SCIENTIFIC COMPUTING – 9 CFU Extended Syllabus – 2024/2025 Prof. Giulio Giunta

Review of basic Numerical Linear Algebra: dot product and orthogonal projection - norms - linear combination of vectors - matrix-vector and matrix-matrix product - bases of linear space and projectors - eigenvalues and eigenvectors - spectral decomposition - power method. Review of basic MATLAB. Class hours: 4. Lab hours: 6.

Matrix factorizations: orthogonal matrices - QR factorization - singular value decomposition (SVD) - applications to data science: the idea of data dimensionality reduction, data analysis in bioinformatics, image analysis and compression, semantic indexing of texts - eigenfaces - principal component analysis - randomized SVD. Lab applications in MATLAB. Class hours: 8. Lab hours: 8.

Solving systems of nonlinear equations: Newton's methods - fixed point method - application to computer graphics. Lab applications in MATLAB. Class hours: 2. Lab hours: 2.

Constrained and unconstrained minimization: gradient, Hessian, Jacobian, Lagrange multipliers - gradient descent method and its variants; moment, ADAM, SGD methods - Newton methods - Levenberg-Marquardt method - nonlinear least squares - applications to data analysis, image processing, maximization of likelihood, robotic arm control, recommendation systems. Lab applications in MATLAB. Class hours: 8. Lab hours: 6.

Google Pagerank algorithm: graphs and matrices - computing the score vector of web pages as a (dominant) eigenvector problem - interpretation of Pagerank as a random process - Markov chains - application of Markov chains to data analysis. Lab applications in MATLAB. Class hours: 4. Lab hours: 4.

Computing derivatives: finite differences, derivation of FD formulas, the choice of the stepsize, differentiation matrices - symbolic differentiation - automatic differentiation, computational graph, forward mode, reverse mode - backpropagation in training neural networks. Lab applications in MATLAB. Class hours: 6. Lab hours: 2

Discrete Fourier Transform and Fourier analysis: DFT and the frequency domain – basic properties of DFT - DTF as a matrix-vector multiplication - FFT algorithms - inverse DFT, short-time DFT and 2D-DFT - Fourier series: basic properties - Fourier coefficients and DFT - aliasing - Gibb's phenomenon - sinc function - Fourier transform: basic properties - inverse FT - How to compute FT - applications to analysis, filtering, and synthesis of audio signals and images. Lab applications in MATLAB. Class hours: 3. Lab hours: 3

Continuous and Discrete Wavelet Transforms: wavelets - CWT and its basic properties - Inverse CWT - 2D-CWT - DWT and wavelet functions - Haar wavelets - properties of DWT - Fast WT and filters - multi-resolution analysis - inverse DWT - 2D-DWT - applications to analysis, filtering, and synthesis of audio signals and images. Lab applications in MATLAB. Class hours: 3. Lab hours: 3