Course of "Automatic Control Systems"
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Block diagrams algebra

Prof. Francesco Montefusco

Department of Economics, Law, Cybersecurity, and Sports Sciences

Università degli Studi di Napoli Parthenope

francesco.montefusco@uniparthenope.it

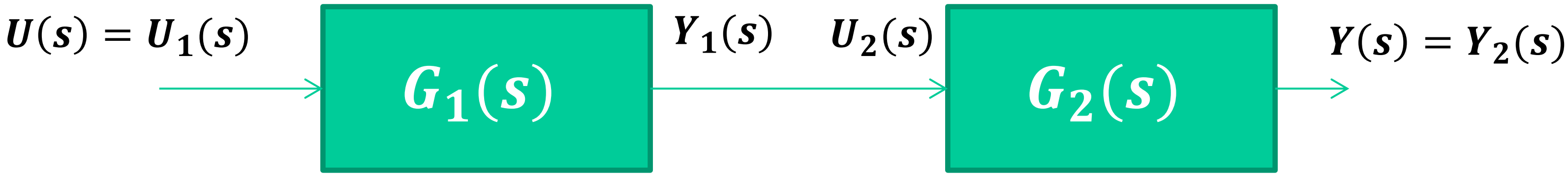
Team code: **mfs9zfr**

Interconnections of LTIs

- In this lesson we consider the ***interconnection problem of linear systems in the Laplace domain***
- Three types of interconnections will be presented:
  - ***Series***
  - ***Parallel***
  - ***feedback***

Series

- Let us consider two transfer functions $G_1(s)$ and $G_2(s)$
- ***The series interconnection between $G_1(s)$ and $G_2(s)$ is represented as***



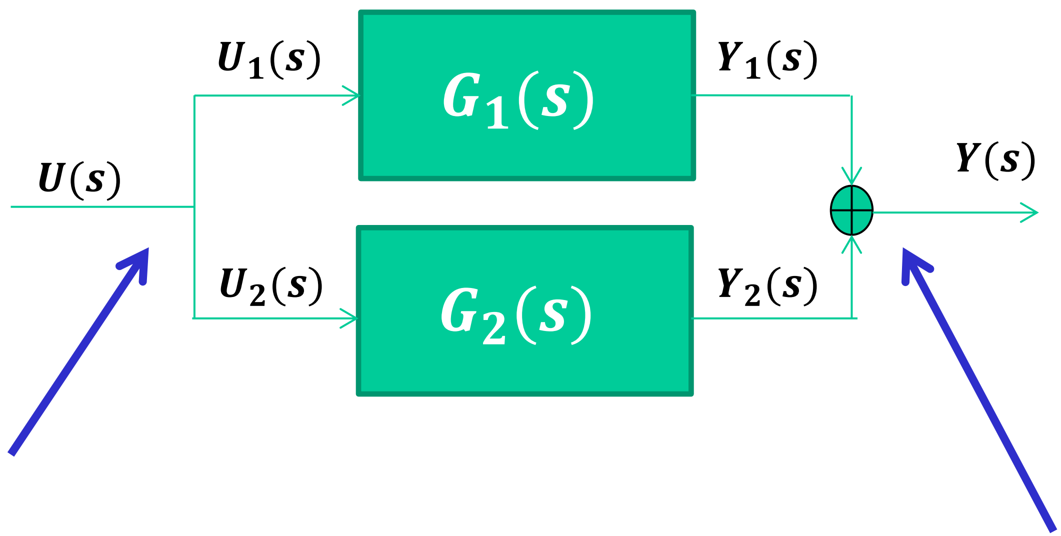
- The relation between $U(s)$ and $Y(s)$ is given by

$$Y(s) = G_2(s)U_2(s) = G_2(s)Y_1(s) = \underbrace{G_2(s)G_1(s)}U(s)$$

Series interconnection

$$G(s) = G_2(s)G_1(s)$$

- Let us consider two transfer functions $G_1(s)$ and $G_2(s)$
- ***The parallel interconnection between $G_1(s)$ and $G_2(s)$ is represented as***



Due to the ***interconnection node***

$$U_1(s) = U_2(s) = U(s).$$

Due to the ***sum node***

$$Y(s) = Y_1(s) + Y_2(s)$$

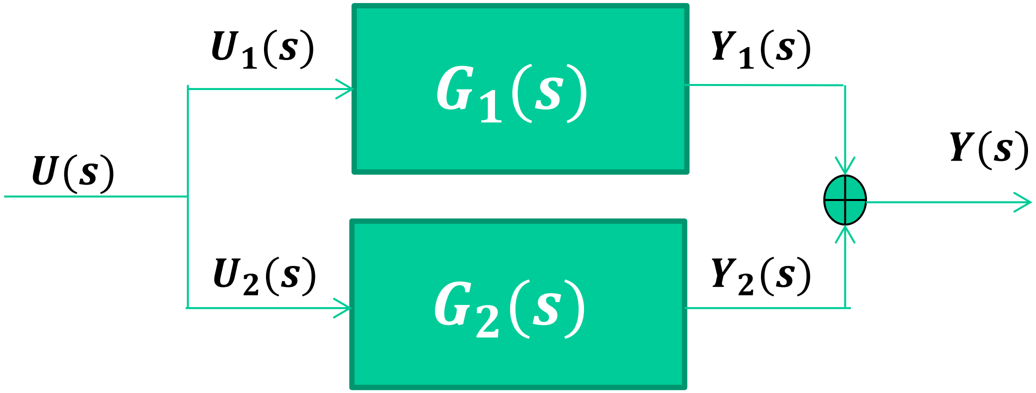
Parallel (2/2)

- The relation between $U(s)$ and $Y(s)$ is given by

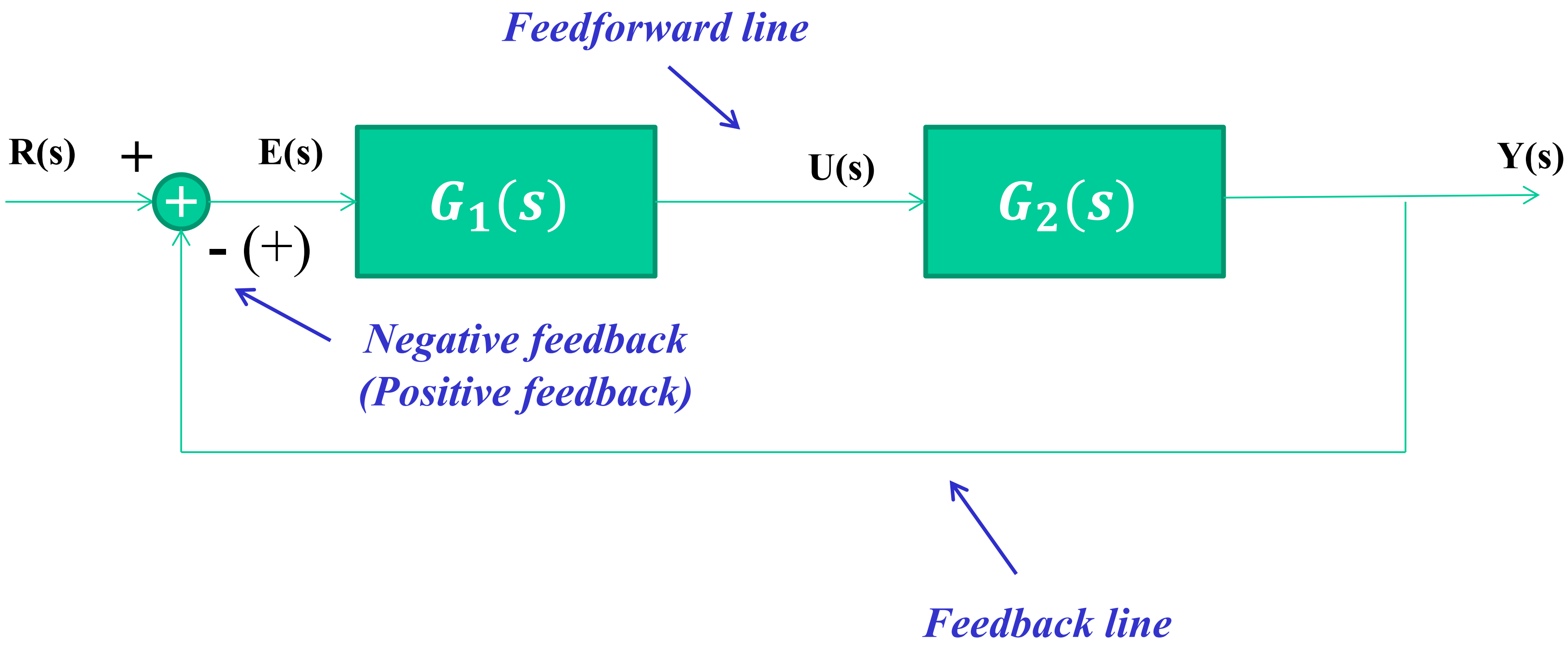
$$Y(s) = Y_1(s) + Y_2(s) = \left(G_1(s) + G_2(s)\right)U(s)$$

Parallel interconnection

$$G(s) = G_1(s) + G_2(s)$$



- Let us consider two transfer functions $G_1(s)$ and $G_2(s)$
- ***The feedback interconnection is represented as***



- In case of SISO system with negative feedback interconnection we have that

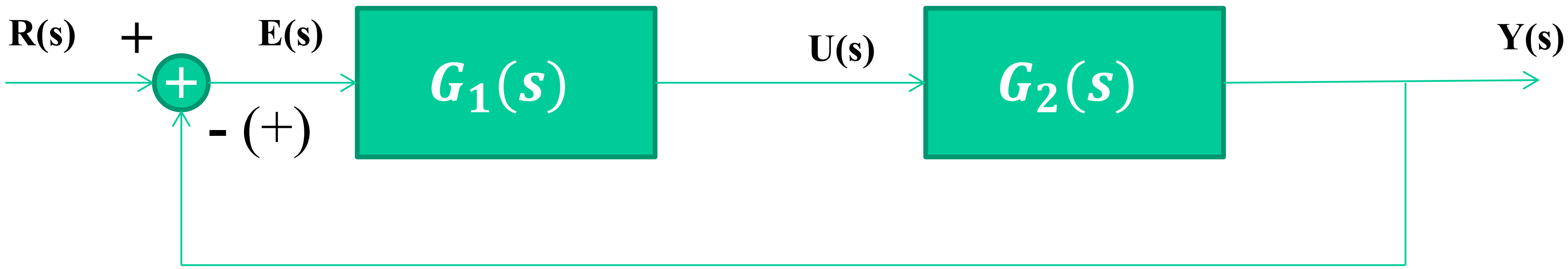
$$Y(s) = G_2(s)G_1(s)E(s) \qquad \textit{Feedforward line}$$

with $E(s) = R(s) - Y(s)$. Hence

$$Y(s) + G_2(s)G_1(s)Y(s) = G_2(s)G_1(s)R(s)$$

$$\Downarrow$$

$$Y(s) = \frac{G_2(s)G_1(s)}{1 + G_2(s)G_1(s)}R(s)$$



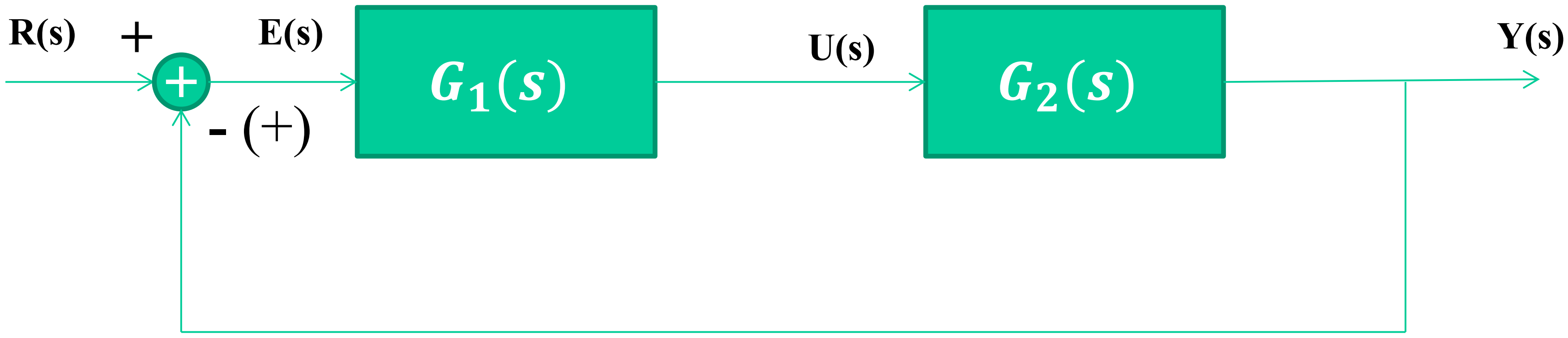
- The relation between $U(s)$ and $Y(s)$ is given by

$$Y(s) = \frac{G_2(s)G_1(s)}{1 + G_2(s)G_1(s)} R(s)$$

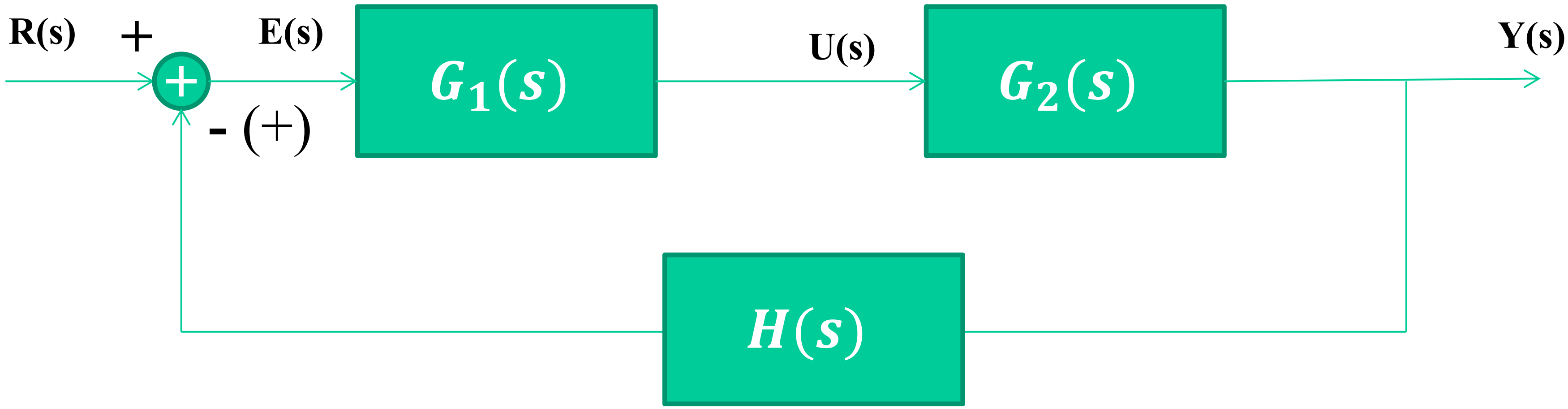
Negative feedback

$$Y(s) = \frac{G_2(s)G_1(s)}{1 - G_2(s)G_1(s)} R(s)$$

Positive feedback



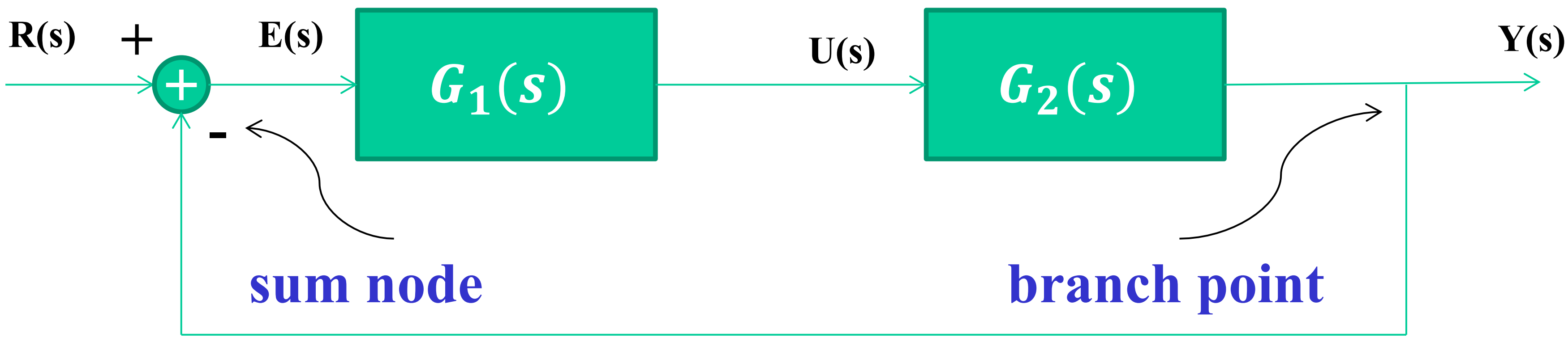
- The feedback interconnection can be also generalized with a block $H(s)$ on the feedback line.



$$Y(s) = \frac{G_2(s)G_1(s)}{1 \underset{(-)}{+} G_2(s)G_1(s)H(s)} R(s)$$

Sum nodes and branch points

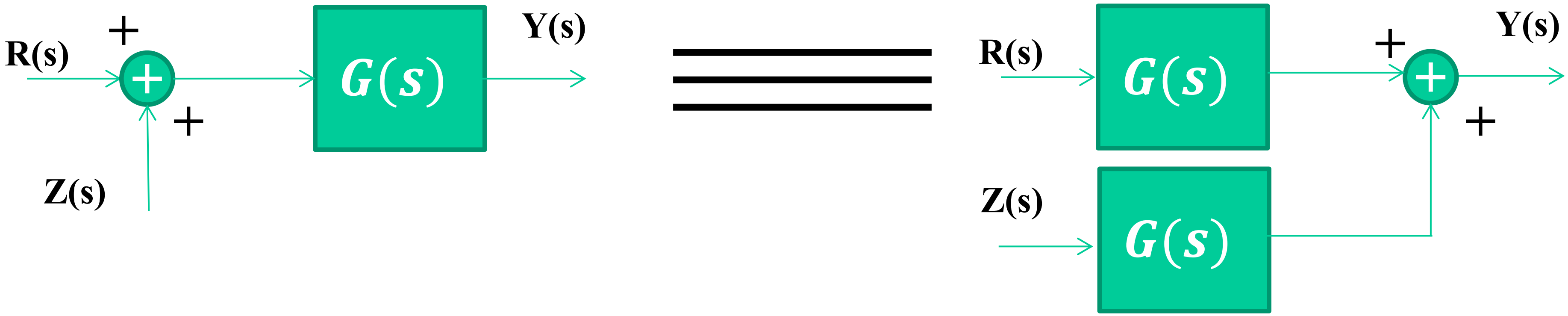
- In the block diagrams we usually find sum nodes and branch points



- It is sometime useful to move these elements in order to simplify the overall diagram

- In the following we present input-output equivalent schemes where the sum node or the branch points have been moved

Sum nodes



Branch points