UNIVERSITÀ DEGLI STUDI DI NAPOLI
PARTHENOPE

Artificial Intelligence

# Search

LESSON 3

prof. Antonino Staiano

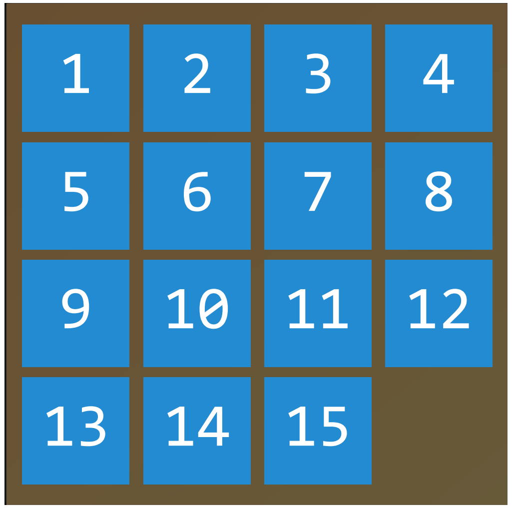
M.Sc. In ''Machine Learning e Big Data'' - University Parthenope of Naples

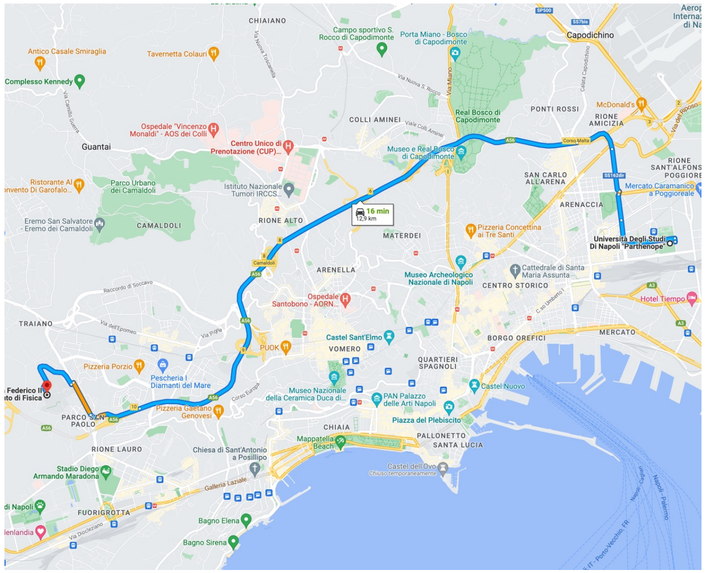
# What a Search Problem is

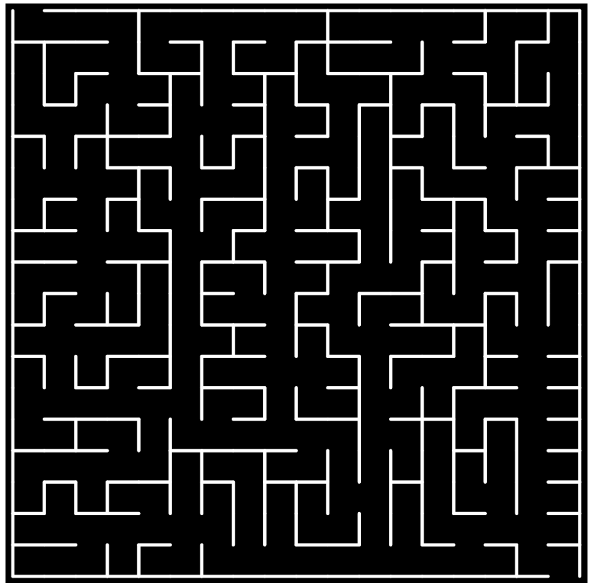
- Search problems involve an agent that
  - is given an initial state and a goal state
  - returns a solution of how to get from the former to the latter
  - uses atomic representations characterized by indivisible states of the world, lacking internal structure
    - Conversely, factored representations divide each state into a predetermined set of attributes, each capable of holding a value
- Example
  - A navigator app uses a typical search process, where the agent (the thinking part of the program) receives your current location and desired destination as input and returns a suggested path based on a search algorithm
  - However, many other search problems exist, like puzzles or mazes

# Designing Agents for Search Problems

- Consider the following problems, and assume that your goal is to design a rational agent (assume a computer program) capable of solving them autonomously
- Let's recall
  - A rational agent is a system that acts rationally, according to a well-defined objective



# Missionaries and Cannibals

- A classic AI toy-problem
  - 3 missionaries and 3 cannibals on one side of a river
  - Goal: cross the river in a boat (or raft) to reach the other side of the river
  - Constraints
    - The boat can only hold two people
    - Do not leave more cannibals than missionaries on either side of the river
  - How can all six cross the river safely?

# Game playing: 15-puzzle

- An array of tiles numbered from 1 to 15 and an empty cell
- Goal:
  - Transform the tiles from an initial configuration into a given desired configuration, by a sequence of moves of a tile into an adjacent empty cell
- A more challenging goal
  - Find the shortest of such sequences
- Example

| | | | |
|---|---|---|---|
| 13 | 10 | 11 | 6 |
| 5 | 7 | 4 | 8 |
| 1 | | 14 | 9 |
| 3 | 15 | 2 | 12 |

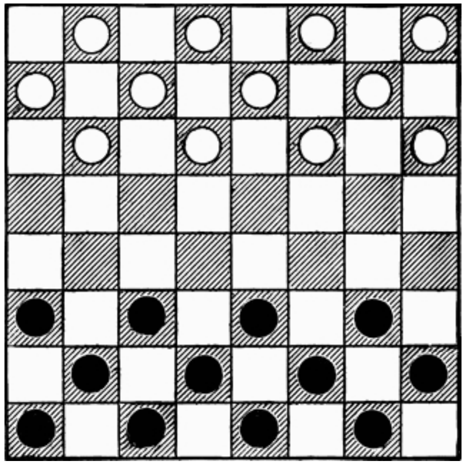
initial configuration

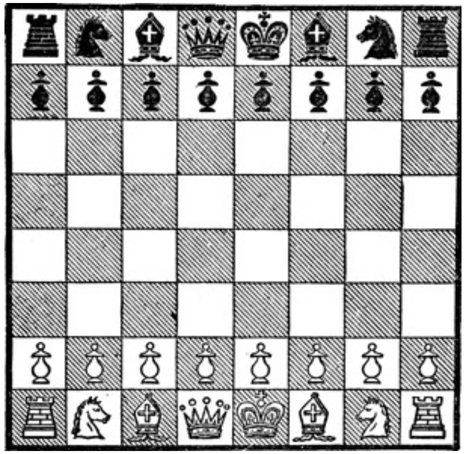
| | | | |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
| 5 | 6 | 7 | 8 |
| 9 | 10 | 11 | 12 |
| 13 | 14 | 15 | |

desired configuration

# Game playing: checkers and chess

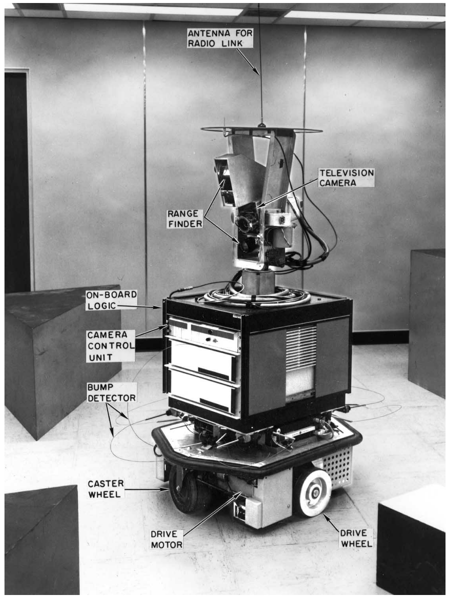
- Two historical problems addressed by many researchers since the early days of AI

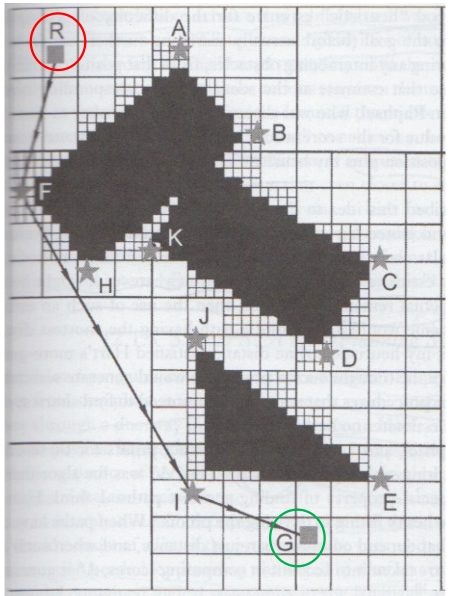




# Robot navigation

- A real-world problem addressed since the '60s

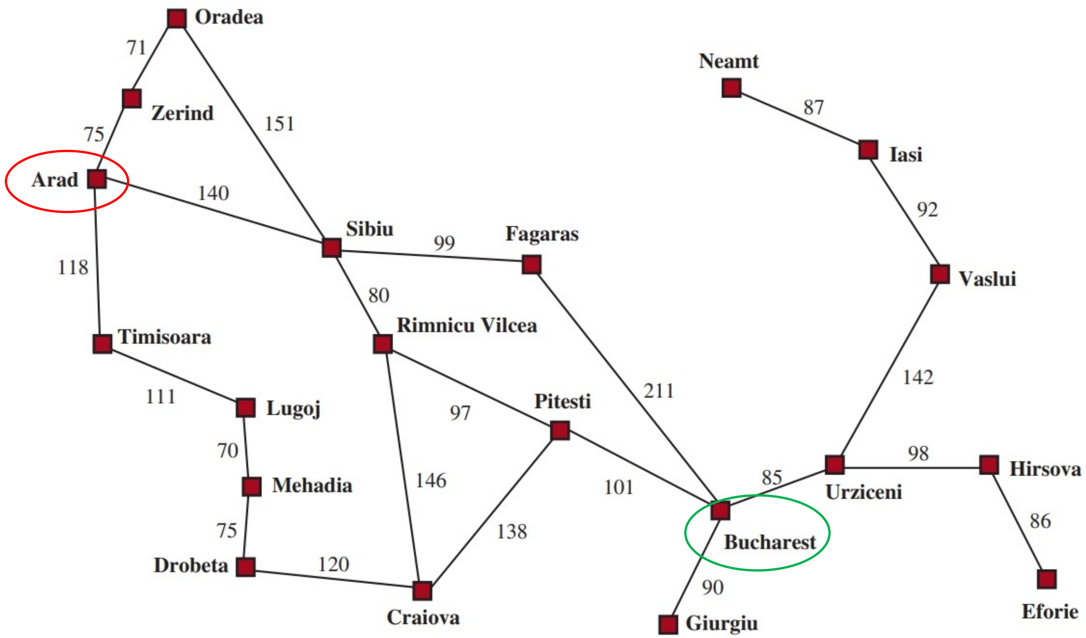



- A robot (left) and a problem to solve (right)
  - Find a route from R to G, possibly the shortest one, avoiding the black obstacles

# Route finding in maps

- Example
  - Finding a route (more challenging, the shortest one) from Arad to Bucharest using the information shown on the map

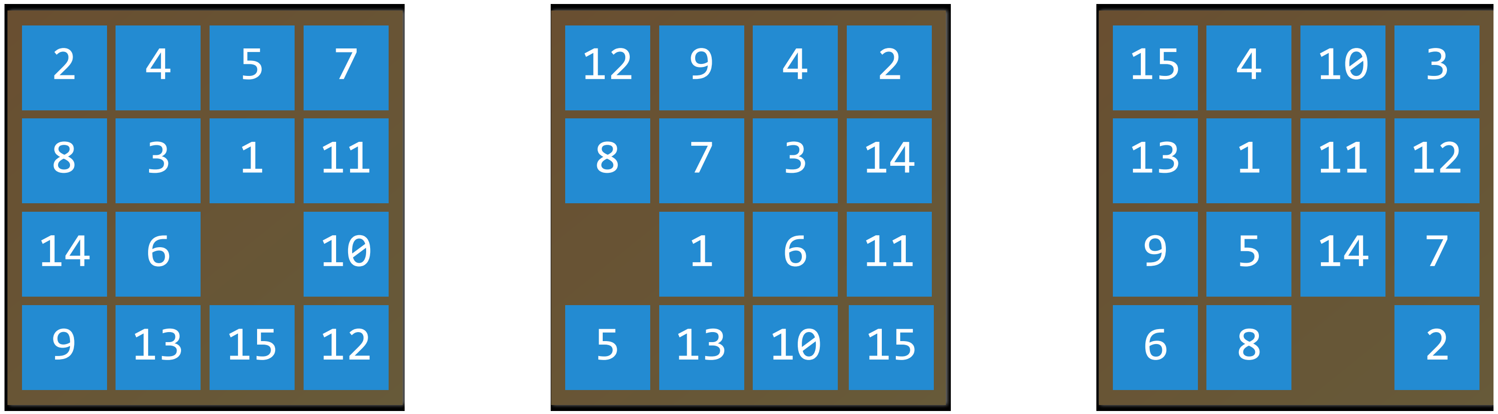


# Searching problems

- The previous problems may seem very different from each other, nonetheless, they share some common characteristics allowing one to solve them using the same approach
- Main characteristic
  - A clear goal can be defined in terms of desired world states
- Given the goal, the task is to find a sequence of actions that will lead to a goal state
- This requires an appropriate definition of the actions and the states to be considered
- The solution to a problem is a sequence of actions leading to a goal state
  - The process of finding a solution is called search

# Problem ingredients

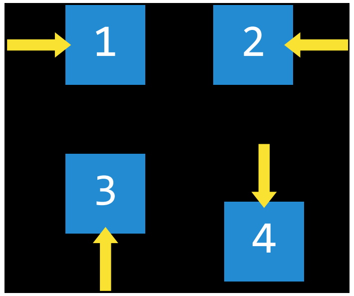
- Agent
  - Entity that perceives its environment and acts upon that environment
- State
  - A configuration of the agent and its environment




- Initial state
  - The state from which the search algorithm starts

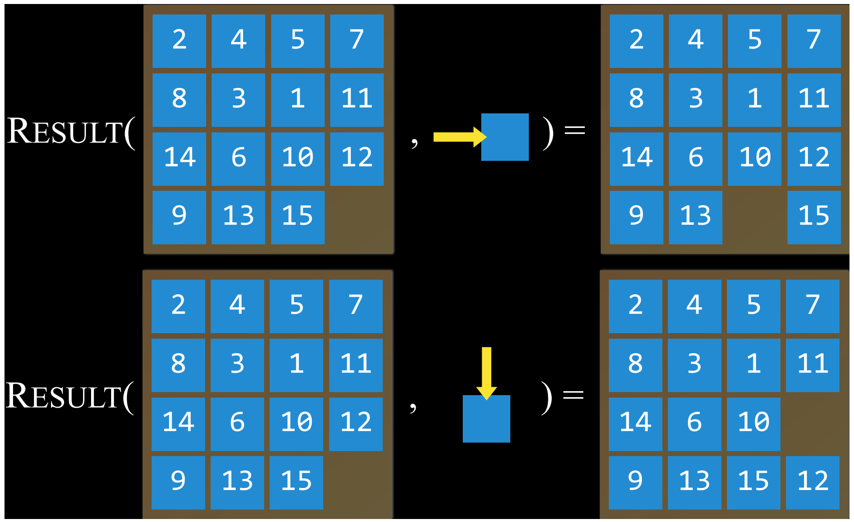
# Problem ingredients

- Actions
  - Choices that can be made in a state
- Actions can be defined as a function
  - ACTIONS(s) returns the set of actions that can be executed in a state s



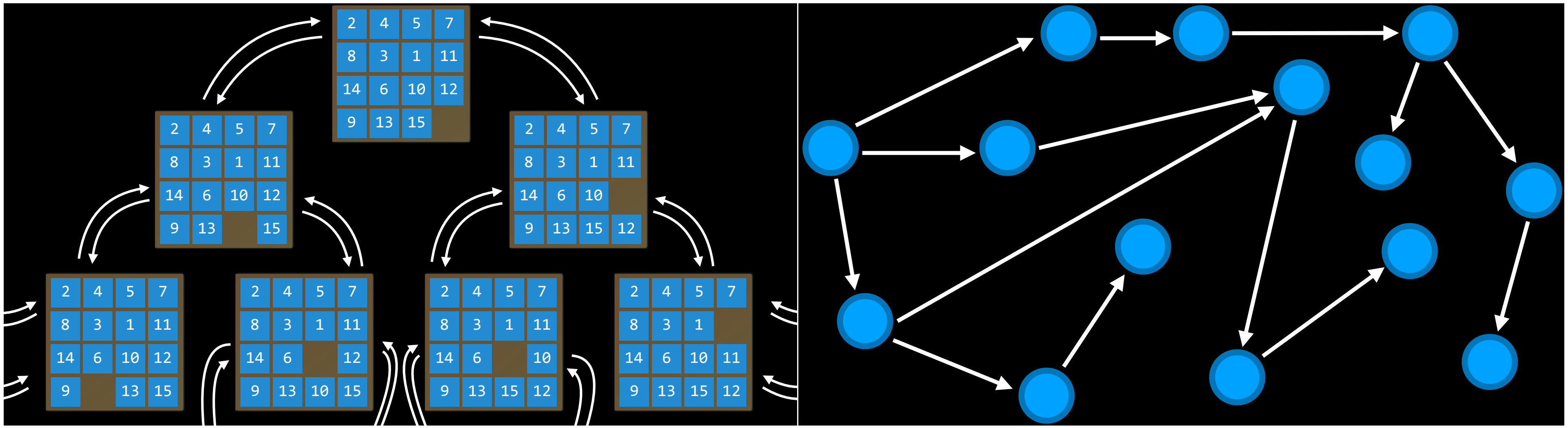
# Problem ingredients

- Transition model
  - A description of what state results from performing any applicable action in any state
- Defined as a function
  - RESULTS(s,a) returns the state resulting from performing action a in state s



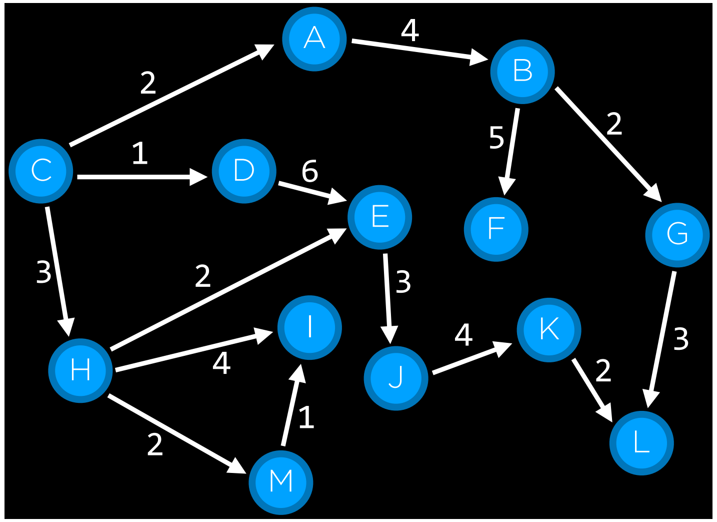
# Problem ingredients

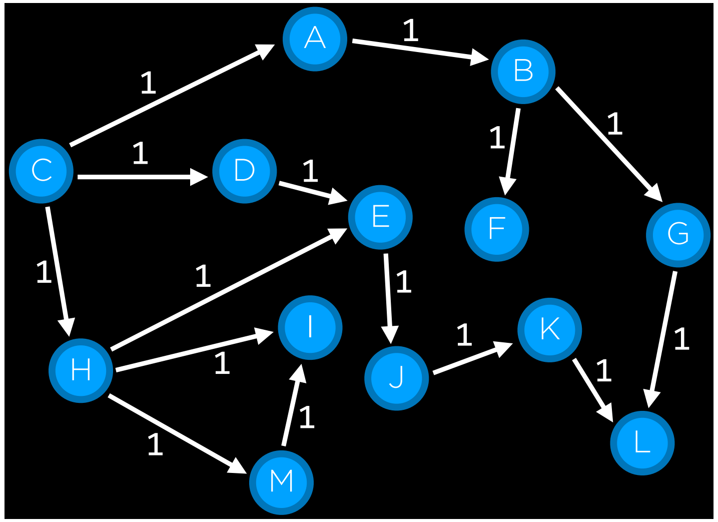
- State space
  - The set of all states reachable from the initial state by any sequence of actions
    - In a 15 puzzle, the state space consists of all the 16!/2 configurations on the board that can be reached from any initial state
    - The state space can be visualized as a directed graph with states, represented as nodes, and actions represented as arrows between nodes



# Problem ingredients

- Goal test
  - Way to determine whether a given state is a goal state
- Path cost
  - Numerical cost associated with a given path



# Example: 15-puzzle

- goal:
  - getting to the desired tile configuration (possibly, by the shortest sequence of moves)
- states:
  - each possible 16!/2 tile configurations
- actions:
  - moving the *n*-th tile ($n$ = 1,...,15) to one of the adjacent cells (two, three or four), if empty

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initial configuration

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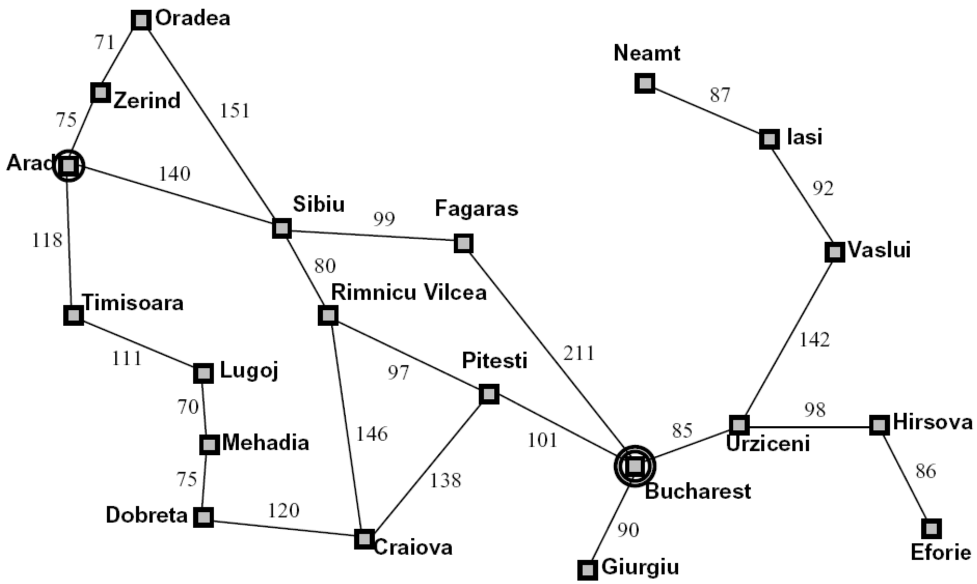
desired configuration

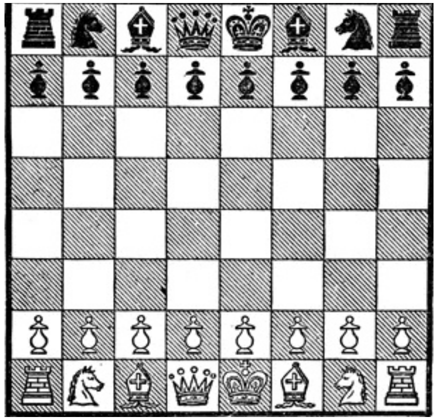
# Example: Route finding on maps

- goal:
  - getting from a given city to a destination (possibly, through the shortest route)
- states:
  - Being in each possible city
- actions:
  - Moving between two adjacent cities



# Example: Chess

- Goal:
  - To checkmate (possible in many chessboard configurations)
- States:
  - Each possible chessboard configuration
- Actions:
  - All legal moves



# Properties of Search Problems

- Static vs dynamic
  - does the environment change over time? Examples: 15-puzzle and chess are static; robot navigation is dynamic if the position of obstacles changes over time
- Fully vs partially observable:
  - is the current state completely known? Examples: 15-puzzle and chess are fully observable; robot navigation is partially observable if sensors are not "perfect"
- Discrete vs continuous sets of states and actions
  - Examples: 15-puzzle and chess are discrete, robot navigation is continuous
- Deterministic vs non-deterministic
  - is the outcome (the resulting state) of any sequence of actions certain. i.e., known in advance? Examples: 15-puzzle is deterministic, chess is not (due to the opponent's move, which is unknown when deciding one's own)

# Real-world Search Problems

- Many challenging real-world problems can be formulated as search problems
  - Traveling salesperson problem
    - Finding the shortest tour that allows one to visit every city on a given map exactly once
  - Route-finding
    - In computer networks airline travel planning, etc.
  - VLSI design
    - Cell layout, channel routing

# Solving Search Problems

- Solution
  - A sequence of actions that leads from the initial state to a goal state
- Optimal solution
  - A solution that has the lowest path cost among all solutions

# Data structures

- In a search process, data is often stored in a node
- Node
  - a data structure that keeps track of
    - A state
    - Its parent node, through which the current node was generated
    - The action that was applied to the state of the parent to get to the current node
    - The path cost from the initial state to this node
- Frontier
  - A mechanism that manages the nodes, that is, the set of nodes to be explored
  - The frontier starts by containing an initial state

# Approach

- Start with a frontier that contains the initial state
- Repeat:
  - If the frontier is empty, then stop, there is no solution
  - Remove a node from the frontier
  - If node contains the goal state, return the solution and stop
    - Else expand node, add resulting nodes to the frontier

# Example: Find a path from A to E

- Start with a frontier that contains the initial state
- Repeat:
  - If the frontier is empty, then no solution
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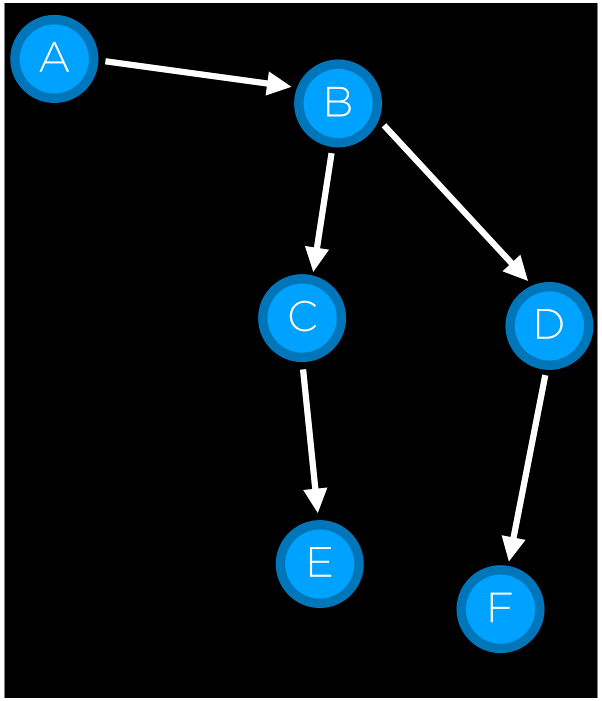
**Frontier**



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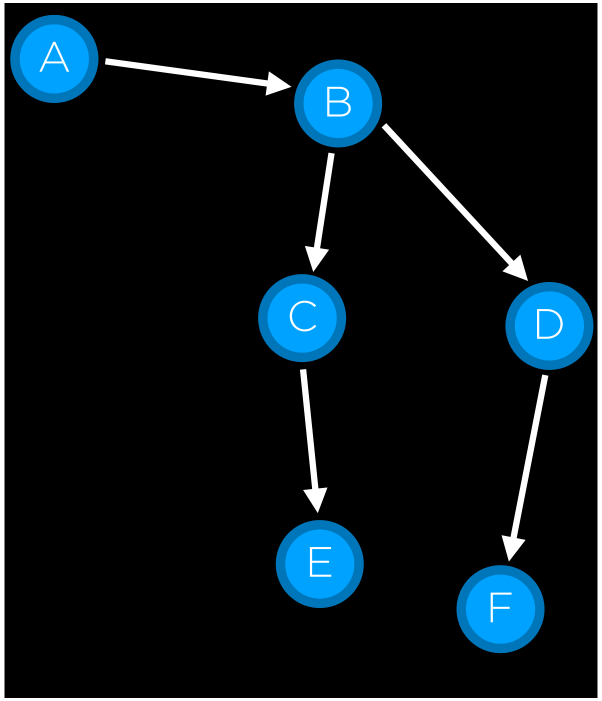

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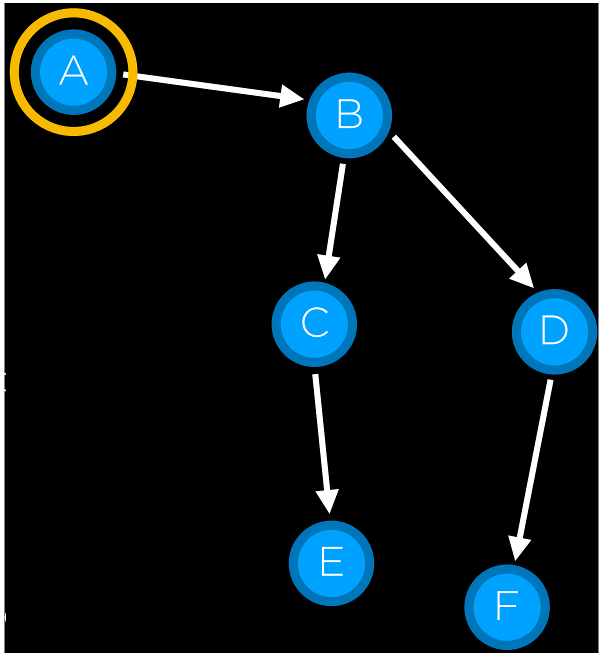

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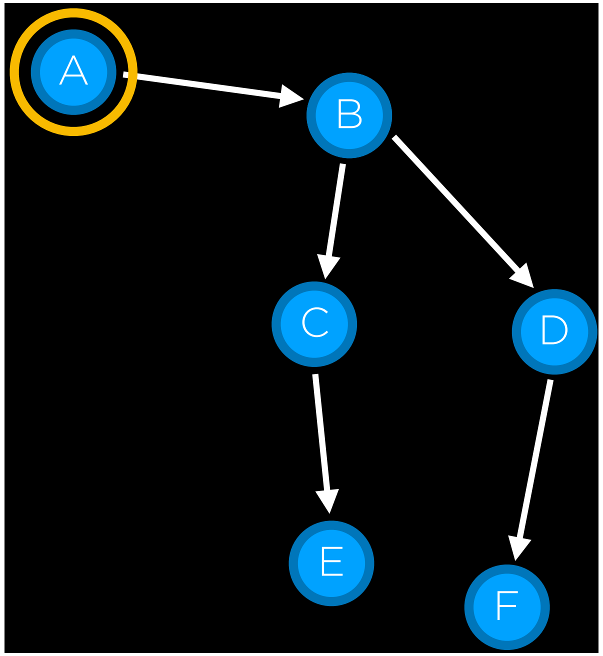

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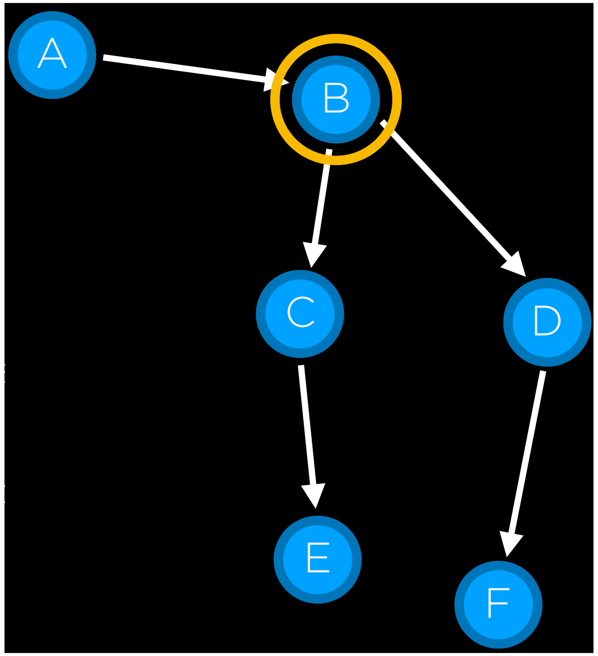

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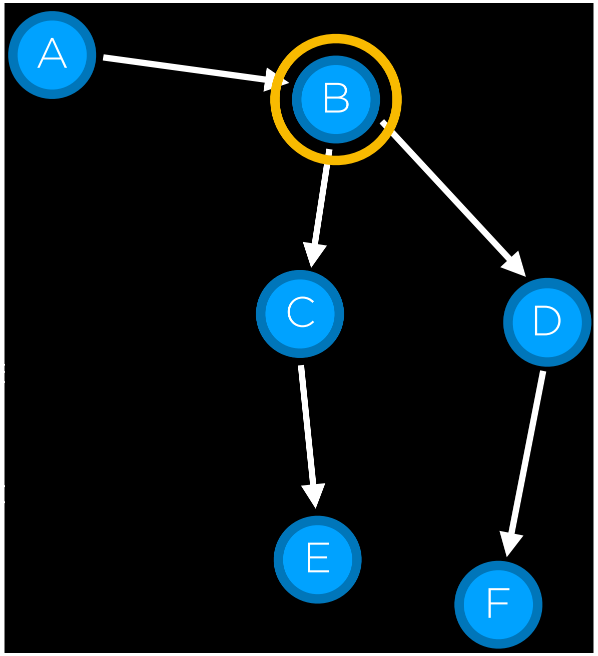

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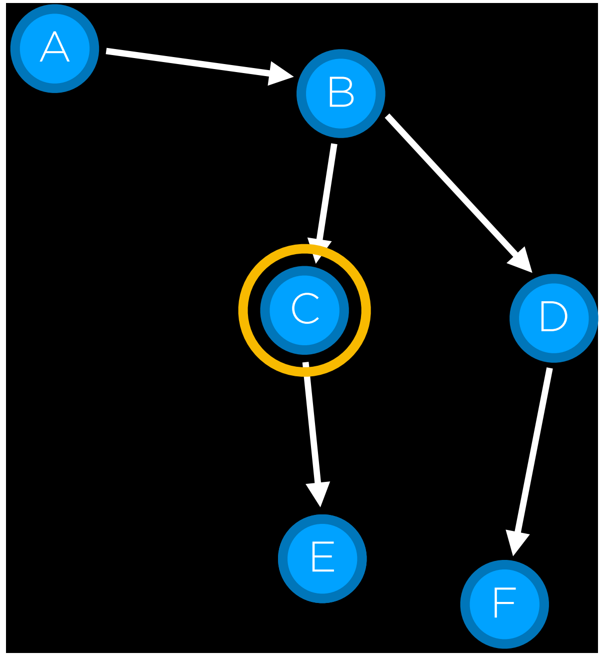

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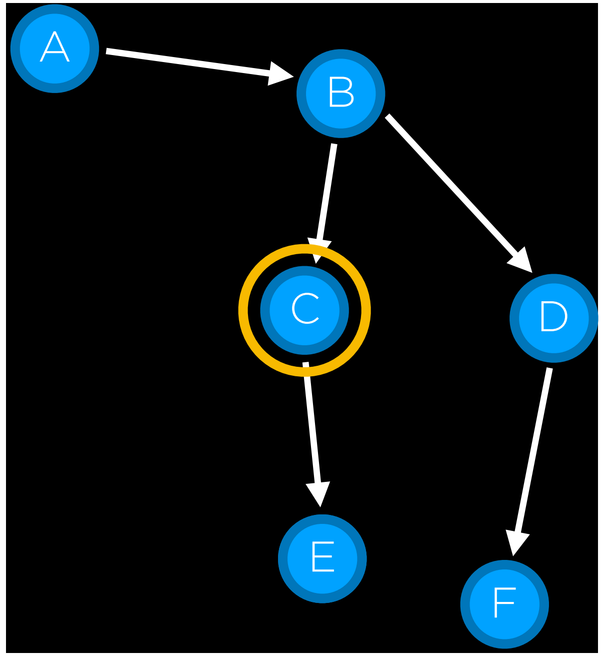

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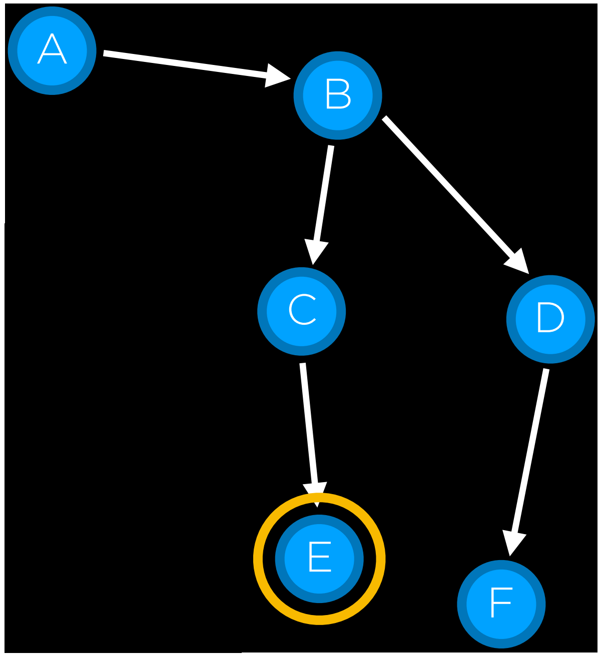

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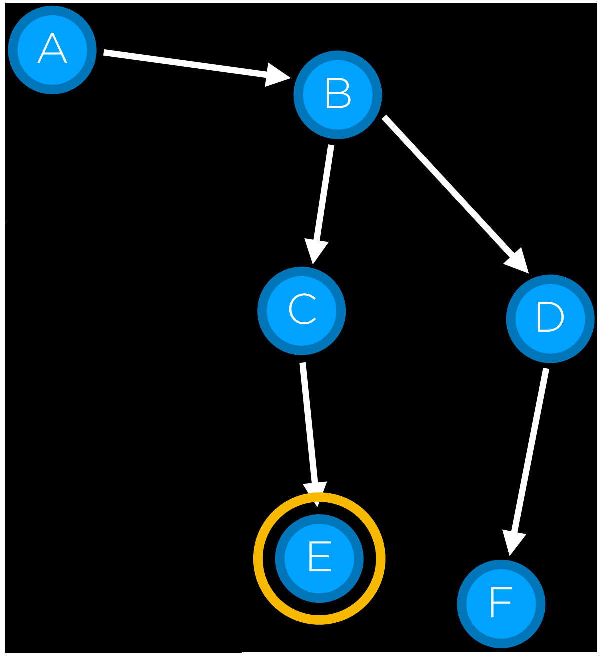
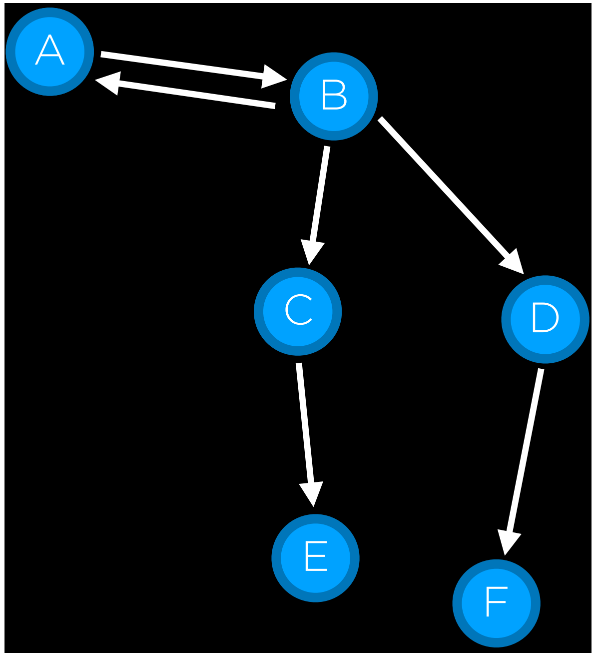


# Any problem here?

- Find a path from A to E

Frontier



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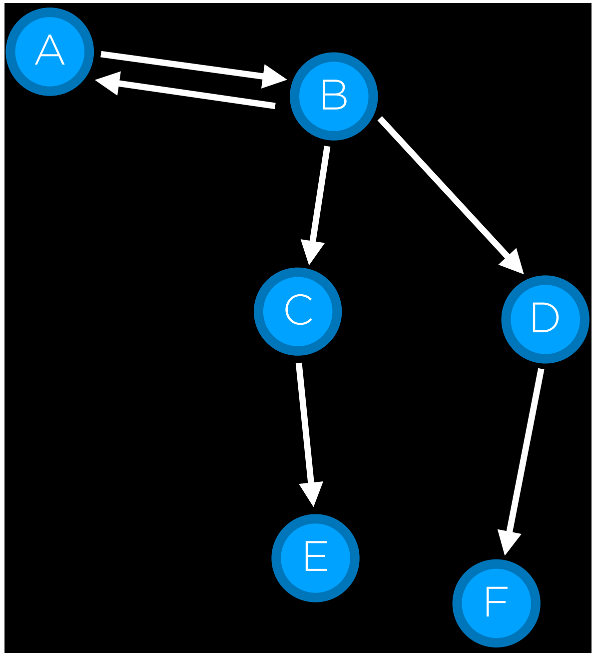

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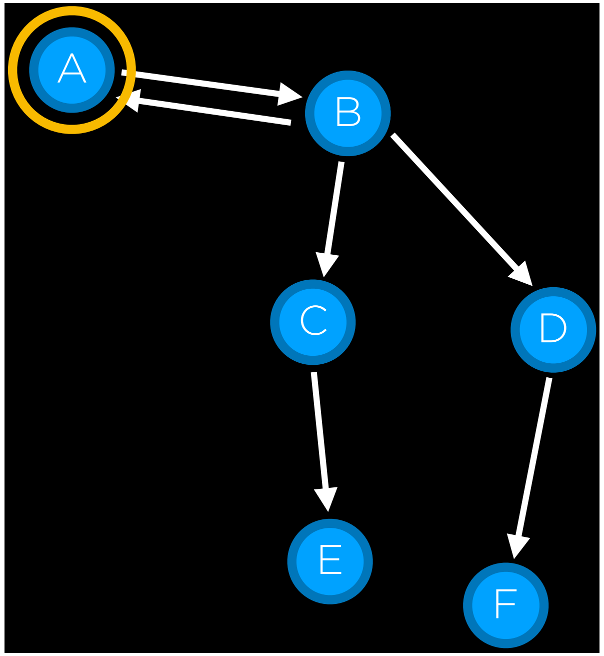

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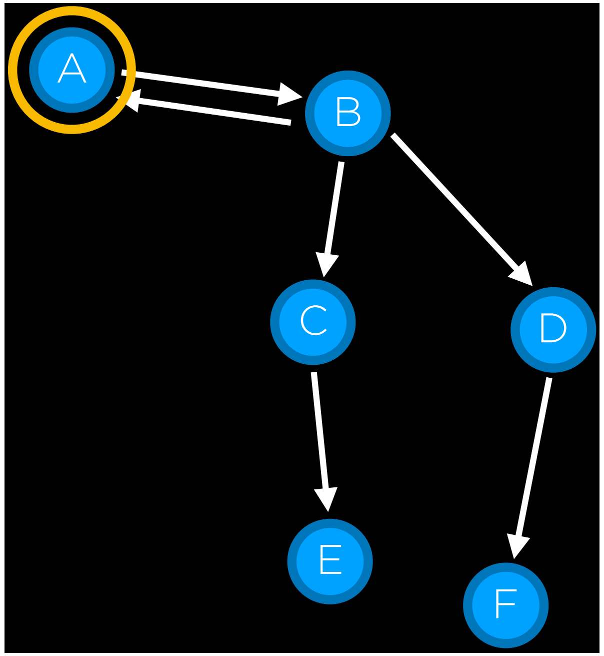

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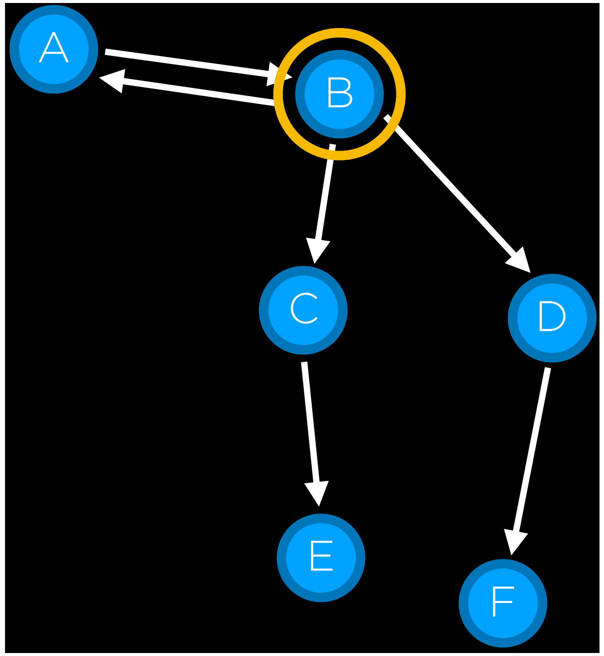

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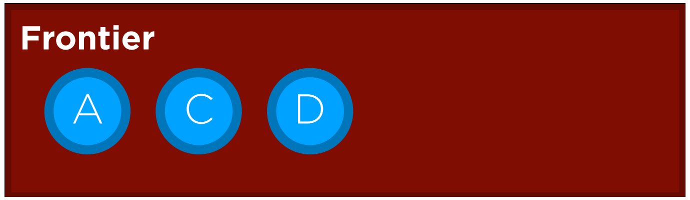


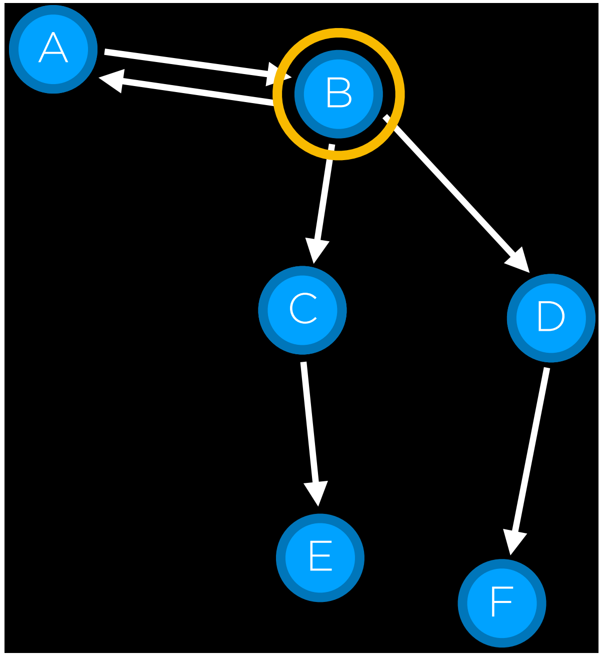

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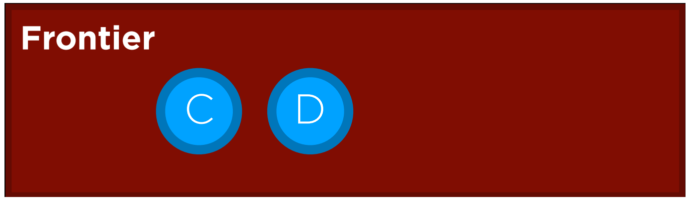


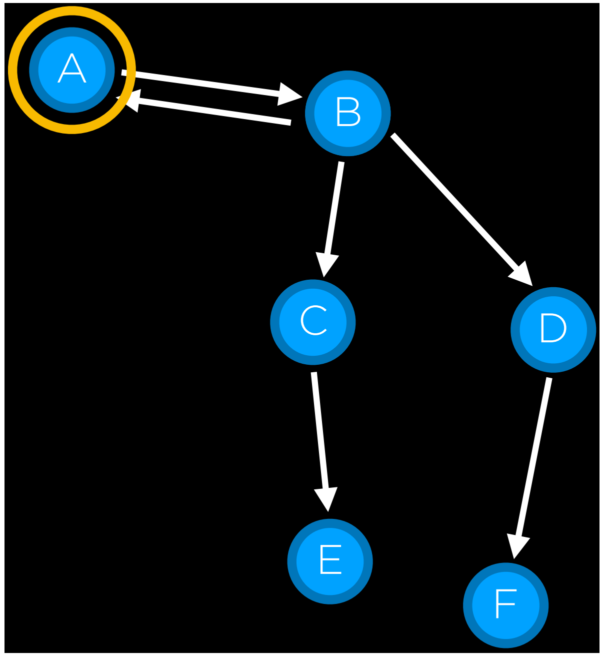

# A Cleaver Approach

- Start with a frontier that contains the initial state
- Start with an empty explored set
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  - Add the node to the explored set
  - Expand node, add resulting nodes to the frontier if they aren't already in the frontier or the explored set

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# Which node should be removed from the frontier?

- The choice of the nodes to be removed impacts the quality of the solution and how fast it is achieved
- There are multiple ways to choose, two of which can be represented by the data structures of
  - stack (in *depth-first* search) and
  - queue (in *breadth-first search*)

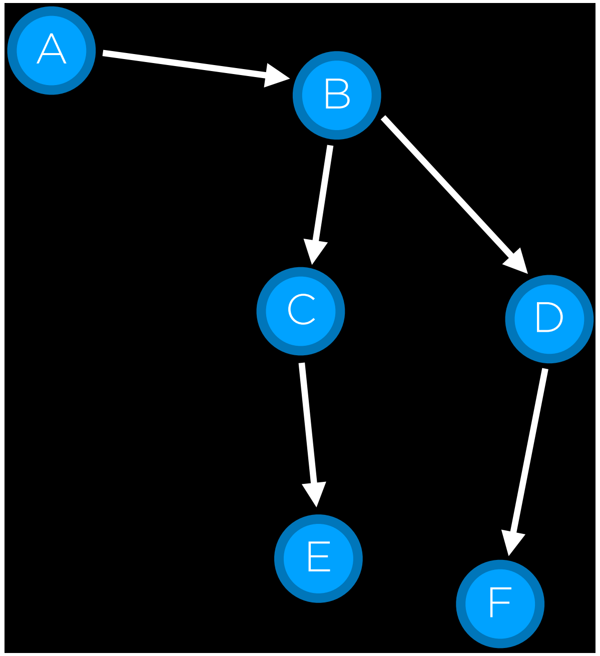
# Depth-First Search

- A *depth-first* search algorithm exhausts every single direction before trying another direction
- In these cases, the frontier is managed as a *stack* data structure
  - *last-in first-out* mode
- After nodes are added to the frontier, the first node to be removed and considered is the last node added
- This results in a search algorithm that goes as deep as possible in the first direction that gets in its way while leaving all other directions for later

# Example: Find a path from A to E

**Frontier**

**Explored Set**

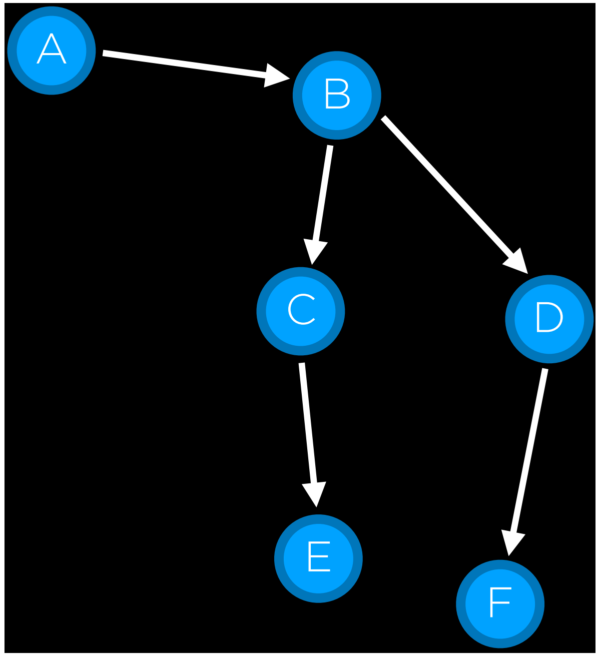


# Example: Find a path from A to E

**Frontier**

A

**Explored Set**

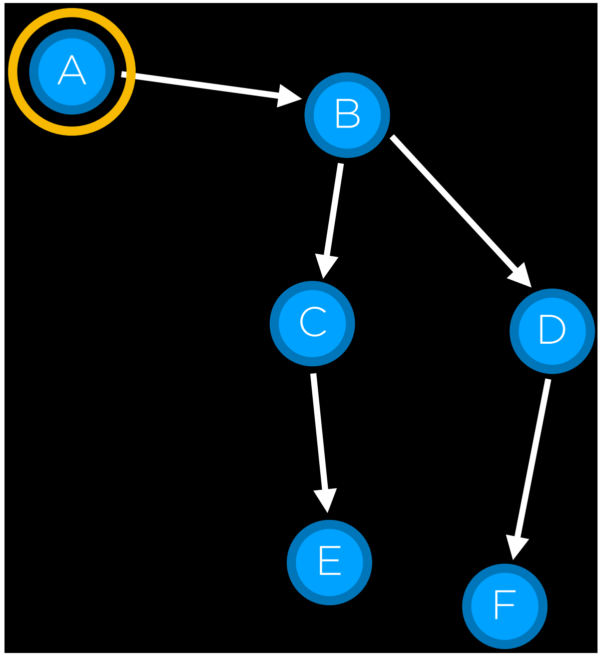


# Example: Find a path from A to E

**Frontier**

**Explored Set**

A



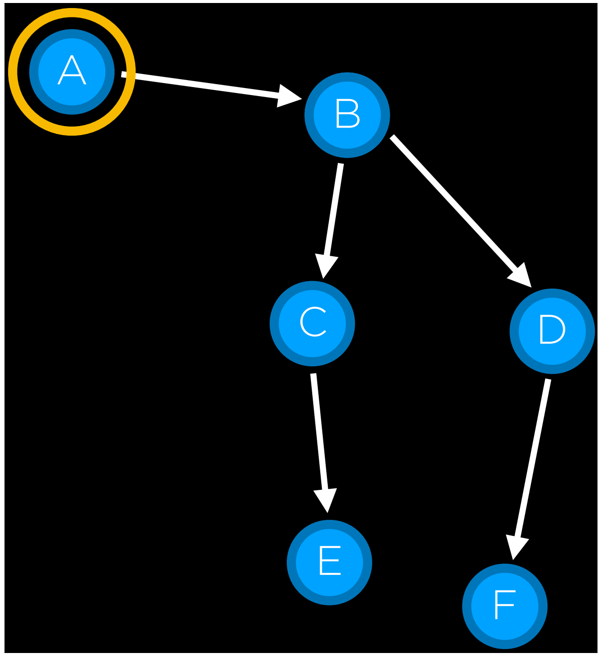
# Example: Find a path from A to E

**Frontier**

B

**Explored Set**

A

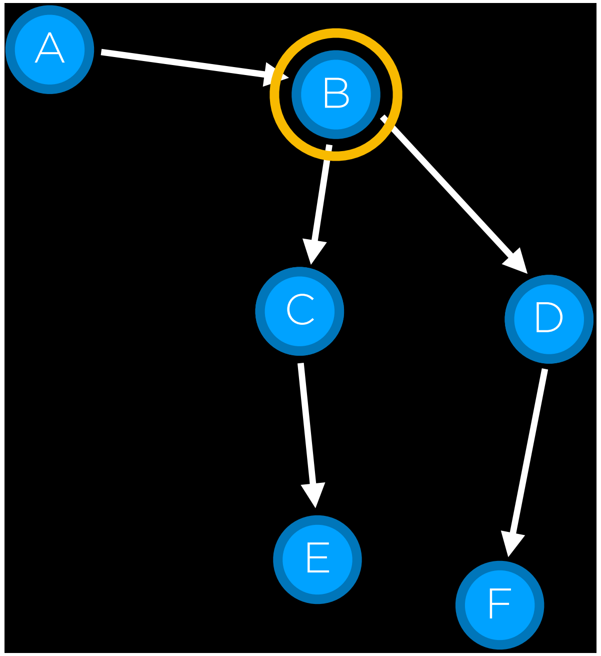


# Example: Find a path from A to E

**Frontier**

**Explored Set**

A B



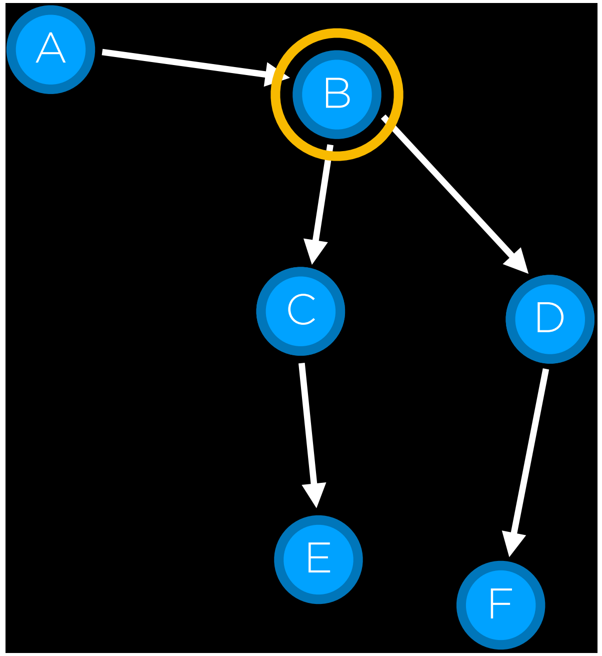
# Example: Find a path from A to E

**Frontier**

C D

**Explored Set**

A B



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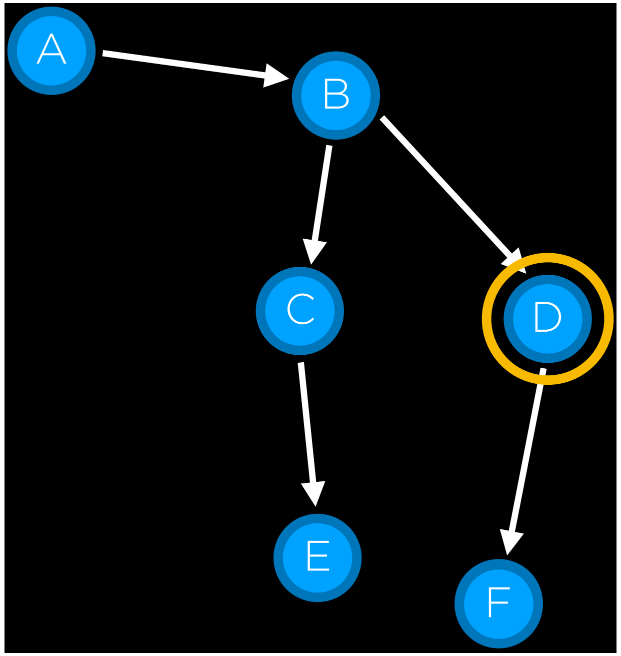


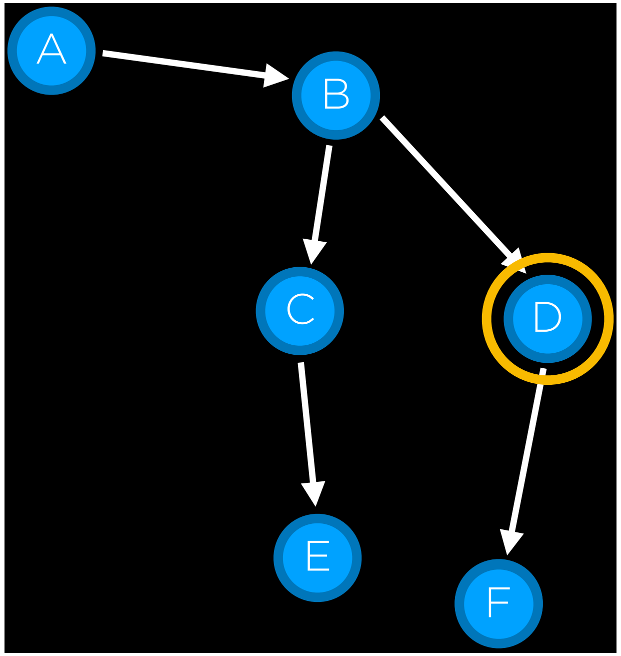
# Example: Find a path from A to E

**Frontier**

C F

**Explored Set**

A B D



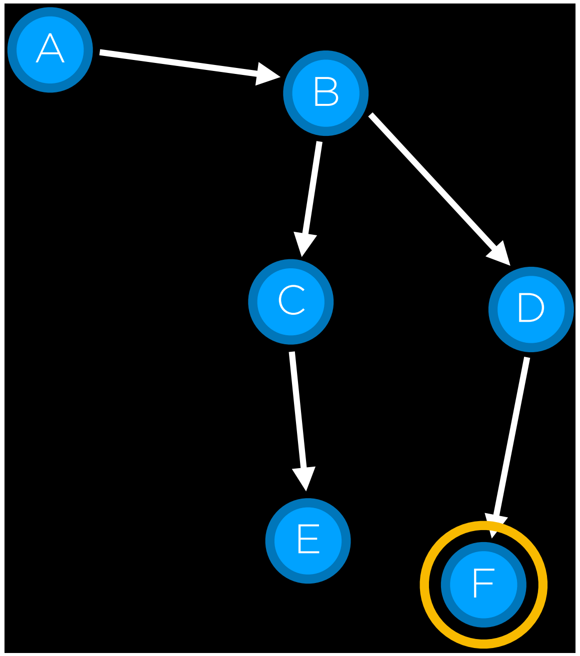
# Example: Find a path from A to E

**Frontier**

C

**Explored Set**

A B D F

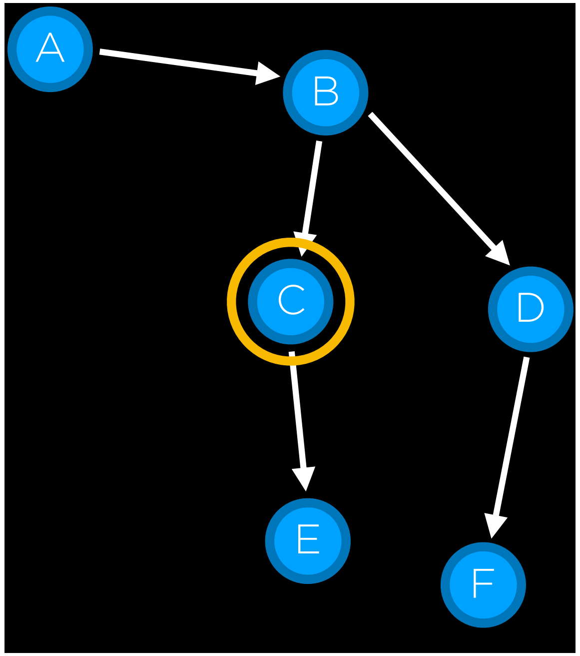


# Example: Find a path from A to E

**Frontier**

**Explored Set**

A B D F C



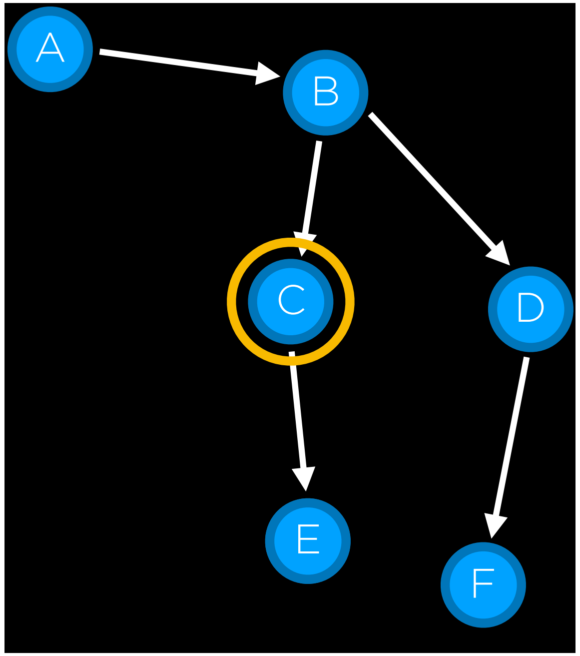
# Example: Find a path from A to E

**Frontier**

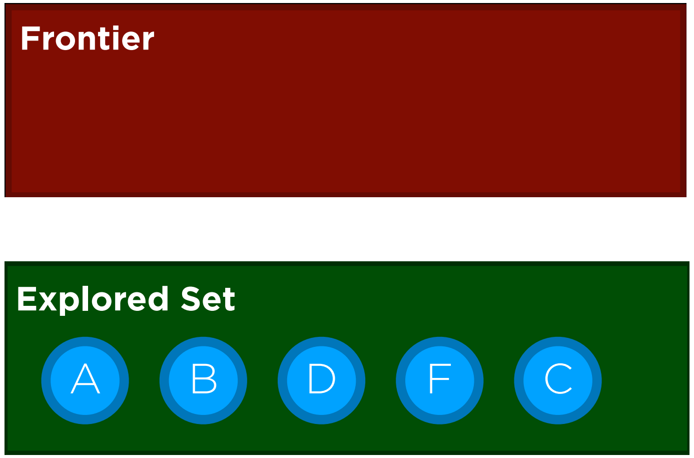
E

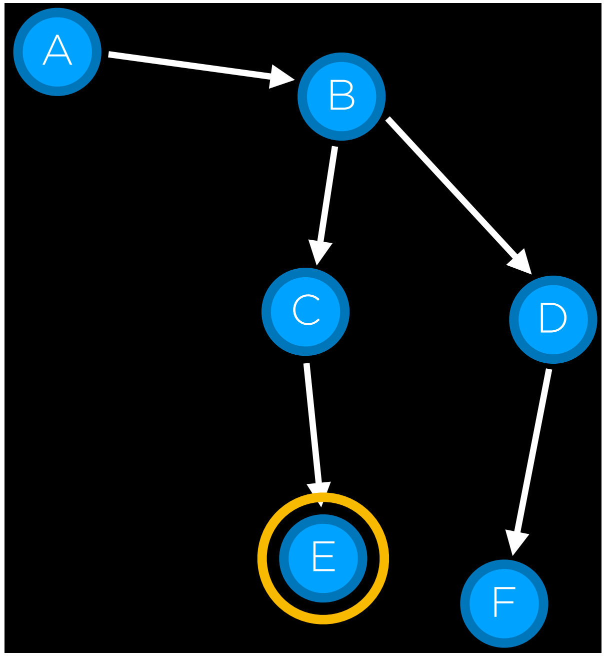
**Explored Set**

A B D F C



# Example: Find a path from A to E



# Depth-First Search

- Pros
  - At best, this algorithm is the fastest
    - If it "lucks out" and always chooses the right path to the solution (by chance), then a *DFS* takes the least possible time to get to a solution
- Cons
  - It is possible that the found solution is not optimal
  - At worst, this algorithm will explore every possible path before finding the solution, thus taking the longest possible time before reaching the solution

# Depth-First Search Code Example

```
# Define the function that removes a node from the frontier and returns it.
def remove(self):
      # Terminate the search if the frontier is empty, because this means that there is no solution.
    if self.empty():
        raise Exception("empty frontier")
    else:
            # Save the last item in the list (which is the newest node added)
        node = self.frontier[-1]
        # Save all the items on the list besides the last node (i.e. removing the last node)
        self.frontier = self.frontier[:-1]
        return node
```

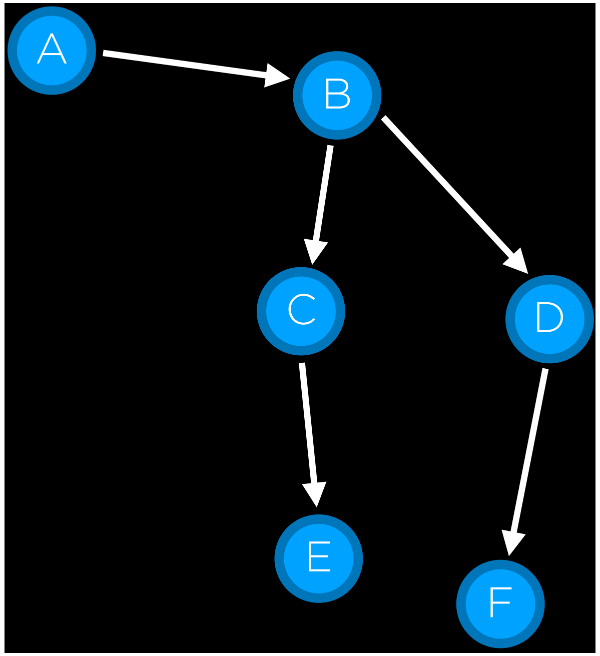
# Breadth-First Search

- The opposite of DFS
- A *BFS* algorithm will follow multiple directions at the same time, taking one step in each possible direction before taking the second step in each direction
- In this case, the frontier is managed as a *queue* data structure
  - *first-in first-out* mode
- All the new nodes add up in line, and nodes are being considered based on which one was added first (first come first served!)
- This results in a search algorithm that takes one step in each possible direction before taking a second step in any one direction

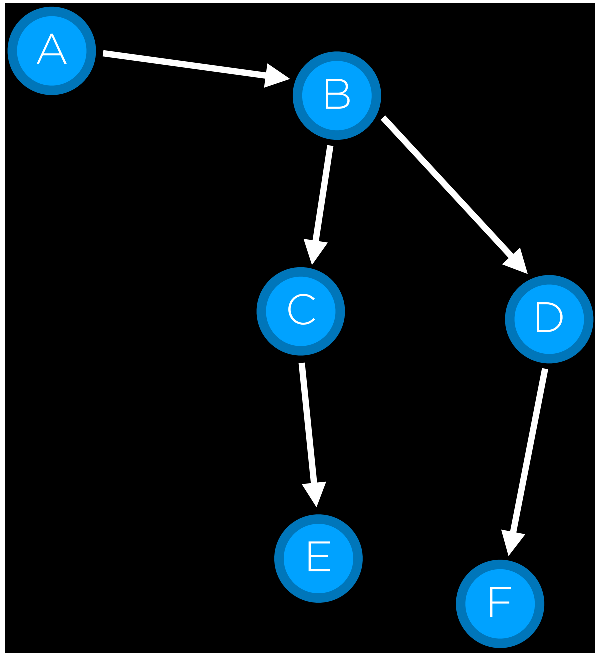
# Example: Find a path from A to E

**Frontier**

**Explored Set**



# Example: Find a path from A to E

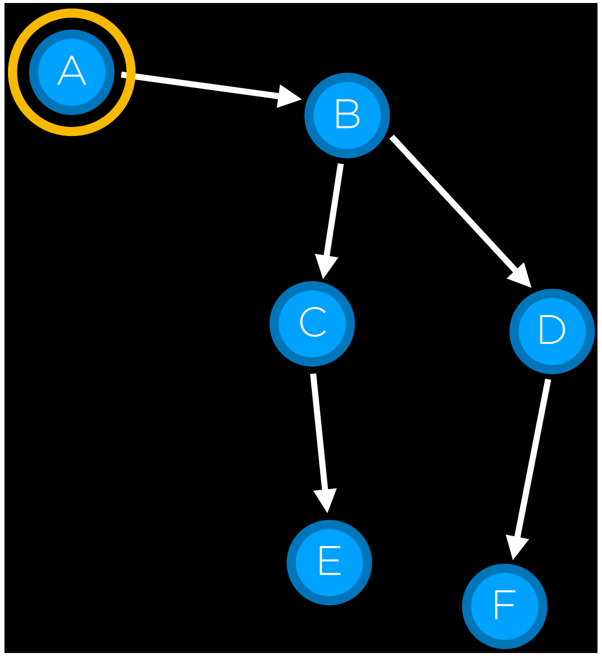



# Example: Find a path from A to E

**Frontier**

**Explored Set**

A



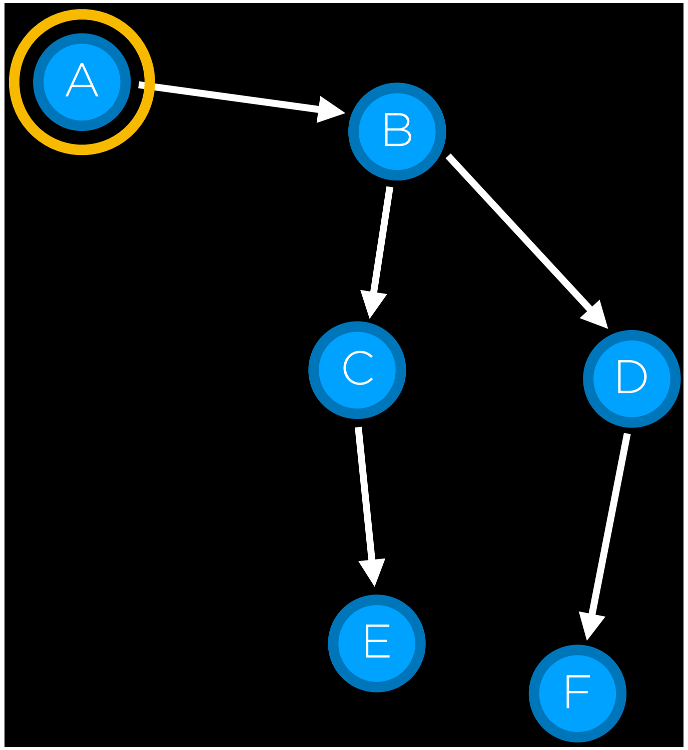
# Example: Find a path from A to E

**Frontier**

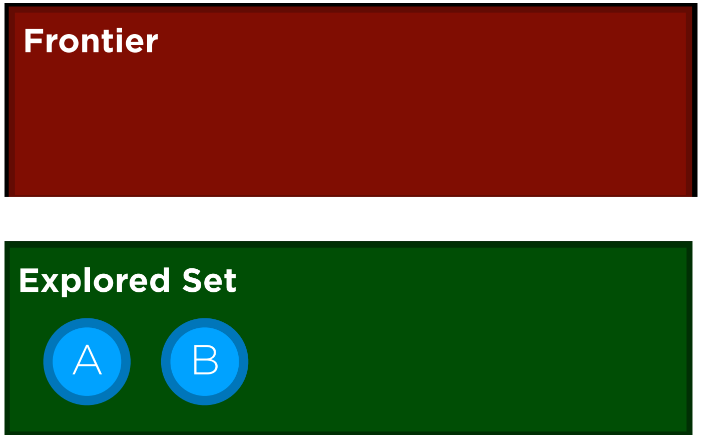
B

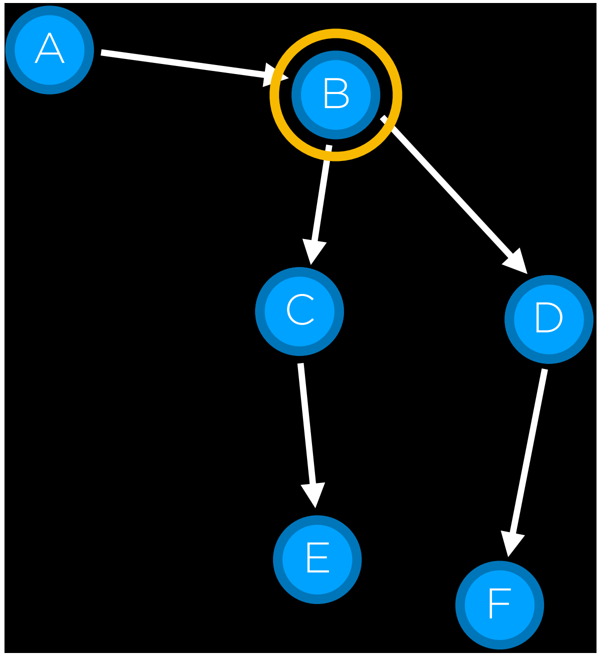
**Explored Set**

A



# Example: Find a path from A to E



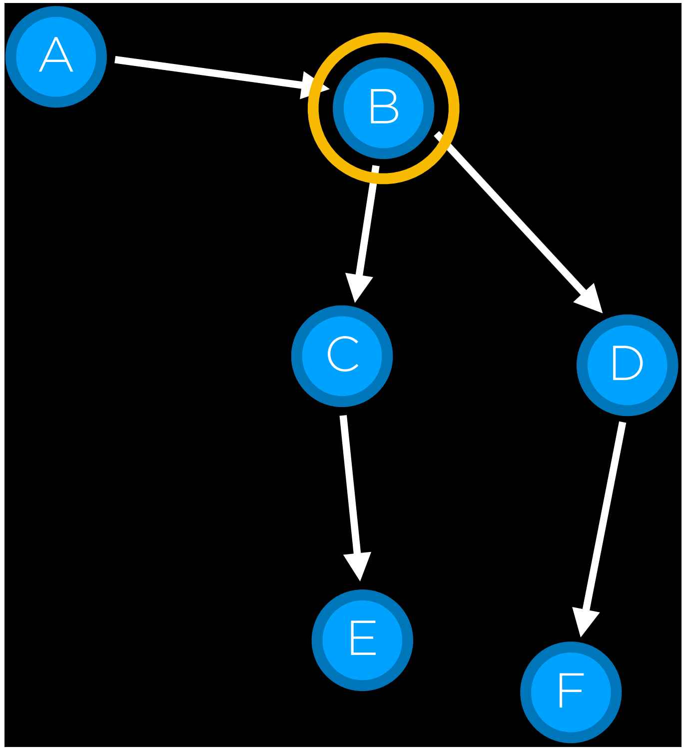
# Example: Find a path from A to E

**Frontier**

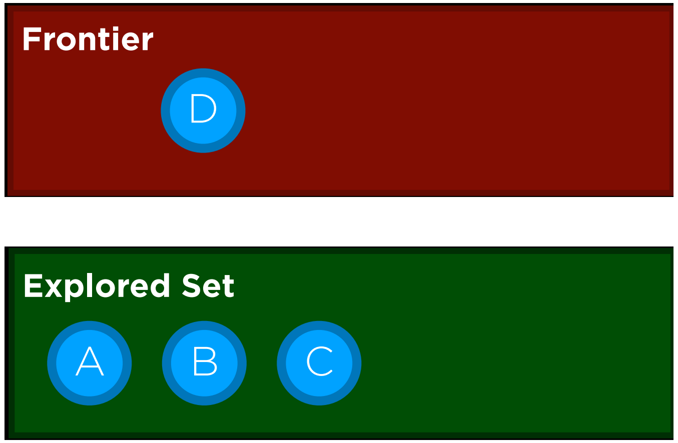
C D

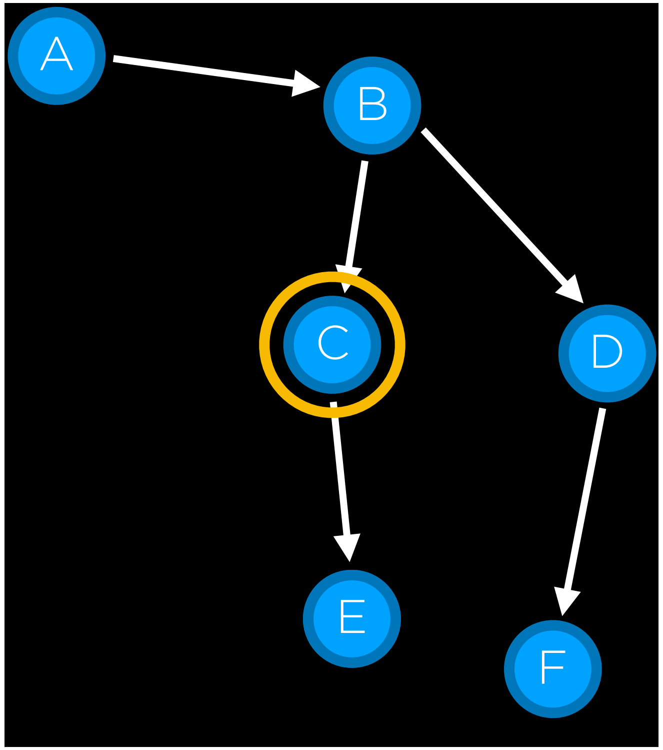
**Explored Set**

A B

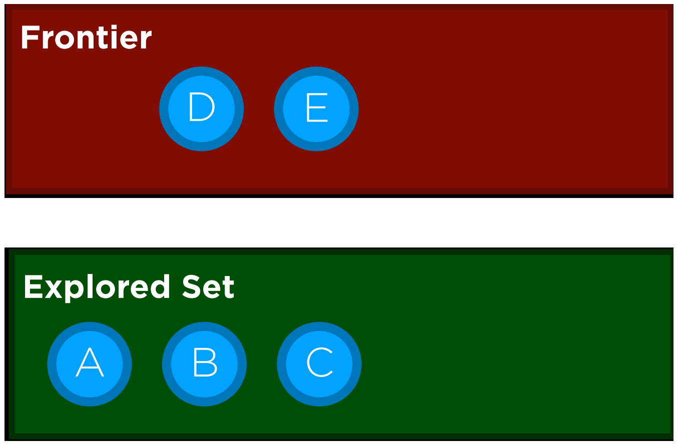


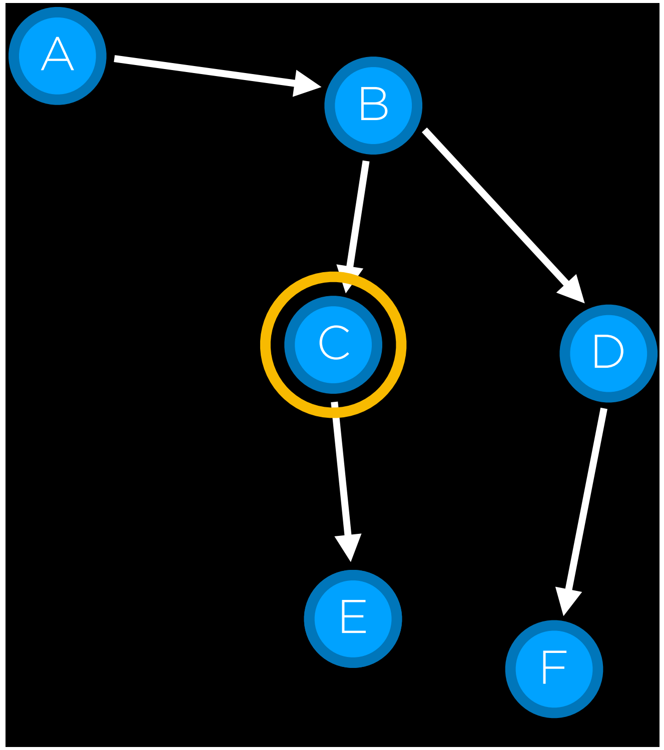
# Example: Find a path from A to E



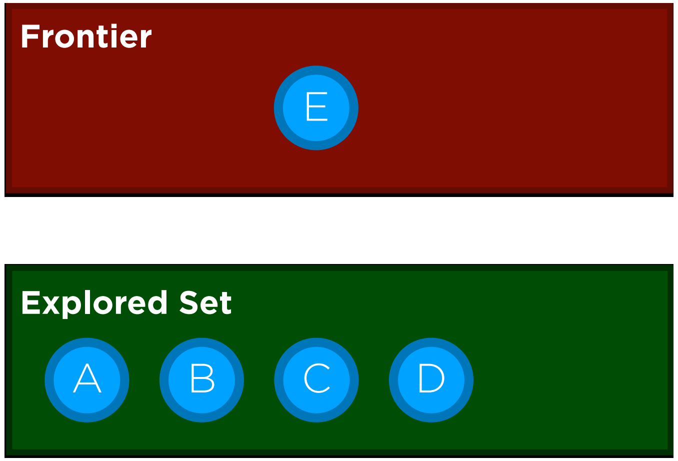



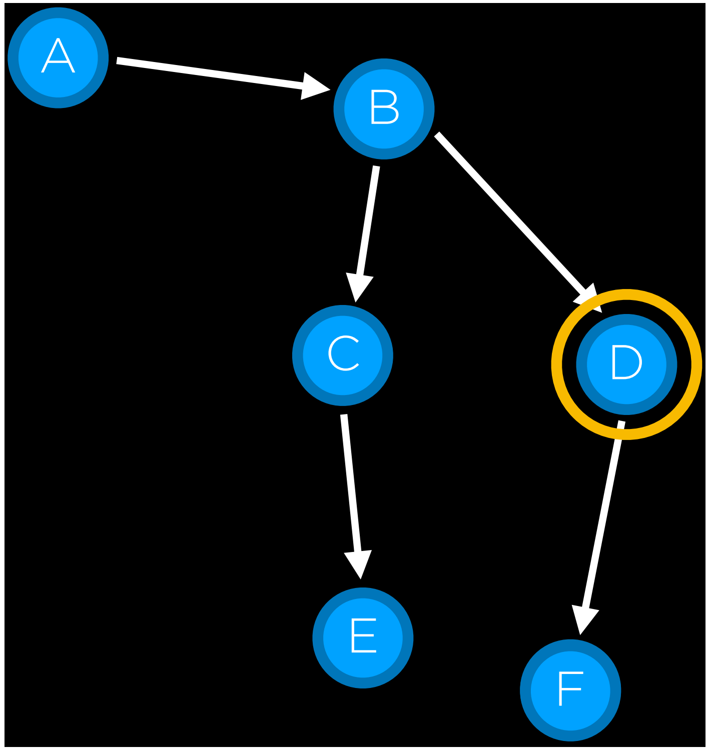
# Example: Find a path from A to E



# Example: Find a path from A to E



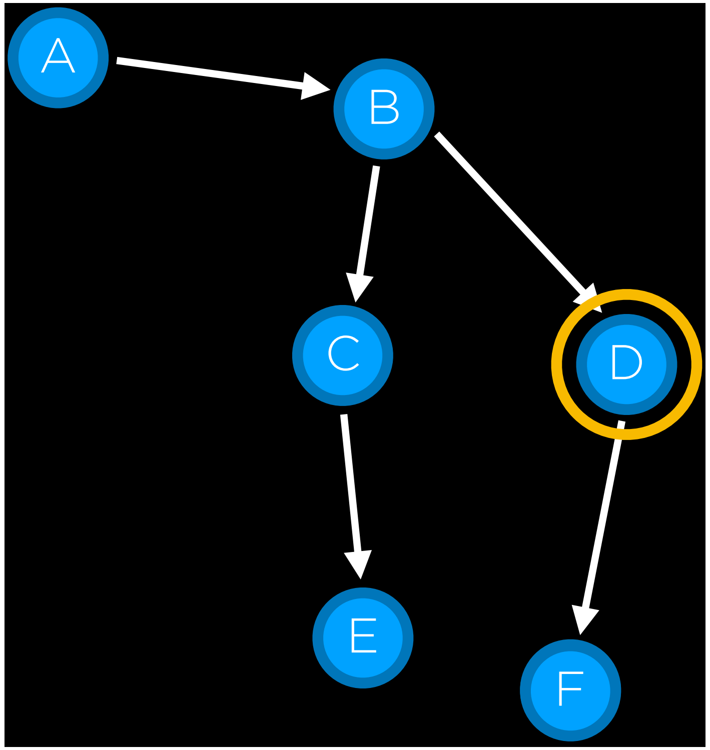
# Example: Find a path from A to E

**Frontier**

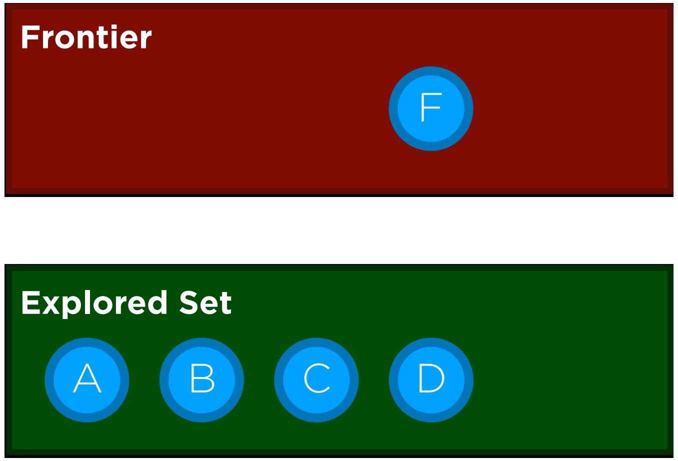
E F

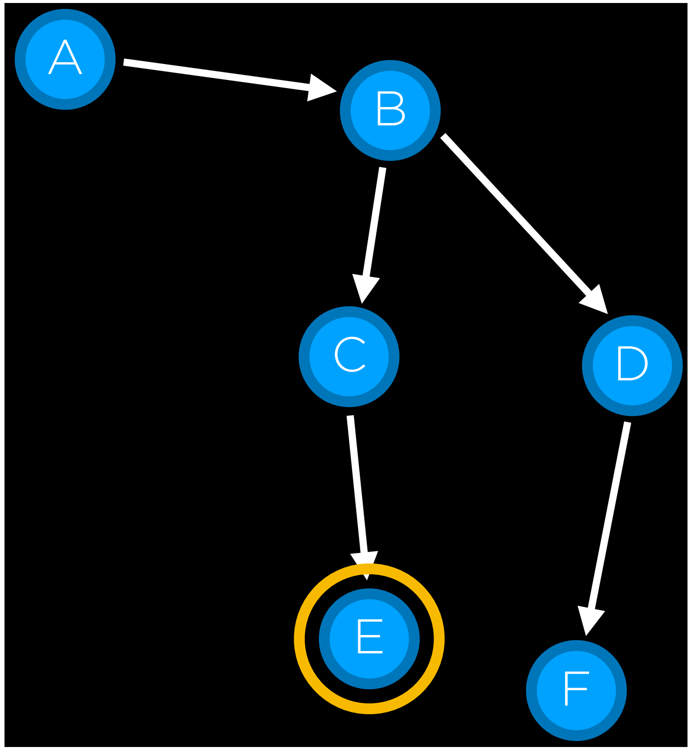
**Explored Set**

A B C D



# Example: Find a path from A to E



# BFS

- Pros
    - This algorithm is guaranteed to find the optimal solution.
- Cons
    - This algorithm is almost guaranteed to take longer than the minimal time to run
    - At worst, this algorithm takes the longest possible time to run

# Breadth-First Search Code Example

```python
# Define the function that removes a node from the frontier and returns it.
def remove(self):
      # Terminate the search if the frontier is empty, because this means that there is no solution.
    if self.empty():
        raise Exception("empty frontier")
    else:
        # Save the oldest item on the list (which was the first one to be added)
        node = self.frontier[0]
        # Save all the items on the list besides the first one (i.e. removing the first node)
        self.frontier = self.frontier[1:]
        return node
```