Course of “Industrial Automation”
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Digitalization

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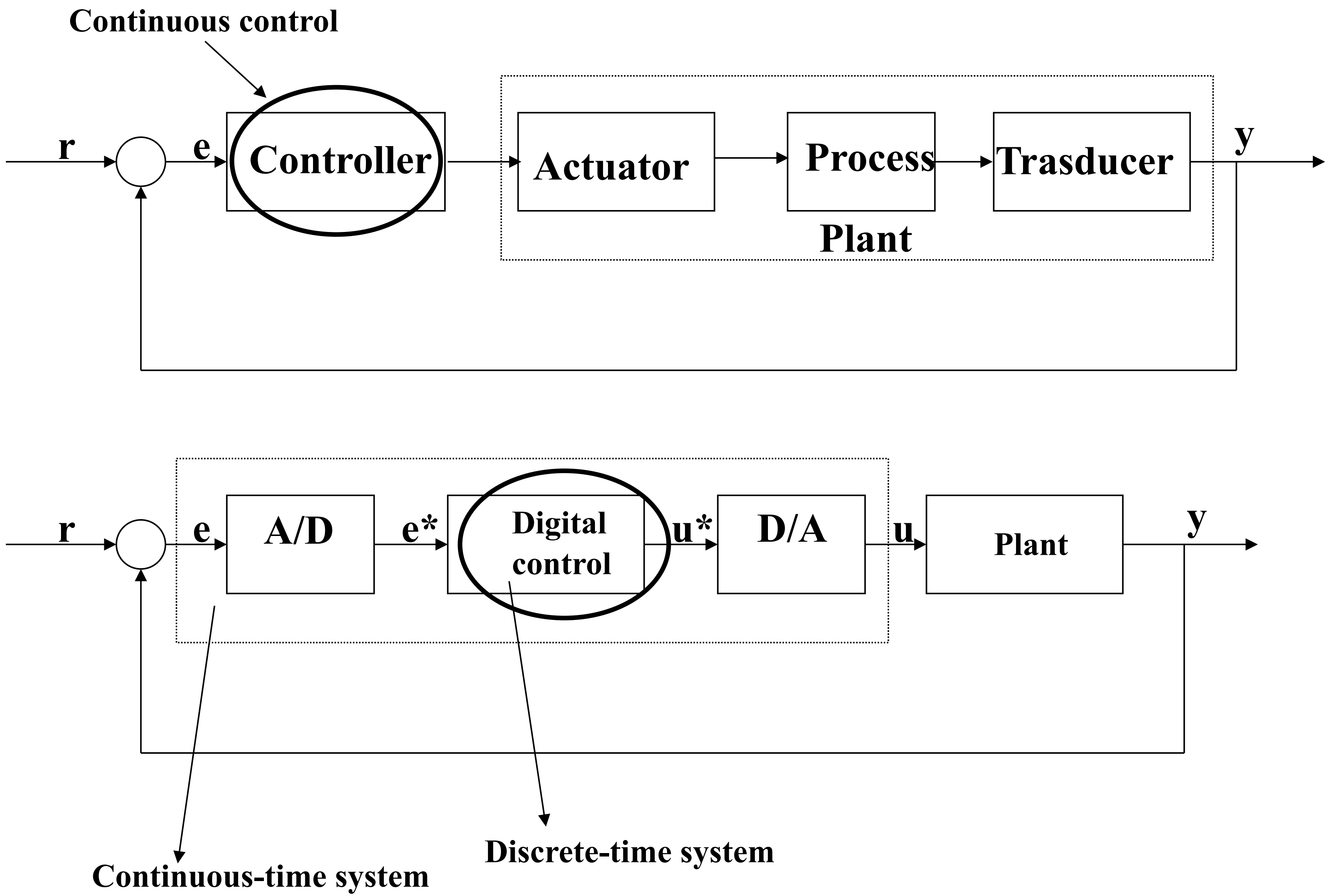
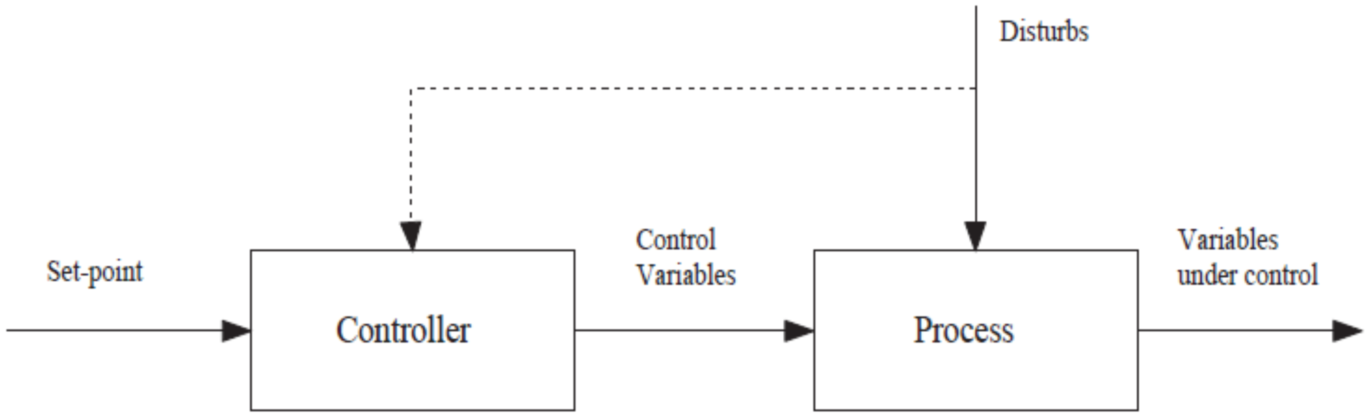
Team code: **vgxlryz**

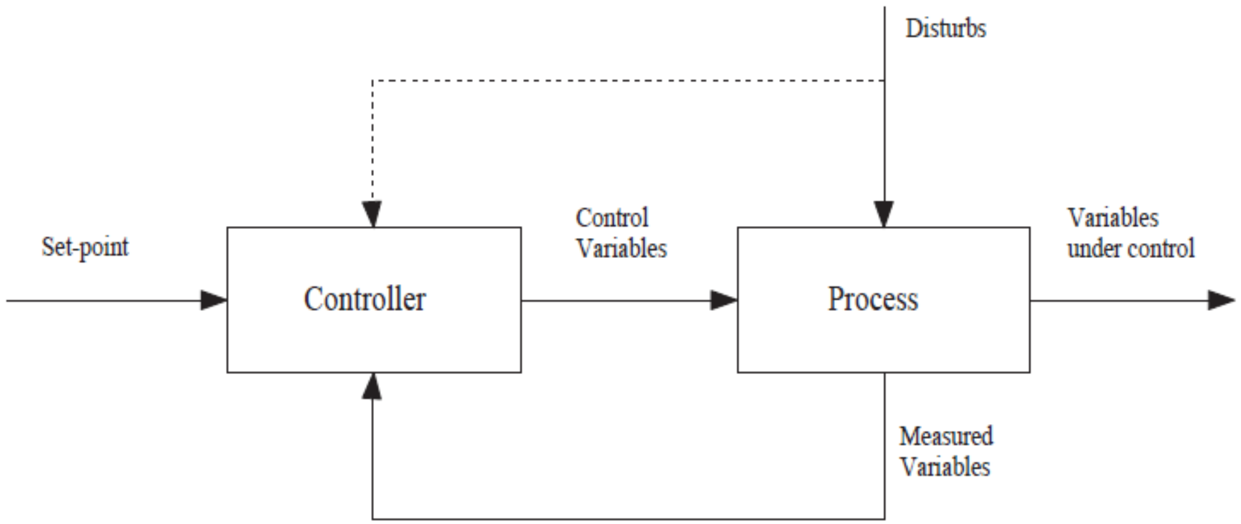
Continuous vs. digital



Continuous control systems



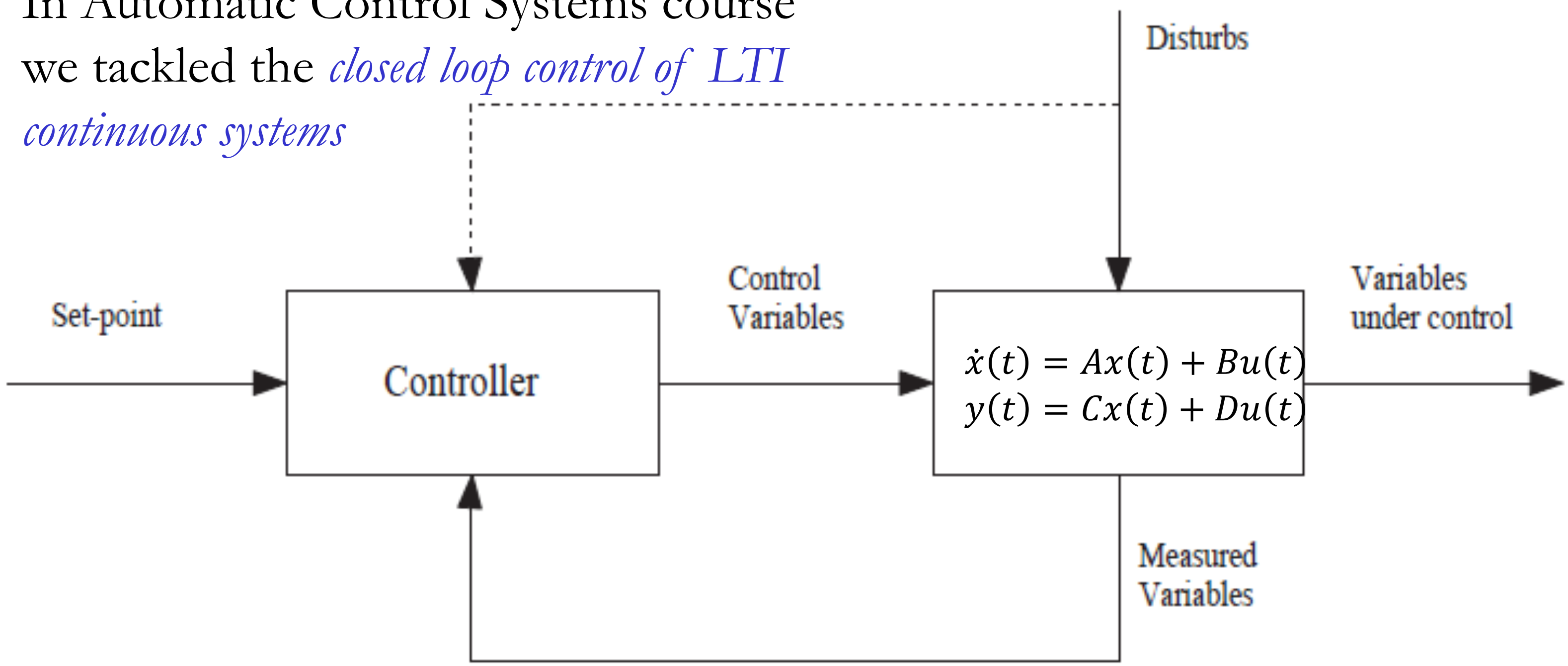
Open loop control strategy → Not robust



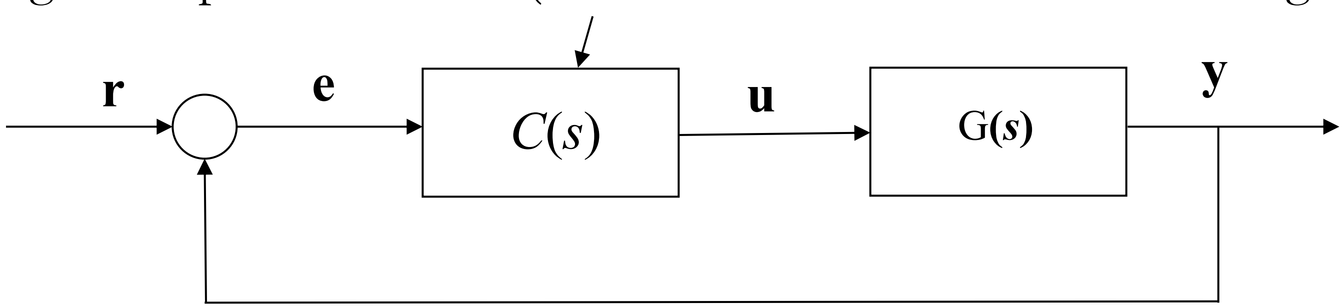
Closed loop control strategy

Continuous control systems

In Automatic Control Systems course we tackled the *closed loop control of LTI continuous systems*



By exploiting the Laplace Trasform (the scheme below without aassuming disturbs)



$$\frac{Y(s)}{R(s)} = W(s) = \frac{C(s)G(s)}{1+C(s)G(s)} = \frac{F(s)}{1+F(s)},$$ with $F(s) = C(s)G(s)$

Controller design: loop shaping

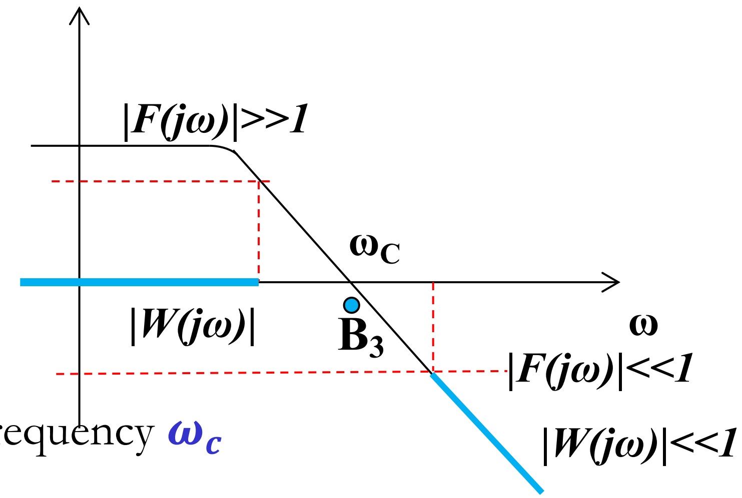
- By assuming at low frequencies

  $F(s) \gg 1 \;\rightarrow\; W(s) \cong 1$

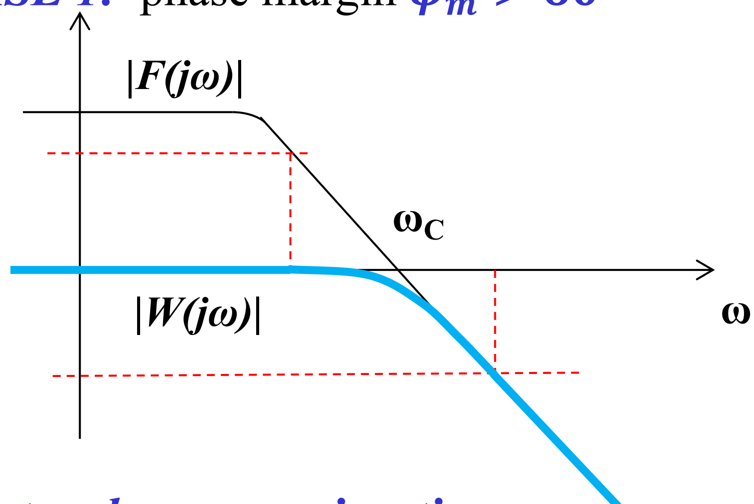
- at high frequencies

  $F(s) \ll 1 \;\rightarrow\; W(s) \cong F(s)$

- Approx. the bandwidth B_3 with the crossing frequency $\boldsymbol{\omega_c}$



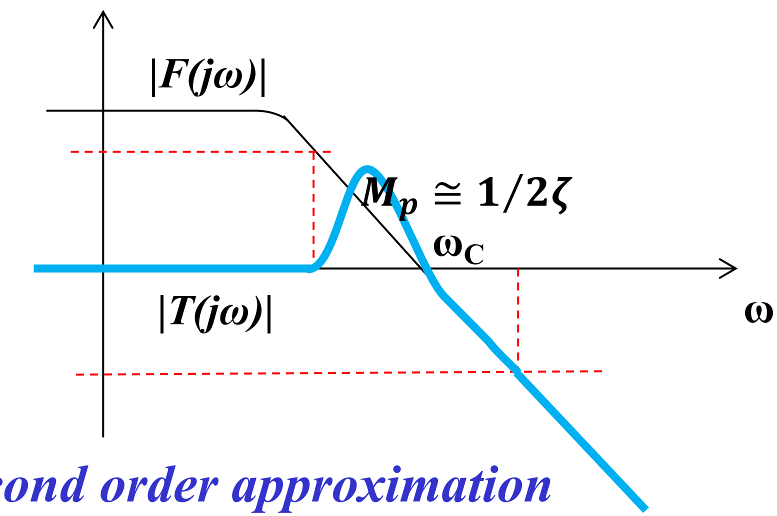
CASE 1: phase margin $\boldsymbol{\varphi_m > 60°}$



First order approximation

$$W_a(s) = \frac{1}{1 + s/\omega_c}$$

CASE 2: phase margin $\boldsymbol{\varphi_m < 60°}$



Second order approximation

$$W_a(s) = \frac{1}{1 + 2\zeta s/\omega_c + s^2/\omega_c^2} \quad \text{and } \zeta \cong \frac{\varphi_m}{100}$$

Continuous controller

The controller *C(s)* in general can be assumed the following form:

$$C(s) = C_1(s)C_2(s),$$

where

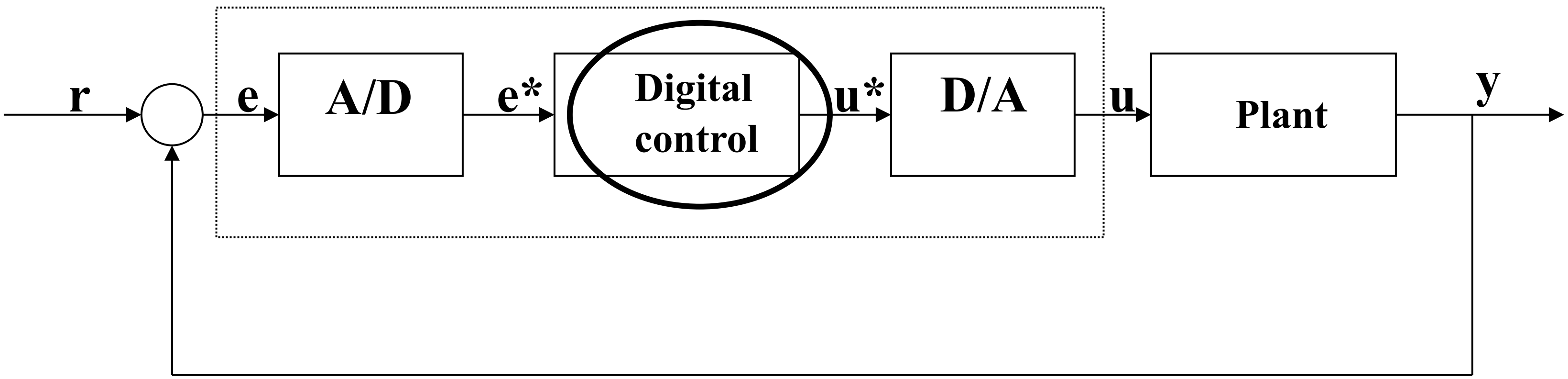
$$C_1\,(s) = \frac{K}{s^{v}}$$ **… to satisfy static performance**

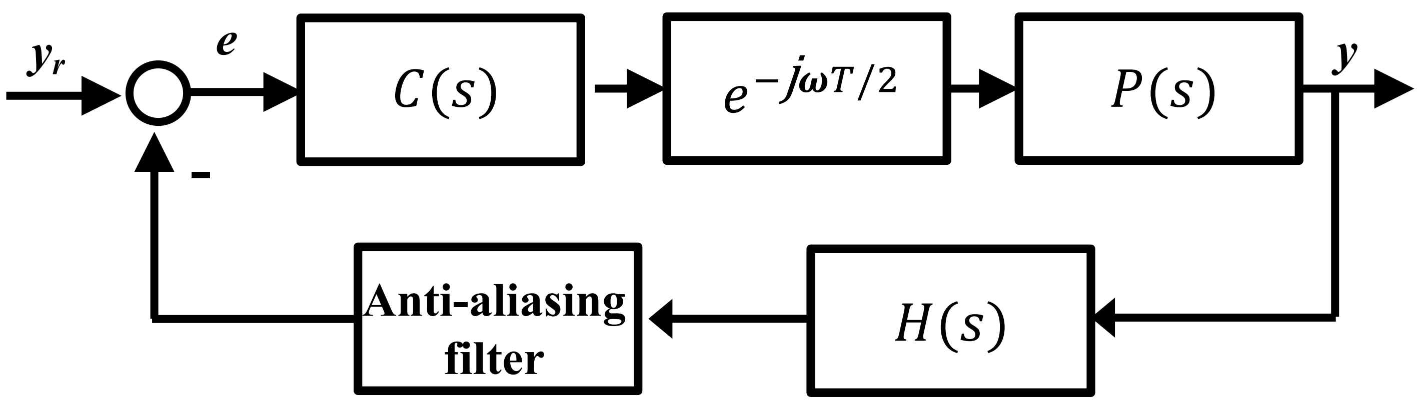
and

$$C_2(s) = \frac{1 + s\tau_1}{1 + s\tau_2}$$ **… a lead-lag compensator to satisfy transient performance, or the cascade of two lead-lag compensators…**

The controller can be realized by using eletronic devises, e.g. analog circuits as operational amplifiers (op amps).

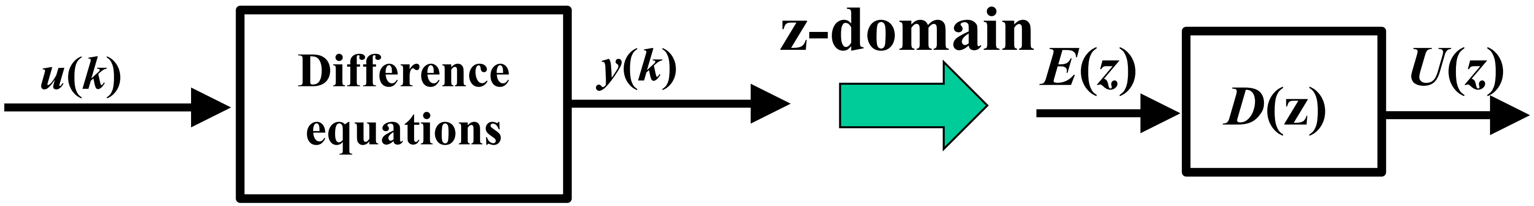
Scheme of the digital control system in continuous-time





From analog to digital

- Implementation of the digital control:



- From $C(s)$ we want to find an equivalent $D(z)$:
- A transformation allows the transition from continuous time to discrete time such that

$$C(s) \cong D(z)$$

same static and dynamic perfomance

$$\boldsymbol{s \longrightarrow z}$$

- Several transformations:

The Laplace trasform of an ideally sampled signal corresponds to the Zeta of the sampled sequence with the sostitution

$$z = e^{sT}$$

By a first-order Padé approximant of the natural logarithm function,

$$s = \frac{2}{T}\frac{z-1}{z+1}$$

Others: By Euler's method,

$s = \frac{z-1}{T}$ (forward rectangular rule) and $s = \frac{z-1}{zT}$ (backward)

Euler's method – Difference equations

- ***y(t)***, **analog** signal; ***T*** is called the **sample period**; ***y(kT***), the **sampled** signal with *k* integer value; it is often written simply as ***y(k)*** - we called this type of variable a **discrete signal**.
- From the definition of a derivative

$$\dot{y} = \lim_{\delta t \to 0} \frac{\delta y}{\delta t}$$

Even if δt is not quite equal to zero

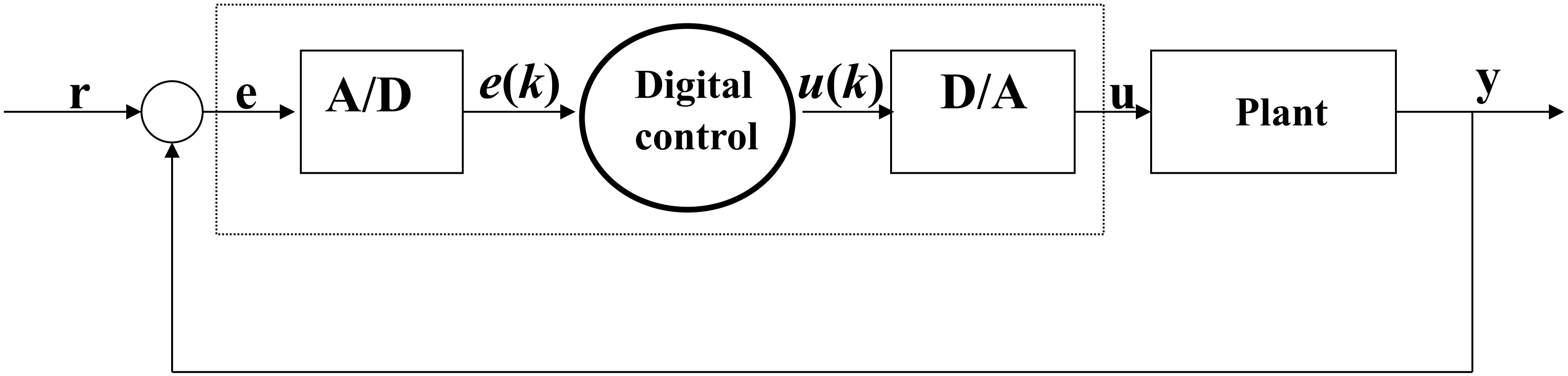
$$\dot{y}(k) = \dot{y}(kT) \cong \frac{y\big((k+1)T\big) - y(kT)}{(k+1)T - kT} = \frac{y(k+1) - y(k)}{T}$$

This approximation can be used in place of all the derivatives that appear in the controller differential equations to arrive at a set of equation (called **difference equations**) that can be solved by a digital computer, repetitevely with time steps of length *T*.

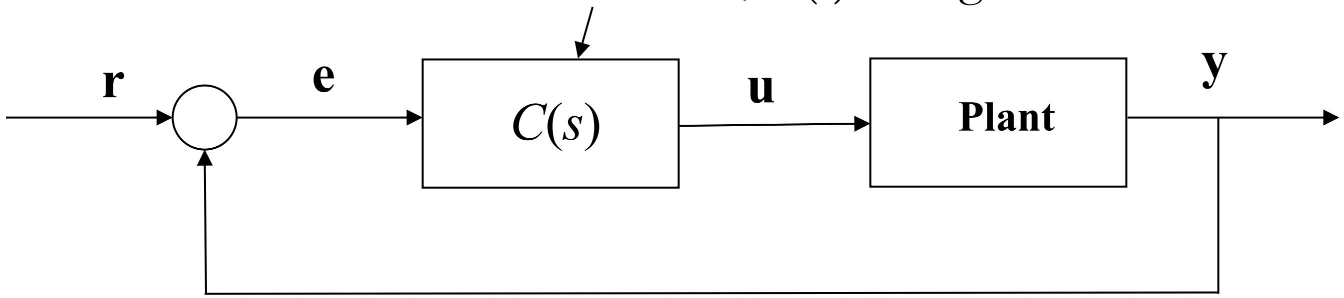
For systems having bandwidths of a few Hertz, sample rate are often on the order of 100 Hz, so that sample peridios are on the order of 10 msec and errors from this approximation can be quite small.

Example – Difference equations by Euler's method

Using Euler's method, find the difference equations to be programmed into the following control system (i.e. digital control block),



for the case where the continuous control, C(s) of figure below



is defined by

$$C(s) = k_0 \frac{s+b}{s+a}$$

Example – Difference equations by Euler's method

$$C(s) = \frac{U(s)}{E(s)} = k_0 \frac{s+b}{s+a} \Rightarrow (s+a)U(s) = k_0(s+b)E(s)$$

The corresponding differential equations

$$\dot{u}(t) + au(t) = k_0\big(\dot{e}(t) + be(t)\big)$$

Using Euler's metyhod

$$\frac{u(k+1) - u(k)}{T} + au(k) = k_0\left(\frac{e(k+1) - e(k)}{T} + be(k)\right)$$

Rearranging

$$u(k+1) = u(k) + T\left[-au(k) + k_0\left(\frac{e(k+1) - e(k)}{T} + be(k)\right)\right]$$

$$\boxed{u(k+1) = (1-aT)u(k) + k_0(bT-1)e(k) + k_0 e(k+1)}$$

Then, the new value of the control, u(k+1), is determined by the value of the control, u(k), and the past and new values of the error signal, e(k) and e(k+1).

Real time controller implementation

$$u(k+1) = (1-aT)u(k) + k_0(bT-1)e(k) + k_0 e(k+1)$$

Implementation

x=0 (initialization of past values for first loop through, i.e. u(k) and e(k))
Define constants:
$a1=1-aT$;
$a2=k_0(bT-1)$;
READ A/D to obtain e (i.e, y and r)
$e=r-y$
$u=x+k_0e$
OUTPUT u to D/A
now compute x for the next loop through
$x=a_1u + a_2e$
go back to READ when T seconds have elapsed since last READ.

Example

Find digital controllers to implement the lead compensator

$$C(s) = 70\frac{s+2}{s+10}$$

for the plant

$$G(s) = \frac{1}{s(s+1)}$$

using different samples rates (for example 20 Hz, …)