Intelligent Signal Processing

Discrete Systems

Angelo Ciaramella

CI & SS Lab

Università Parthenope
DiST
Dipartimento di Scienze e Tecnologie

Automatic control systems

- Automated controls play an essential role in the technological progress of human civilization
  - E.g., washing machines, refrigerators, ovens, automatic pilots of airplanes, robots, etc.

- A real world problem can be described by a System
  - Complex
    - e.g., planetary system
  - Simple
    - e.g., control system of the water temperature

Automatic control systems

- Element
  - it is the smallest part of a system that may be treated as an entity
- Block
  - set of items that can be grouped for describing a I/O relation

Systems

- Control system
  - set of physical components arranged to control themselves or other systems
- Automatic control system
  - regulates itself without the need of a human intervention
- Open Loop Control System
  - the control action is independent from the exit
- Closed Loop Control System
  - the control action is influenced by the exit

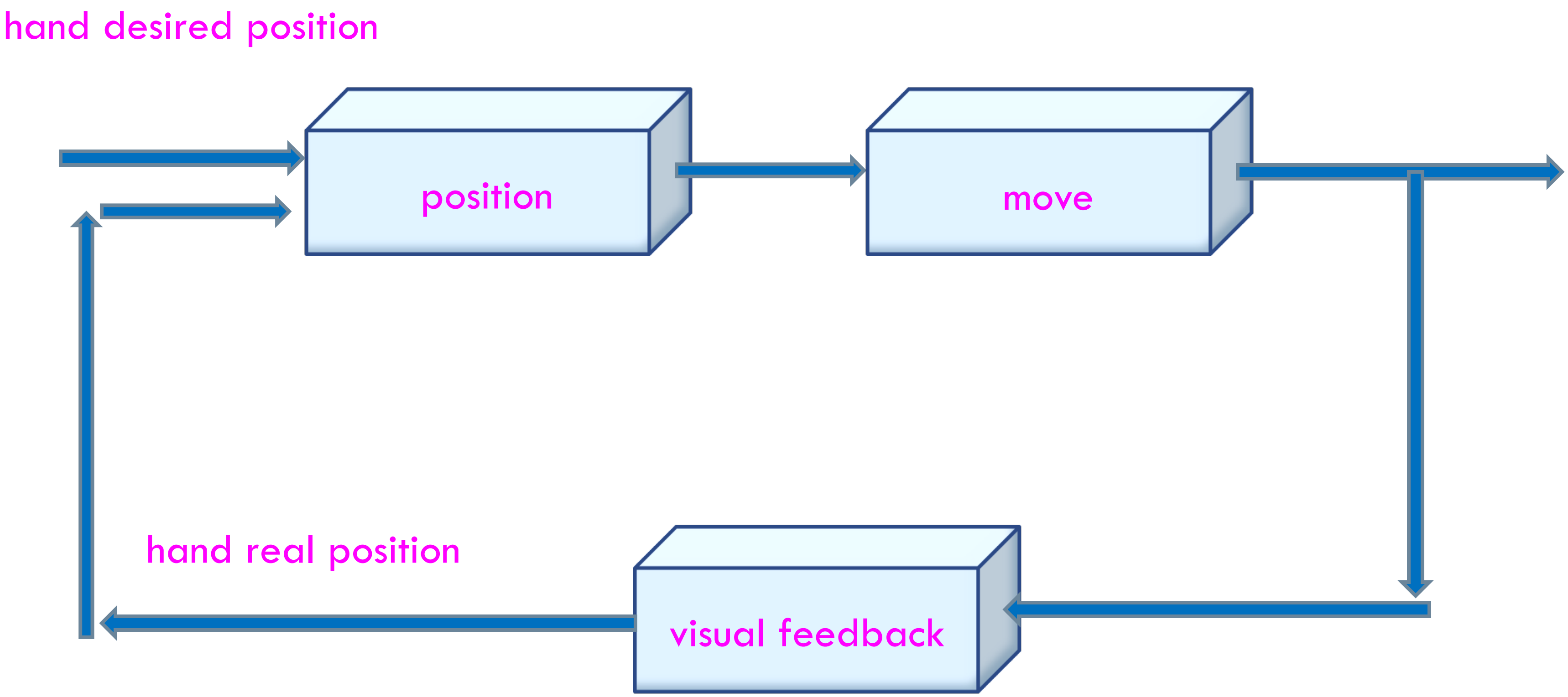
Systems

- Feedback
  - In a closed loop control system, the output signal is brought back into the input
    - e.g., driving
- SISO system
  - Single-Input and Single-Output input and output variables
- MIMO system
  - Multi-Input and Multi-Output input and output variables

Systems

- Linear System
  - the I/O relations can be represented by linear functions
    - e.g., Ohm's law (V = R x I)
- Time-Invariant System
  - system with constant coefficients
    - e.g., system spring-mass and dissipation
- Time-Variant System
  - system with variable coefficients
    - e.g., rocket burning fuel

Example: taking a glass …



System for taking a glass.

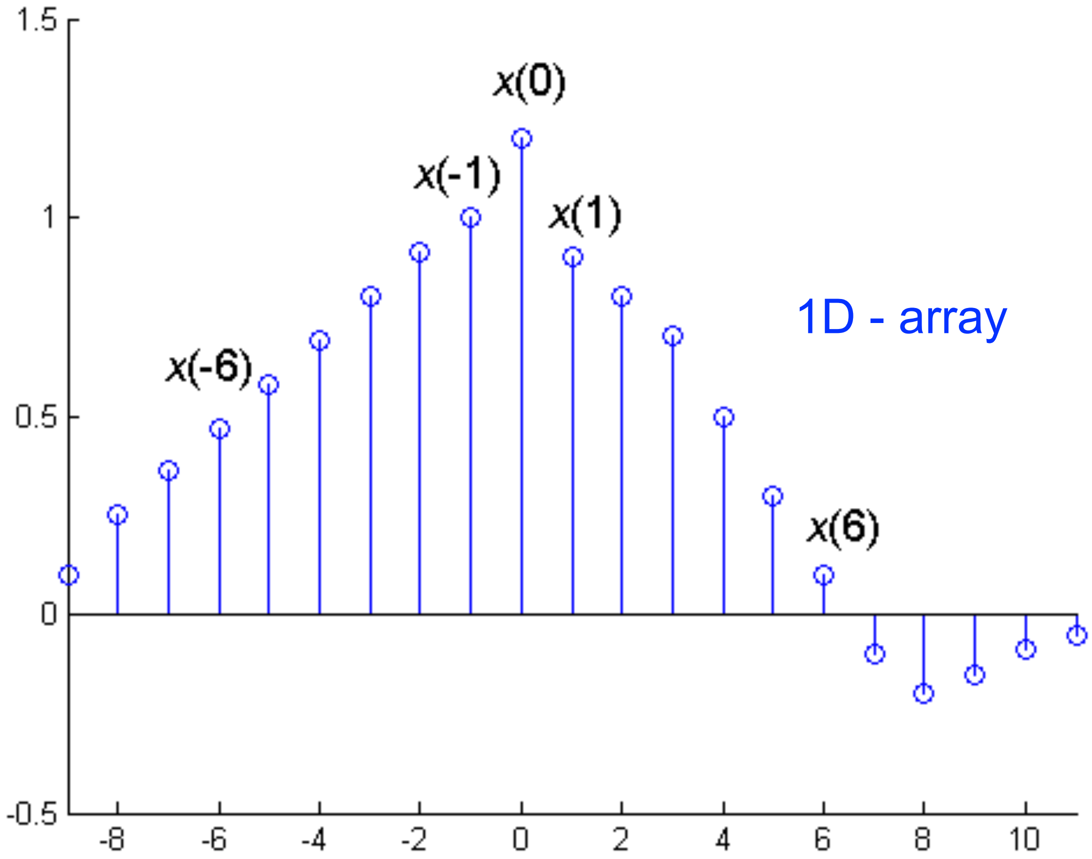
Discrete sequences

- Discrete-Time Systems process signals represented by sequences
  - obtained directly from a system
  - after digitizing Time-Invariant systems

- Sequence **x**

$$\mathbf{x} = \{x(n)\} \qquad -\infty < n < \infty$$

Discrete sequences

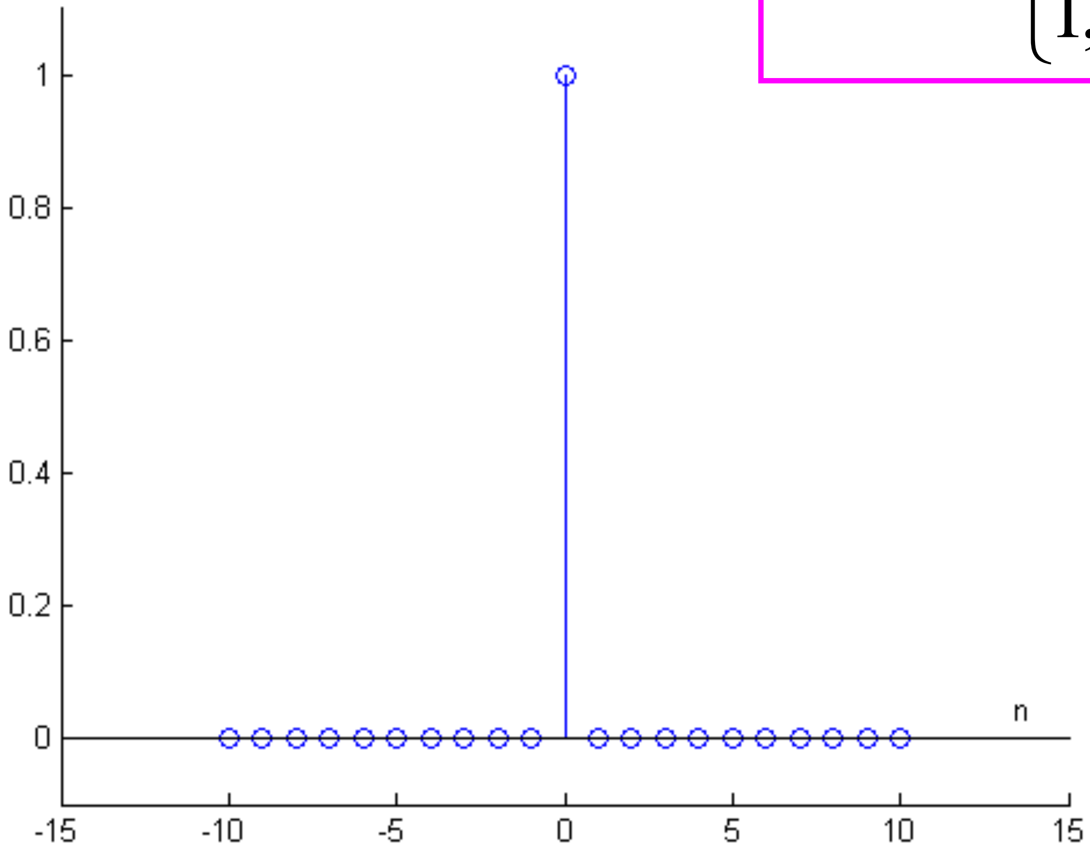


Discrete sequences

- Types of sequences
  - unit impulse signal
  - unit step function
  - real values exponential function
  - sinusoidal function

Unit impulse signal

$$\delta(n) = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$$

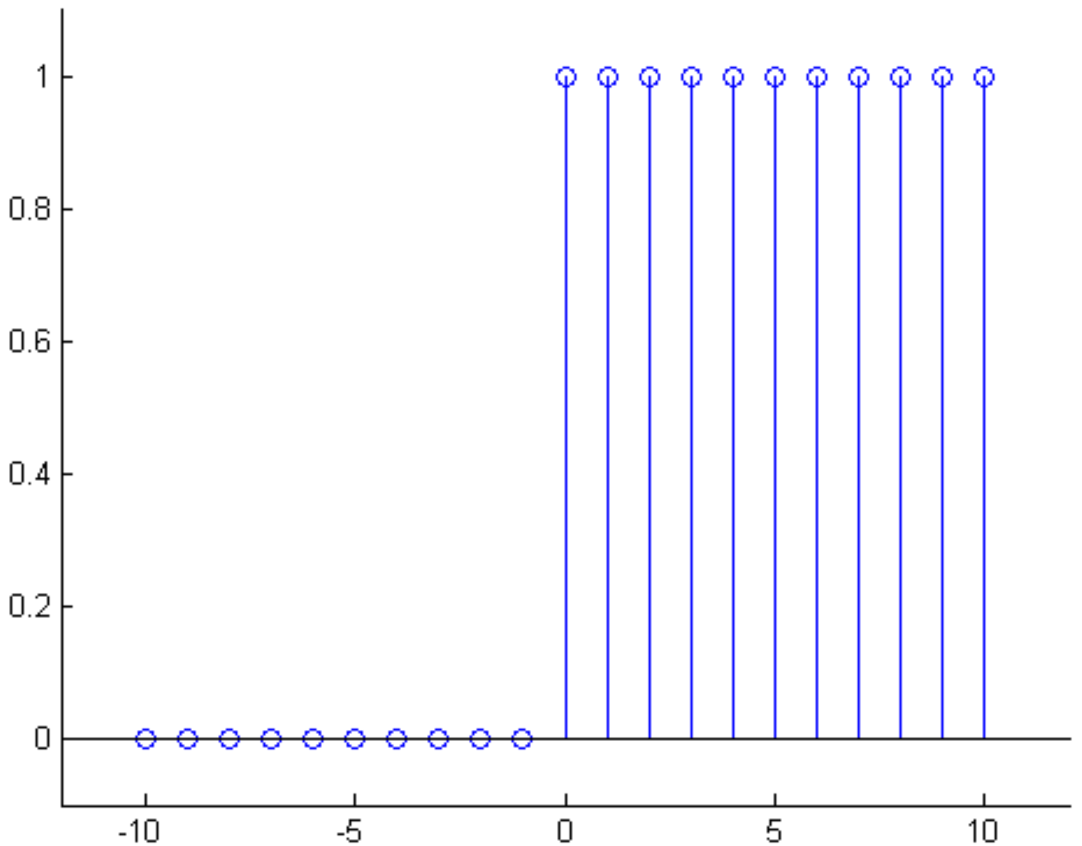


Unit step function

$$u(n) = \sum_{k=-\infty}^{n} \delta(k)$$

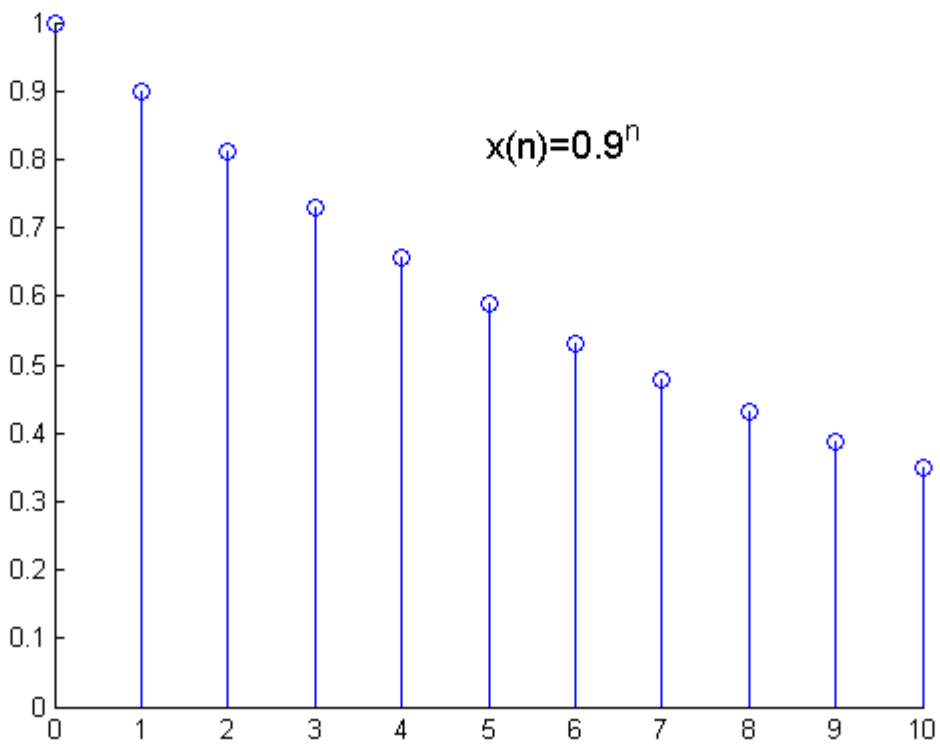
$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

$$\delta(n) = u(n) - u(n-1)$$



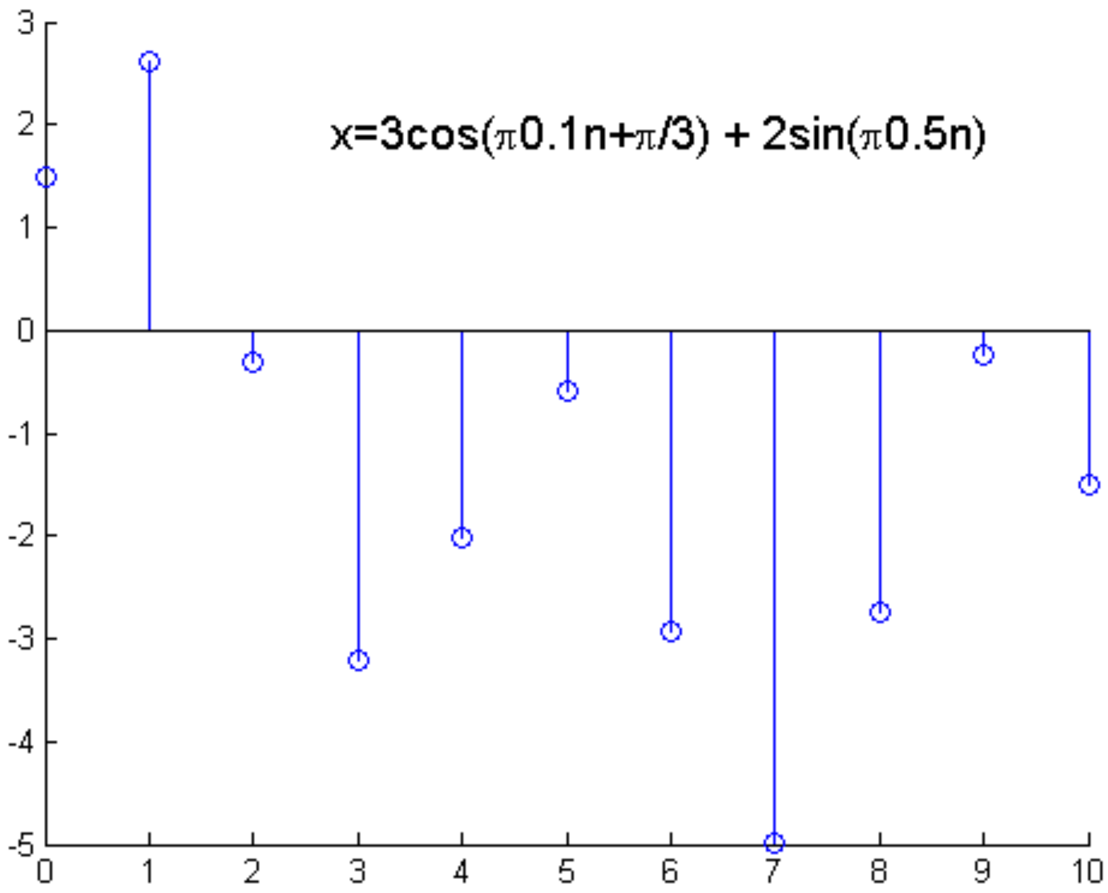
Real exponential function

$$x(n) = a^n \qquad \forall n \quad a \in \Re$$



Sinusoidal function

$$x(n) = A\cos(\omega n + \phi) \qquad \forall n$$



Basics operations

- Periodicity

$$x(n) = x(n+N) \qquad \forall n$$

- Energy

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 \qquad \forall n$$

- Operations

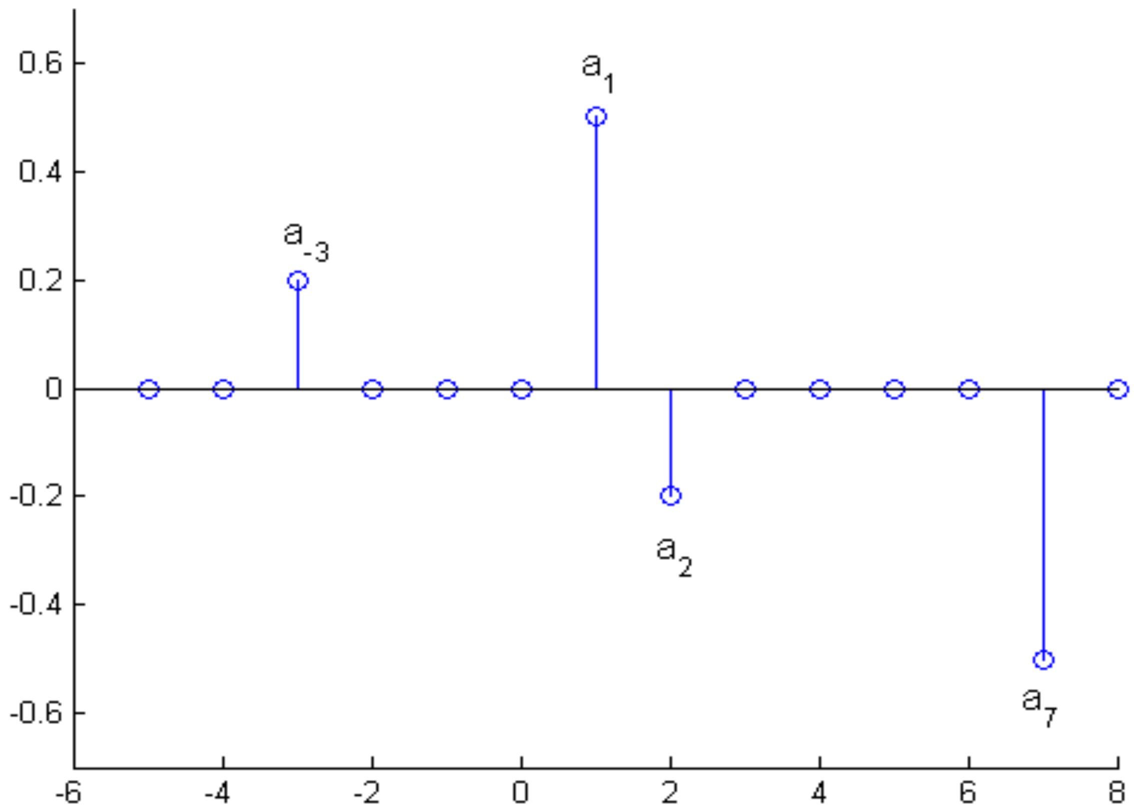
$$x \cdot y = \{x(n)y(n)\}$$ product

$$x + y = \{x(n) + y(n)\}$$ summation

$$\alpha \cdot x = \{\alpha x(n)\}$$ scalar product

Unit impulse and sequences

$$y(n)=\sum_{k=-\infty}^{\infty}x(k)\delta(n-k)$$

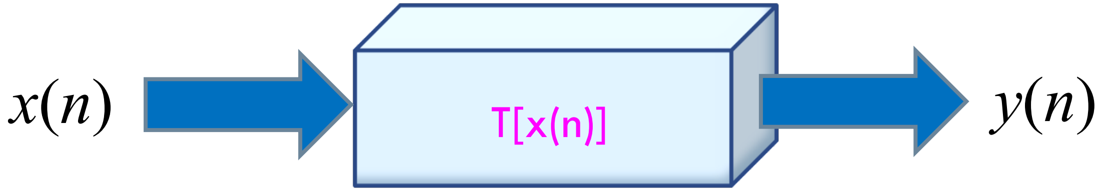


$$x(n)=a_{-3}\delta(n+3)+a_1\delta(n-1)+a_2\delta(n-2)+a_7\delta(n-7)$$

Systems

- Mathematically
  - unique transformation mapping an input sequence x(*n*) into an output y(*n*)

$$y(n) = T\left[x(n)\right]$$



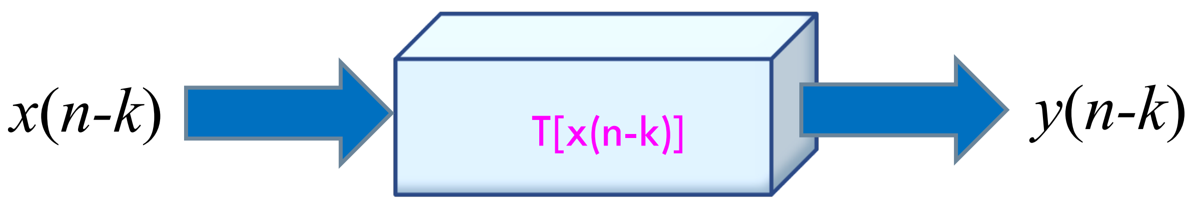
LTI systems

- Linear Time-Invariant (LTI) theory
  - comes from applied mathematics
  - has direct applications in
    - NMR spectroscopy, seismology, circuits, signal processing, control theory, and other technical areas

- It investigates the response of a linear and time-invariant system to an arbitrary input signal

Time-invariant systems

- Time-Invariant condition
  - If y(n) is the response to x(n) then y(n-k) is the response to x (n-k)
  - k is a positive or negative integer

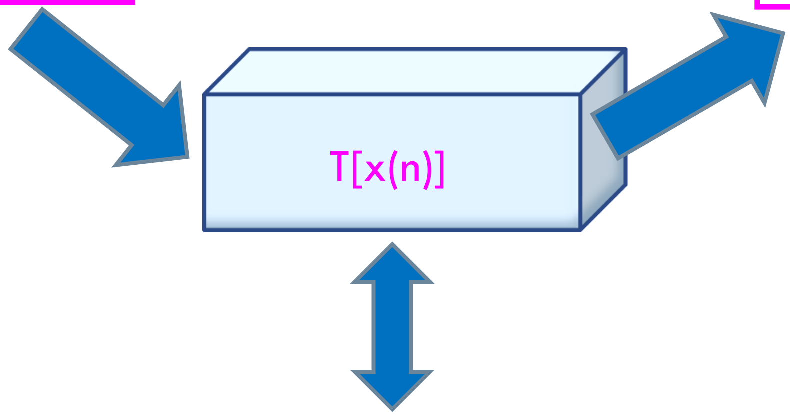


Time-Invariant System

Linear Systems

- The class of Linear Systems is defined by the principle of superposition

$$x(n) = ax_1(n) + bx_2(n)$$



$$y(n) = ay_1(n) + by_2(n)$$

$$T[ax_1(n) + bx_2(n)] = aT[x_1(n)] + bT[x_2(n)]$$

Linear System

Impulse Response

- A linear system can be completely characterized by its Impulse Response

Convolution

$$y(n) = T[x(n)] = T\left[\sum_{k=-\infty}^{\infty} x(k)\delta(n-k)\right]$$

$$y(n) = \sum_{k=-\infty}^{\infty} T[x(k)\delta(n-k)] = \sum_{k=-\infty}^{\infty} x(k)T[\delta(n-k)] = \sum_{k=-\infty}^{\infty} x(k)h_k(n)$$

Impulse Response, $h_k(n) = h(k-n)$

Impulse Response

- The convolution operation is denoted as

$$y(n) = x(n) * h(n)$$

- Equivalently we can write

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k) = h(n) * x(n)$$

Linear difference equation

- We consider a system with M inputs and N outputs

$$\sum_{k=0}^{N} a_k y(n-k) = \sum_{r=0}^{M} b_r x(n-r)$$

Equilibrium

- The output y at time n corresponds to

$$y(n) = -\sum_{k=1}^{N} \frac{a_k}{a_0} y(n-k) + \sum_{r=0}^{M} \frac{b_r}{a_0} x(n-r)$$

Infinite Impulse Response (IIR) Filter

FIR filter

- If the output depends only from the inputs

$$y(n) = -\sum_{k=1}^{N} \frac{a_k}{a_0} y(n-k) + \sum_{r=0}^{M} \frac{b_r}{a_0} x(n-r)$$

Finite Impulse Response (FIR) Filter

$$h(n) = \begin{cases} \dfrac{b_n}{a_0} & n = 0,1,\dots,M \\ 0 & \text{altrove} \end{cases}$$