Course of
"Automatic Control Systems"
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Classification of closed loop systems w.r.t a proportional control action

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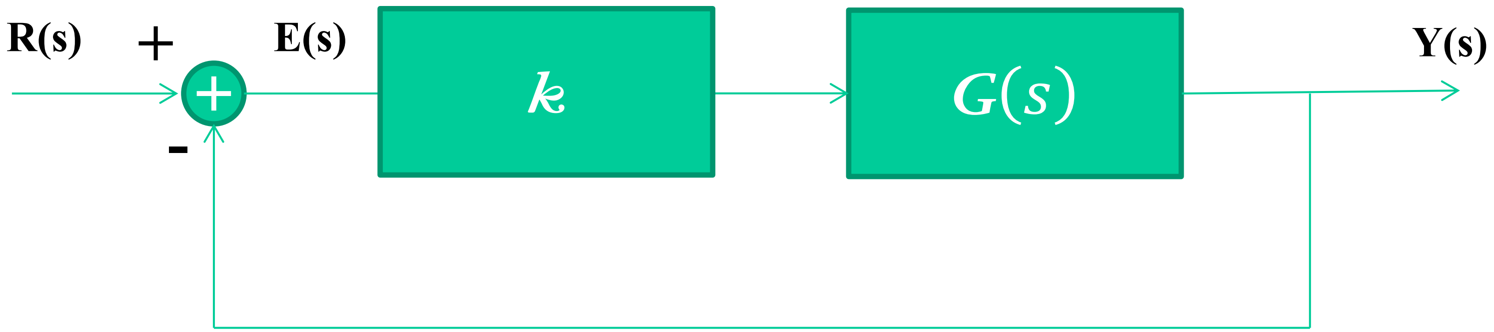
Stability of the closed loop systems

- Let us consider a closed loop system with a proportional control action

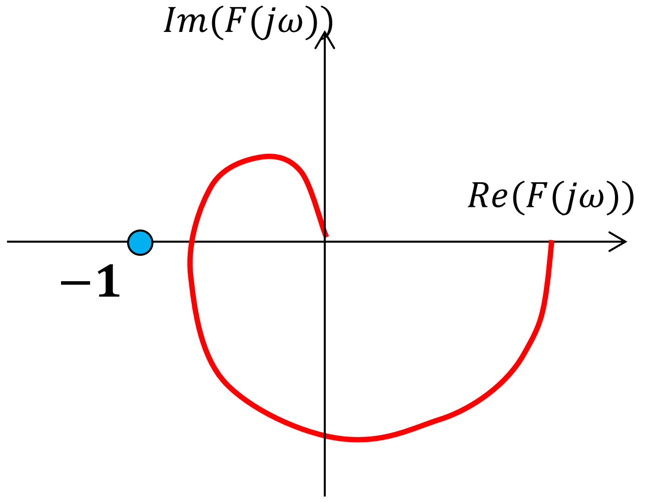


- In the following we will classify the closed loop systems depending on their stability behavior when k varies from 0 to $+\infty$

Regular stability of closed loop systems

- ***The stability*** of a closed loop system $T(s)$ ***is said to be regular*** when
  - T(s) is unstable for high values of the proportional gain k
  - T(s) is asymptotically stable for low values of the proportional gain k

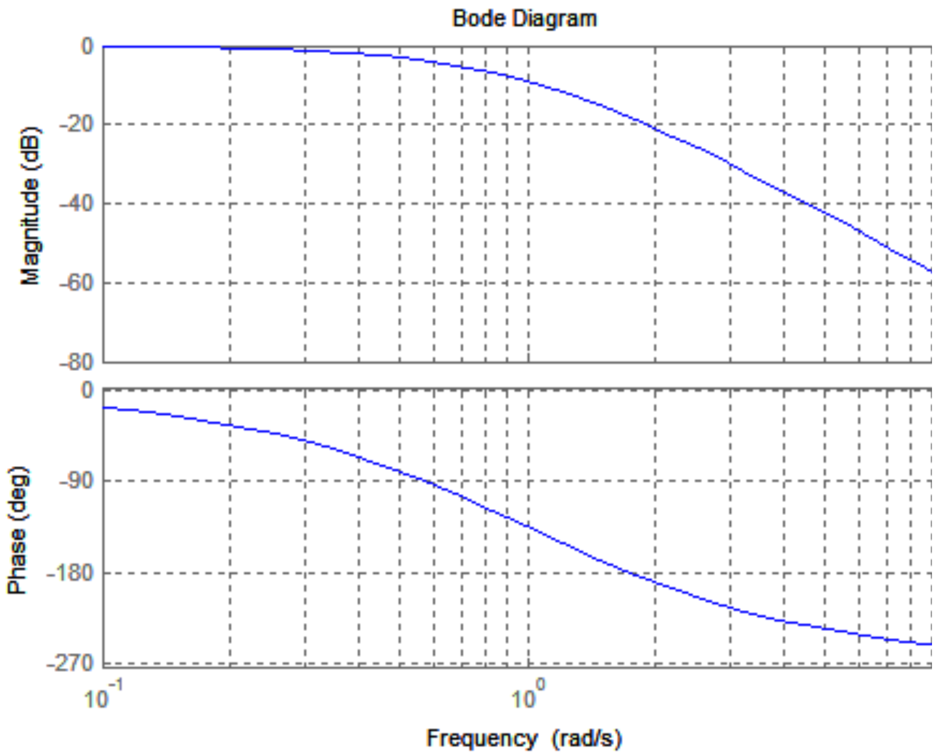
- Closed loop systems with regular stability are usually characterized by an open loop function $F(s)$ with:
  - No poles with positive real part
    ($T(s)$ asymp. Stable iff $\overleftarrow{\mathcal{N}} = 0$)
  - A Nyquist plot that intersect only once the negative x-axis

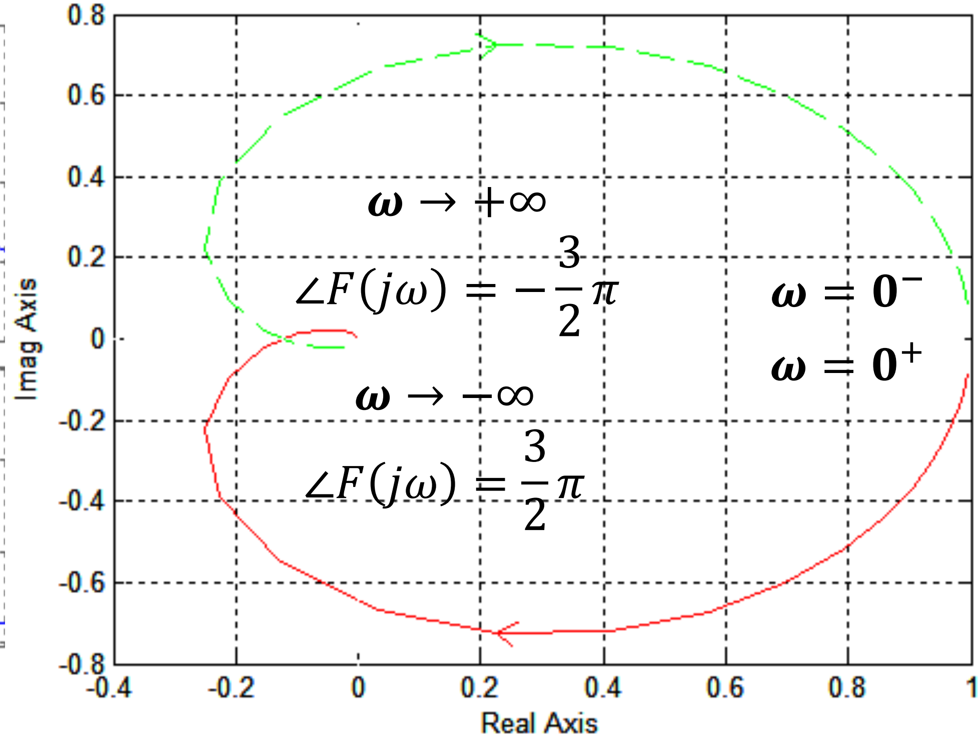


There exists a <u>critical gain $\bar{k}$</u> such that the closed loop system is unstable for <u>$k > \bar{k}$</u>

Regular stability of closed loop systems

- Let us consider the transfer function $F(s) = \frac{1}{(1+s)^3} \;\rightarrow\; n_{p^+}\big(F(s)\big) = 0$





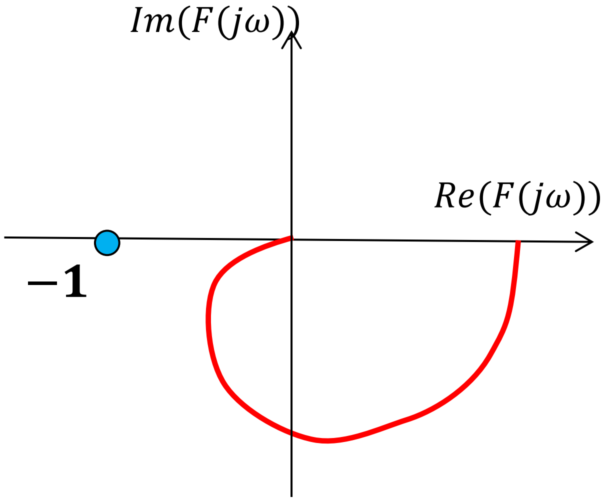
$\overleftarrow{\mathcal{N}} = 0,\qquad \textit{the critical gain is } \bar{k} \approx 8$

$T(s)$ *asymptotically stable for* $k < 8$ $\qquad$ $T(s)$ *unstable for* $k > 8$

Inherent stability of closed loop systems

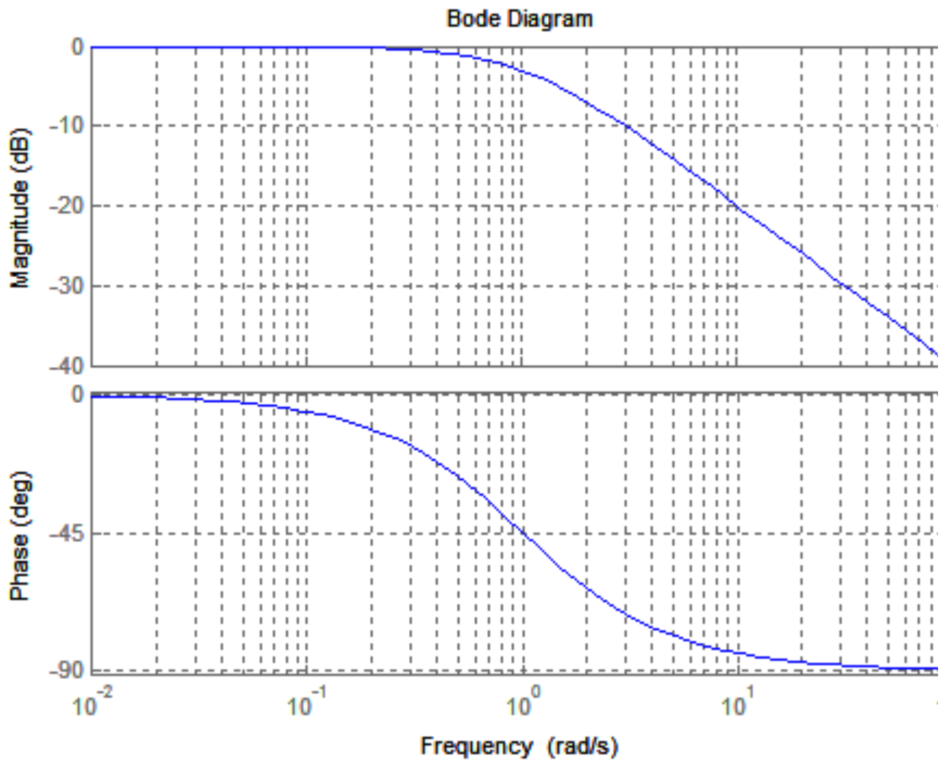
- ***The stability*** of a closed loop system $T(s)$ ***is said to be inherent*** when
  - ***$T(s)$ is asymptotically stable for all possible k***

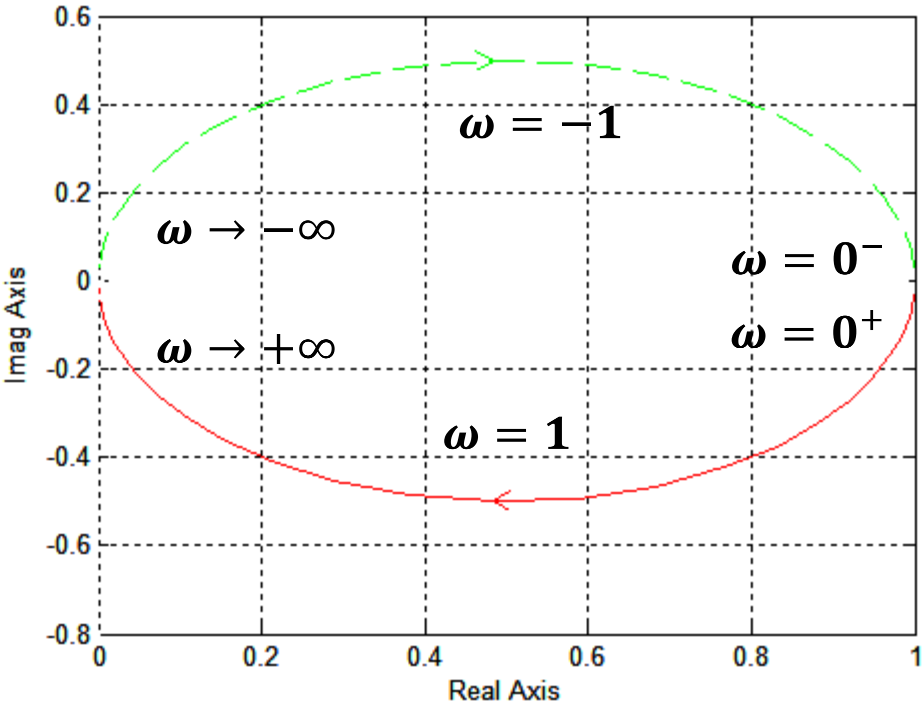
- Inherently stable closed loop systems are usually characterized by an open loop function $F(s)$ with:
  - No poles with positive real part
    ($T(s)$ asymp. Stable iff $\overleftarrow{\mathcal{N}} = 0$)
  - A Nyquist plot that doesn't intersect the negative x-axis



The closed loop system is asymptotically stable regardless the value of the proportional gain

- Let us consider the transfer function $F(s) = \frac{1}{1+s} \rightarrow n_{p^+}\big(F(s)\big) = 0$



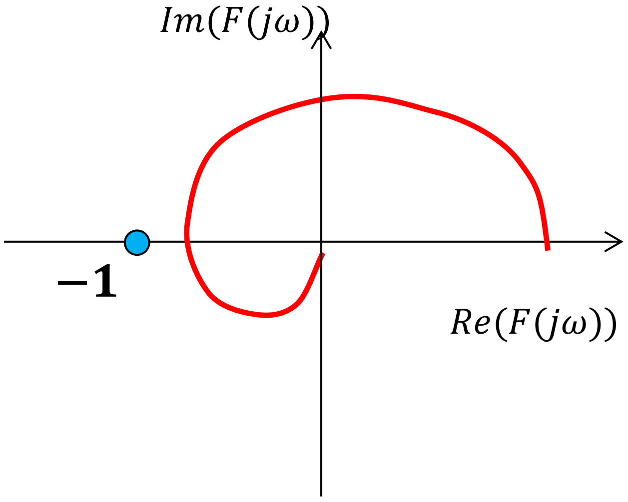


$$\overleftarrow{\mathcal{N}} = 0 \;\; \textit{for all possible values of } k$$

Paradoxical stability of closed loop systems

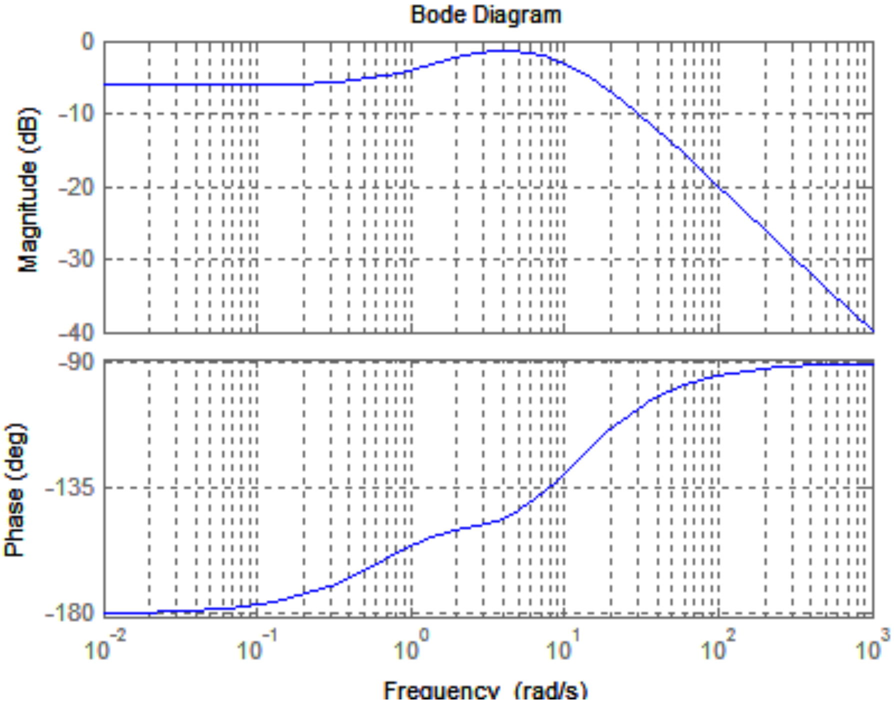
- ***The stability*** of a closed loop system $T(s)$ ***is said to be paradoxical*** when
  - $\boldsymbol{T(s)}$ ***is unstable for low*** values of the proportional gain $\boldsymbol{k}$
  - $\boldsymbol{T(s)}$ ***is asymptotically stable for high*** values of the proportional gain $\boldsymbol{k}$

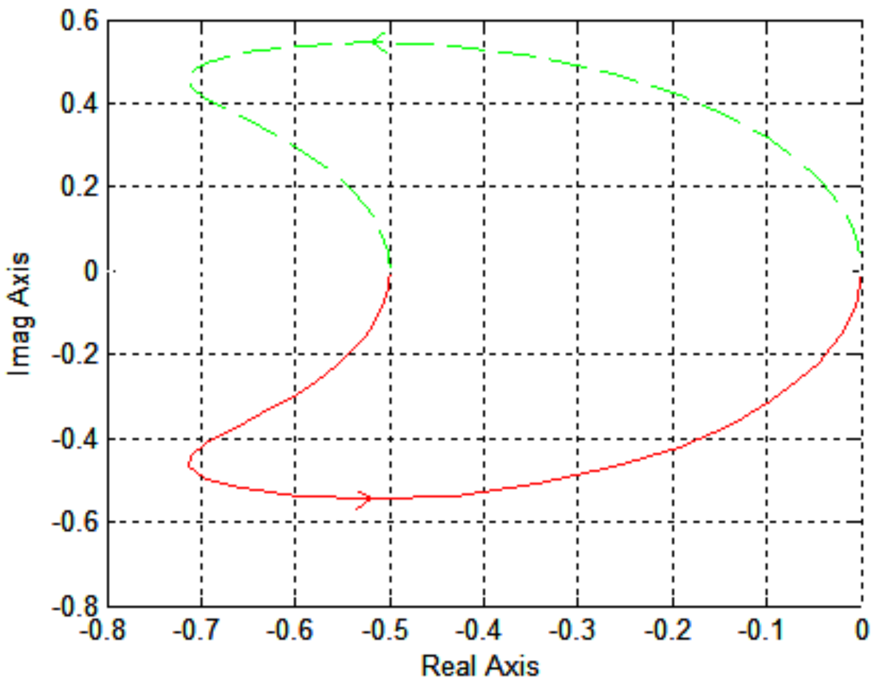
- Paradoxically stable closed loop systems are usually characterized by an open loop function $F(s)$ with:
  - Poles with positive real part
    ($T(s)$ asymp. Stable iff $\overleftarrow{\mathcal{N}} > \mathbf{0}$)
  - A Nyquist plot that intersect only once the negative x-axis



There exists a <u>critical gain $\bar{\mathbf{k}}$</u> after which the closed loop system becomes asymptotically stable

- Let us consider the transfer function $F(s) = \frac{10(1+s)}{(2+s)(s-10)} \;\rightarrow\; n_{p^+}(F(s)) = 1$



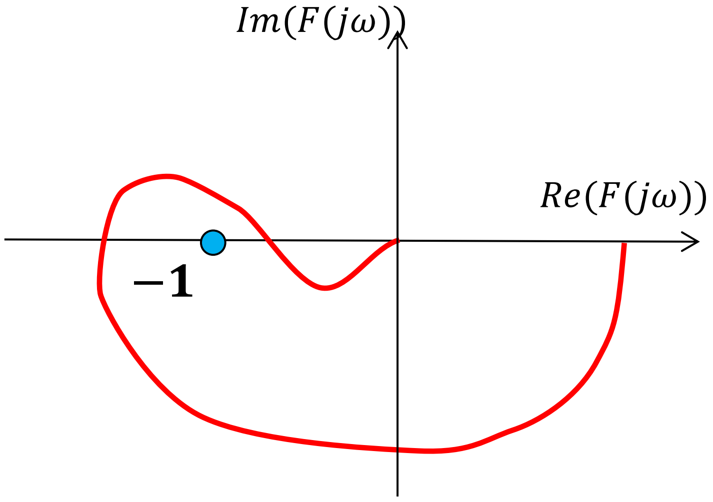


$\overleftarrow{\mathcal{N}} = 1 \quad \boldsymbol{for\ k\ greater\ that\ the\ critical\ gain\ \bar{k} = 2}$

$\boldsymbol{T(s)\ asymptotically\ stable\ \ for\ \ k > 2}$ $\qquad$ $\boldsymbol{T(s)\ unstable\ \ for\ \ k < 2}$

- ***The stability*** of a closed loop system $T(s)$ ***is said to be conditioned*** when
  - $\boldsymbol{T(s)}$ ***is asymptotically stable for a limited interval of*** $\boldsymbol{k}$

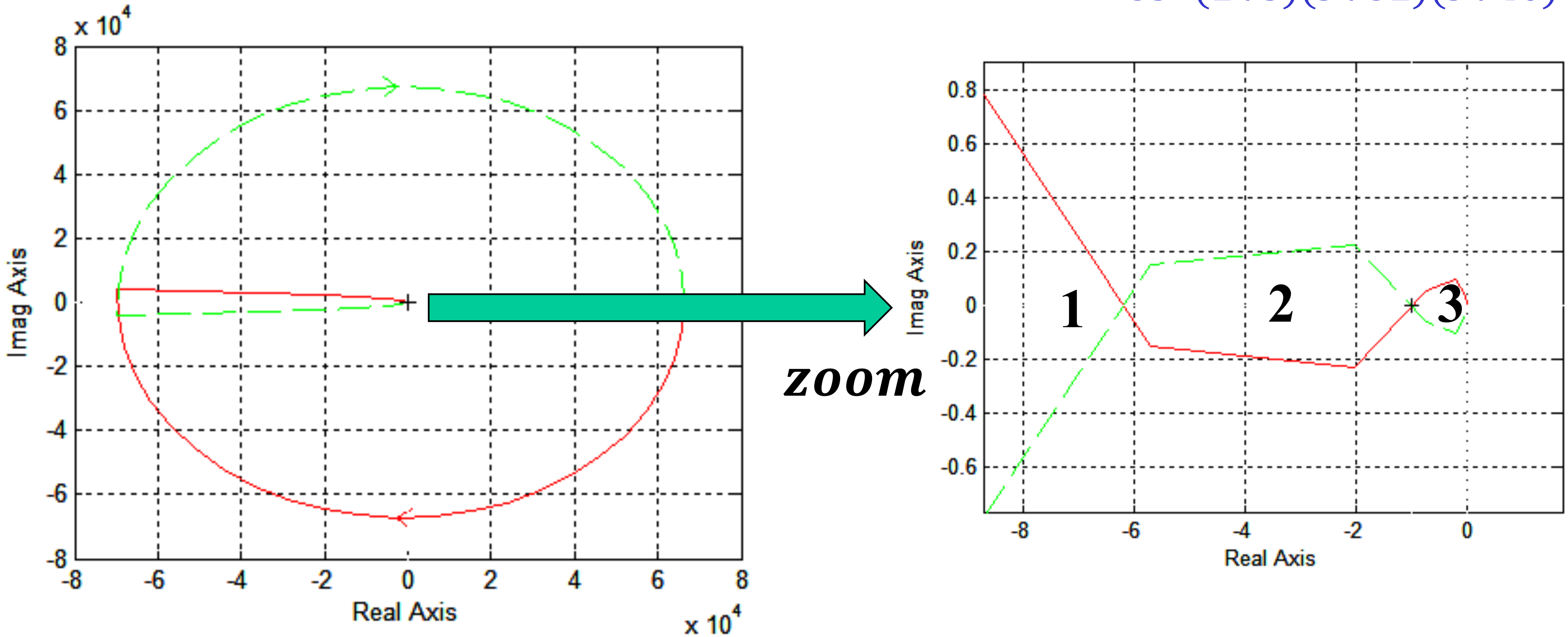
- Conditionally stable closed loop systems are usually characterized by an open loop function $F(s)$ with:
  - No poles with positive real part ($T(s)$ asymp. Stable iff $\overleftarrow{\mathcal{N}} = 0$)
  - A Nyquist plot that intersect the negative x-axis more than one time



Conditional stability of closed loop systems

- Let us consider the transfer function

$$F(s) = \frac{10000(s+3)(s+9)}{3s^2(1+s)(s+32)(s+40)}$$

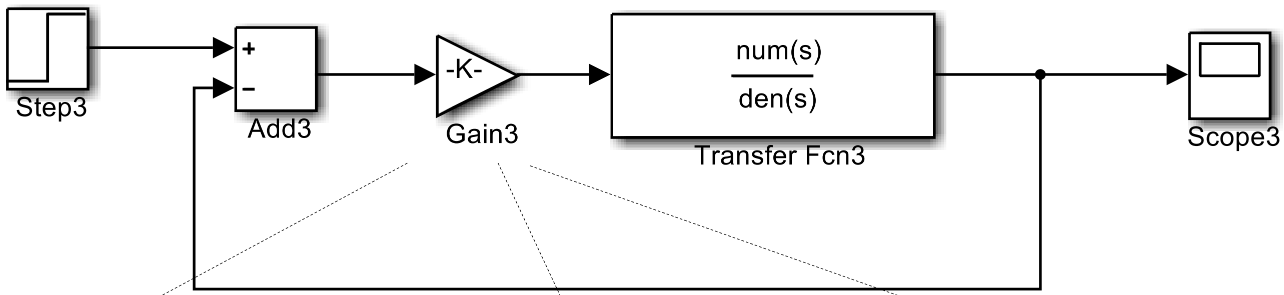


- For $k \leq k_1$ the critical point moves in the first circle $\overleftarrow{N} = -2 \;\rightarrow\; T(s)$ unstable
- For $k_1 < k < k_2$ the critical point moves in the second circle $\overleftarrow{N} = 0 \;\rightarrow\; T(s)$ asymp. stab.
- For $k > \; k_2$ the critical point moves in the third circle $\overleftarrow{N} = -2 \;\rightarrow\; T(s)$ unstable

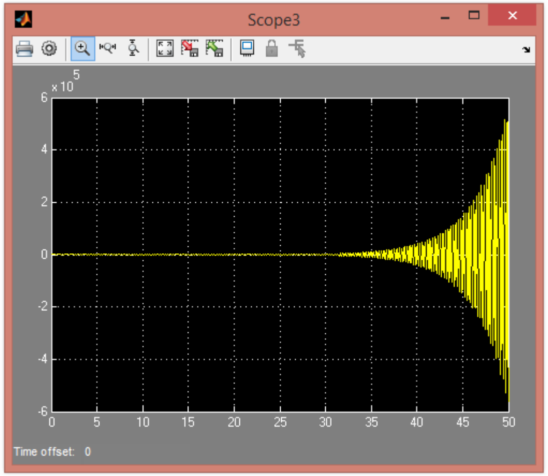
Conditional stability of closed loop systems

- Let us consider the transfer function

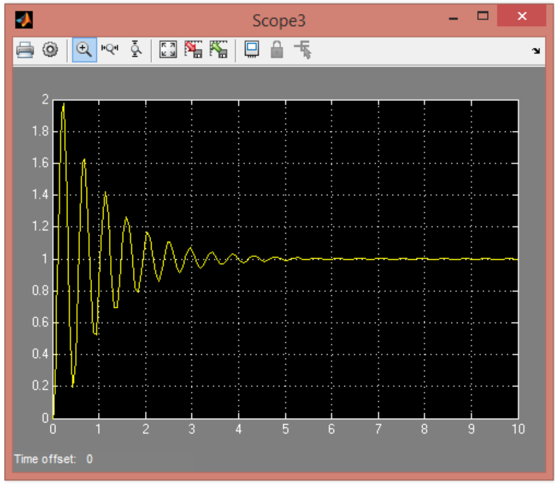
$$F(s) = \frac{10000(s+3)(s+9)}{3s^2(1+s)(s+32)(s+40)}$$



$k < k_1$



$k_1 < k < k_2$



$k > k_2$