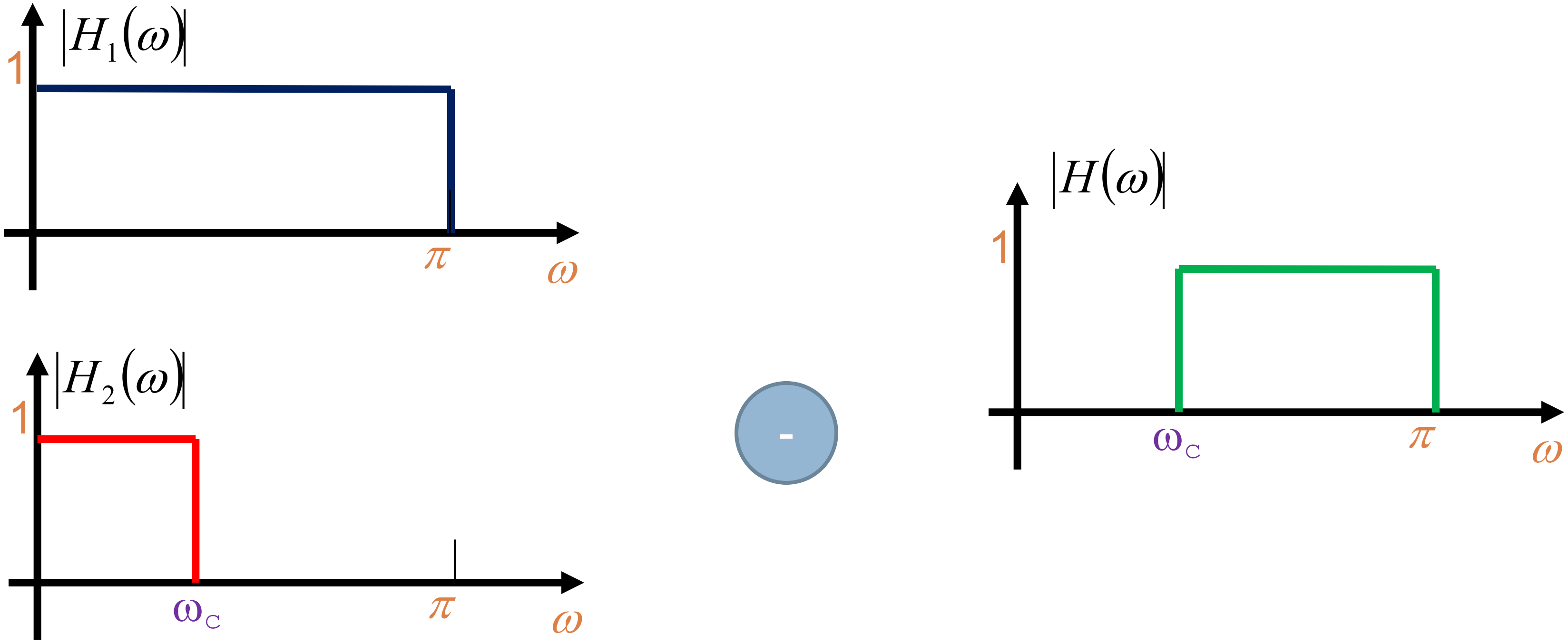
d)

band-pass filter

band-pass filter



high-pass filter



band-stop filter