Course of "Automatic Control Systems" 2023/24

PID controller

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PID controller

- A PID controller is characterized by a Proportional-Integral-Derivative control actions with respect to the tracking error $e(t) = r(t) - y(t)$.

- A PID can be written in the time domain as

$$u(t) = K_P e(t) + K_I \int e(t)dt + K_D \frac{de(t)}{dt}$$

even if it is usually defined in the form

$$u(t) = K_P \left(e(t) + \frac{K_I}{K_P} \int e(t)dt + \frac{K_D}{K_P} \frac{de(t)}{dt} \right)$$

$$= K_P \left(e(t) + \frac{1}{T_I} \int e(t)dt + T_D \frac{de(t)}{dt} \right)$$

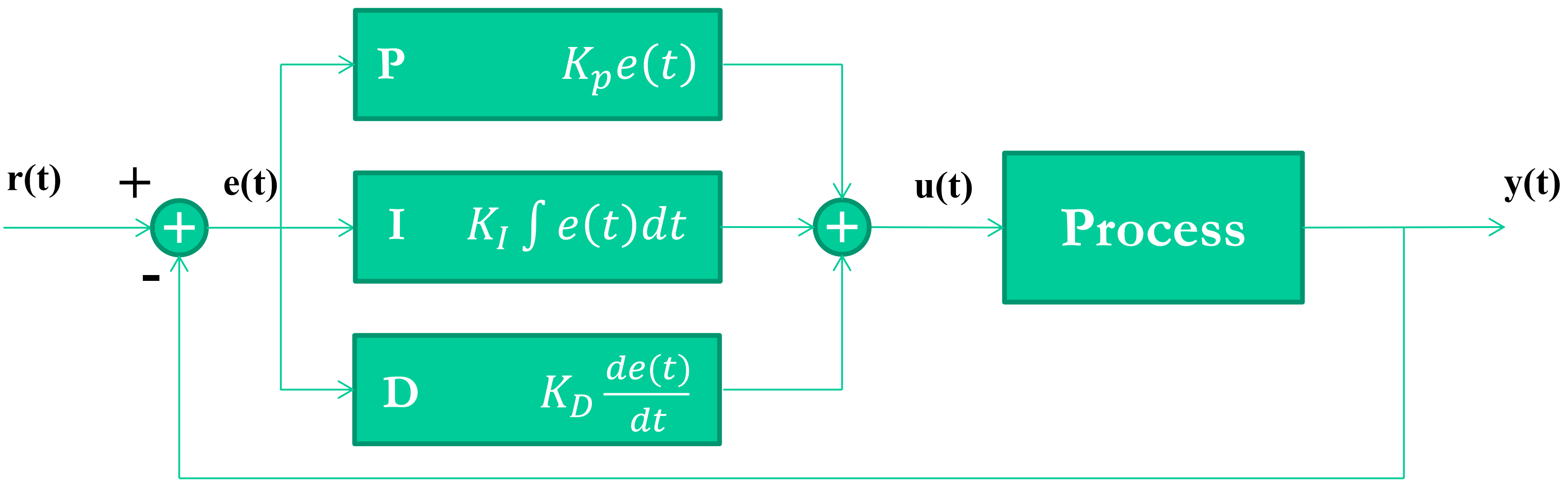
where $T_I = \frac{K_P}{K_I}$ (Integral time) and $T_D = \frac{K_D}{K_P}$ (Derivative time)

PID controller in the time domain

- A PID controller in the time domain form,

$$u(t) = K_P e(t) + K_I \int e(t)dt + K_D \frac{de(t)}{dt}$$

can be represented as

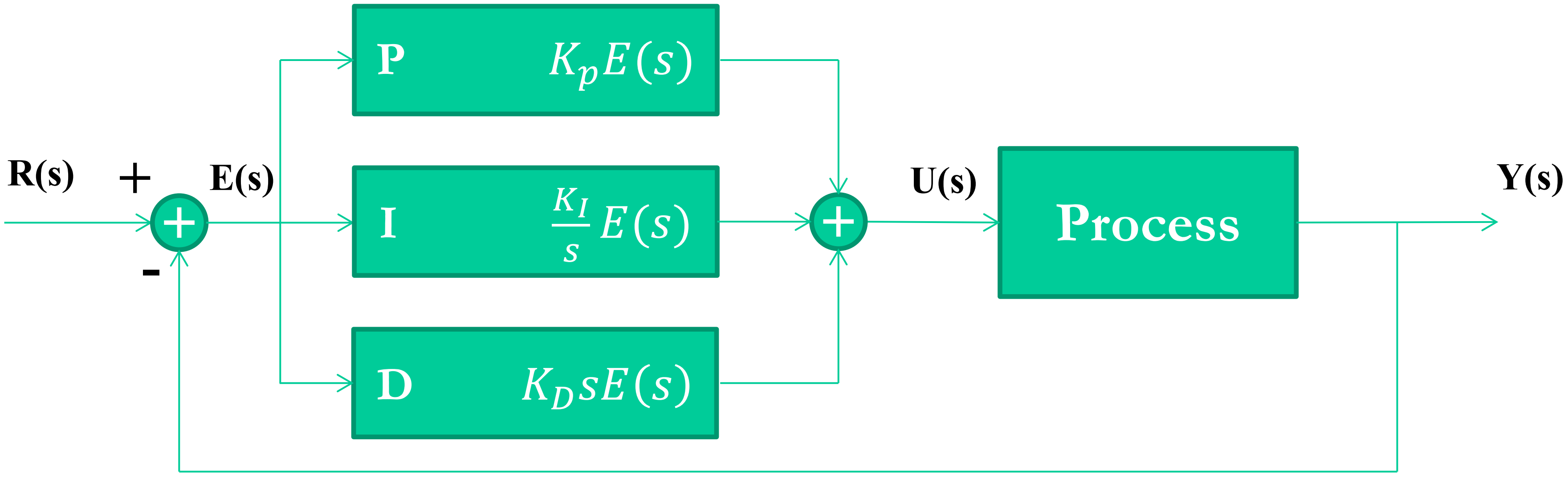


PID controller in the Laplace domain

- A PID controller defined in the Laplace domain as

$$U(s) = K_p E(s) + \frac{K_I}{s} E(s) + K_D s E(s)$$

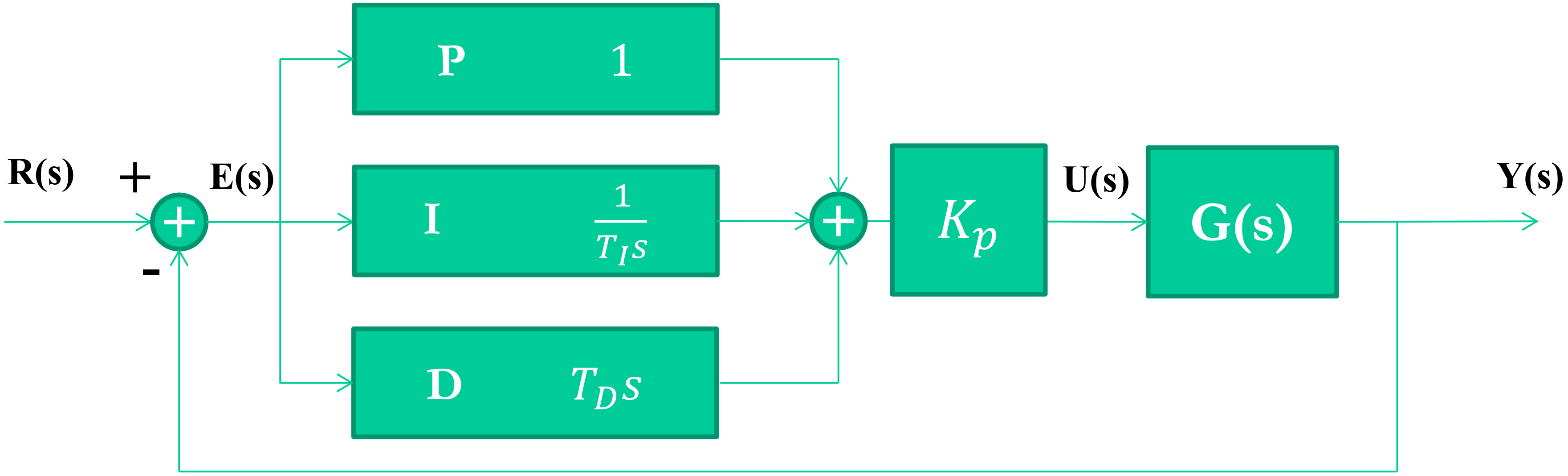
can be represented as



PID controller in the Laplace domain

- Or by $u(t) = K_p\left(e(t) + \frac{1}{T_I}\int e(t)dt + T_D\frac{de(t)}{dt}\right)$

$$U(s) = K_p\left(E(s) + \frac{1}{T_I s}E(s) + T_D s E(s)\right)$$



PID controller

- Usually, only a subset of the possible PID control actions are implemented.
- In particular we have
  - ***Proportional controller (P)***
  - ***Integral controller (I)***
  - ***Proportional-Integral controller (PI)***
  - ***Proportional-Derivative controller (PD)***
  - ***Proportional-Integral-Derivative controller (PID)***

P controller

- The P controller can be written as

$$u(t) = K_p e(t) \rightarrow U(s) = K_p E(s) \rightarrow \frac{U(s)}{E(s)} = K(s) = K_p$$

- **P controllers are used to reduce the steady-state error** when

  - the integral action is not required for the steady-state performance

  - the bandwidth can be increased without violating the requirements

  - the phase margin can be reduced without violating the requirements

P controller: steady-state effect

- Assuming that the steady-state error to polynomial signal is finite, increasing the proportional gain it is possible to reduce the steady-state error with respect to reference and disturb polynomial signals.

Order k / Type n	Step R_0/s	Ramp R_0/s^2	Quadratic R_0/s^3
$n=0$	$\frac{R_0}{1+F_0}$	∞	∞
$n=1$	**0**	$\frac{R_0}{F_0}$	∞
$n=2$	**0**	**0**	$\frac{R_0}{F_0}$

$$F_0 = K_p G_0$$

where G_0 is the static gain of the plant to be controlled (hp, plant without poles in the origin)

I controller

- The integral (I) action can be written as

$$u(t) = K_I \int e(t)dt \rightarrow U(s) = \frac{K_I E(s)}{s} \rightarrow \frac{U(s)}{E(s)} = K(s) = \frac{K_I}{s}$$

- **I controllers are used to reduce or eliminate the steady-state error** when

  - the phase margin can be reduced of **90°** without violating the requirements

I controller: steady-state effect

- Assuming that the steady-state error to polynomial signal is finite

$$order\ of\ the\ polynomial\ signal = type\ of\ F(s)$$

adding an integral action it is possible to eliminate the steady-state error with respect to reference and disturb polynomial signals.

Order k / Type n	Step R_0/s	Ramp R_0/s^2	Quadratic R_0/s^3
$n = 0$	$\frac{R_0}{1+F_0}$	∞	∞
$n = 1$	0	$\frac{R_0}{F_0}$	∞
$n = 2$	0	0	$\frac{R_0}{F_0}$

PI controller

- The proportional-integral controller in the Laplace domain can be written as

$$U(s) = K_P E(s) + \frac{K_I}{s} E(s) = \left(K_P + \frac{K_I}{s}\right) E(s) \rightarrow U(s) = K_p \left(1 + \frac{K_I}{K_P s}\right) E(s)$$

$$\rightarrow \frac{U(s)}{E(s)} = K(s) = K_p \left(1 + \frac{1}{T_I s}\right) E(s)$$

$$K(s) = \frac{K_P}{T_I} \frac{(1 + T_I s)}{s} = K_I \frac{\left(1 + \frac{K_P}{K_I} s\right)}{s}$$

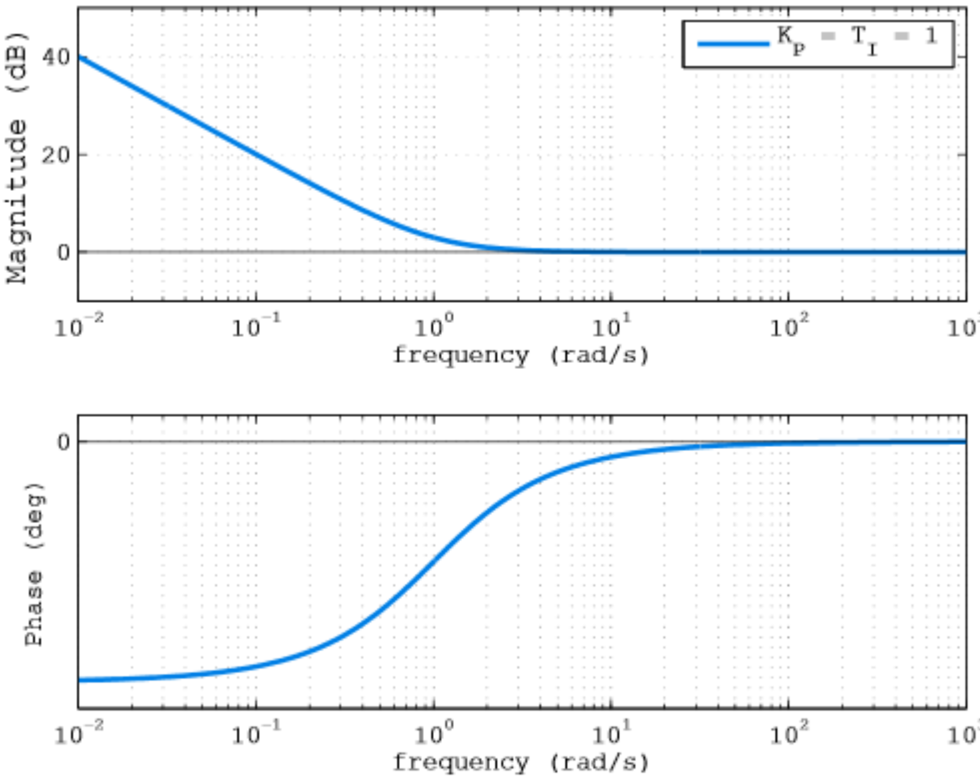
- The PI controllers are composed by
  - a gain $\frac{K_P}{T_I} = K_I \ (T_I = \frac{K_P}{K_I})$
  - a pole in the origin
  - a zero in $z = -\frac{1}{T_I}$

PI controller

- PI controllers are used to improve the steady-state performance of the system

- Due to the presence of the zero, a PI controller performs better than a pure integral controller in terms of transient requirements

- Indeed, the zero can reduce or eliminate the phase lag at the crossing frequency ω_c



PD controller

- The proportional-derivative controller in the Laplace domain can be written as

$$U(s) = K_p E(s) + K_d s E(s) \rightarrow U(s) = \left(K_p + K_d s\right) E(s) = K_p \left(1 + \frac{K_d}{K_p}\right) E(s)$$

$$\Downarrow$$

$$K(s) = K_P (1 + T_D s)$$

$$T_D = \frac{K_d}{K_p}$$

- In the present form, the controller has an improper transfer function due to the presence of the ideal derivative action.

PD controller

- In the common practice, a real derivative action is implemented

$$K(s) = K_P\left(1 + \frac{T_D s}{1 + \frac{T_D}{N} s}\right) = K_P + \frac{K_D s}{1 + \frac{K_D}{K_P N} s}$$

with $N \gg 1$. In general, typical values of N are from 5 to 20.

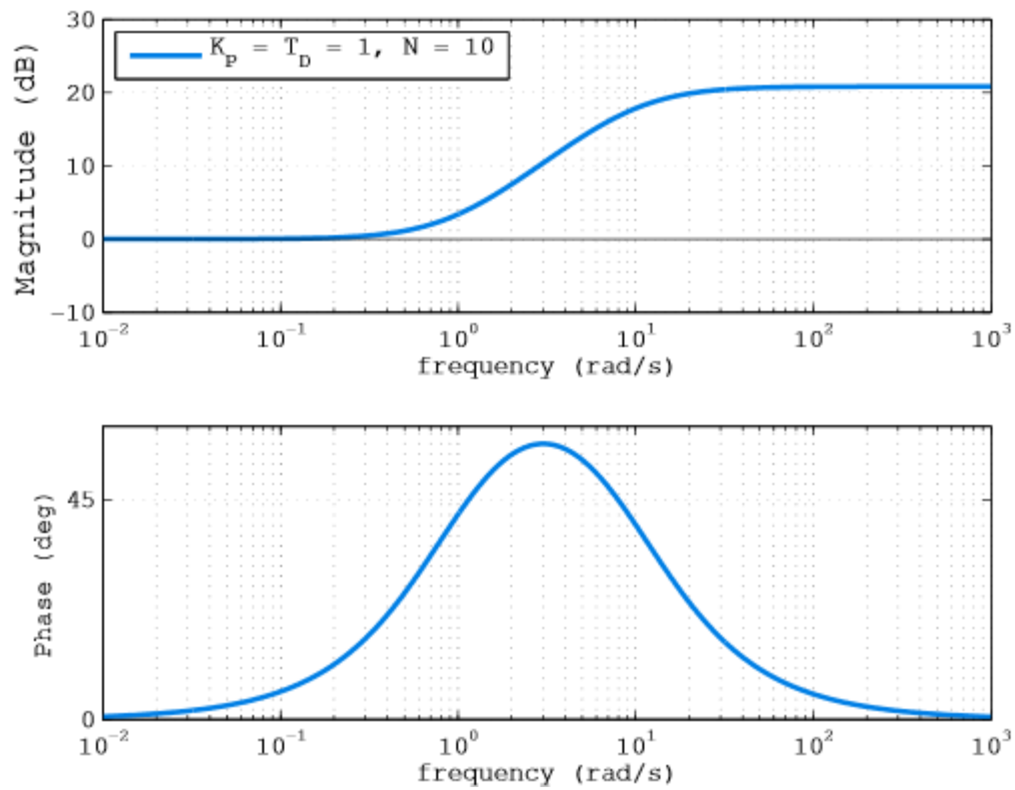
- Taking into account that $T_D + \frac{T_D}{N} \cong T_D$, a real PD controller is in the form

$$K(s) = K_P \frac{(1 + T_D s)}{\left(1 + \frac{T_D}{N} s\right)}$$

PD controller

- PD controllers has the same steady-state performance of the proportional controller

- However, due to the presence of a no-null zero and pole, it can also guarantee an increment of the phase margin



Ideal PID controller

- The proportional-integral-derivative controller in the Laplace domain can be written as

$$U(s) = K_P E(s) + \frac{K_I}{s} E(s) + K_D s E(s) \rightarrow U(s) = \left(K_P + \frac{K_I}{s} + K_D s\right) E(s)$$

$$\downarrow$$

$$\frac{U(s)}{E(s)} = K(s) = K_P + \frac{K_I}{s} + K_D s = \frac{K_D s^2 + K_P s + K_I}{s}$$

Or $K(s) = K_P\left(1 + \frac{K_I}{K_P s} + \frac{K_D}{K_P} s\right) = K_P\left(1 + \frac{1}{T_I s} + T_D s\right) = \frac{K_P}{T_I} \frac{(1+T_I s+T_I T_D s^2)}{s}$

- Also, in this case a pole at high frequencies have to be added for the physical implementation.

Real PID controller

- By implementing the real derivative action as $K_{D_r} = \frac{K_D s}{1+\frac{K_D}{K_P N}s} = \frac{K_P T_D s}{1+\frac{T_D}{N}s}$

$$K(s) = K_P + \frac{K_I}{s} + \frac{K_D s}{1 + \frac{K_D}{K_P N} s}$$

Or

$$K(s) = K_P \left(1 + \frac{1}{T_I s} + \frac{T_D s}{1 + \frac{T_D}{N} s}\right)$$

Also in this case a pole at high frequencies have to be added for the physical implementation.

PID controller

- PID controllers are used to improve both the steady-state and transient performance of the system

- Due to the pole in the origin, they are able
  - to reduce or eliminate the steady-stare error

- Due to the PD action, they are able to
  - increase or reduce the phase margin of the system