Course of
"Automatic Control Systems"
2024/25

Control Systems - Introduction

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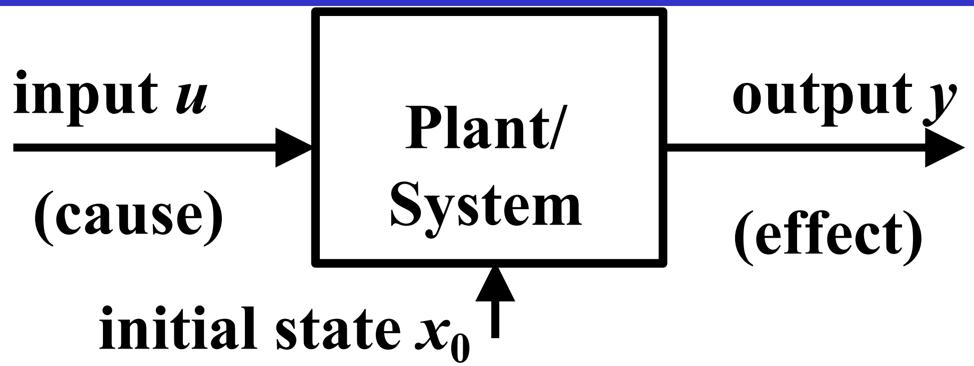
Mathematical model of a system



- Mathematical model that allows us to describe the evolution of the variables (inputs and outputs) starting from a certain initial instant t_0.
- The inputs act from the outside on the system and, therefore, represent the causes acting on the system.
- The outputs represent the responses to these causes, i.e. the effects.
- The causes of the system are not only the inputs but also the initial conditions of the system at instant t_0. These conditions represent initial state or memory of the system, i.e. they contain all the past evolution of the system ($t < t_0$) to determine the evolution ($t \geq t_0$), given the input stress to the system for $t \geq t_0$

Block diagrams algebra

- **By considering linear time-invariant systems evolving from zero initial conditions, the transfer function completely identifies the input-output behavior of the system.**

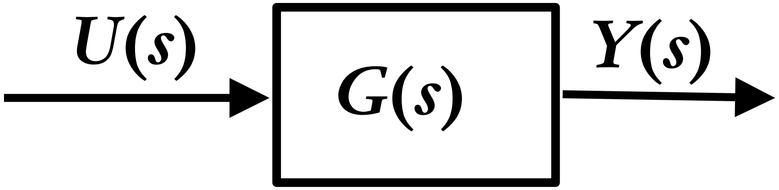
$G(s)$: transfer function

$$\boldsymbol{Y(s) = \underbrace{C(sI - A)^{-1}x_0}_{Y_{free}} + \underbrace{\left(C(sI - A)^{-1}B + D\right)U(s)}_{Y_{forced}}}.$$

Indeed, if x_0=0,

$$\boldsymbol{Y(s) = Y_f(s) = G(s)U(s)}.$$

- **To represent complex linear systems, the so-called "block diagram algebra" is used, which exploits the representation of the system through its transfer function.**

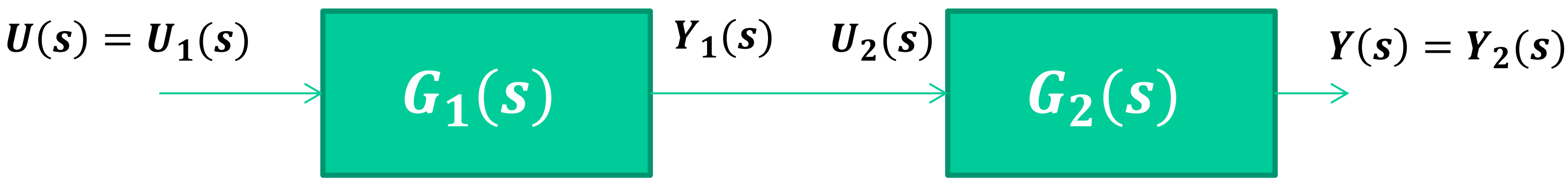


Interconnections of LTIs

- Three types of interconnections will be presented:
  - ***Series***
  - ***Parallel***
  - ***Feedback***

Series (1/2)

- Let us consider two transfer functions $G_1(s)$ and $G_2(s)$
- ***The series interconnection between $G_1(s)$ and $G_2(s)$ is represented as***



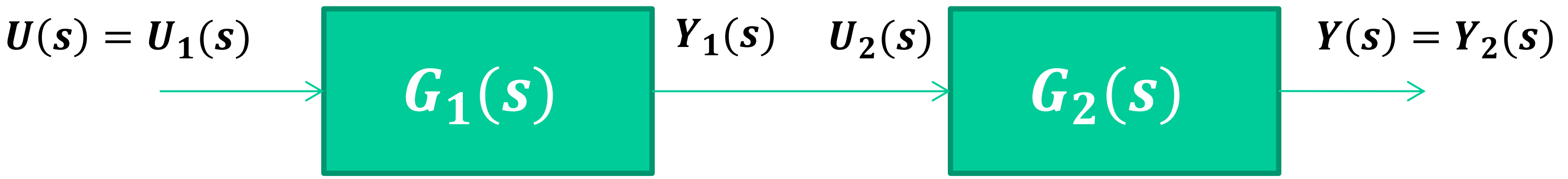
- The relation between $U(s)$ and $Y(s)$ is given by

$$Y(s) = G_2(s)U_2(s) = G_2(s)Y_1(s) = \underbrace{G_2(s)G_1(s)}U(s)$$

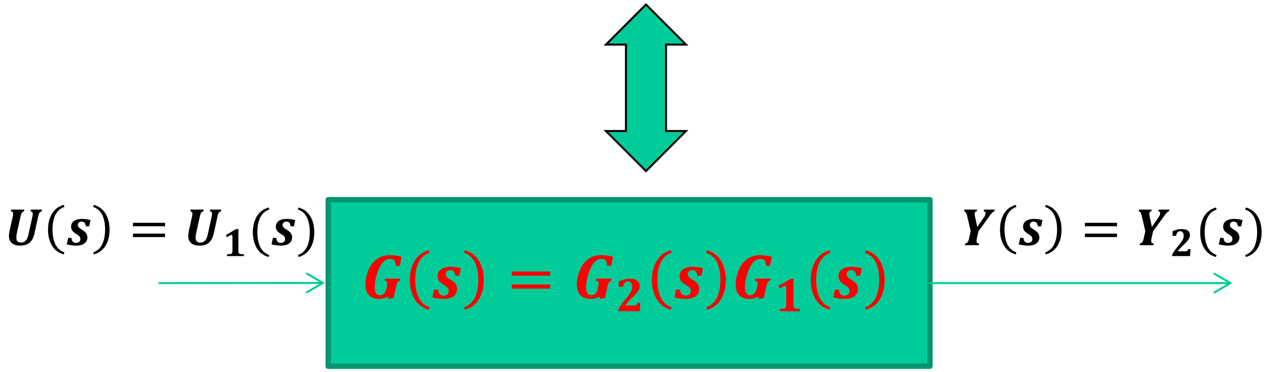
Series interconnection

$$G(s) = G_2(s)G_1(s)$$

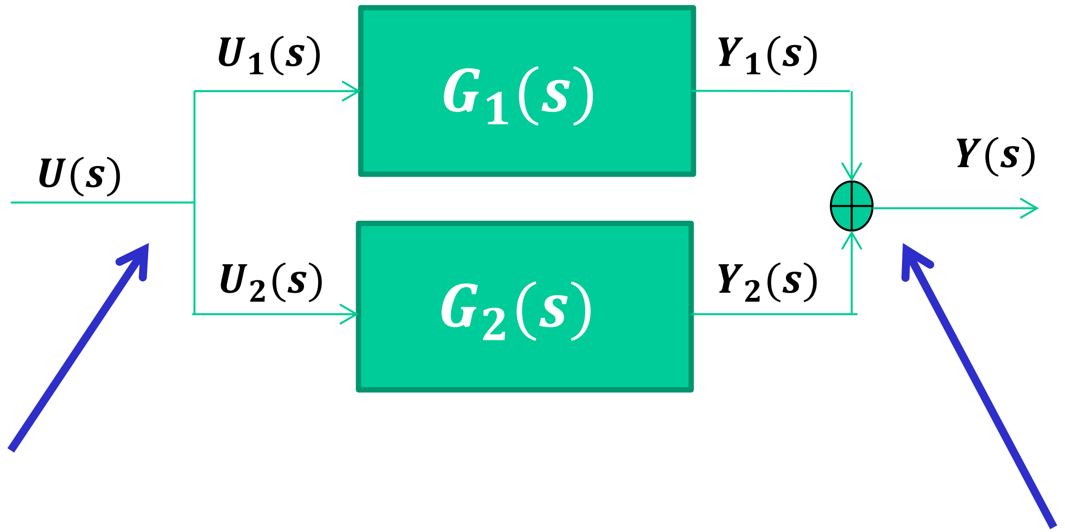
The series interconnection between $G_1(s)$ and $G_2(s)$:



$$Y(s) = G_2(s)U_2(s) = G_2(s)Y_1(s) = G_2(s)G_1(s)U(s)$$



- Let us consider two transfer functions $G_1(s)$ and $G_2(s)$
- ***The parallel interconnection between $G_1(s)$ and $G_2(s)$ is represented as***



Due to the ***interconnection node***

$$U_1(s) = U_2(s) = U(s).$$

Due to the ***sum node***

$$Y(s) = Y_1(s) + Y_2(s)$$

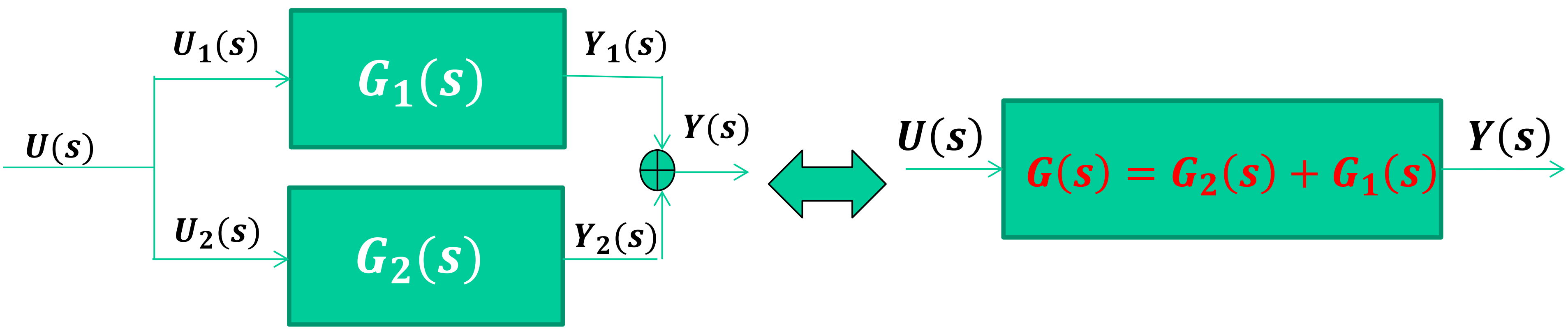
Parallel (2/2)

- The relation between $U(s)$ and $Y(s)$ is given by

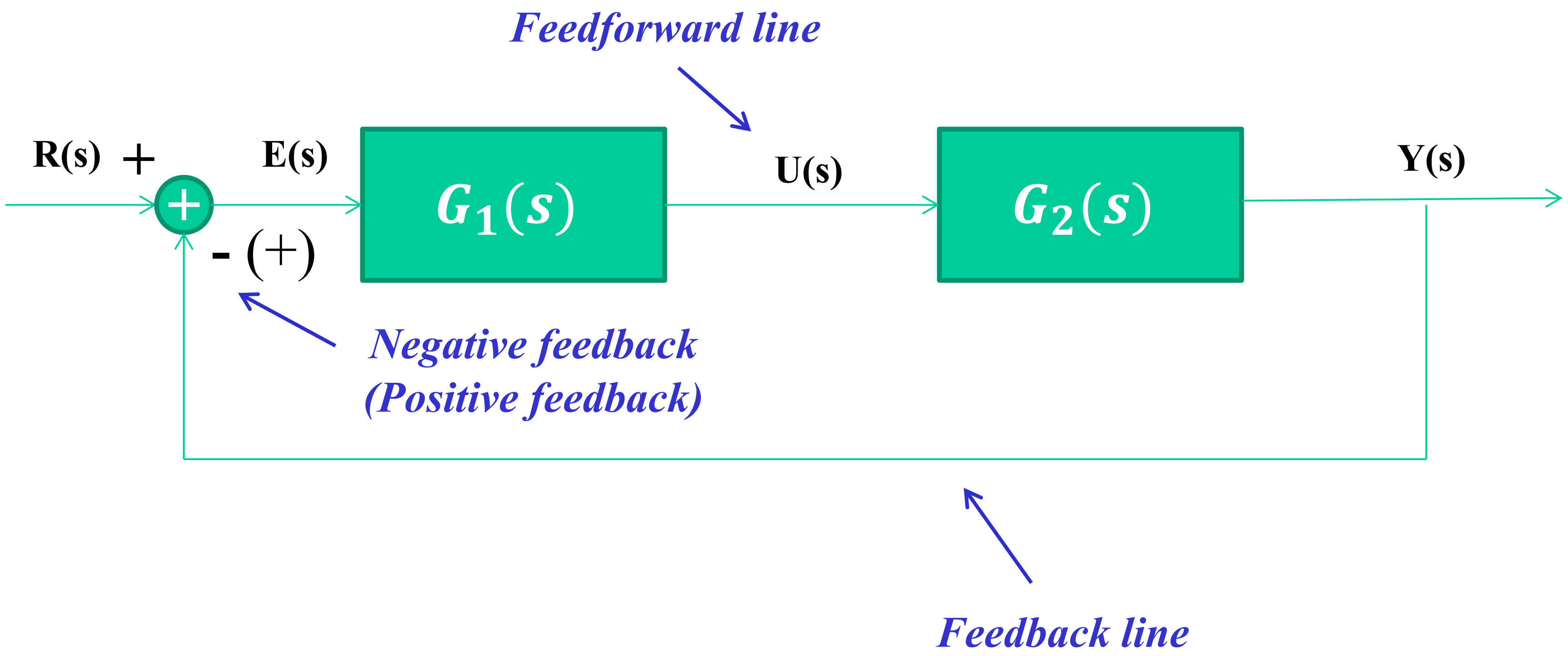
$$Y(s) = Y_1(s) + Y_2(s) = \left(G_1(s) + G_2(s)\right)U(s)$$

Parallel interconnection

$$G(s) = G_1(s) + G_2(s)$$



- Let us consider two transfer functions $G_1(s)$ and $G_2(s)$
- ***The feedback interconnection is represented as***



- In case of SISO system with negative feedback interconnection we have that

$$Y(s) = G_2(s)G_1(s)E(s) \qquad \textit{Feedforward line}$$

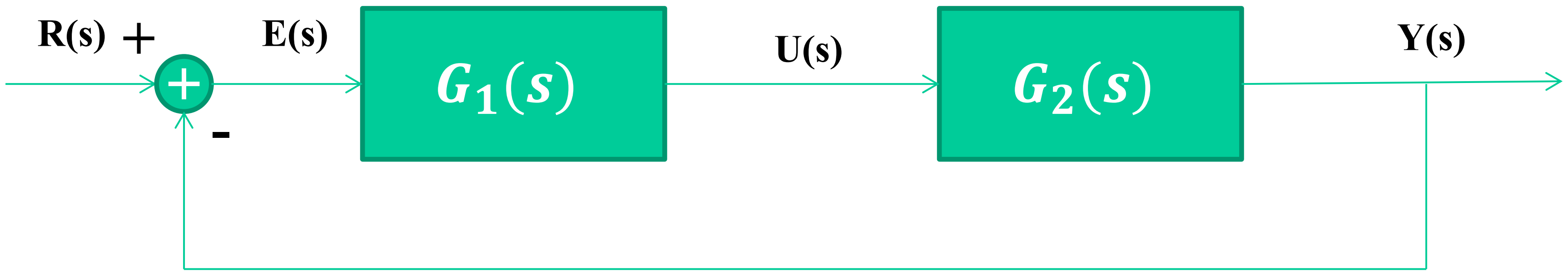
with $E(s) = R(s) - Y(s)$. Hence

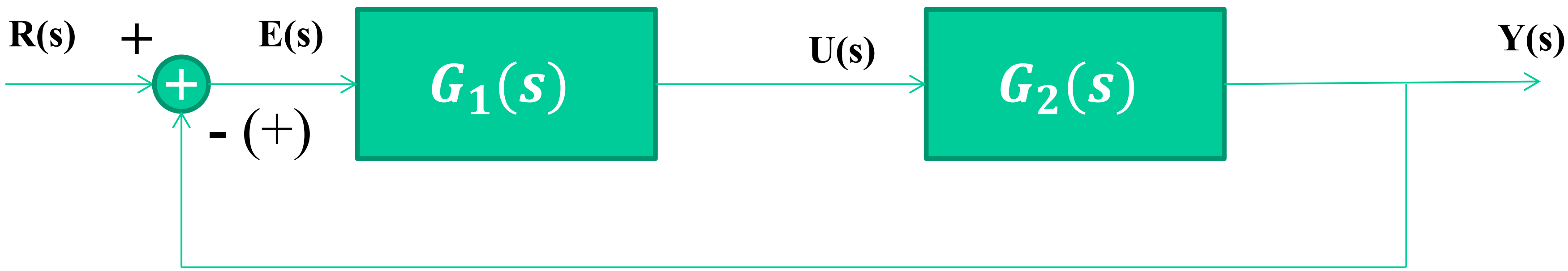
$$Y(s) = G_2(s)G_1(s)R(s) - G_2(s)G_1(s)Y(s)$$

$$Y(s) + G_2(s)G_1(s)Y(s) = G_2(s)G_1(s)R(s)$$

$$Y(s)\big(1 + G_2(s)G_1(s)\big) = G_2(s)G_1(s)R(s)$$

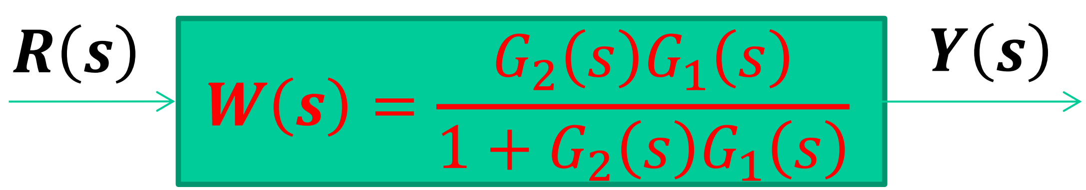
$$Y(s) = \frac{G_2(s)G_1(s)}{1 + G_2(s)G_1(s)}R(s)$$





$$Y(s) = \frac{G_2(s)G_1(s)}{1 + G_2(s)G_1(s)} R(s)$$

$W(s)$



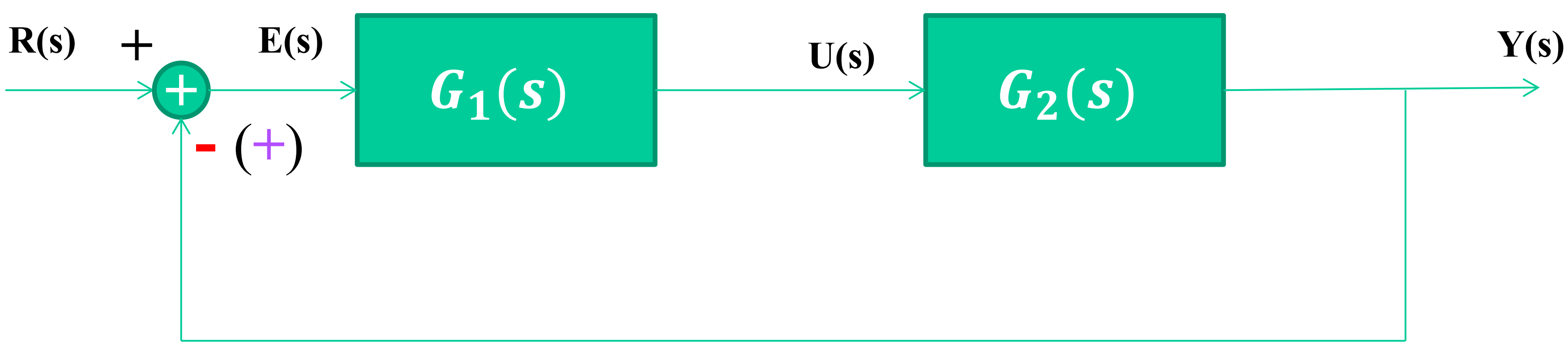
- The relation between $U(s)$ and $Y(s)$ is given by

$$Y(s) = \underbrace{\frac{G_2(s)G_1(s)}{1 + G_2(s)G_1(s)}}_{W(s)} R(s)$$

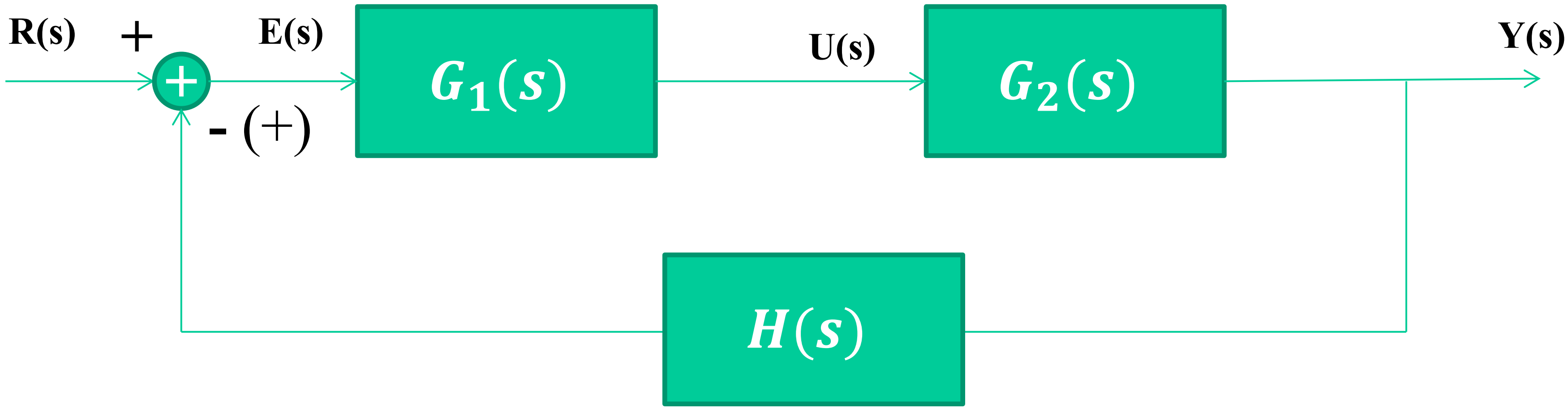
Negative feedback

$$Y(s) = \underbrace{\frac{G_2(s)G_1(s)}{1 - G_2(s)G_1(s)}}_{W(s)} R(s)$$

Positive feedback



- The feedback interconnection can be also generalized with a block $H(s)$ on the feedback line.

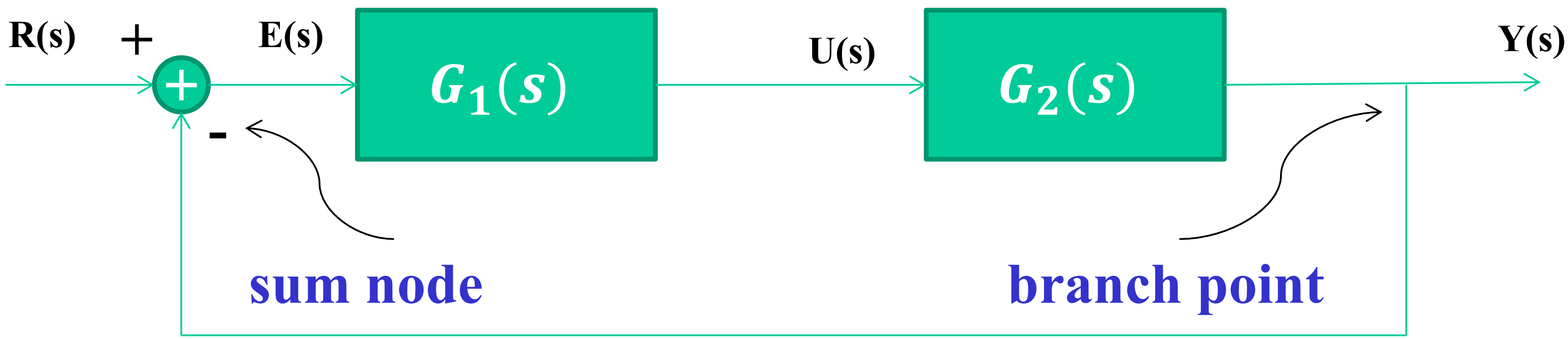


$$Y(s) = \frac{G_2(s)G_1(s)}{1 \underset{(-)}{+} G_2(s)G_1(s)H(s)} R(s)$$

$W(s)$

Sum nodes and branch points

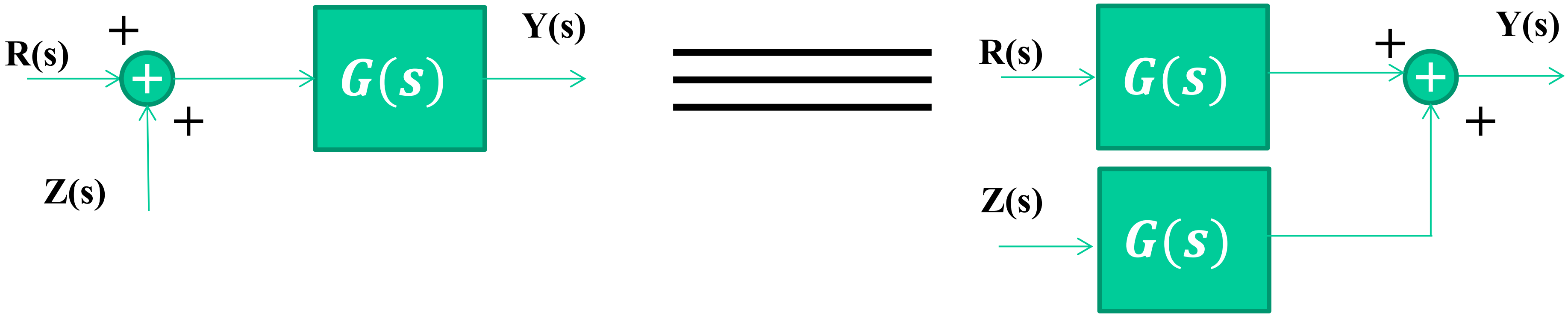
- In the block diagrams we usually find sum nodes and branch points



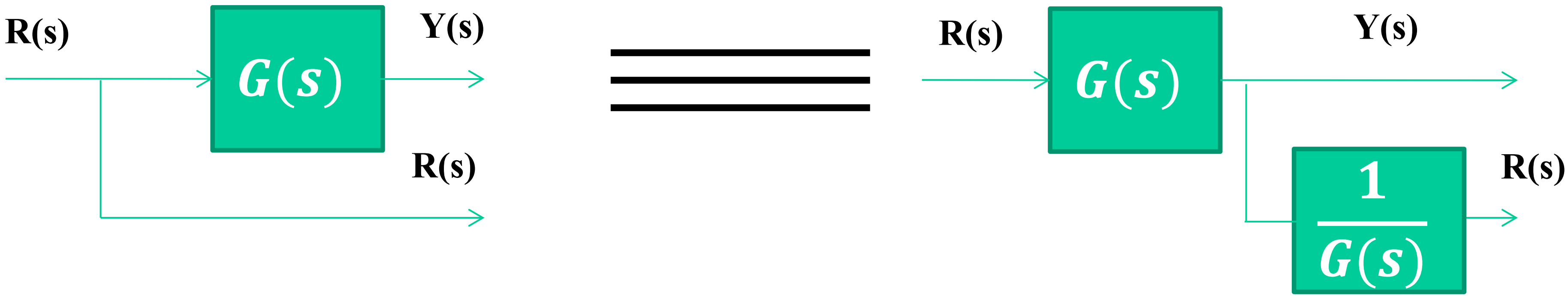
- It is sometime useful to move these elements in order to simplify the overall diagram

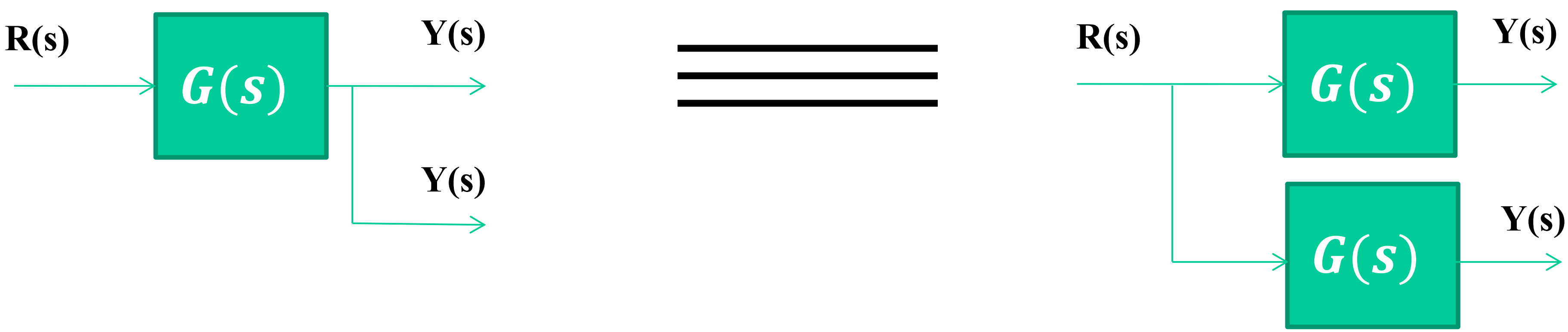
- In the following we present input-output equivalent schemes where the sum node or the branch points have been moved

Sum nodes



Branch points





What is a control system?

- ***Controlling a system means imposing a desired operating mode on it.***

- This is generally expressed by the request to maintain an output variable $y(t)$, called **controlled output**, as close as possible to a signal $r(t)$, called **reference signal**, which specifies the desired behavior of the system output over time

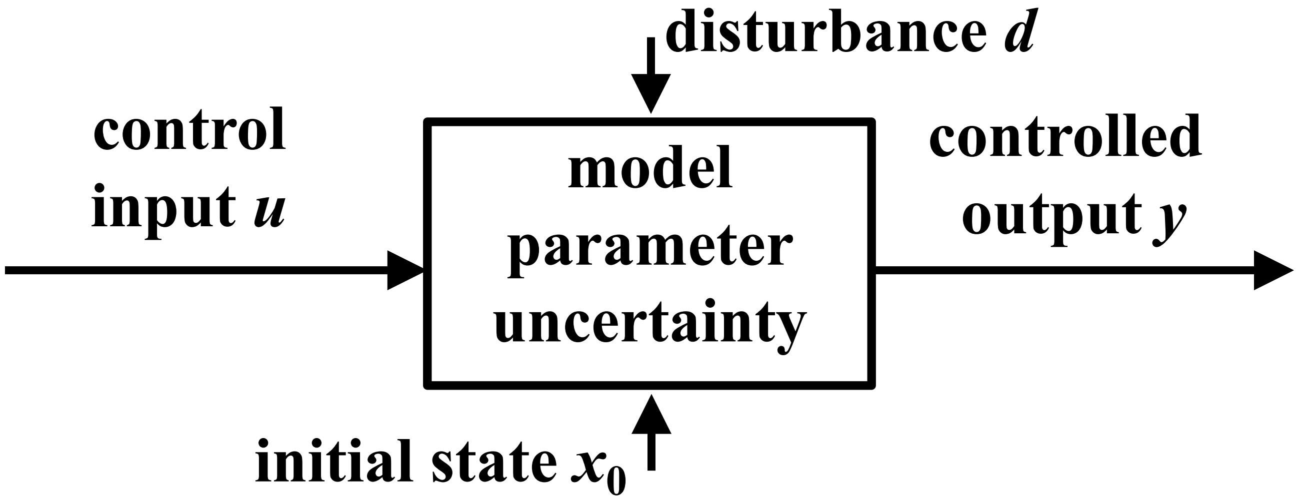
- This aim is achieved by acting on the system through an input signal $u(t)$ called **control input**.

- In Automatic Control Systems, the instant-by-instant determination of the control signal capable of ensuring good tracking of the reference by the controlled output is delegated to a devise called **controller**.

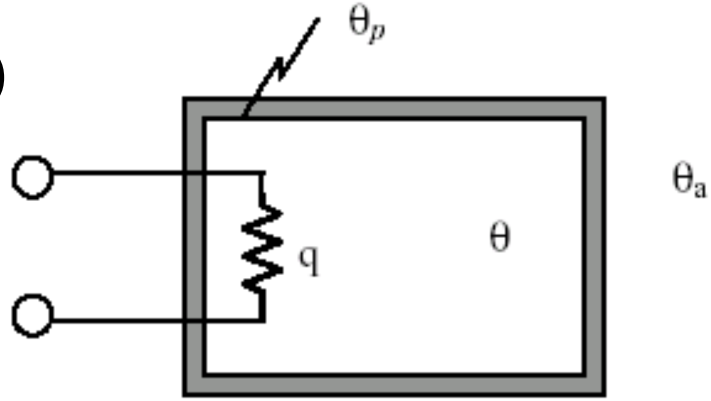
- Such determination is generally not simple given the nature of the input-output relationship of the system to be controlled.

Issues

- Other issues that make this task even more complex are the following:
  - the initial state $\boldsymbol{x}_0$, when the control system is turned on, is generally unknown;
  - the mathematical model of the system to be controlled is often affected by uncertainty in the value of some parameters $\boldsymbol{p}$ and, sometimes, is even only partially known;
  - usually, the behavior of the variable to be controlled, $\boldsymbol{y}$ is not determined only by the control input but also depends on other input variables, which cannot be manipulated, called disturbance inputs or disturbances, d(t).



Example - electric oven model – resistance heating

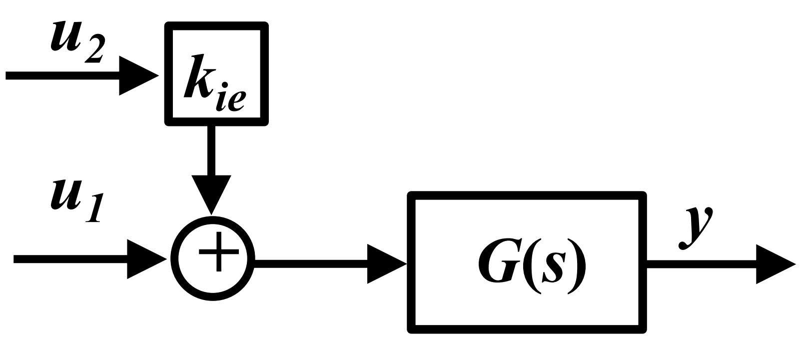


- Energy conservation: $C_f\dot{\theta} = q - k_{ie}(\theta - \theta_a)$
- In order to control the temperature θ, it is needed to «tune» q, the power to be supplied in a such way $\theta = \theta_r$ (θ_r =***desired temperature***).
- By assuming the following parameters: $C_f = 500$ $Ws/°C$ e $k_{ie} = 5$ $W/°C$.
- In the Laplace domain ($u_1 = q$, $u_2 = \theta_a$, $y = \theta$):

$$C_f sY(s) + k_{ie}Y(s) = U_1(s) + k_{ie}U_2(s)$$

$$\Rightarrow Y(s) = \frac{1/k_{ie}}{1 + \frac{C_f}{k_{ie}}s}\left(U_1(s) + k_{ie}U_2(s)\right)$$

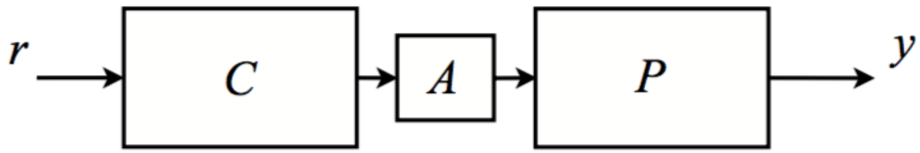
$\Leftarrow$ ***G(s)***



- **At steady state ($\dot{\theta}$=0), if θ_r = 220°C and θ_a = 20°C, then $q = k_{ie}(\theta - \theta_a)$ = 1 KW, (at least for $3t = C_f / k_{ie}$=300 s).**

Open-loop control system

- In an open-loop control system, controller ***C*** sends a signal to an actuating devise (or actuator) ***A*** which can modify the state of a process ***P*** to obtain the desired output response ***y***.



- If the dynamics of the process ***P*** are perfectly known and the control system is not subject to external (or environmental) disturbances, then the output ***y*** could in theory be made to perfectly track any desired reference signal ***r*** using the following controller (open-loop control):

$C = (AP)^{-1}$ ⟹ $Y(s) = P(s)A(s)C(s)R(s) = (PA)(AP)^{-1}R = r.$

- To be feasible, the controller must be causal (i.e., a system where the output depends on past and current inputs but not future inputs, in the domain of Laplace - the variable ***s*** - the degree of the denominator of the t.f. must be greater than or equal to the degree of the numerator).
- By denoting with ***G=PA***, the t.f. of the actuator-process series, then the controller **C=*G*$^{-1}$** will be feasible if the order of the numerator of ***G*** is equal to the order of the denominator.

Numerical example: open-loop control system

Let us to design an open loop control system for the system defined by the following tf,

$$G(s) = \frac{1}{s+2},$$

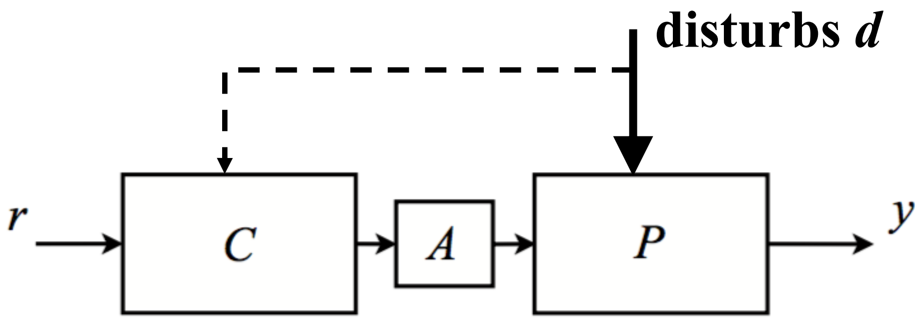
and verify the unit step response ($\boldsymbol{r(t) = 1(t)}$).

In this case we cannot employ a controller with tf as the inverse of G ($\boldsymbol{C=G^{-1}}$), but we must add a filter $\boldsymbol{Q(s)}$ in such a way that $\boldsymbol{C(s)=G(s)^{-1} \cdot Q(s)}$ is feasable. Furthermore, the $\boldsymbol{Q(s)}$ poles must be chosen witout affecting the control system requirements, i.e. the $\boldsymbol{Q(s)}$ poles must be at least 5/6 times greater than the $\boldsymbol{Q(s)}$ poles, and the gain of $\boldsymbol{Q(s)}$ must be (ideally) unitary in the frequency range in which the system operates.

$$C(s) = G(s)^{-1}Q(s) = (s+2)\frac{15}{s+15}.$$

Open-loop control system

- Then, in an ***open loop control strategy***, the controller inputs are the set point (reference signal) at each instant of time and, in some cases, information concerning the disturbs



- These information, together with the knowledge of the mathematical model of the system, allows the control system to elaborate the control law.
- If the knowledge of the system is precise, there aren't any uncertain parameters and the disturbs assume their nominal value, then an open loop controller will work correctly. ***However, an open loop control strategy is not robust.***
- ***Indeed, in case of uncertainties and disturbs (not be controlled, usually time variant and uncertain), robust control strategies are used to guarantee a limited loss of performance of the controller*****.**

Plant uncertainty - 1

- Assume the parametric variations (i.e., uncertainties) of the process-actuator series:
$$G(s) = P(s)A(s).$$

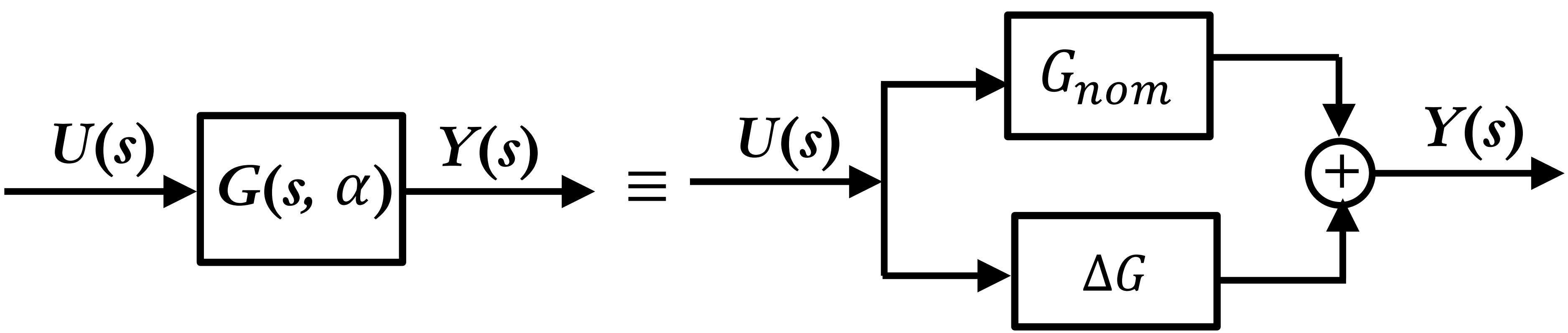
  $G(s) = G(s, \alpha)$, where α is a parameter.

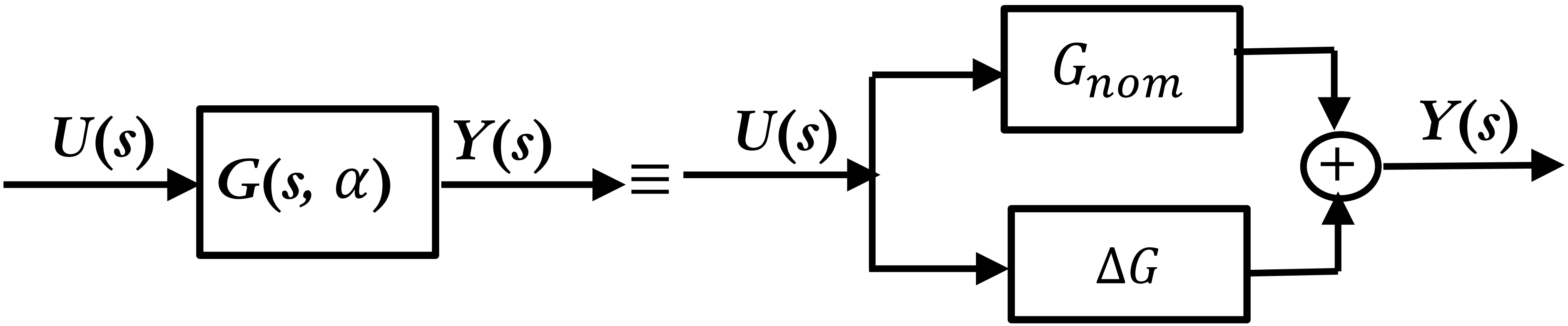
- By Taylor expansion of $\boldsymbol{G(s)}$ wrt $\boldsymbol{\alpha}$ around the nominal values $\boldsymbol{\hat{\alpha}}$ (***nom.***) and assuming the expansion at first order:

$$G(s, \alpha) \cong G(s, \hat{\alpha}) + \frac{\partial G}{\partial \alpha}|_{\alpha=\hat{\alpha}} d\alpha = G_{nom}(s) + \Delta G(s),$$

  where ΔG takes into account the uncertainty due to α.

- By using block diagrams:





$$Y = Y_{nom}(s) + \Delta Y(s).$$

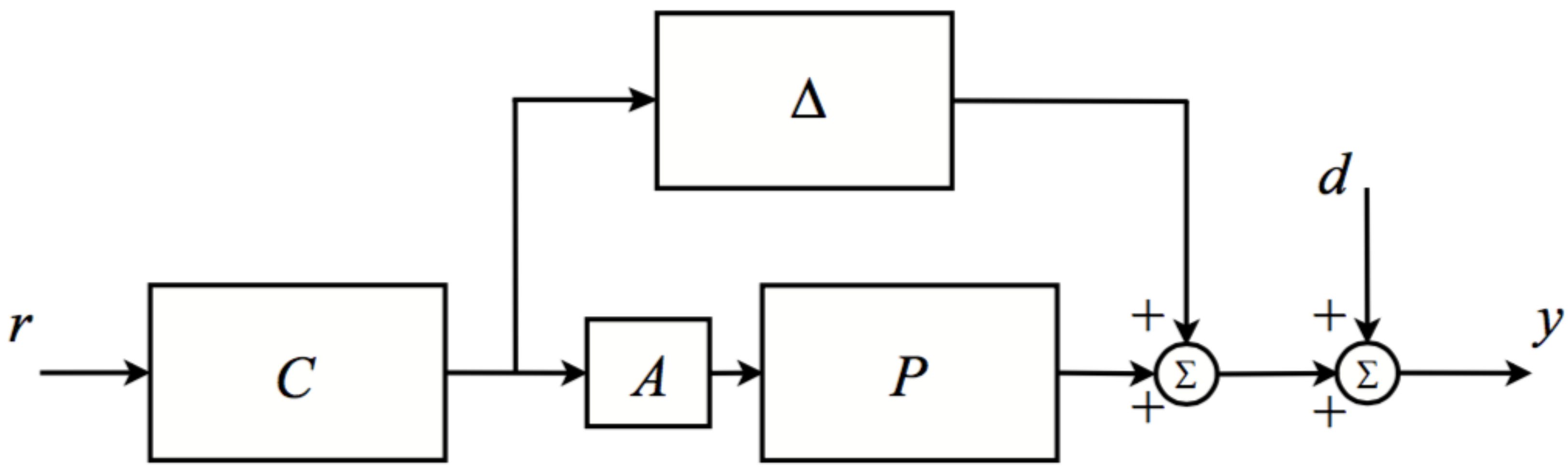
Therefore the variation of the output compared to the nominal output results:

$$\frac{\Delta Y}{Y_{nom}} = \frac{\Delta G\ U}{G_{nom} U} = \frac{\Delta G}{G},$$

and, the variations will be small only if ΔG is low.

Limitations of the open-loop control system

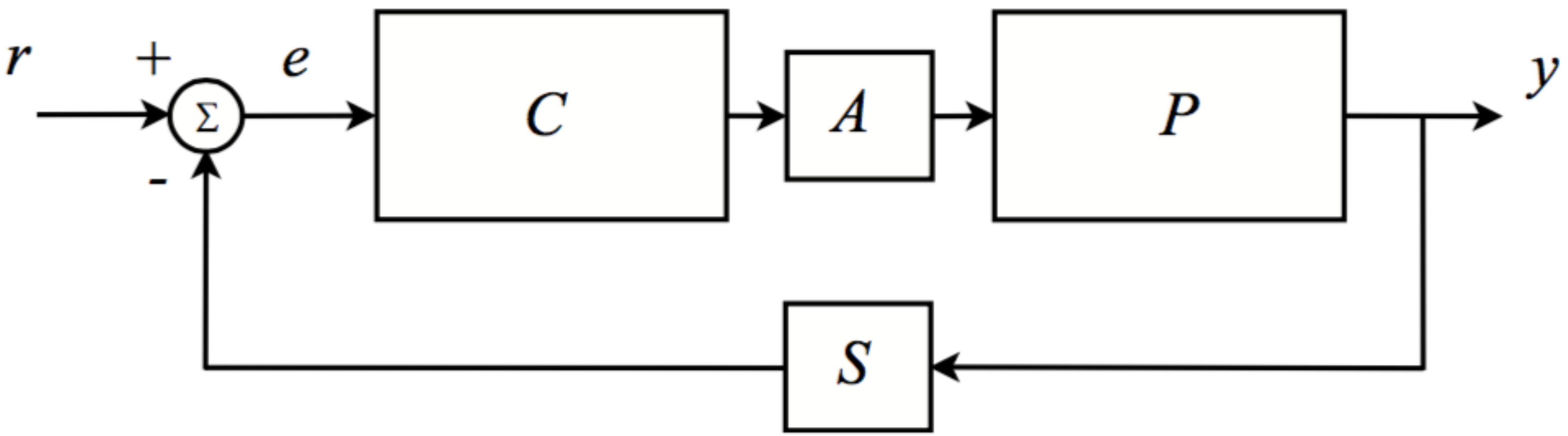
Open loop control is not possible in most real cases: it is the inevitable presence of uncertainty in both the dynamics of the process to be controlled, and the environment in which it operates (the presence of disturbances), that necessitates the use of feedback control (open-loop control system is used in combination with closed loop).



$$Y = D + (PA + \Delta)CR.$$

Since Δ and *d* are unknown, it is impossible to design an open-loop controller such that ***y*=*r*.**

Closed-loop feedback control system

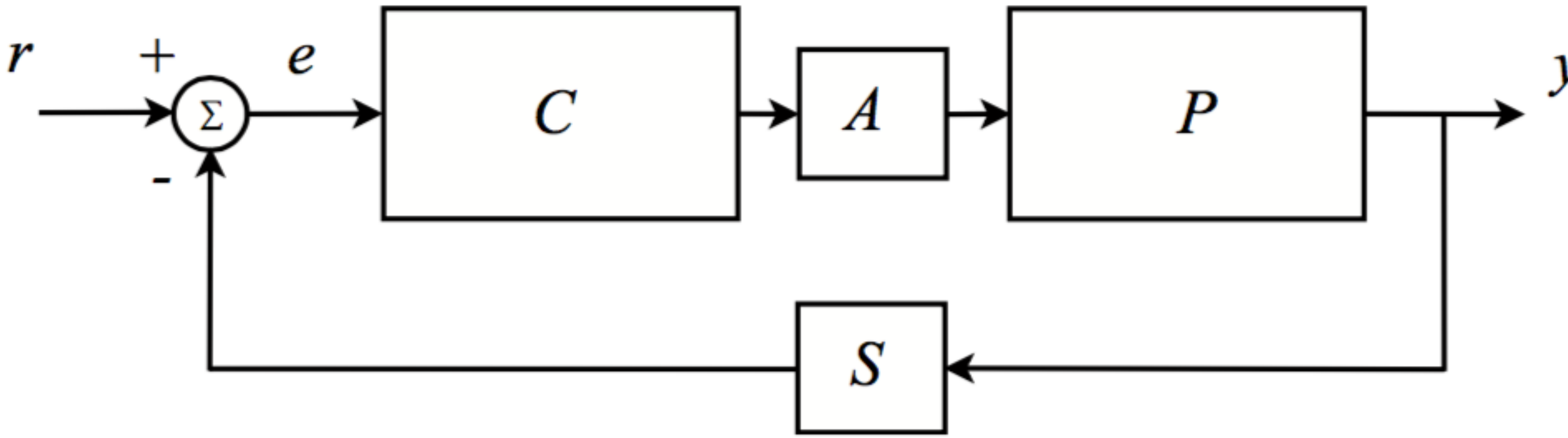


A closed-loop feedback control system uses a sensor ***S*** to continuously «feed back» a measurement of the actual output of the system.

This signal is compared with the desired output of the system(reference signal) to generate an **error signal**.

The error signal is the input of the controller, which, based on this signal, in turn generates a **control signal** which is the input to the plant

Closed-loop feedback control system



$$Y(s) = P(s)A(s)C(s)\big(R(s) - Y(s)S(s)\big)$$

S=1, G=PA

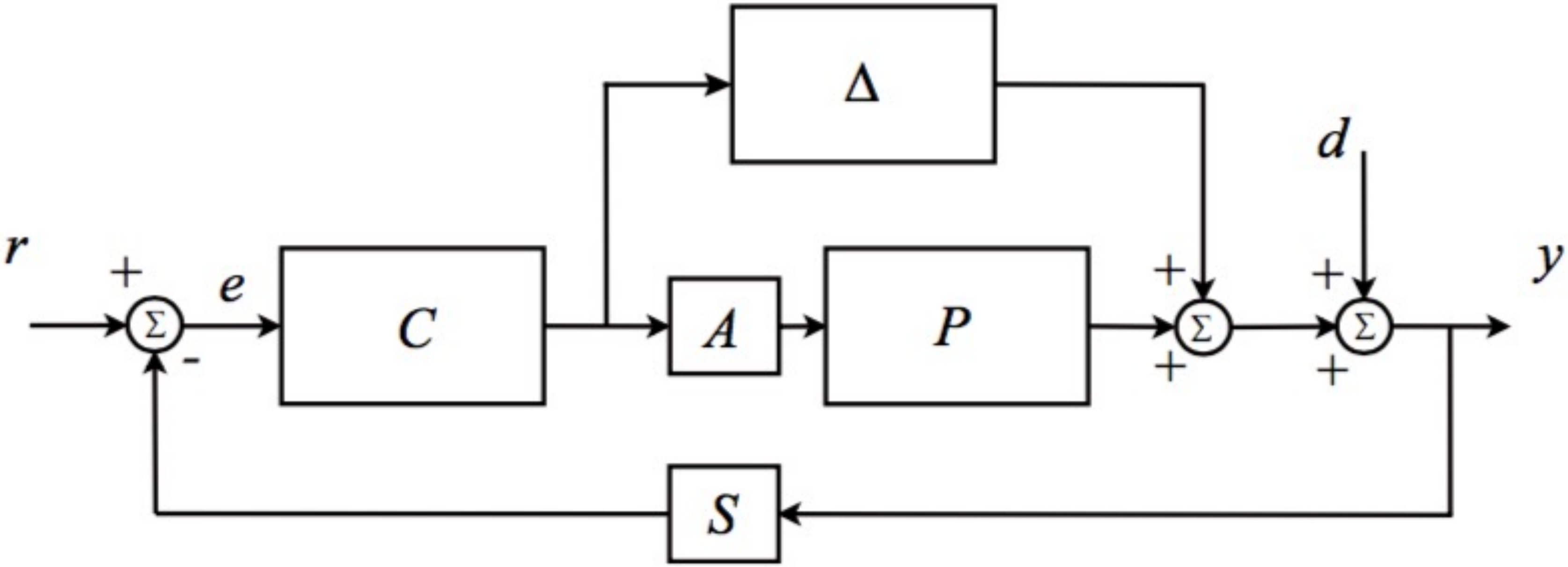
$$Y(s)(1 + G(s)C(s)) = G(s)C(s)R(s) \Rightarrow Y(s) = \frac{G(s)C(s)}{1 + G(s)C(s)}R(s)$$

$W(s)$

- **If $|CG| \gg 1, \Rightarrow |W| \approx 1 \Leftrightarrow y \approx r.$**

Closed-loop feedback control system with plant uncertainty and disturbances - 1

By considering the effects of the plant uncertainty and disturbances on this system,



Assuming that S=**1** (excellent quality of sensor/transducer, which is not affected by noise),

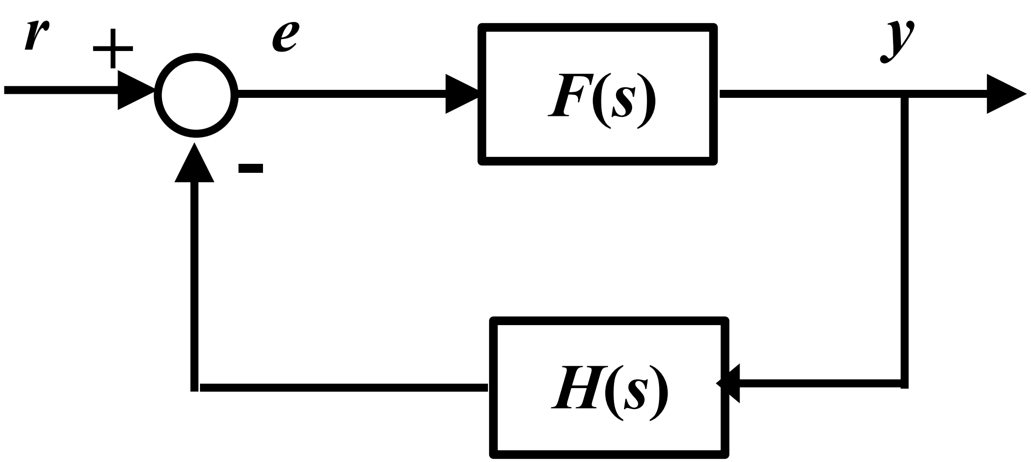
Closed-loop feedback control system with plant uncertainty and disturbances - 2

$$Y(s) = \frac{1}{1 + \big(G(s) + \Delta(s)\big)C(s)} D(s) + \frac{\big(G(s) + \Delta(s)\big)C(s)}{1 + \big(G(s) + \Delta(s)\big)C(s)} R(s)$$

In this case $\left|\big(G(s) + \Delta(s)\big)C(s)\right| \gg \mathbf{1}$, the control system is now able to attenuate the effects of disturbance of ***d*** and the uncertainty Δ on ***y*, indeed**

$\boldsymbol{C} \ \rightarrow \infty, \boldsymbol{y} \approx r$, **for any finite values of *d*** and **Δ.**

Sensitivity analysis of feedback control system wrt uncertainty



***F*(*s*):** controller-process-actuator series

***H*(*s*):** trasducer/sensor

Uncertainties wrt to ***F*** or ***H***

- **In the case of uncertainty on *F*,**

$$W(s) = \frac{F(s)}{1 + F(s)H(s)} \Rightarrow \Delta W = \frac{\partial W}{\partial F}\frac{\partial \boldsymbol{F}}{\partial \alpha} d\alpha = \frac{1 + FH - FH}{(1 + \boldsymbol{F}H)^2}\Delta \boldsymbol{F}(s)$$

$$= \frac{\Delta \boldsymbol{F}(s)}{(1 + \boldsymbol{F}H)^2}$$

By defining the sensitivity coefficient of the closed-loop system wrt to ***F* *uncertainty***

$$S_W^F = \frac{\Delta W}{W} = \frac{1}{1 + FH}\frac{\Delta \boldsymbol{F}}{F} \Rightarrow$$

Low value of S_W^F, by $|FH| \gg 1$ (operating on *F*, i.e., the controller ***C***).

Ideal control performance

- **In the case of uncertainty on *H*,**

$$W(s) = \frac{F(s)}{1 + F(s)H(s)} \Rightarrow \Delta W = \frac{\partial W}{\partial H}\frac{\partial H}{\partial \alpha} d\alpha = -\frac{F^2}{(1 + \boldsymbol{F}H)^2}\Delta H$$

By defining the sensitivity coefficient of the closed-loop system wrt to ***H uncertainty***

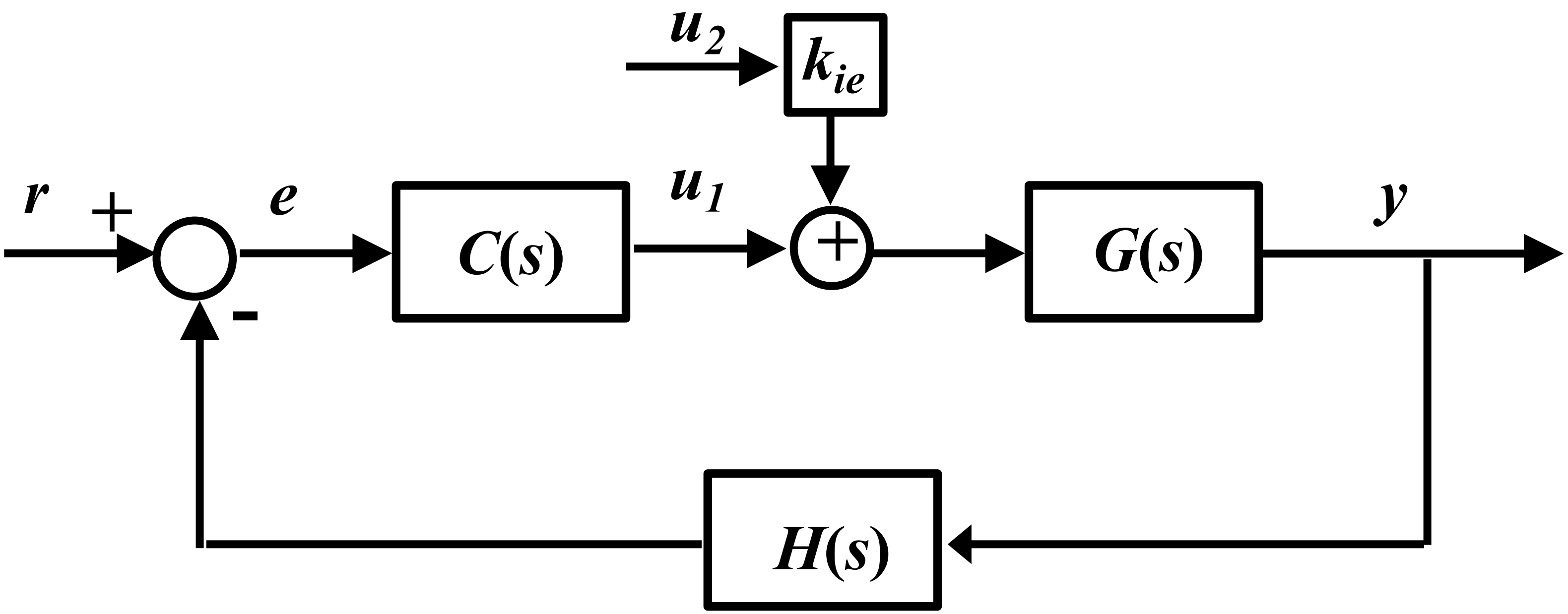
$$S_W^H = \frac{\Delta W}{W} = -\frac{FH}{1 + FH}\frac{\Delta H}{H} \Rightarrow$$

Since $|FH| \gg 1$, $S_W^H = \frac{\Delta H}{H}$, that is, the system is maximally sensitive to parametric variations of ***H***. Therefore, it is necessary to use excellent quality transducers/sensors.

Power of feedback – ensure robustness

- **Robust control** of a negative feedback control system (low sensitivity to parametric variations of F and H in hp):
  - $|\boldsymbol{FH}| \gg 1$
  - $\boldsymbol{H}$ low sensitive w.r.t parametric variation (of excellent quality).

Temperature regulation of an oven by closed-loop feedback control system



$$C(s) = k_C; H(s) = 1;$$

$$Y(s) = Y_r(s) + Y_d(s) = \frac{k_C G(s)}{1 + k_C G(s)} R(s) + \frac{k_{ie} G(s)}{1 + k_C G(s)} U_2(s).$$

Closed-loop control system issues

- It is important to note that closed-loop systems, in addition to obvious advantages, can also present serious drawbacks.

- In reality, these results were obtained by considering only systems in which both the plant and the controller are static systems, that is, they are in a stationary regime.

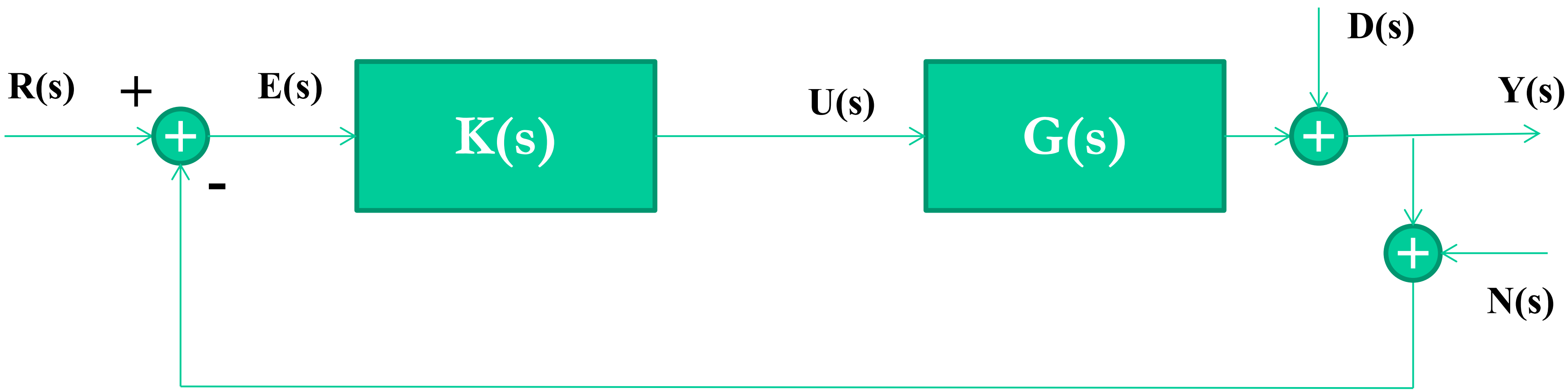
- By assuming dynamic systems, closed-loop systems present a drawback to which extreme attention must be paid during the design: **as the controller gain constant increases, many closed-loop systems tend to exhibit unstable behavior**.

- In other words, the controlled output no longer tends to follow the reference signal but moves away indefinitely from it.

- This behavior must absolutely be avoided because, in addition to leading to unsatisfactory behavior of the control system, it would also lead to an increase in the value of all the internal signals of the system, including the control signal, and this would cause serious damage to the system if no action were taken in time.

Design of a closed-loop control system

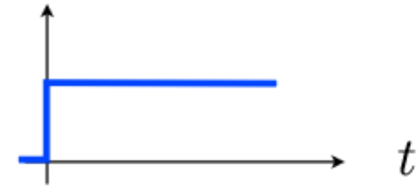
- In a correct formulation of a control problem, it is necessary to define
  - the class of input signals
    - Reference inputs
    - Disturbances
    - Noise
  - the "limits" to be considered acceptable on the error signal in the various operating conditions;
  - the limitations on the control signal.



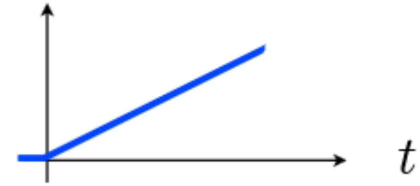
Polynomial reference signals (of order k)

- *A polynomial canonic signal of order* $\boldsymbol{k}$ is defined as $\boldsymbol{r(t) = \frac{t^k}{k!} 1(t)}$.

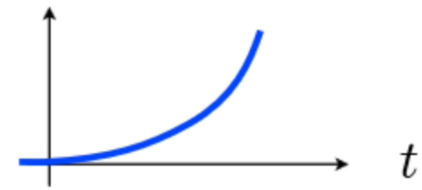
order 0 (step function) $\quad \boldsymbol{1(t)}$



order 1 (ramp function) $\quad \boldsymbol{t \cdot 1(t)}$



order 2 (quadratic function) $\quad \boldsymbol{\frac{t^2}{2} \cdot 1(t)}$



Step signal is assumed **as reference**, when the system works with piecewise constant reference signals. These control systems are commonly called "**regulation systems**". **Ramp signal** is assumed **as reference**, when the system works with reference signals that vary over time. These control systems are commonly called "**servo systems**".

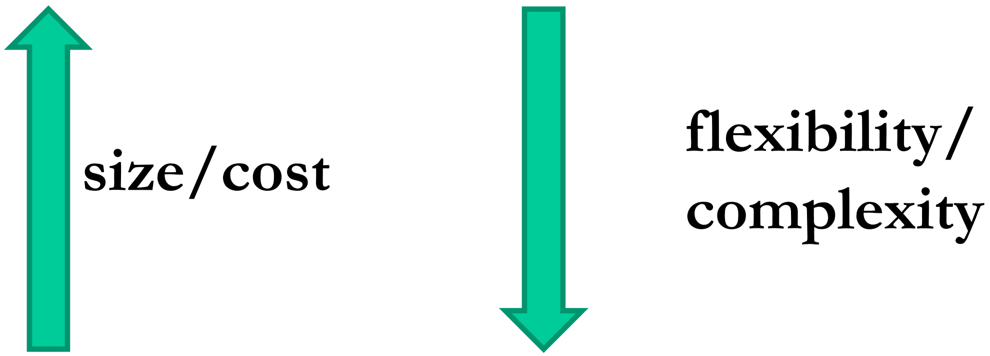
Class of disturbance signals

- Similar considerations as for reference signals: it is common to refer to a step or constant disturbance, or to a ramp disturbance.
- In some applications, however, it needs to deal with different types of disturbances, such as periodic disturbances.
- Consider, for example, the problem of positioning a marine platform, in which the wave motion, which exhibits a periodic/pseudo-periodic trend, works as a disturbance. In this case, it is common to characterize the disturbance as a sinusoidal signal.

Control requirements

- The closed loop control requirements are the following:
  - ***Stability - Robust stability***
  - ***Steady-state performances***
    - ***maximum permissible error w.r.t. to the class of reference signal to be tracked***
    - ***rejection of the disturbs***
    - ***insensibility to the noise***
  - ***Transient performances***
    - ***these are usually expressed in terms of tracking properties of the step reference signal by evaluating two feature parameters (of the step response of the closed-loop control system:***
      - ***Dynamic precision*** (overshoot, oscillation period …)
      - ***Time response*** (settling time, rise time, peak time, …)

- In the past, controllers were made in “analog domain” with the following technologies
  - ***Mechanical***
  - ***Hydraulic or pneumatic***
  - ***Electromechanical***
  - ***Electrical***
  - ***Electronic***



- By employing electric and electronic devises, it was possible to separate the “intelligence/control” part from the “power/actuating” part, increasing the flexibility of the controller.

- The most recent development is controllers based on digital technology, which have spread rapidly in the most varied applications.

Digital control: advantages

- The advantages exhibited by digital controllers are:
  - **Low cost**
  - **Flexibility**
  - **Possibility to implement complex control rules**
  - **Integration** of the control system functions with other types of characteristics (supervision, diagnostics, etc.)

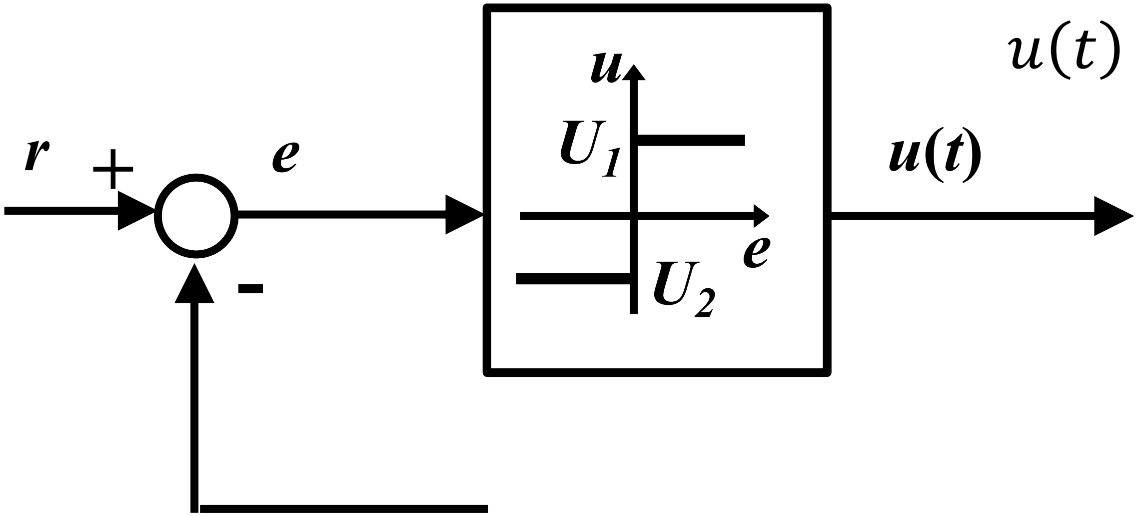
Digital control: disadvantages

- The disadvantages exhibited by digital controllers are:
  - **More difficult and complex design**
  - **Less robust stability**
  - **Possibility of unexpected shutdowns due to SW bugs**
  - **Need to use electrical energy**

Implementation of control systems

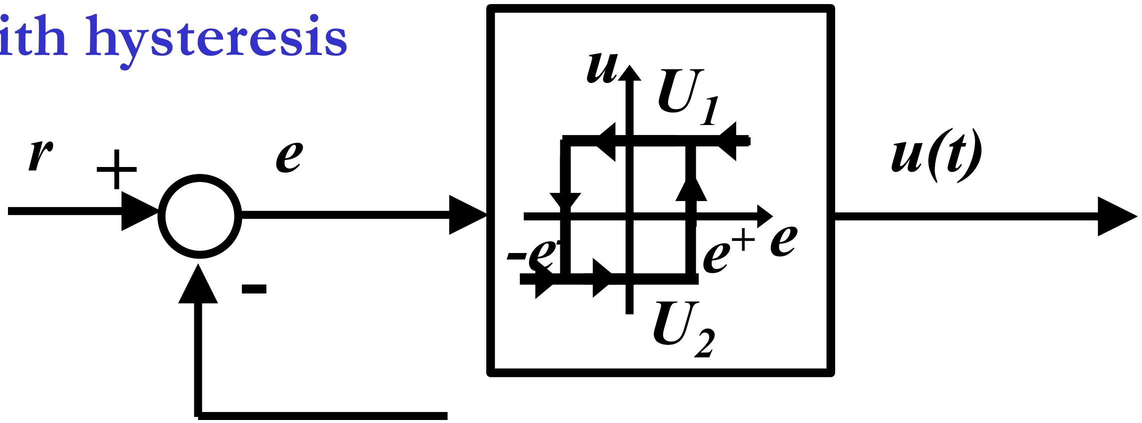
- Different control systems:
  - ***ON/OFF***
  - ***Continuous control systems (analog)***
  - ***Digital control systems***
  - ***Programmable Logic controller***

On/Off controller

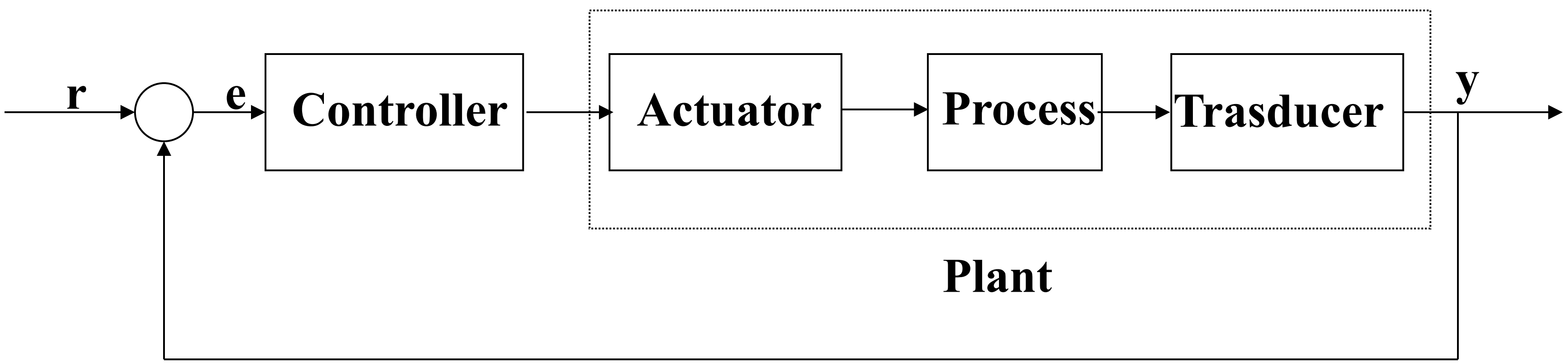


$$u(t) = \begin{cases} U_1 & \text{se} \quad e(t) > 0 \\ U_2 & \text{se} \quad e(t) < 0 \end{cases}$$

Relay with hysteresis



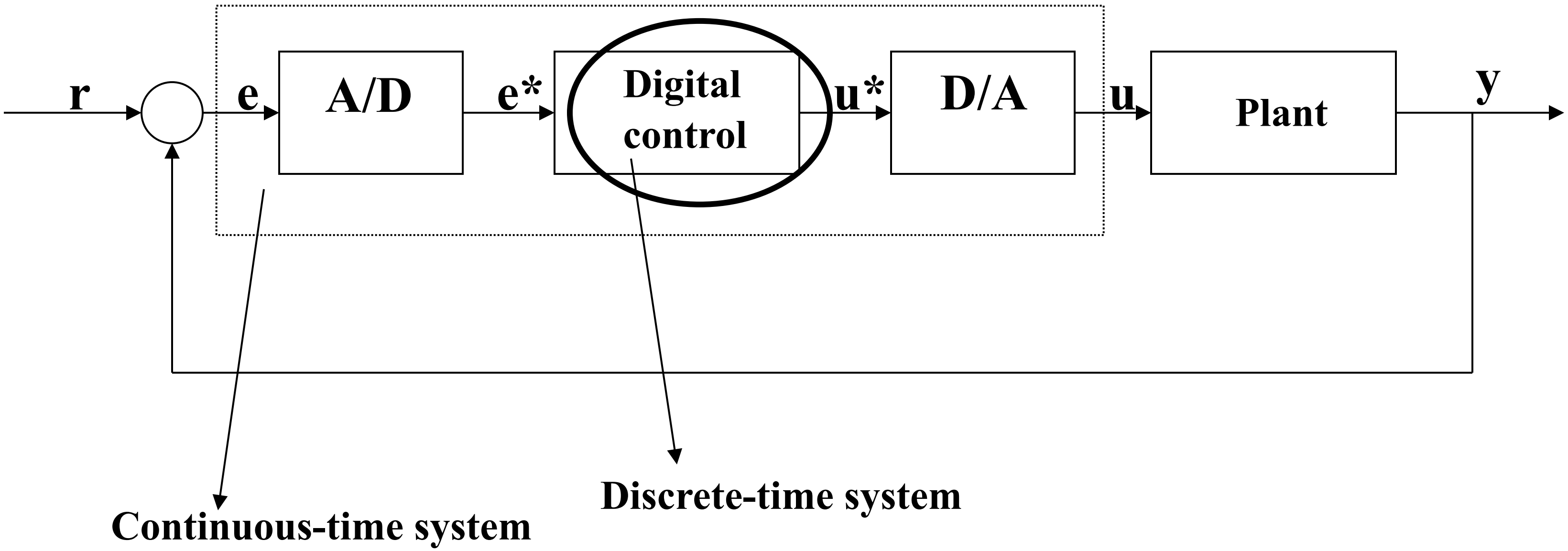
Continuous control system



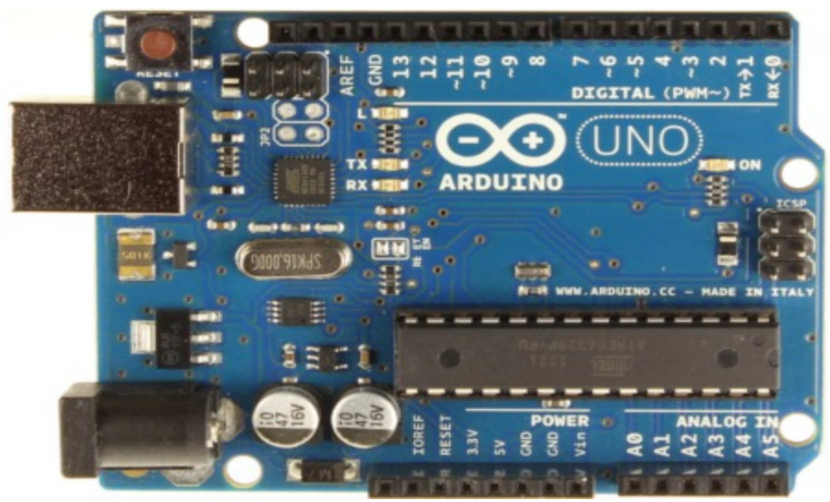
Implementation of ***C*(*s*)**

- Past: analog electronic technology (op amps), hydraulic technology, pneumatic technology
- Present: digital technology (microprocessor systems)

Digital control system

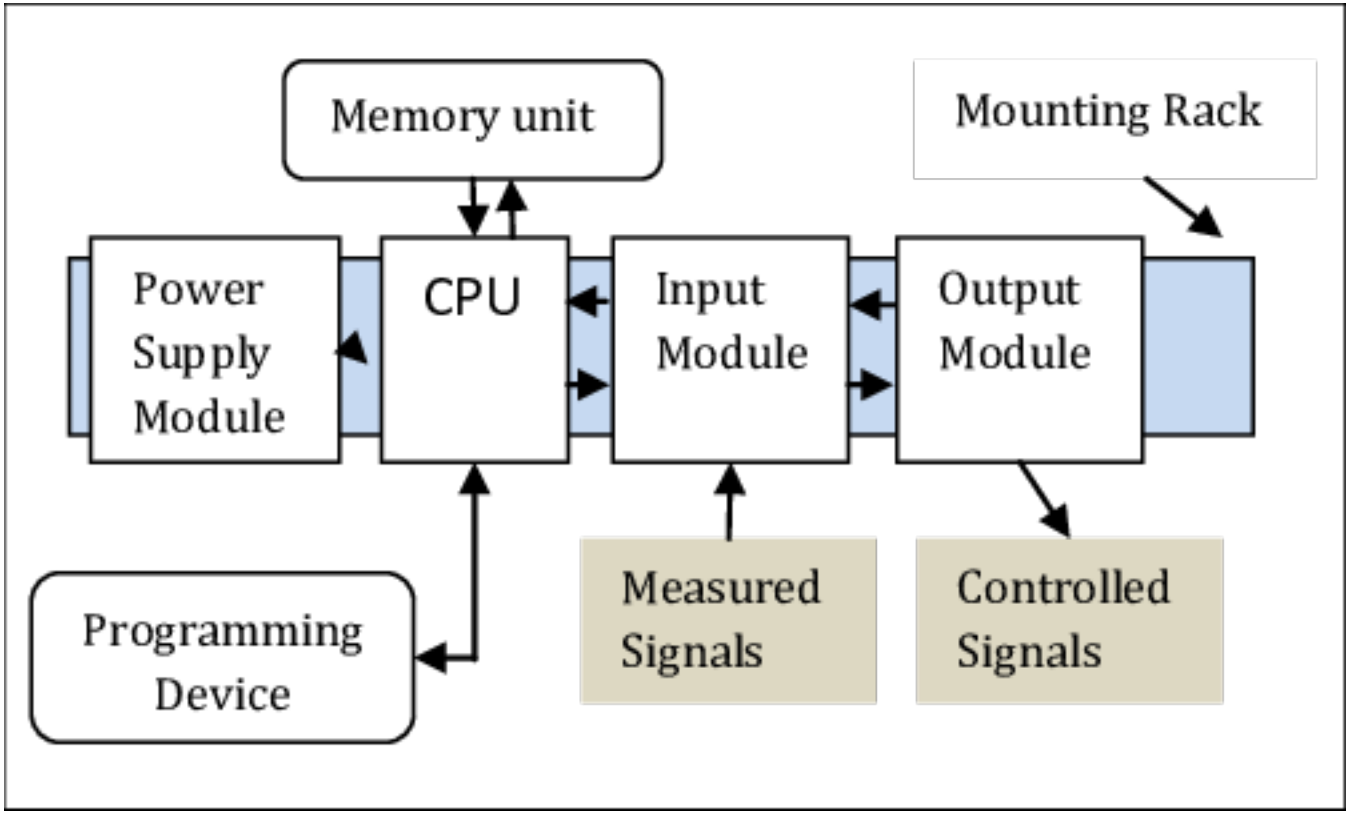


Single-board microcontroller



Implementation of ***C*(*z*)**

- The digital controller is basically a processor/ a digital calculator, i.e. a discrete-time system, which must be interfaced appropriately with the process to be controlled.
- ***C*(*z*)** is an algorithm (sums, products, . . .) that can be implemented in any programming language



A software processes the input data and decides the actions to be implemented so that the controlled system works properly.