Course of "Automatic Control Systems" 2023/24

Open-loop vs closed-loop

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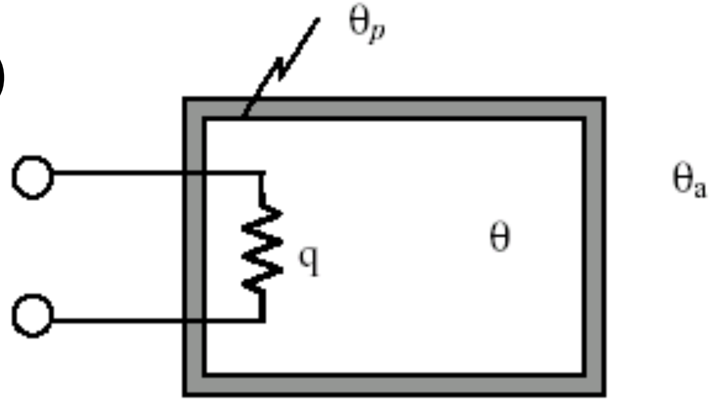
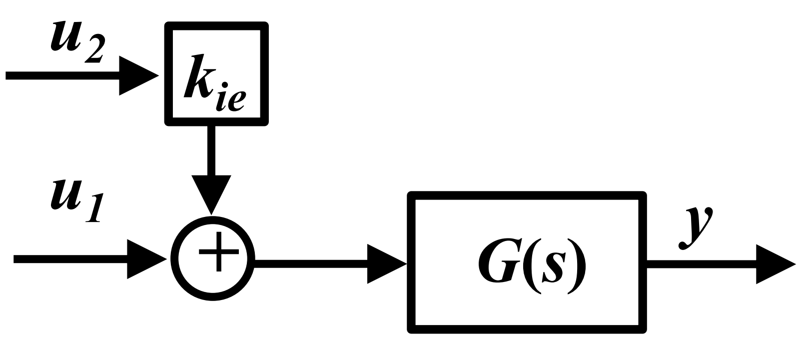
Example - electric oven model – resistance heating

- Energy conservation: $C_f\dot{\theta} = q - k_{ie}(\theta - \theta_a)$
- In order to control the temperature θ, it is needed to «tune» q, the power to be supplied in a such way $\theta = \theta_r$ (θ_r =***desired temperature***).
- By assuming the following parameters: $C_f = 500$ $Ws/°C$ e $k_{ie} = 5$ $W/°C$.
- In the Laplace domain ($u_1 = q$, $u_2 = \theta_a$, $y = \theta$):



$$C_f sY(s) + k_{ie}Y(s) = U_1(s) + k_{ie}U_2(s)$$

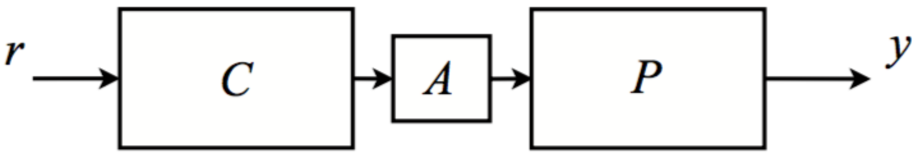
$$\Rightarrow \quad Y(s) = \underbrace{\frac{1/k_{ie}}{1 + \frac{C_f}{k_{ie}}s}}_{G(s)} \left(U_1(s) + k_{ie}U_2(s)\right)$$



- **At steady state ($\dot{\theta}$=0), if θ_r = 220°C and θ_a = 20°C, then $q = k_{ie}(\theta - \theta_a)$ = 1 KW, (at least for 3*t* = C_f / k_{ie}=300 s).**

Open-loop control system

- In an open-loop control system, controller ***C*** sends a signal to an actuating devise (or actuator) ***A*** which can modify the state of a process ***P*** to obtain the desired output response ***y***.



- If the dynamics of the process ***P*** are perfectly known and the control system is not subject to external (or environmental) disturbances, then the output ***y*** could in theory be made to perfectly track any desired reference signal ***r*** using the following controller (open-loop control):

$$C = (AP)^{-1} \quad \Longrightarrow \quad Y(s) = P(s)A(s)C(s)R(s) = (PA)(AP)^{-1}R = r.$$

- To be feasible, the controller must be causal (i.e., a system where the output depends on past and current inputs but not future inputs, in the domain of Laplace - the variable ***s*** - the degree of the denominator of the t.f. must be greater than or equal to the degree of the numerator).
- By denoting with ***G*=*PA***, the t.f. of the actuator-process series, then the controller **C**=$\boldsymbol{G^{-1}}$ will be feasible if the order of the numerator of ***G*** is equal to the order of the denominator.

Numerical example: open-loop control system

Let us to design an open loop control system for the system defined by the following tf,

$$G(s) = \frac{1}{s+2},$$

and verify the unit step response (***r*(*t*) = 1(*t*)**).

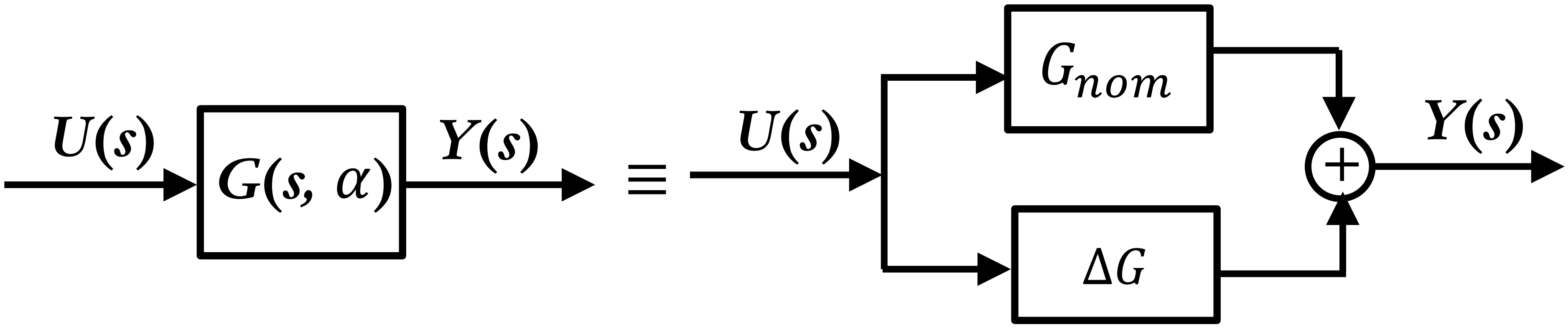
In this case we cannot employ a controller with tf as the inverse of G ($\boldsymbol{C}$=$\boldsymbol{G^{-1}}$), but we must add a filter ***F*(*s*)** in such a way that **C(s)=G(s)·F(s)** is feasable. Furthermore, the **F(s)** poles must be chosen witout affecting the control system requirements, i.e. the **F(s)** poles must be at least 5/6 times greater than the **G(s)** poles, and the gain of **F(s)** must be (ideally) unitary in the frequency range in which the system operates.

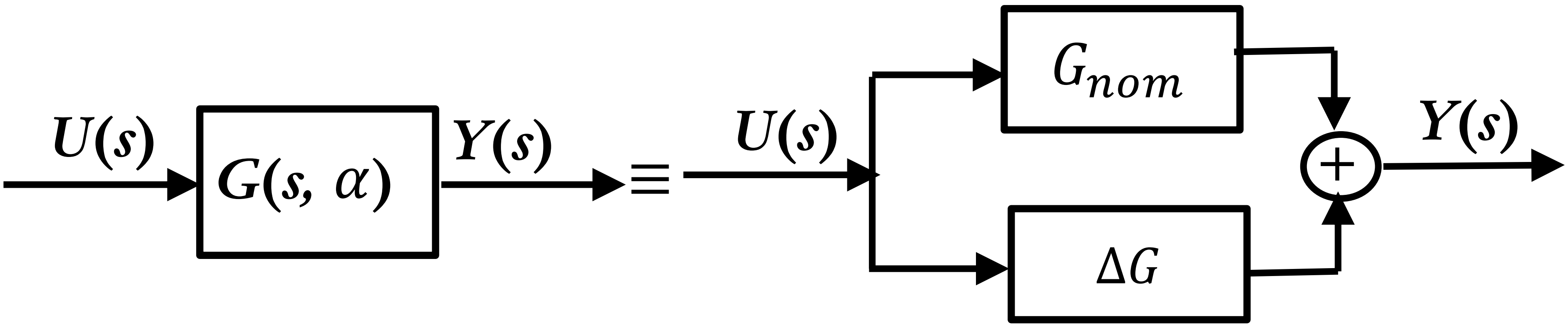
$$C(s) = G(s)^{-1}F(s) = (s+2)\frac{15}{s+15}.$$

- Assume the parametric variations (i.e., uncertainties) of the process-actuator series:
$$G(s) = P(s)A(s).$$
$G(s) = G(s, \alpha)$, where α is a parameter.

- By Taylor expansion of $\boldsymbol{G(s)}$ wrt $\boldsymbol{\alpha}$ around the nominal values $\boldsymbol{\hat{\alpha}}$ (***nom.***) and assuming the expansion at first order:
$$G(s,\alpha) \cong G(s,\hat{\alpha}) + \frac{\partial G}{\partial \alpha}\Big|_{\alpha=\hat{\alpha}} d\alpha = G_{nom}(s) + \Delta G(s),$$
where ΔG takes into account the uncertainty due to α.

- By using block diagrams:





$$Y = Y_{nom}(s) + \Delta Y(s).$$

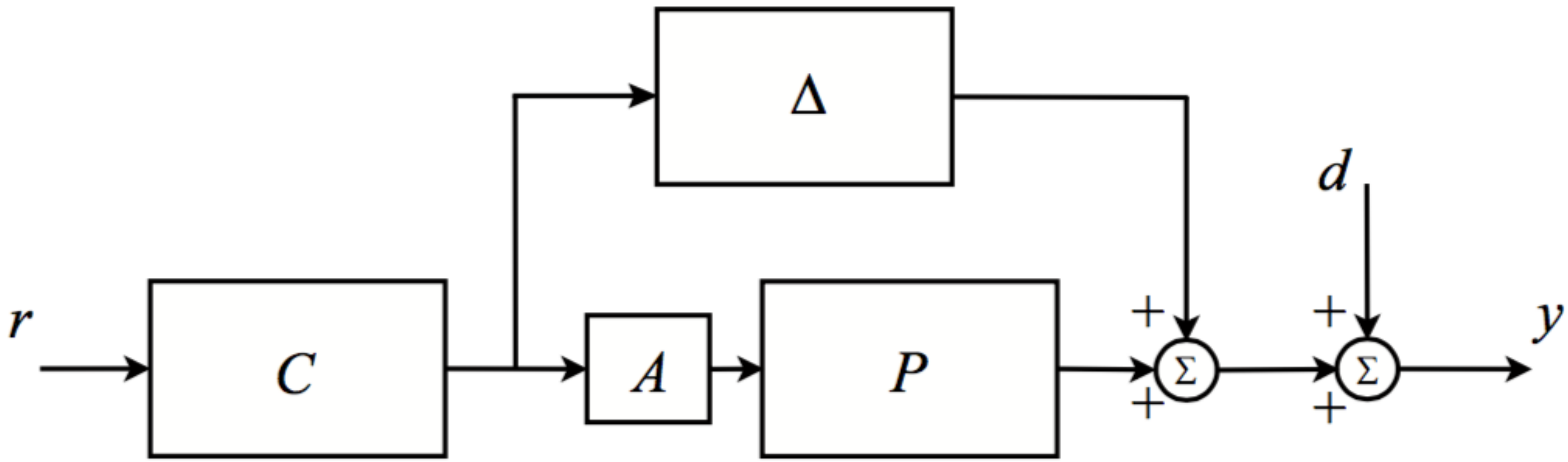
Therefore the variation of the output compared to the nominal output results:

$$\frac{\Delta Y}{Y_{nom}} = \frac{\Delta G\ U}{G_{nom} U} = \frac{\Delta G}{G},$$

and, the variations will be small only if ΔG is low.

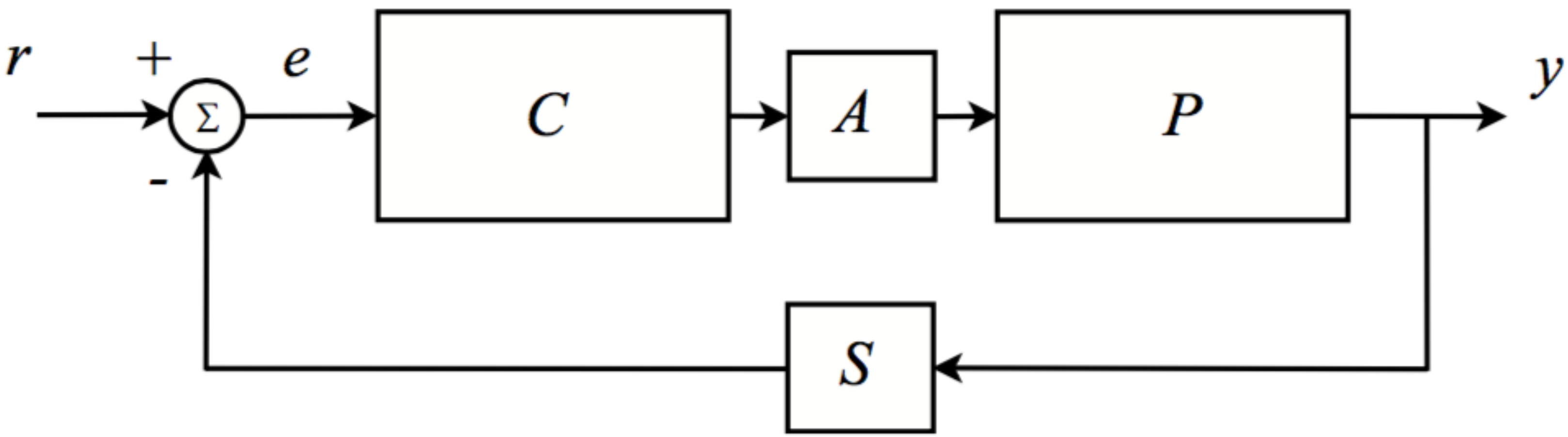
Limitations of the open-loop control system

Open loop control is not possible in most real cases: it is the inevitable presence of uncertainty in both the dynamics of the process to be controlled, and the environment in which it operates (the presence of disturbances), that necessitates the use of feedback control (open-loop control system is used in combination with closed loop).



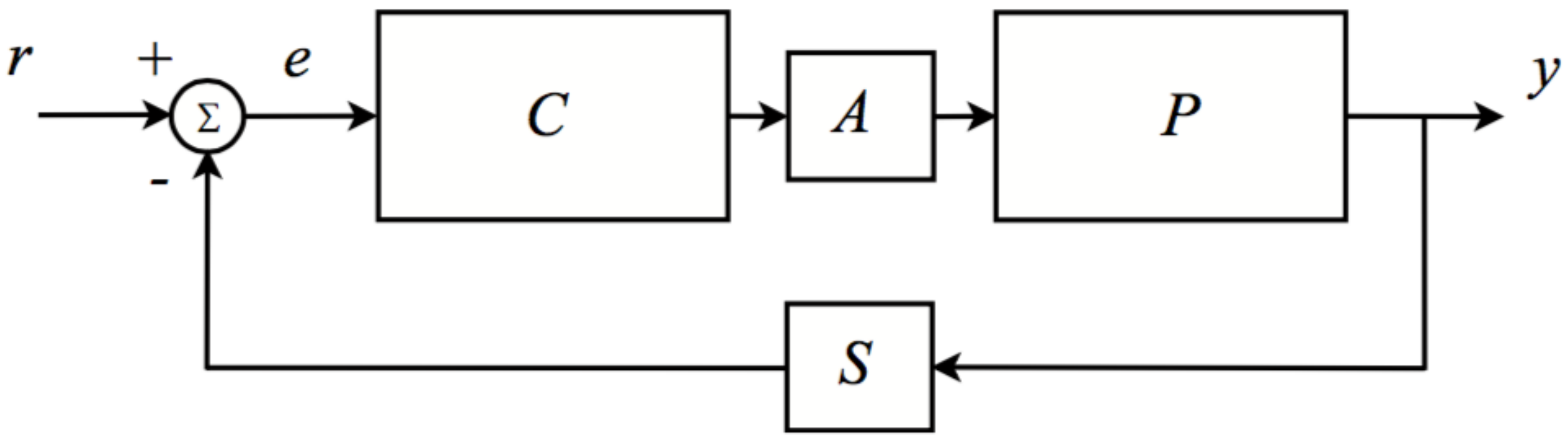
$$Y = D + (PA + \Delta)CR.$$

Since Δ and d are unknown, it is impossible to design an open-loop controller such that ***y=r.***



A closed-loop feedback control system uses a sensor ***S*** to continuously «feed back» a measurement of the actual output of the system.
This signal is compared with the desired output of the system(reference signal) to generate an **error signal**.
The error signal is the input of the controller, which, based on this signal, in turn generates a **control signal** which is the input to the plant

Closed-loop feedback control system



S=1, G=PA

$$Y(s) = P(s)A(s)C(s)\big(R(s) - Y(s)S(s)\big)$$

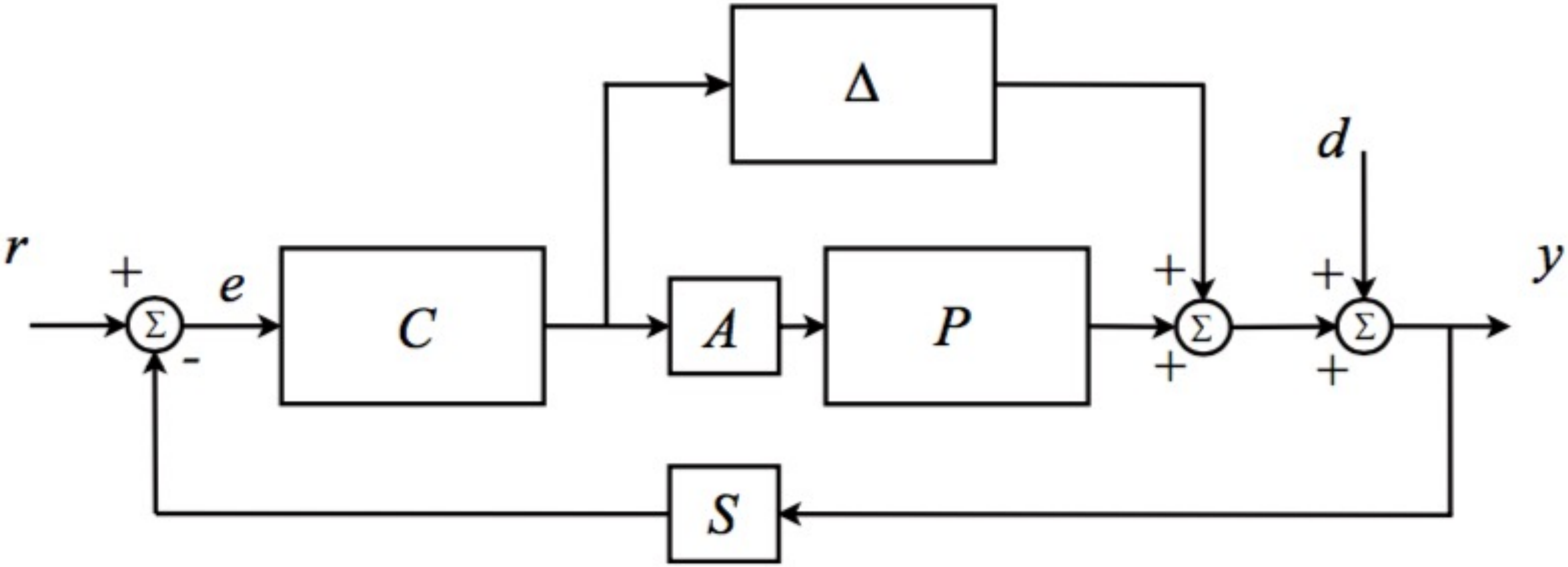
$$Y(s)(1 + G(s)C(s)) = G(s)C(s)R(s) \Rightarrow Y(s) = \frac{G(s)C(s)}{1 + G(s)C(s)} R(s)$$

***W*(*s*)**

- **If *GC*≫1, *y*≈*r*.**

Closed-loop feedback control system with plant uncertainty and disturbances - 1

By considering the effects of the plant uncertainty and disturbances on this system,



Assuming that ***S*=1** (excellent quality of sensor/transducer, which is not affected by noise),

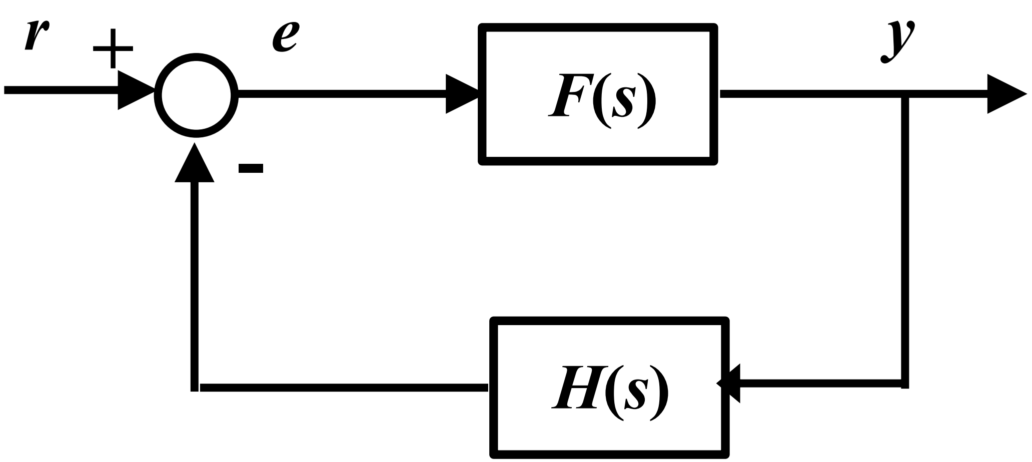
Closed-loop feedback control system with plant uncertainty and disturbances - 2

$$Y(s) = \frac{1}{1 + \big(G(s) + \Delta(s)\big)C(s)} D(s) + \frac{\big(G(s) + \Delta(s)\big)C(s)}{1 + \big(G(s) + \Delta(s)\big)C(s)} R(s)$$

In this case $\big(G(s) + \Delta(s)\big)C(s) \gg$ **1, i.e., C** $\gg$ **1,** the control system is now able to attenuate the effects of disturbance of ***d*** and the uncertainty Δ on ***y*, indeed**

$\boldsymbol{C} \ \rightarrow \infty, \boldsymbol{y} \approx r$, **for any finite values of *d*** and **Δ.**

Sensitivity analysis of feedback control system wrt uncertainty



***F*(*s*):** controller-process-actuator series

***H*(*s*):** trasducer/sensor

Uncertainties wrt to ***F*** or ***H***

- **In the case of uncertainty on *F*,**

$$W(s) = \frac{F(s)}{1 + F(s)H(s)} \Rightarrow \Delta W = \frac{\partial W}{\partial F}\frac{\partial \boldsymbol{F}}{\partial \alpha}d\alpha = \frac{1 + FH - FH}{(1 + \boldsymbol{F}H)^2}\Delta \boldsymbol{F}(s)$$

$$= \frac{\Delta \boldsymbol{F}(s)}{(1 + \boldsymbol{F}H)^2}$$

By defining the sensitivity coefficient of the closed-loop system wrt to ***F* uncertainty**

$$S_W^F = \frac{\Delta W}{W} = \frac{1}{1 + FH}\frac{\Delta \boldsymbol{F}}{F} \Rightarrow$$

Low value of S_W^F, by $FH \gg 1$ (operating on *F*, i.e., the controller ***C***).

Ideal control performance

- ***FH* ≫1** must be satisfied in a frequency range in which the spectrum of parametric variations of ***F*** is allocated**.**

- **In the case of uncertainty on *H*,**

$$W(s) = \frac{F\,(s)}{1 + F(s)H(s)} \;\Rightarrow\; \Delta W = \frac{\partial W}{\partial H}\frac{\partial H}{\partial \alpha} d\alpha = -\frac{F^2}{(1+\boldsymbol{F}H)^2}\Delta H$$

By defining the sensitivity coefficient of the closed-loop system wrt to ***H uncertainty***

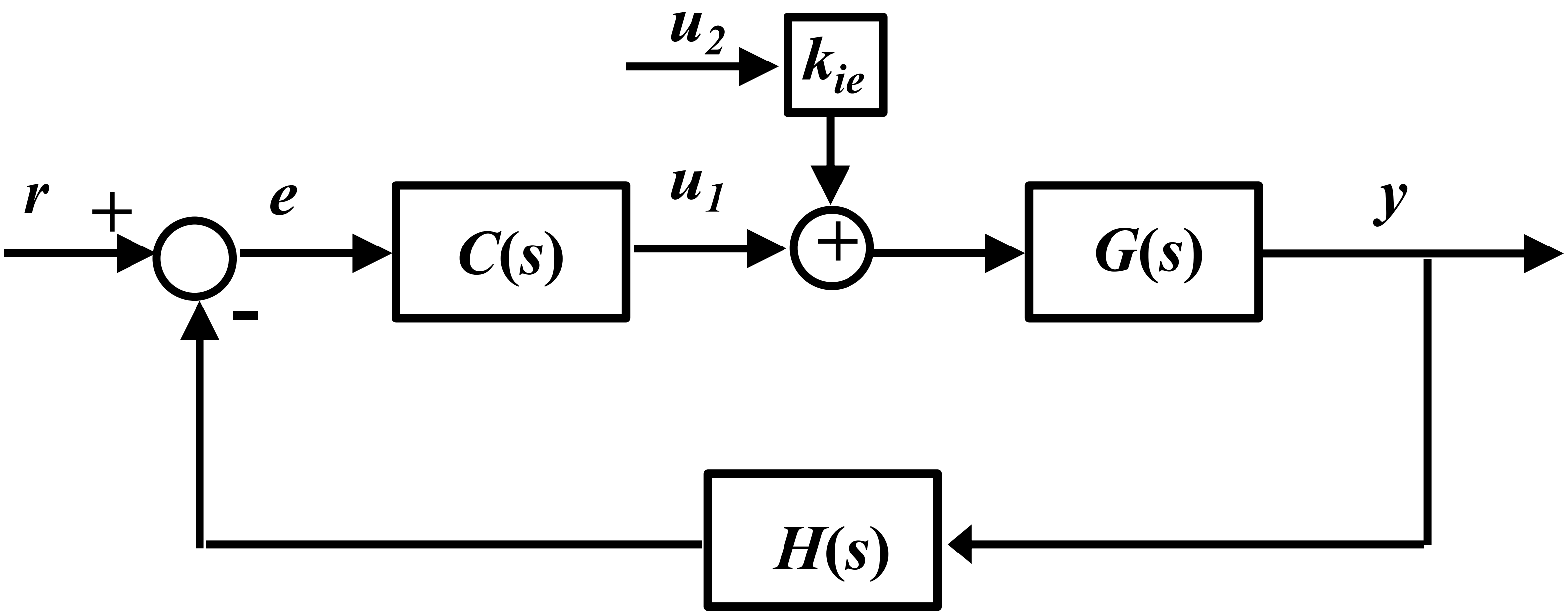
$$S_W^H = \frac{\Delta W}{W} = -\frac{FH}{1+FH}\frac{\Delta H}{H} \;\Rightarrow$$

Since $FH \gg 1$, $S_W^H = \frac{\Delta H}{H}$, that is, the system is maximally sensitive to parametric variations of ***H***. Therefore, it is necessary to use excellent quality transducers/sensors.

Power of feedback – ensure robustness

- **Robust control** of a negative feedback control system (low sensitivity to parametric variations of F and H in hp):
  - ***FH*** $\gg 1$
  - ***FH*** with a frequency range in which the spectrum of parametric variations of F is allocated
  - ***H*** low sensitive w.r.t parametric variation (of excellent quality).

Temperature regulation of an oven by closed-loop feedback control system



$$C(s) = k_C; H(s) = 1;$$

$$Y(s) = Y_r(s) + Y_d(s) = \frac{k_C G(s)}{1 + k_C G(s)} R(s) + \frac{k_{ie} G(s)}{1 + k_C G(s)} U_2(s).$$