Course of "Industrial Automation"

z-Transform

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z-Transform definition

- Given a sequence of discrete values, $f(0), \ldots, f(k), \ldots$, the z-transform is defined by

$$\mathcal{Z}[f(k)] = F(z) = \sum_{k=0}^{+\infty} z^{-k} f(k)$$

where k is an integer variable and z a complex variable

- the series needs to converge for some z, i.e. $r < |z| < R_0$, i.e. r and R_0 bounds on the magnitude of z, for which the series converges

- ***The zeta-Transform is a bilateral only if the function f(k) is null for $k \leq 0$***

z-Transform of common signals

- Unit pulse(Kronecker delta)

$$f(k) = \delta(k) = \begin{cases} 1 & se\ k = 0 \\ 0 & se\ k \neq 0 \end{cases}$$

- z-Transform of the unit pulse

$$\mathcal{Z}[\delta(k)] = 1$$

- The unti step

$$f(k) = 1(k) = \begin{cases} 1 & se\ k \geq 0 \\ 0 & se\ k < 0 \end{cases}$$

- z-Transform of the unit step

$$\mathcal{Z}[1(k)] = \frac{z}{z-1}$$

z-Transform main properties (1/3)

- ***Linearity***

$$\mathcal{Z}[\alpha f(k) + \beta g(k)] = \alpha F(z) + \beta G(z)$$

- ***Time advance (of one unit)***

$$\mathcal{Z}[f(k+1)1(k)\,] = zF(z) - zf(0)$$

- ***Time delay (of one unit)***

$$\mathcal{Z}[f(k-1)1(k-1)\,] = \frac{1}{z}F(z)$$

z-Transform main properties (2/3)

- ***Convolution of time sequences***

$$\mathcal{Z}[f(k) * g(k)] = F(z)G(z)$$

- ***Differentiation in the z-domain***

$$\mathcal{Z}[kf(k)] = -z\frac{d}{dz}F(z)$$

z-Transform main properties (3/3)

- ***Initial value theorem***

$$f(0) = \lim_{z \to \infty} F(z)$$

- ***Final value theorem***

if $\lim_{k \to \infty} f(k)$ is finite, then

$$\lim_{k \to \infty} f(k) = \lim_{z \to 1} \left(\frac{z-1}{z} F(z) \right)$$

Selected set of signals

- *Ramp signal* $k1(k)$
- *Polynomial inputs*
- *Power functions*
- *Sinusoidal inputs*

Selected Laplace transforms

- ***Ramp***

$$\mathcal{Z}[k1(k)] = \frac{z}{(z-1)^2}$$

- ***Power***

$$\mathcal{Z}[a^k 1(k)] = \frac{z}{z-a}$$

$$\mathcal{Z}[ka^k 1(k)] = \frac{az}{(z-a)^2}$$

Selected Laplace transforms

- ***Sinusoidal inputs***

  - ***sine***

$$\mathcal{Z}[\sin(k\theta)1(k)] = \frac{z\sin\theta}{z^2 - 2z\cos\theta + 1}$$

  - ***cosine***

$$\mathcal{Z}[\cos(k\theta)1(k)] = \frac{z\,(\mathrm{z} - \cos\theta)}{z^2 - 2z\cos\theta + 1}$$