UNIVERSITÀ DEGLI STUDI DI NAPOLI
PARTHENOPE

Artificial Intelligence

Adversarial Search

LESSON 8

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Adversarial Search

- Game theory
- Optimal Decisions in Games
  - Minimax decisions
  - $\alpha - \beta$ pruning
  - Monte Carlo Tree Search
  - Games of chance
  - Limitations of game search algorithms

Monte Carlo Tree Search

- For more complex games, like Go, alpha-beta search with limited depth remains an unfeasible way to walk
- A possible effective and different alternative is Monte Carlo Tree Search (MCTS)
  - It doesn't use a heuristic evaluation function
  - Starting from a state, its value is estimated as an average utility over several simulations (also called playouts) of the game
  - The simulation chooses alternating moves for the players until a terminal position is reached

Playout Policy

- How do we choose what moves to make during the playout?
- Playout policy helps to decide what moves to make during the playout
  - It biases the moves toward good ones
    - Learned by self-play by using neural networks (e.g., Go)
    - Game-specific heuristics (e.g., Chess, Othello)
      - For instance, “consider capture moves” or “take the corner square”

Pure Monte Carlo Search

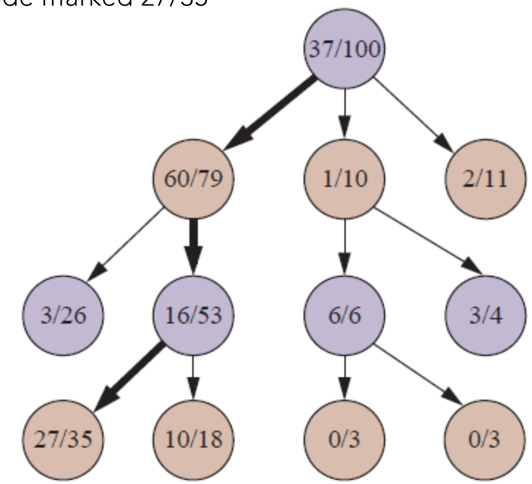
- Given a playout policy
  - From what positions do we start the playouts?
  - How many playouts do we allocate to each position?
- Pure Monte Carlo search
  - N simulations starting from the current state of the game
  - Track among the possible moves the one with the highest win percentage
  - As N increases, this strategy could converge to optimal play
    - However, in general, it is not enough
- A selection policy is needed to selectively focus the computational resources on the important parts of the game tree
  - A trade-off between exploration and exploitation

MCTS

- Iteratively maintains a search tree and grows it at each step, performing
  - Selection
    - Starting from the root, chooses, according to the selection strategy, a move to a successor node
    - Repeat the process going down the tree to a leaf
  - Expansion
    - Grows the search tree by generating a new child of the selected node
  - Simulation
    - Performs a playout from the newly generated child node
  - Back-propagation
    - The result of the simulation updates all the nodes going up to the root
- The four steps are repeated for a set number of iterations, or until the allotted time has expired
  - then return the move with the highest number of playouts

MCTS steps: Selection

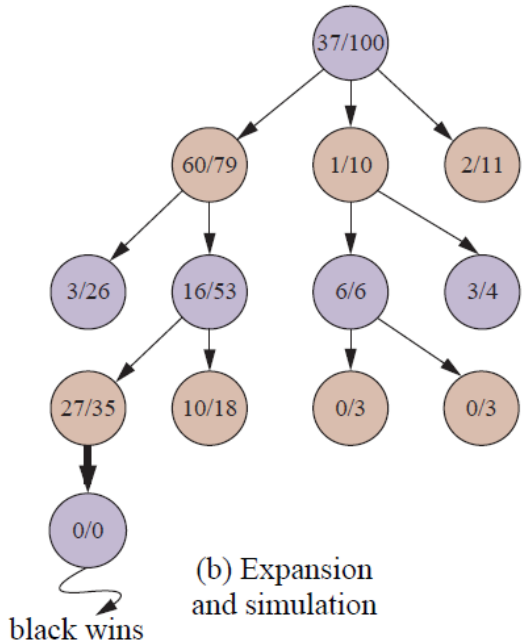
- A search tree with the root representing a state where white has just moved, and white has won 37 out of the 100 playouts
  - The thick arrow shows the selection of a move by black that leads to a node where black has won 60/79 playouts
    - This is the best win percentage among the three moves, so selecting it is an example of exploitation
    - But it would also have been reasonable to select the 2/11 node for exploration purposes
    - Selection continues to the leaf node marked 27/35



(a) Selection

MCTS steps: Expansion and Simulation

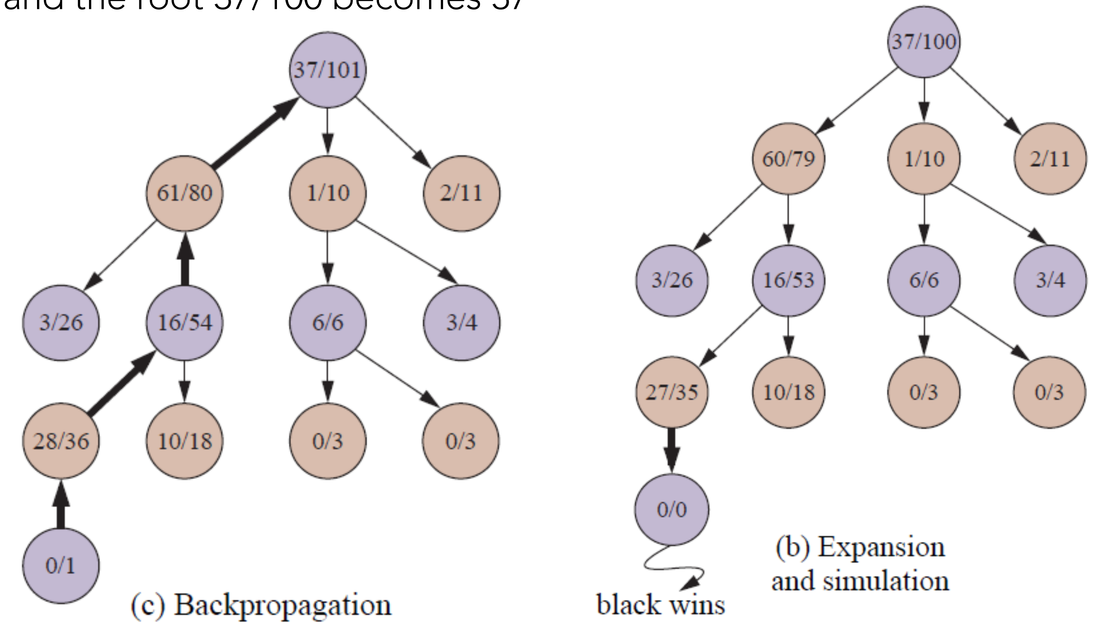
- The Expansion step grows the search tree by generating a new child of the selected node
  - the new node marked with 0/0
- Simulation moves for both players according to the playout policy
  - The moves are not recorded in the search tree
    - The simulation here results in a win for black



(b) Expansion and simulation

MCTS steps: Backpropagation

- The simulation results are used to update all the search tree nodes going up to the root
- Black nodes are incremented in both the number of wins and the number of playouts
  - 27/35 becomes 28/36 and 60/79 becomes 61/80
- Since white lost, the white nodes are incremented in the number of playouts only
  - 16/53 becomes 16/54 and the root 37/100 becomes 37



(c) Backpropagation

(b) Expansion and simulation

Selection Policy

- Upper Confidence Bounds to Trees (UCT)
  - Ranks each possible move according to the Upper Confidence Bound (UCB1) formula

$$UCB1(n) = \frac{U(n)}{N(n)} + C \times \sqrt{\frac{\log N(\textsc{Parent}(n))}{N(n)}}$$

  - where
    - U(n) is the total utility of all playouts that went through node n
    - N(n) is the number of playouts through node n
      - The first term is the average utility, the exploitation term
    - PARENT(n) is the parent node of n in the tree

UCT MCTS Algorithm

```
function MONTE-CARLO-TREE-SEARCH(state) returns an action
  tree ←  NODE(state)
  while IS-TIME-REMAINING() do
       leaf  ←  SELECT(tree)
       child ←  EXPAND(leaf )
       result ←  SIMULATE(child)
       BACK-PROPAGATE(result, child)
  return the move in ACTIONS(state) whose node has highest number of playouts
```

- The time to compute a playout is linear in the depth of the game tree
  - because only one move is taken at each choice point

On Monte Carlo Search

- Monte Carlo search has an advantage over alpha–beta for games where
  - the branching factor is very high (e.g, Go)
  - when it is difficult to define a good evaluation function
  - It relies on the aggregate of many playouts and thus is not as vulnerable to a single error
- MCTS and evaluation functions could be combined
  - playout for a certain number of moves, then truncate the playout and apply an evaluation function
- Monte Carlo search can be applied to brand-new games
  - No experience to consider for defining an evaluation function
  - No additional information and only the game rules
  - Good policies can be learned using neural networks trained by self-play alone

State-of-the-art: Checkers and Othello

- 1951
  - First computer player by Christopher Strachey
- 1994
  - The computer program Chinook ends the 40-year reign of human champion Marion Tinsley
    - Library of opening moves from grandmasters
    - deep search algorithm
    - A good move evaluation function (based on a linear model)
    - A database for all positions with 8 pieces or fewer
      - 443,748,401,247 positions
- Othello:
  - human champions refuse to compete against computers, which are too good
    - Programs have been at superhuman level since 1997

State-of-the-art: Chess

- 1997
  - Deep Blue defeats human champion Gary Kasparov in six-game match
    - alpha–beta search
    - 200.000.000 position evaluations per second
    - Very sophisticated evaluation function
    - Undisclosed methods for extending some lines of search up to 40 ply
  - Modern programs (e.g., Stockfish or AlphaZero) are better

State-of-the-art: Go

- For long, Go was considered as the Holy Grail of AI due to the size of its game tree
  - On a 19x19 grid, the number of legal positions is about 2×10^{170} , b >300
  - This results in about 10^{800} games, considering a length of 400 or less
- 2010-2014
  - Using Monte Carlo tree search and machine learning, computer players reach low dan levels
- 2015-2018
  - Google DeepMind invents AphaGo (pattern knowledge bases to suggest plausible moves)
    - 2015: AlphaGo beats Fan Hui, The European Go Champion
    - 2016: AlphaGo beats Lee Sedol (4-1), a 9-dan grandmaster
    - 2017: AlphaGo beats Ke Jie, 1st world human player
      - AlphaGo combines MCTS and Deep Learning with extensive training, both from human and computer play
  - In 2018, AlphaZero surpassed Alphago by learning through self-play without any expert human knowledge and without access to any past games

Types of Games

	deterministic	chance
perfect information	chess, checkers, go, othello	Backgammon, monopoly
imperfect information	battleships, blind tictactoe	bridge, poker, scrabble

Non-deterministic (Stochastic) Games

- In real life, many unpredictable external events can put us into unforeseen situations
- Games that mirror this unpredictability are called stochastic games
  - Include a random element, such as
    - Explicit randomness: rolling dice
    - Unpredictable opponents: ghosts respond randomly
    - Actions may fail: when moving a robot, the wheels might slip

Non-deterministic (Stochastic) Games

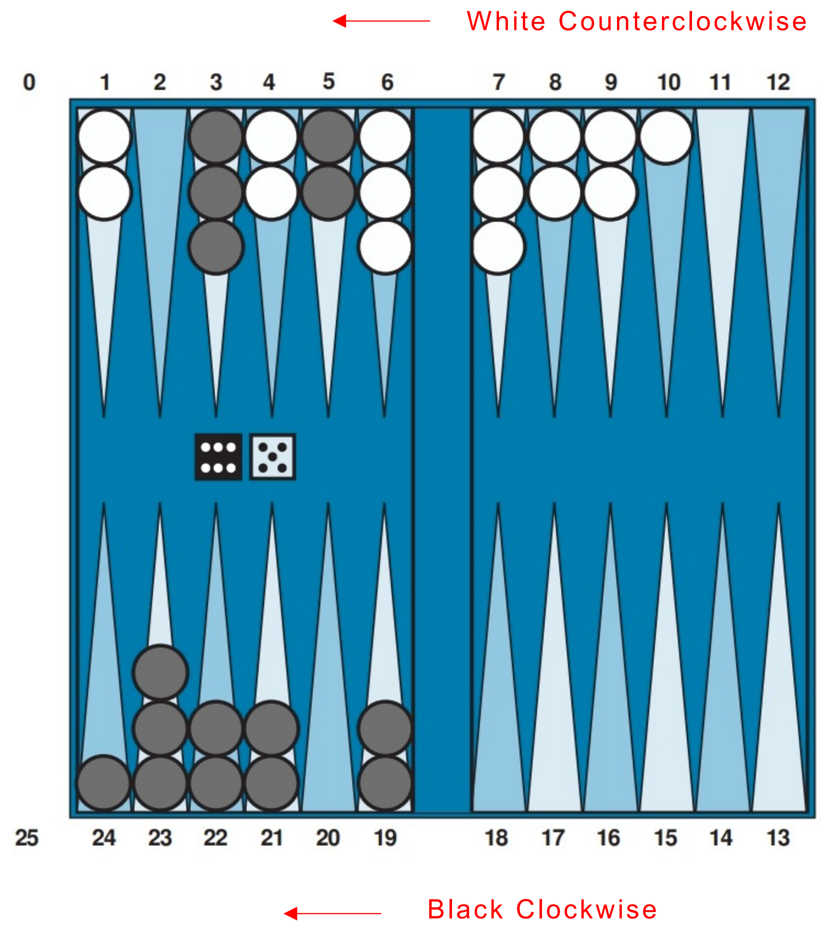
- In a game tree, this random element can be modeled with chances nodes
  - map a state-action pair to the set of possible outcomes, along with their respective probability
- This is equivalent to considering the environment as an extra random agent player that moves after each of the other players

Non-deterministic (Stochastic) Games

- Backgammon is an example that combines luck and skill
- The goal of the game is to move all one's pieces off the board
  - Black moves clockwise toward 25
  - White moves counterclockwise toward 0
  - A piece can move to any position unless multiple opponent pieces are there
    - if there is one opponent, it is captured and must start over

Non-deterministic games: Backgammon

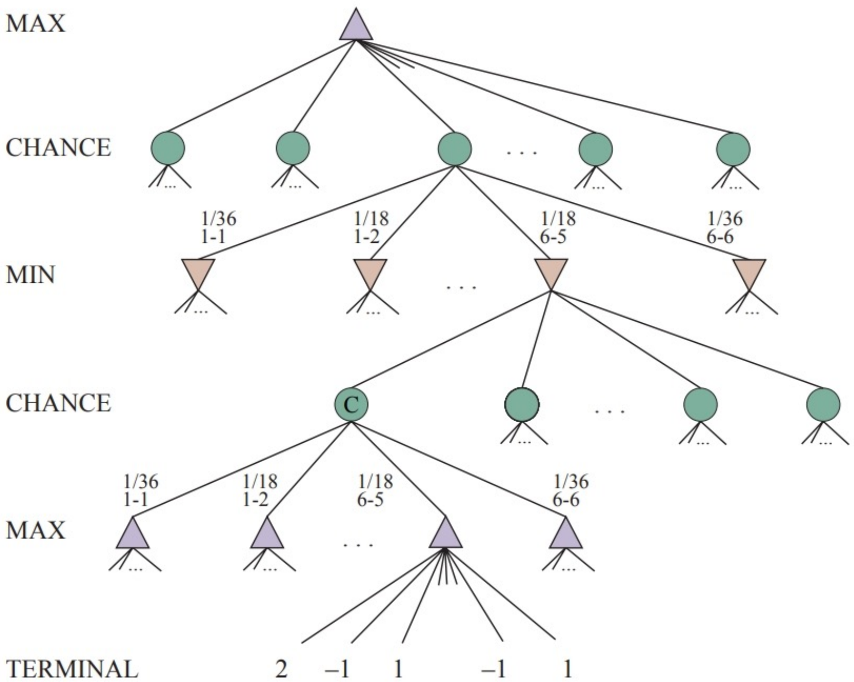
- Black has rolled 6–5 and must choose among four legal moves:
  - (5–11,5–10)
  - (5–11,19–24)
  - (5–10,10–16)
  - (5–11,11–16)
- (5–11,11–16) means
  - move one piece from position 5 to 11 and then move a piece from 11 to 16
- Now, Black knows what moves can be made but does not know what White is going to roll and thus does not know what White's legal moves will be
- so Black cannot construct a standard game tree



Non-deterministic games in general

- A game tree in backgammon must include chance nodes in addition to MAX and MIN nodes
- In non-deterministic games, chance introduced by dice, card-shuffling

- The branches leading from each chance node denote the possible dice rolls
  - each branch is labeled with the roll and its probability
  - There are 36 ways to roll two dice, each equally likely
    - 6–5 is the same as a 5–6, there are only 21 distinct rolls
    - The six doubles (1–1 through 6–6) each have a probability of 1/36, so P(1–1) = 1/36
    - The other 15 distinct rolls each have a 1/18 probability



ExpectMiniMax

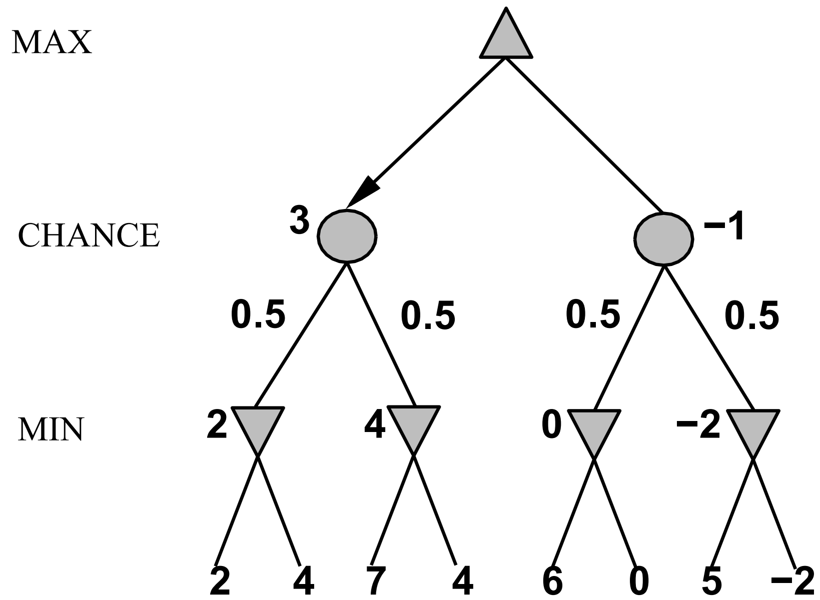
- We want to pick the move that leads to the best position
  - However, positions do not have definite minimax values
  - We can only calculate the expected value of a position: the average over all possible outcomes of the chance nodes
  - Expectminimax value
    - Generalization of the minimax value for deterministic games

$$\textsc{Expectiminimax}(s) = \begin{cases} \textsc{Utility}(s, \quad) & \text{if } \textsc{Is-Terminal}(s) \\ \max_a \textsc{Expectiminimax}(\textsc{Result}(s,a)) & \text{if } \textsc{To-Move}(s) = \textsc{max} \\ \min_a \textsc{Expectiminimax}(\textsc{Result}(s,a)) & \text{if } \textsc{To-Move}(s) = \textsc{min} \\ \sum_r P(r) \textsc{Expectiminimax}(\textsc{Result}(s,r)) & \text{if } \textsc{To-Move}(s) = \textsc{chance} \end{cases}$$

    - r is a possible dice roll
    - RESULTS(s,r) same state as s with r as the result of the dice roll

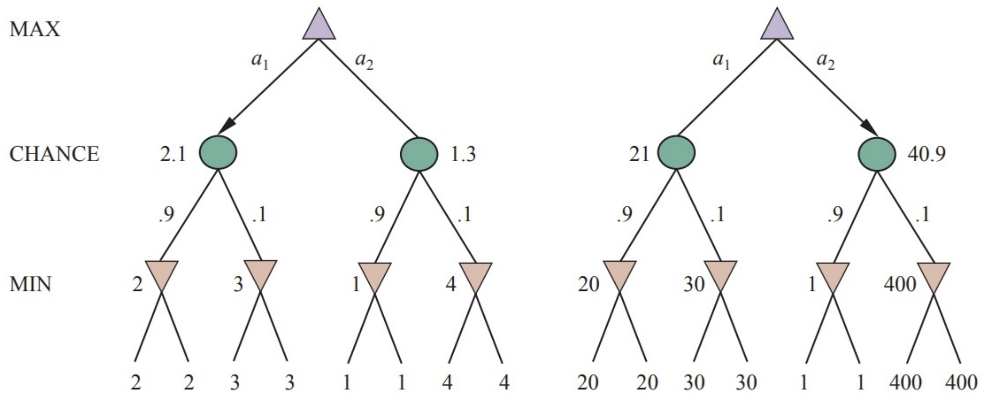
Non-deterministic games in general

- Simplified example with coin-flipping:



Evaluation Functions

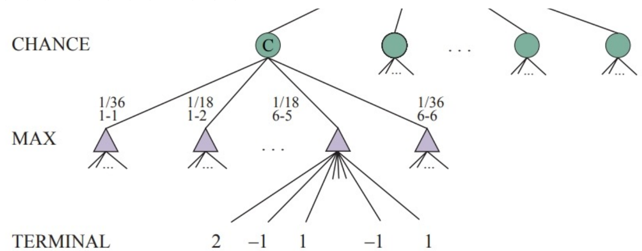
- As for minimax, the value of *expectminimax* ma be approximated by stopping the recursion early and using an evaluation function
  - The chance nodes, however, need to be involved in evaluating the value of the positions



  - The program behaves differently if some of the evaluation values change, even if the preference order remains the same
    - So, the evaluation function must return values that are a positive linear transformation of the expected utility of a state

Non-Deterministic Games in Practice

- Because expectiminimax considers all the possible dice-roll sequences, it will take $O(b^m n^m)$
  - where n is the number of distinct rolls
- Even for small depths d, it would be unfeasible to look ahead very far
  - In backgammon n is 21 and b is usually around 20, but in some situations can be as high as 4000 for dice rolls that are doubles
- However, alpha-beta pruning can be applied also to game trees with chance nodes
- if we put bounds on the possible values of the utility function, then we can arrive at bounds for the average without looking at every number
  - For example, if all utility values are between −2 and +2; then the value of leaf nodes is bounded
    - We can place an upper bound on the value of a chance node without looking at all its children



Partially Observable Games

- In deterministic partially observable games, uncertainty about the state of the board arises entirely from a lack of access to the opponent's choices
- This class includes games such as Battleship
  - each player's ships are placed in locations hidden from the opponent
- In stochastic partially observable games, the missing information is generated by the random dealing of cards
  - bridge, whist, hearts, and poker

Card Games

- At first sight, it might seem that these card games are just like dice games
  - the cards are dealt randomly and determine the moves available to each player, but all the “dice” are rolled at the beginning!
  - it suggests an algorithm:
    - treat the start of the game as a chance node with every possible deal as an outcome and then use the EXPECTIMINIMAX formula to pick the best move
  - Note that in this approach the only chance node is the root node
  - Then, the game becomes fully observable

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  - Then, the game becomes fully observable
    - sometimes called averaging over clairvoyance
    - it assumes that once the actual deal has occurred, the game becomes fully observable to both players

Common Sense Example

- Day 1
  - Road A leads to a small heap of gold pieces
  - Road B leads to a fork:
    - take the left fork and you'll find a mound of jewels
    - take the right fork and you'll be run over by a bus
- Day 2
  - Road A leads to a small heap of gold pieces
  - Road B leads to a fork:
    - take the left fork and you'll be run over by a bus
    - take the right fork and you'll find a mound of jewels
- Day 3
  - Road A leads to a small heap of gold pieces
  - Road B leads to a fork:
    - guess correctly and you'll find a mound of jewels
    - guess incorrectly and you'll be run over by a bus

Property Analysis

- The intuition that the value of an action is the average of its values in all actual states is WRONG
- With partial observability, the value of an action depends on the information state or belief state the agent is in
  - optimal play requires reasoning about the current and future belief states of each player
- Can generate and search a tree of information states
- This leads to rational behaviors such as
  - Acting to obtain information
  - Signaling to one's partner
  - Acting randomly to minimize information disclosure

Stochastic and/or Partially Observable Games in Practice

- Backgammon
  - BKG (1980) used a manually constructed evaluation function and searched at depth 1 only
    - First program to defeat a human world champion
  - TD-Gammon (1995) learned its evaluation function using NNs trained by self-play
- Poker
  - Game theory (2015) to determine the exact optimal strategy for a version of poker with just two players
  - In 2017, champion poker players were beaten at heads-up (two players) no-limit Texas hold 'em in two separate matches against the programs Libratus and DeepStack
  - In 2019, Pluribus defeated top-ranked professional human players in Texas hold 'em games with six players
- Bridge
  - GIB program, based on Monte Carlo simulation, won the computer championship and did surprisingly well against expert human players
  - In the 21st century, the computer bridge championship has been dominated by two commercial programs, JACK and WBRIDGE5

Limitations of Game Search Algorithms

- Alpha–beta search vulnerable to errors in the heuristic function
- Waste of computational time for deciding the best move where it is obvious (meta-reasoning)
  - Both alpha-beta and MCST
- The reasoning is done on individual moves
  - Humans reason on abstract levels
- Possibility to incorporate Machine Learning into the game search process

Summary

- Minimax algorithm: selects optimal moves by a depth-first enumeration of the game tree
- Alpha–beta algorithm: greater efficiency by eliminating subtrees
- Evaluation function: a heuristic that estimates the utility of state
- Monte Carlo tree search (MCTS): no heuristic, play game to the end with rules and repeated multiple times to determine optimal moves during playout