Course of
"Automatic Control Systems"
2023/24

Example of controller design

Prof. Francesco Montefusco

Department of Economics, Law, Cybersecurity, and Sports Sciences

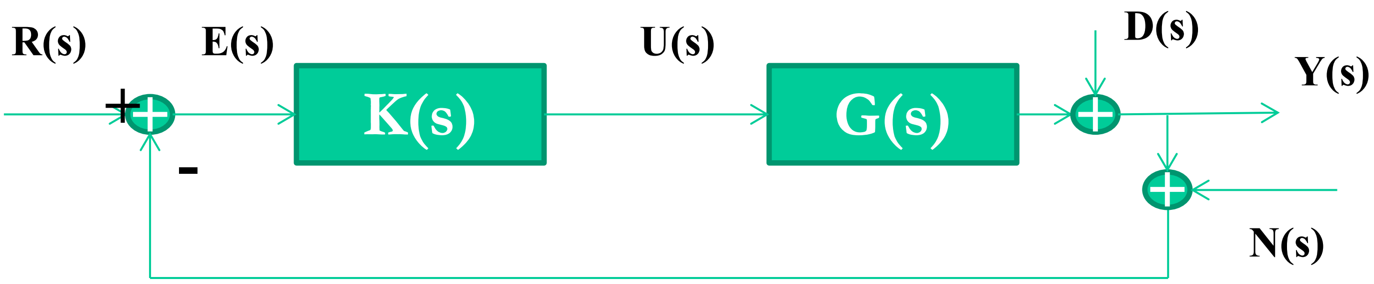
Università degli Studi di Napoli Parthenope

francesco.montefusco@uniparthenope.it

Team code**: mfs9zfr**

Example of controller design

- Let us consider a closed loop system in the form



where

$$G(s) = \frac{30}{(s+10)(s+3)}$$

Requirements

Let us consider the following requirements:

1. $e_\infty = 0$ for a reference signal $r(t) = 1 \cdot 1(t)$

2. Attenuation $\geq 95\%$ for multi-frequency noise in the range $[100 \quad +\infty]$ rad/s

3. Overshoot $s \leq 20\%$

4. Settling time $t_{s1\%} \leq 0.5s$

1. $e_\infty = 0$ for a reference signal $r(t) = 1 \cdot 1(t)$

- Steady-state requirement for polynomial reference signal.
- To assure a null steady state error for a polynomial signal of order 0 it is necessary that $F(s)$ is of type 1, that is $F(s)$ has a pole in the origin.
- Taking into account that the plant transfer function $G(s)$ doesn't contain poles in the origin, the steady-state part of the controller is

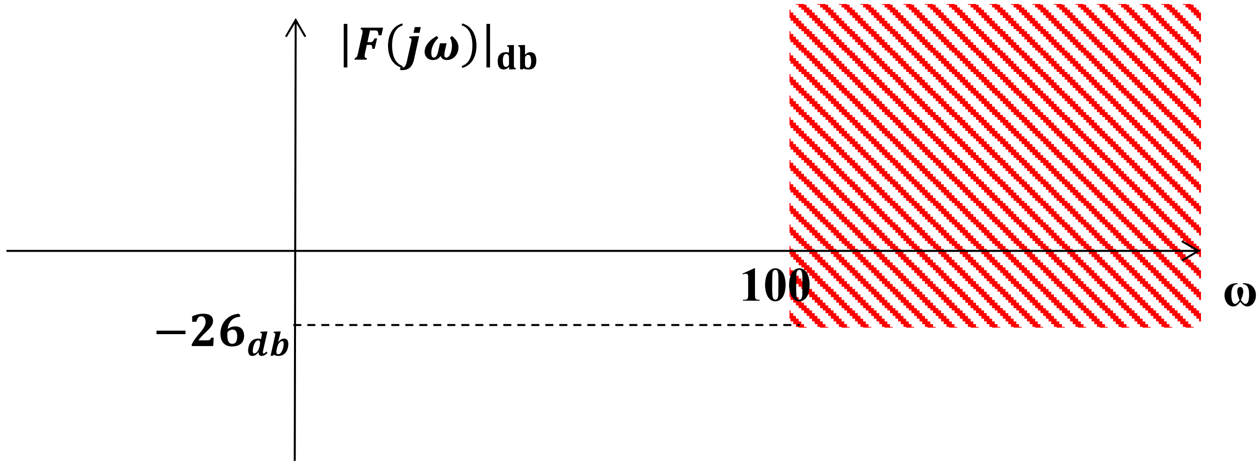
$$K'(s) = \frac{k_0}{s}$$

where k_0 is a free parameter

2. Attenuation $\geq 95\%$ for multi-frequency noise in the range $[100 \quad +\infty]$ rad/s

- Steady-state requirement for multi-frequency noise
- It implies that

$$|T(s)| = \left|\frac{F(s)}{1+F(s)}\right| \leq 0.05 \quad \rightarrow \quad |F(s)| \leq 0.05 \quad \rightarrow \quad |F(s)|_{db} \leq -26_{db}$$



Transient spec. on the overshoot

3. Overshoot $s \leq 20\%$

- Transient requirement on the overshoot

- Taking into account that $s = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$, we have that

$$s \leq 20\% \quad \rightarrow \quad \zeta \geq 0.45 \quad \rightarrow \quad \varphi_m \cong 100\zeta \geq 45°$$

- Hence the complementary sensitivity function can be approximated by a second order system in the form

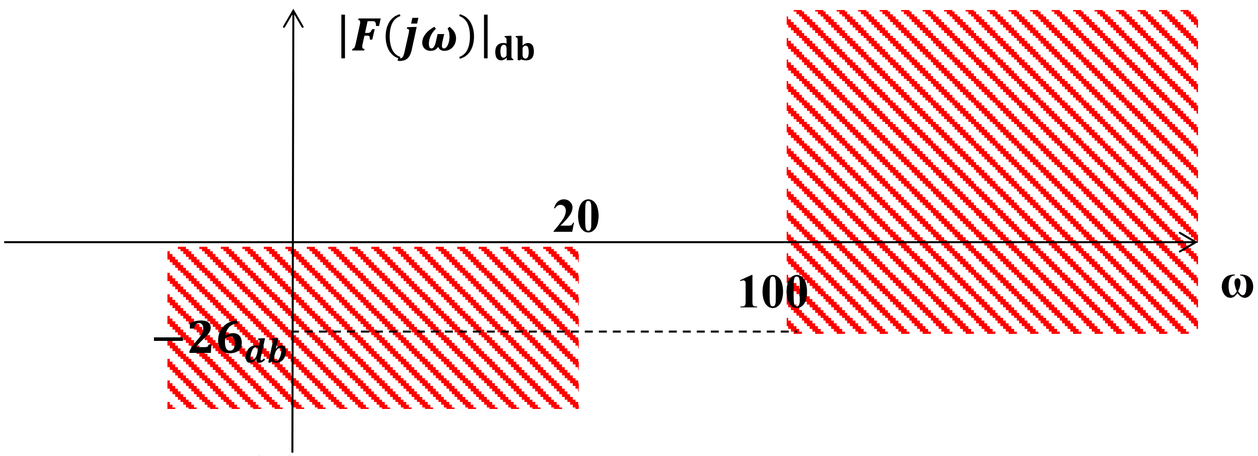
$$T_a(s) = \frac{1}{1 + \frac{2\zeta}{\omega_n}s + \frac{s^2}{\omega_n^2}}$$

where $\zeta = 0.45$ and ω_n depends on the settling time requirement

4. Settling time $t_{s1\%} \leq 0.5s$

- Transient requirement on the settling time
- Taking into account that the settling time at 1% for a second order system is defined as $t_{s1\%} \cong \frac{4.6}{\zeta\omega_n}$, we have that

$$t_{s1\%} \leq 0.5 \quad \rightarrow \quad \frac{1}{\omega_n} \leq \frac{0.5\zeta}{4.6} \quad \rightarrow \quad \omega_c = \omega_n \geq \frac{4.6}{0.5 \cdot 0.45} = 20$$



- **The transfer function $F(s)$ should have a crossing frequency $\omega_c > 20$ and a phase margin $\varphi_m > 45°$.**

- Design the controller $K(s)$ so that the open loop function $F(s) = K(s)G(s)$ satisfies the previous requirements

- The controller will be in the form

$$K(s) = K'(s) \cdot K''(s)$$

where

- $K'(s)$ have been designed according to the steady-state requirements concerning polynomial reference and/or disturbs

$$K'(s) = \frac{1}{s}$$

- $K''(s)$ have to be designed according to the steady-state multi-frequency requirements and the transient requirements

Step2: Controller design

- In order to design $K''(s)$, let us consider the Bode diagrams of the function

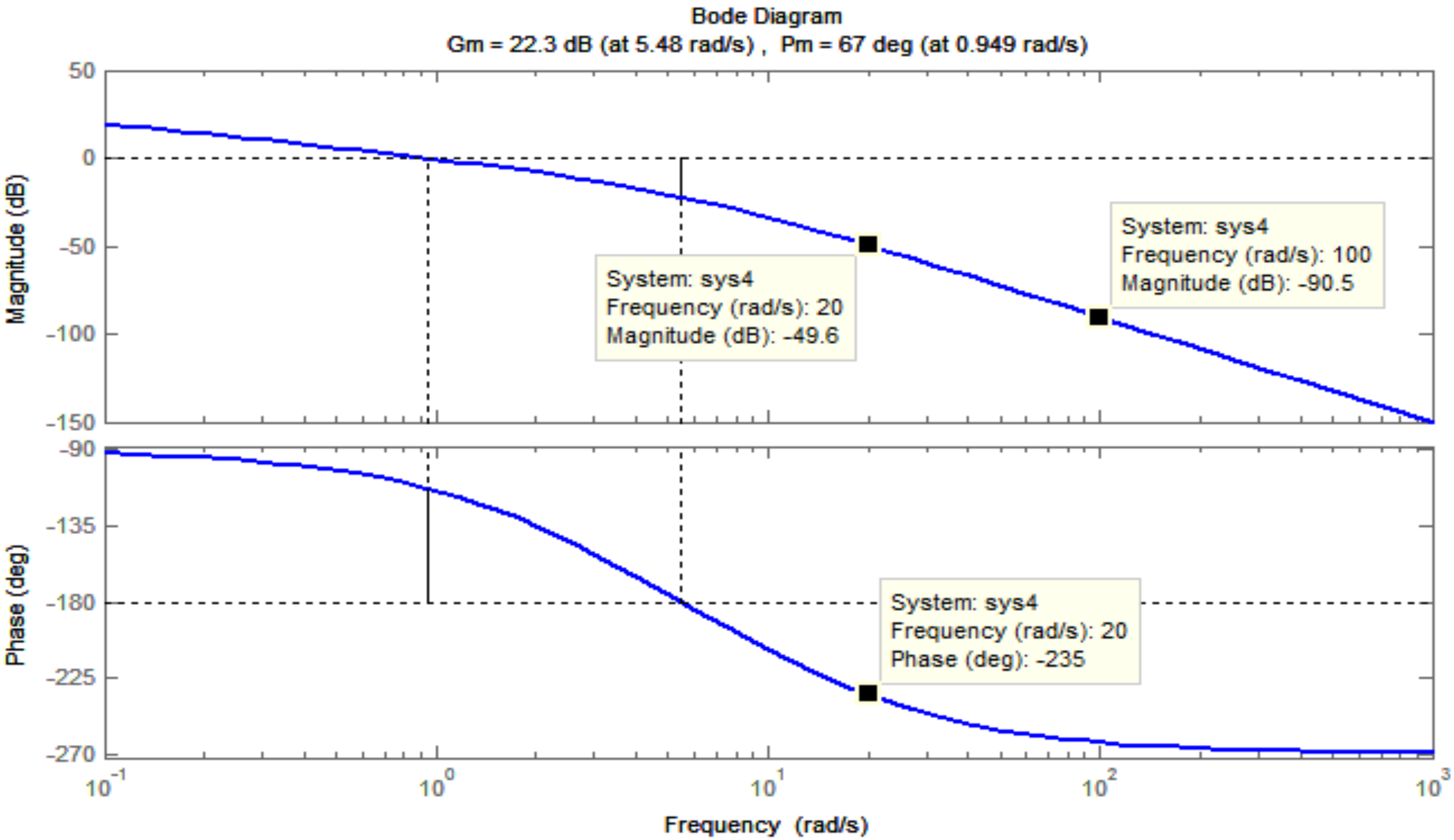
$$F'(s) = K'(s) \cdot G(s) = \frac{30}{s(s+10)(s+3)} = \frac{1}{s\left(1+\frac{s}{10}\right)\left(1+\frac{s}{3}\right)}$$

assuming $k_0 = 1$.

The transfer function is characterized by

- A unitary constant term
- A pole in the origin
- Two real poles in $p_1 = -10$ $(\frac{1}{\tau_1} = 10)$ and $p_2 = -3$ $(\frac{1}{\tau_2} = 3)$

Controller design



- $F'(s)$ magnitude amplification of 50_{db} to have a crossing frequency $\omega_c = 20$rad/s
- $F'(s)$ phase lead of $100°$ to have $\varphi_m = 45°$

Controller design

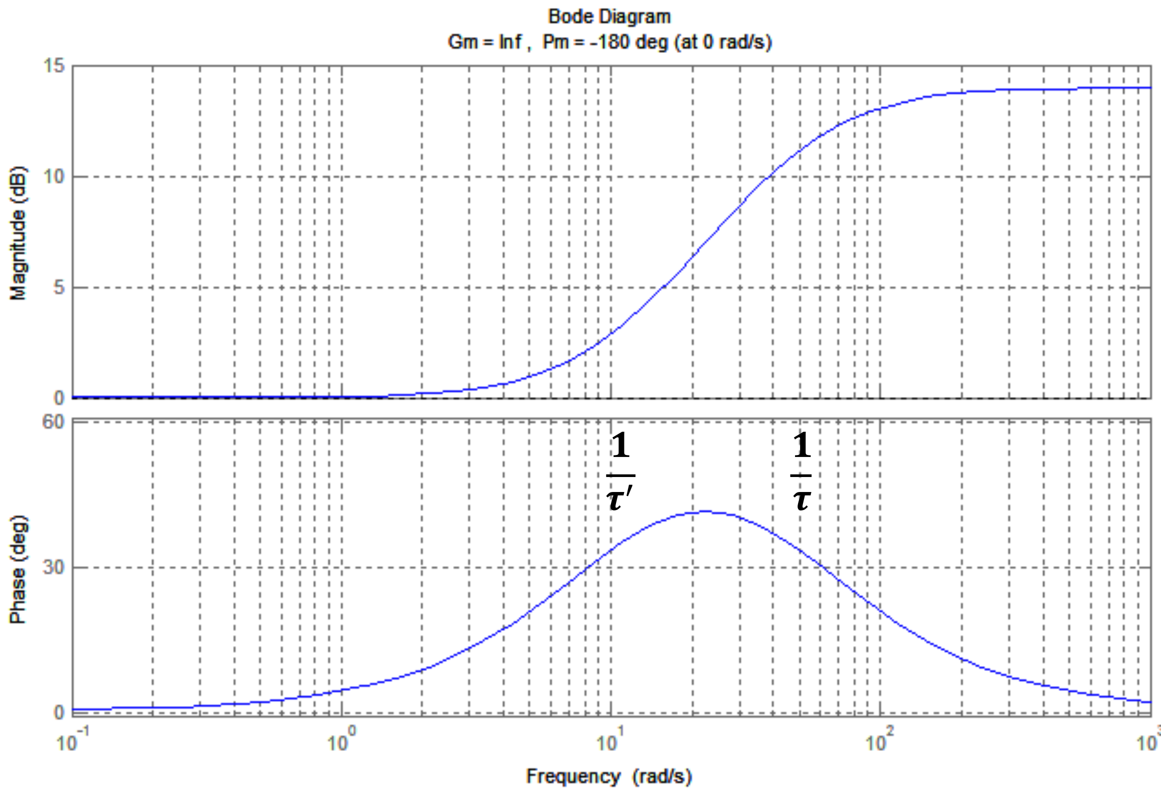
- The magnitude amplification can be easily achieved with a gain $k_0 = 50_{db}$, however it doesn't affect the phase

- To achieve both a magnitude amplification and a phase lead we can add a control structure composed by 1 pole and 1 zero in the form

$$K''(s) = \frac{(1+s\tau')}{(1+s\tau)}$$

with $\tau' > \tau$.

Maximum phase lead= 90°



Controller design

- In our case, we require $100°$ phase lead hence an additional zero is needed. In particular we will add:
  - **Two zeros in $z_1 = -2$ and $z_2 = -10$**
    - $150°$ phase lead in $\omega = 20$ rad/s ($\varphi_m \cong 100$)
    - 26_{db} magnitude amplification in $\omega = 20$ rad/s
    - 54_{db} magnitude amplification in $\omega = 100$ rad/s
  - **Gain $k_0 = 20$**
    - 26_{db} magnitude amplification ($\omega_c \cong 20$ rad/s and $|F(j100)|_{db} \cong -10$)
  - **Pole in $p = -20$**
    - $45°$ phase lag in $\omega = 20$ rad/s ($\varphi_m \cong 55°$)
    - 3_{db} magnitude attenuation in $\omega = 20$ rad/s ($\omega_c \cong 20$ rad/s)
    - 14_{db} magnitude attenuation in $\omega = 100$ rad/s ($|F(j100)|_{db} \cong -24$)

Controller design

- The controller is in the form

$$K(s) = K'(s)K''(s) = \frac{20\left(1+\frac{s}{2}\right)\left(1+\frac{s}{10}\right)}{s\left(1+\frac{s}{20}\right)}$$

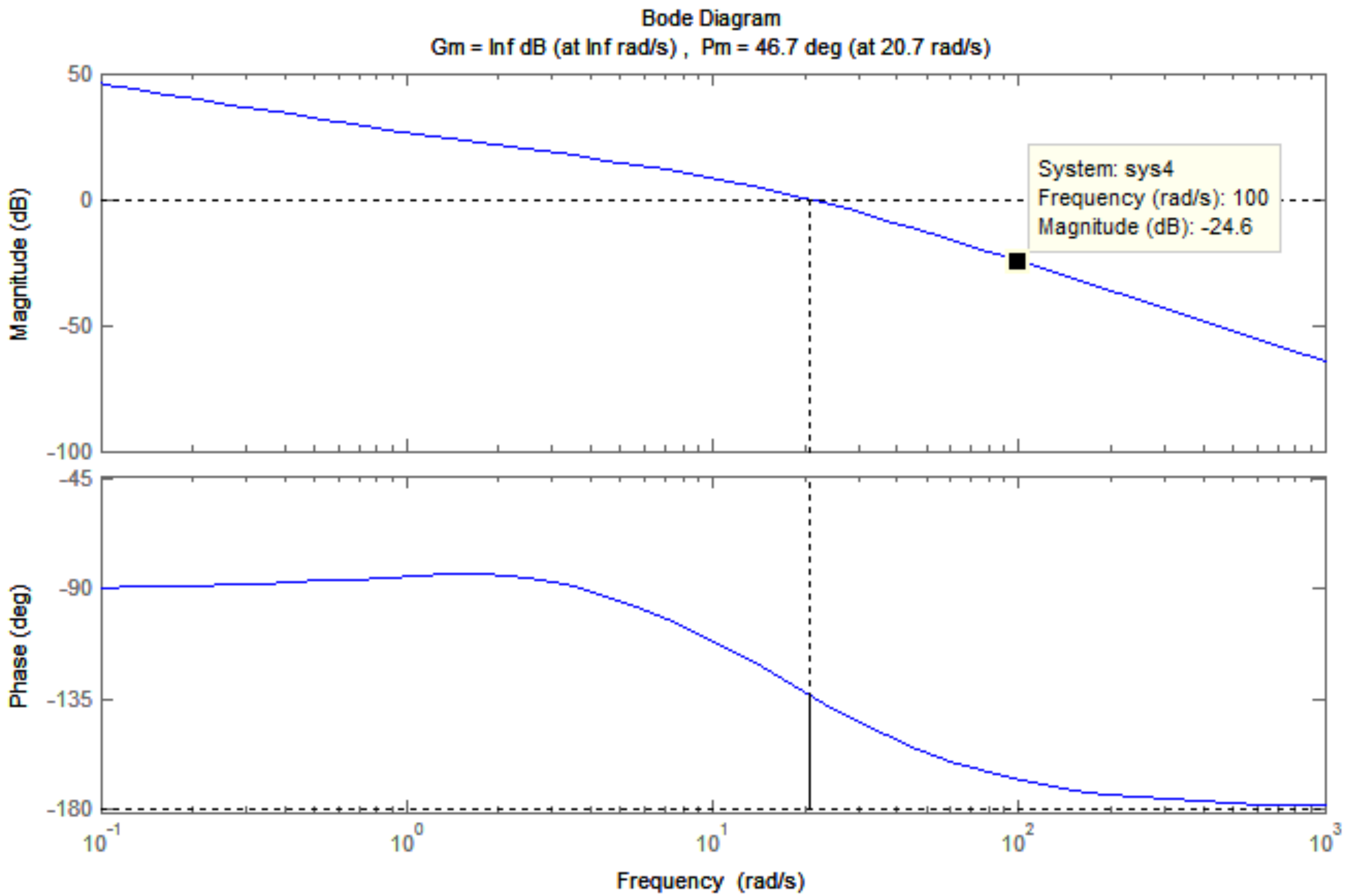
- The pole in $\mathrm{p} = -20$ was necessary also for the physical feasibility of the controller that can not be in-proper.

- In the following slide the Bode diagrams of

$$F(s) = K'(s) \cdot K''(s) \cdot G(s)$$

are reported.

Controller design



Validation: step response