Course of "Automatic Control Systems"
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Control system requirements

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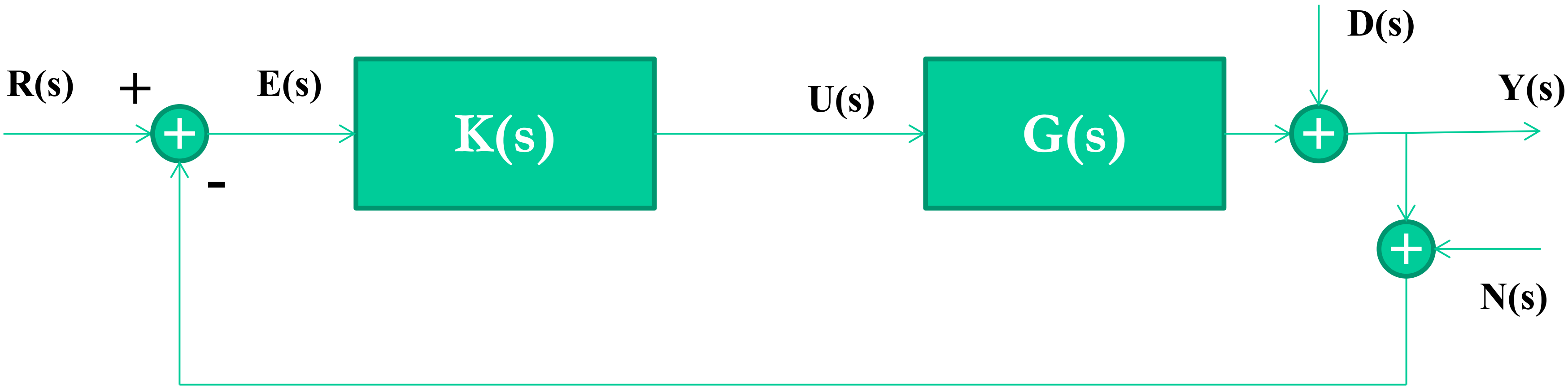
Closed loop transfer function

- A SISO closed loop control system in the Laplace domain can be indicated as



- G(s) plant to be controlled
- K(s) controller
- R(s) reference
- Y(s) controlled output
- U(s) control variable
- E(s) tracking error
- D(s) disturb
- N(s) measurement noise

Closed loop function

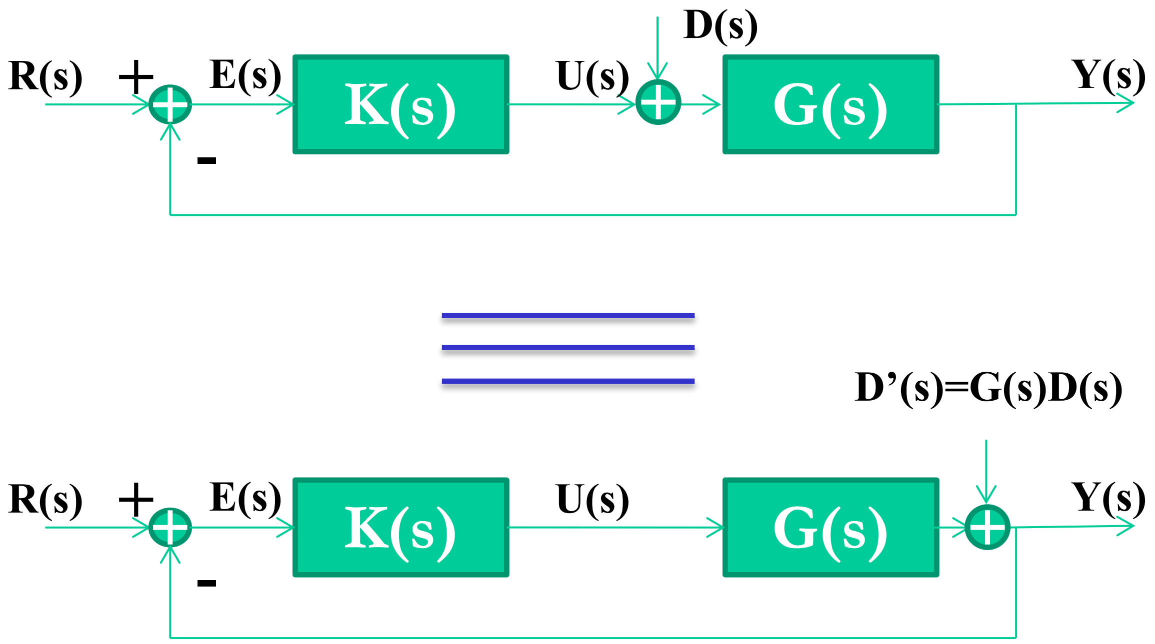
$$W(s) = \frac{Y(s)}{R(s)} = \frac{G(s)K(s)}{1 + G(s)K(s)}$$

(N(s)=0; D(s)=0)

Closed loop function

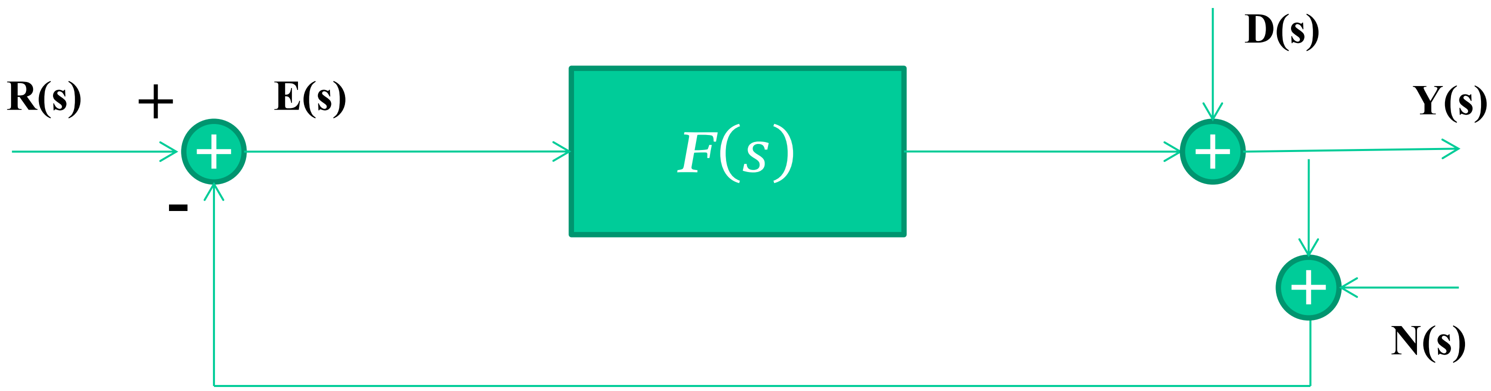
- The ***transfer function*** $\boldsymbol{G(s)}$ usually contains the plant and the actuator and the sensors dynamics.

- In the previous scheme, the ***disturb signal*** $\boldsymbol{D(s)}$ is additive on the output. In other cases, it could also be summed to the plant input.



Open loop function $F(s)$

- The transfer function given be the series of controller $K(s)$ and plant $G(s)$ is called ***Open Loop (O.L.) function*** $F(s) = G(s)K(s)$



- The O.L. function $F(s)$ assumes a main role in the control theory

- Indeed, it is easier to design a controller $K(s)$ able to modify as desired $F(s)$ instead of closed loop function $W(s) = \frac{G(s)K(s)}{1+G(s)K(s)}$

- It makes important to transform the closed loop requirements in terms of $F(s)$ constraints.

Control requirements

- The closed loop control requirements can be divided in four classes:
  - ***Stability***
  - ***Robust stability***
  - ***Steady-state performances***
  - ***Transient performances***
- The control requirements must be verified taking into account the limits of the actuators**.**
- In this lesson we will introduce the main parameters usually used to quantify the set of requirements
- Then we will care about how to transform the requirements in terms of open loop function $F(s)$ constraints

Stability

- The asymptotic stability of the nominal closed loop system is the most important property to be guaranteed.

- The asymptotic stability of the closed loop system implies that all the poles of the transfer function

$$W(s) = \frac{G(s)K(s)}{1 + G(s)K(s)} = \frac{F(s)}{1 + F(s)}$$

have negative real part.

- The poles of $W(s)$ are the roots of the polynomial

$$\text{num}\big(F(\boldsymbol{s})\big) + \text{den}\big(F(\boldsymbol{s})\big)$$

Stability

- However, it is difficult to design the controller $K(s)$ such that the roots of

$$\text{num}\big(F(\boldsymbol{s})\big) + \text{den}\big(F(\boldsymbol{s})\big)$$

have negative real parts

- Routh criterion is useful for the system analysis but not for the control design

- The ***Nyquist criterion*** provides a necessary and sufficient condition for the stability of the closed loop system related to the behavior of the open loop transfer function $F(s)|_{s=j\omega}$.

Robust Stability

- Due to ***model uncertainties*** (uncertain parameters, model simplification, linearization), it is usually required a controller assuring the asymptotic stability of the C.L. system with a certain 'safety margin'.

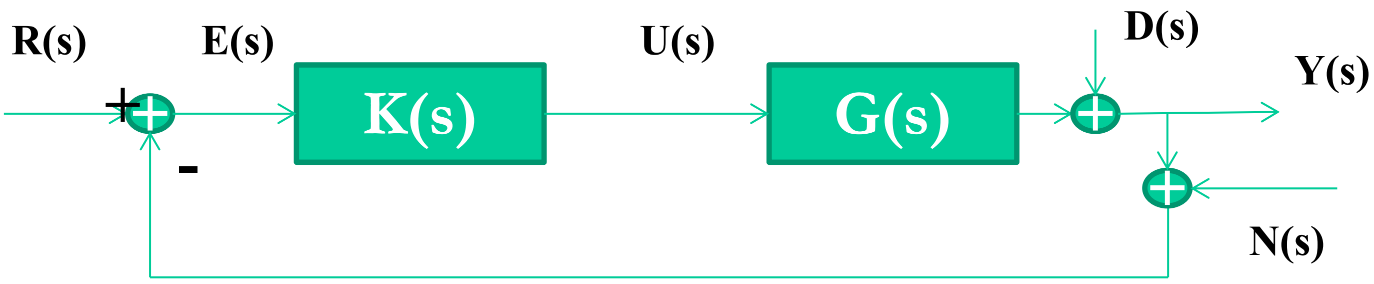
- This concept is called ***robust stability*** of the closed loop system.

- The robust stability of the closed loop system depends on the class and the range of uncertainties.

- The ***stability margins (gain and phase margins)*** relate the robust stability of the closed loop system to the behavior of the open loop transfer function $F(s)|_{s=j\omega}$

Closed loop tracking performances

- The performance of the closed loop system are evaluated in terms of
  - ***Tracking of the reference input***
  - ***Rejection of the disturbs***
  - ***Insensibility to the noise***



- When the stability of the C.L. system is guaranteed, the responses of the system can be divided in a transient and a steady-state parts.
- The ***steady-state performance*** cares about the steady-state behavior of the closed loop system while the ***transient performance*** cares about the tracking of the reference signal during the transient phase

Steady-state performance

- The steady-state performance depends on the class of input signals $R(s), D(s), N(s)$ and the type of transfer function $F(s)$

Tracking of the reference input $R(s)$

- Null or bounded steady-state error to ***polynomial inputs*** (step, ramp,…)
- Null or bounded error to ***sinusoidal inputs*** at fixed frequency

- ***Rejection of the disturbs D(s)***
  - Null or bounded steady-state error to ***polynomial inputs***
  - Bounded error to ***multi-frequency sinusoidal inputs***

- ***Insensibility to the noise $N(s)$***
  - Bounded error to ***multi-frequency sinusoidal inputs***

Due to the superposition principle, the three requirements are treated separately

Transient performance

- The ***transient performance*** are usually expressed in terms of tracking properties of the reference signal $R(s)$.

- $R(s)$ is usually assumed as a polynomial signal of order 0 (step)

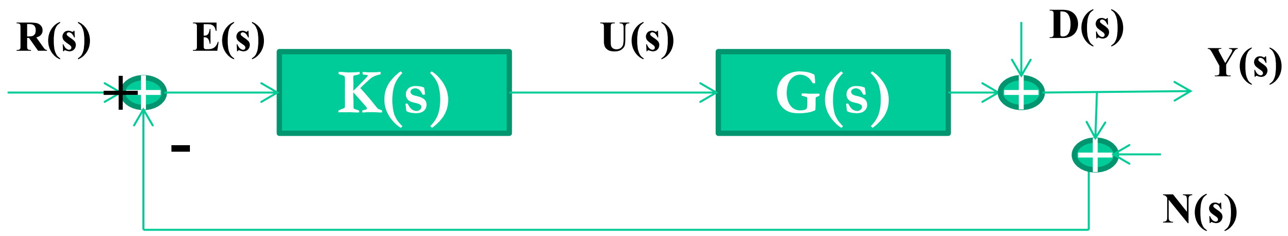
- The transient performance can be classified in
  - ***Dynamic precision performance*** (overshoot, oscillation period …)
  - ***Time response performance*** (rise time, peak time, settling time…)

- The rejection of the disturbs is usually not included among the transient performance because the transfer functions $R(s) \rightarrow Y(s)$ and $D(s) \rightarrow Y(s)$ have the same poles (excluding poles-zeros cancellation).

Closed loop functions

- From the previous analysis, it turns out that the closed loop performance depends on the relations between the input and outputs on the systems

- The closed loop system has three inputs $\boldsymbol{R(s), D(s), N(s)}$ and three outputs $\boldsymbol{E(s), U(s), Y(s)}$.



- The dynamic relations between inputs and outputs of the systems are expressed by 9 transfer functions.

Closed loop functions

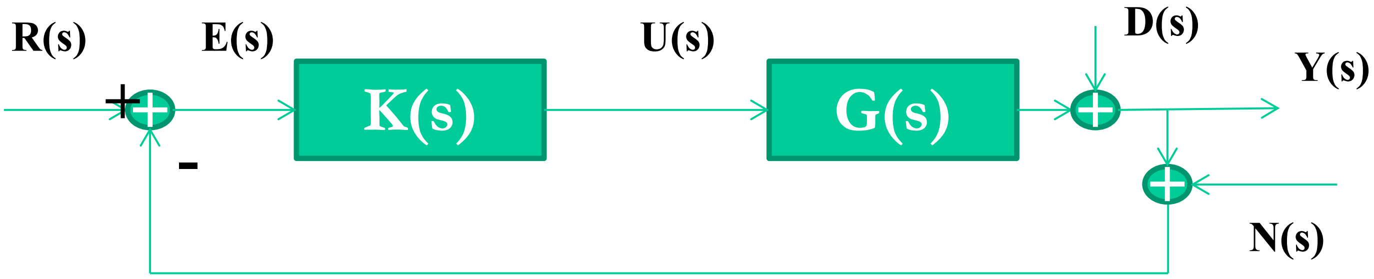
- The 9 transfer functions connecting inputs and outputs depends by three main functions

$$\begin{pmatrix} Y(s) \\ E(s) \\ U(s) \end{pmatrix} = \begin{pmatrix} \boldsymbol{T(s)} & \boldsymbol{S(s)} & * \\ * & * & * \\ \boldsymbol{Q(s)} & * & * \end{pmatrix} \begin{pmatrix} R(s) \\ D(s) \\ N(s) \end{pmatrix}$$

- $\boldsymbol{T(s) = \frac{G(s)K(s)}{1+G(s)K(s)}}$ ***COMPLEMENTARY SENSITIVITY FUNCTION***
- $\boldsymbol{S(s) = \frac{1}{1+G(s)K(s)}}$ ***SENSITIVITY FUNCTION***
- $\boldsymbol{Q(s) = \frac{K(s)}{1+G(s)K(s)}}$ ***CONTROL SENSITIVITY FUNCTION***

Closed loop functions

- The nine input-output relations can be written as functions of $\boldsymbol{T(s)}, \boldsymbol{S(s)}, \boldsymbol{Q(s)}$



$$\begin{pmatrix} Y(s) \\ E(s) \\ U(s) \end{pmatrix} = \begin{pmatrix} \boldsymbol{T(s)} & \boldsymbol{S(s)} & -\boldsymbol{T(s)} \\ \boldsymbol{S(s)} & -\boldsymbol{S(s)} & -\boldsymbol{S(s)} \\ \boldsymbol{Q(s)} & -\boldsymbol{Q(s)} & -\boldsymbol{Q(s)} \end{pmatrix} \begin{pmatrix} R(s) \\ D(s) \\ N(s) \end{pmatrix}$$

Ideal control performance

- The performance of the closed loop system have been classified in
  - ***Tracking of the reference input*** → $\boldsymbol{T(s) = 1}$
  - ***Rejection of the disturbs*** → $\boldsymbol{S(s) = 0}$
- This is in accordance with the fact that

$$\boldsymbol{T(s) + S(s) = \frac{G(s)K(s)}{1 + G(s)K(s)} + \frac{1}{1 + G(s)K(s)} = 1}$$

Ideal control performance

- However this choice has two main drawbacks

1. $Q(s) = T(s)G^{-1}(s) = G^{-1}(s)$

   If the plant is strictly proper, $\mathrm{Q(s)}$ is improper and it causes a very high or infinity request of the control input for $\mathrm{t} \to 0$ (initial value theorem)

2. $Y(s) = T(s)R(s) + S(s)D(s) - T(s)N(s)$,

   The noise is not filtered by the system

Real control performance

- To overcome these problems the control theory takes advantage to the fact that the input signals have usually different intervals of frequency
  - ***Reference and disturbs at low frequencies***
  - ***Noise at high frequencies***
- So, a rule of thumb is to choose
  - $\boldsymbol{T(s) = 1}$ **and** $\boldsymbol{S(s) = 0}$ **at low frequencies**
  - $\boldsymbol{T(s) = 0}$ **at high frequencies**
- In this way also the problem related to the input signal is reduced.