Intelligent Signal Processing

z Transform

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z Transform

- Continuous time systems
  - The Laplace Transform can be considered a generalization of the Fourier Transform

- Discrete time systems
  - The z Transform can be considered a generalization of the Discrete Time Fourier Transform

z Transform

- z Transform
  - use a generic complex number
  - when $z = e^{j\omega}$ a DTFT is obtained
  - contains further details on the nature of the signal

- The z Transform (bilateral) of a sequence $x(n)$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$X(z) = \sum_{n=0}^{\infty} x(n) z^{-n}$$

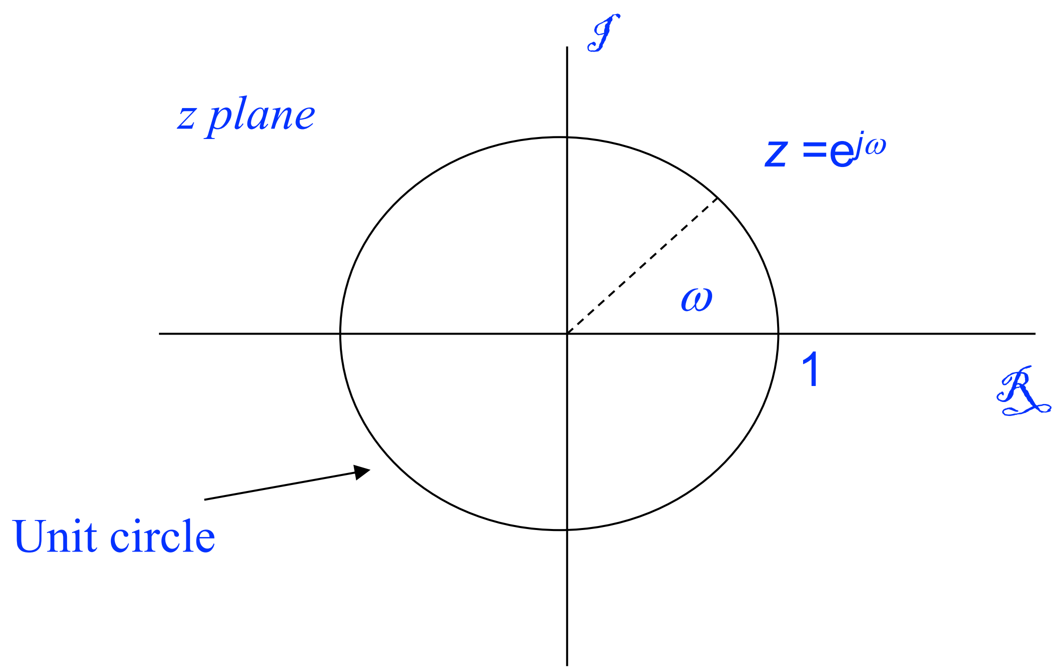
right unilateral

z Transform and DTFT

- Setting $z = re^{j\omega}$ we obtain

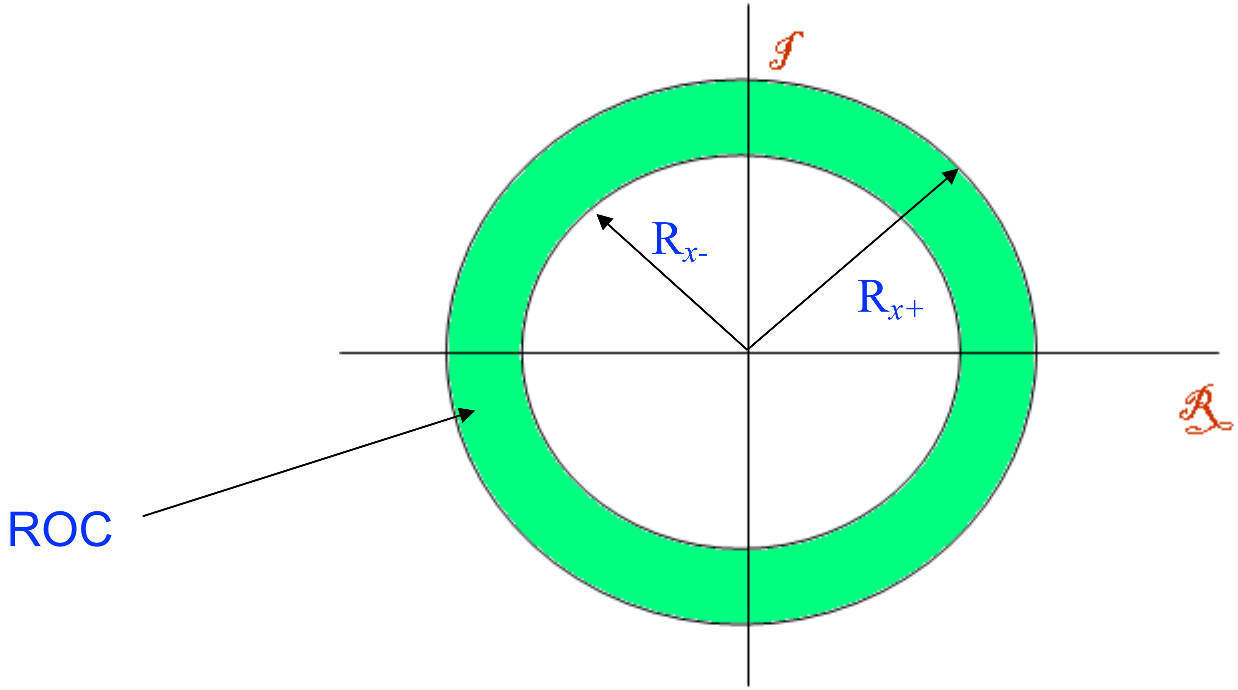
$$X(re^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)\left(re^{j\omega}\right)^{-n} = \sum_{n=-\infty}^{\infty} x(n) r^{-n} e^{-j\omega n}$$

- For $r = 1 \quad (|z| = 1)$ the z Transform becomes the DTFT



Region of Convergence

- Given a sequence x(n) the set of z values for which the z Transform converges is named
  - Region of Convergence



Region of Convergence

- Properties
  - The outer boundary is a circle or can be extended to infinity
  - The inner border is a circle and can be extended to become the origin

- If the ROC
  - includes the unit circle, this implies convergence of the z-transform also the Fourier transform converges
  - does not include the unit circle the Fourier transform it is not absolutely convergent

Zeros and poles

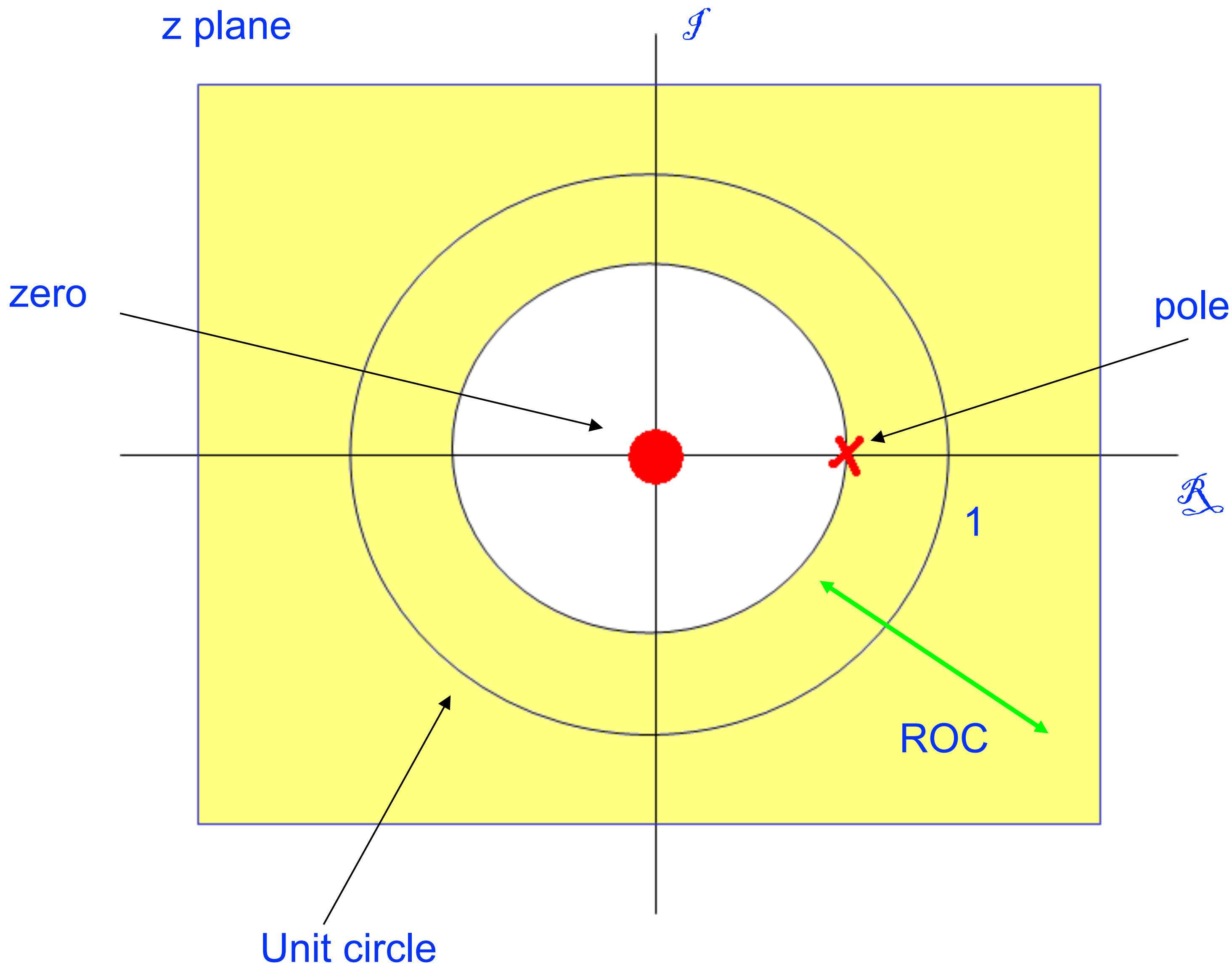
- An important class is the rational function (polynomials ratio in z)

$$X(z) = \frac{P(z)}{Q(z)}$$

- the zeros of the system are roots of the numerator polynomial

- the poles of the system are roots of the denominator polynomial

Example of ROC



Convergence properties

- The ROC must be a connected region
- The ROC is a ring or a disc in the z-plane centered at the origin
- The Fourier Transform of x(n) converges absolutely if and only if the ROC of the z Transform of x(n) comprises the unit circle
- The ROC does not contain any pole and is bounded by poles or zeros or infinite
- If x(n) is a sequence of finite duration the ROC is the entire z-plane except for possible $z = 0$ and $z = +\infty$
- If x (n) is the monolateral right the ROC is the outside of a circle (pole amplitude increased up to (possibly) $+\infty$)
- If x (n) is monolateral left the ROC is the inside of a circle (pole different from zero with lower amplitude up (possibly) to 0)
- If x (n) is the two-sided ROC consists of a ring in the z plane limited from the inside and from the outside by a pole and in accordance with the property 3 contains no pole