

Master Degree in Information Technology Engineering for Health and Communication: Health Curriculum

Electromagnetic interactions and diagnostics

LAB SESSION

MODELING DIELECTRIC PROPERTIES



EXERCISE 1

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EXERCISE 1

Relative Dielectric Constant of Pure Water:

Compute in Matlab environment the real and imaginary parts of the pure water dielectric constant using the double-Debye model. Applicable range of input parameters:

- $0 \leq T (^{\circ}\text{C}) \leq 30$
- $0 \leq f (\text{GHz}) \leq 1000$
- $0 \leq S (\text{psu}) \leq 40$

Guidelines:

- Account for two cases: water temperature set to 0 °C and 20 °C
- Limit the resulting plots in the frequency range 1 – 50 GHz
- Show real and imaginary parts on a single bi-logarithmic plot
- Evaluate the relaxation frequencies



EXERCISE 1

Relative Dielectric Constant of Pure Water:

$$\varepsilon = \varepsilon_{w\infty} + \frac{\varepsilon_{w0} - \varepsilon_{w1}}{(1 + j2\pi f\tau_{w1})} + \frac{\varepsilon_{w1} - \varepsilon_{w\infty}}{(1 + j2\pi f\tau_{w2})}$$

$$\left\{ \begin{array}{l} \varepsilon_{w0} = 87.85306e^{-0.00456992 \cdot T} \\ \varepsilon_{w1} = a_4 e^{-a_5 \cdot T} \\ \varepsilon_{w\infty} = a_{16} + a_{17} \cdot T \\ \tau_{w1}(\text{ns}) = a_8 e^{\frac{a_{10}}{T+a_{11}}} \\ \tau_{w2}(\text{ns}) = a_{12} e^{\frac{a_{14}}{T+a_{15}}} \end{array} \right.$$

$$\begin{array}{l} a_1 = 0.46606917\text{E} - 02 \\ a_2 = -0.26087876\text{E} - 04 \\ a_3 = -0.63926782\text{E} - 05 \\ a_4 = 0.63000075\text{E} + 01 \\ a_5 = 0.26242021\text{E} - 02 \\ a_6 = -0.42984155\text{E} - 02 \\ a_7 = 0.34414691\text{E} - 04 \\ a_8 = 0.17667420\text{E} - 03 \\ a_9 = -0.20491560\text{E} - 06 \\ a_{10} = 0.58366888\text{E} + 03 \\ a_{11} = 0.12684992\text{E} + 03 \\ a_{12} = 0.69227972\text{E} - 04 \\ a_{13} = 0.38957681\text{E} - 06 \\ a_{14} = 0.30742330\text{E} + 03 \\ a_{15} = 0.12634992\text{E} + 03 \\ a_{16} = 0.37245044\text{E} + 01 \\ a_{17} = 0.92609781\text{E} - 02 \\ a_{18} = -0.26093754\text{E} - 01 \end{array}$$

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EXERCISE 1

Relative Dielectric Constant of Pure Water:

```
function [epsr epsi] = RelDielConst_PureWater(t,f)

%Model parameters
a=[0.63000075e1 0.26242021e-2 0.17667420e-3 0.58366888e3 0.12634992e3 ...
0.69227972e-4 0.30742330e3 0.12634992e3 0.37245044e1 0.92609781e-2];

%Equation parameters
epsS = 87.85306*exp(-0.00456992*t);
eps0ne = a(1)*exp(-a(2)*t);
tau1 = a(3)*exp(a(4)/(t+a(5)));
tau2 = a(6)*exp(a(7)/(t+a(8)));
epsInf = a(9) + a(10)*t;

%Complex permittivity evaluation
eps = ((epsS-eps0ne)./(1+1i*2*pi.*f.*tau1)) + ((eps0ne-epsInf)./(1+1i*2*pi.*f.*tau2)) + epsInf;
%Extraction of real and imaginary components of relative permittivity
epsr = real(eps);
epsi = -imag(eps);

end
```



EXERCISE 1

Relative Dielectric Constant of Pure Water:

```
clc, clear all, close all
```

```
f1 = linspace(1,50,20000);
```

```
t = %INPUT WATER TEMPERATURE%
```

```
[dc_w0 loss_fact_w0] = RelDielConst_PureWater(t,f1);
```

```
figure(), loglog(f1,dc_w0,'b'), grid on, hold on, loglog(f1,loss_fact_w0,'r'),  
axis([1 50 1 100]), xlabel('Frequency (GHz)'), ylabel('Pure water dielectric  
constant @ 0 °C'), legend('Relative permittivity','Loss factor')
```

```
[LF_m f_m] = max(loss_fact_w0);  
f_c = f1(f_m)
```



EXERCISE 2

Relative Dielectric Constant of Saline Water:

Compute in Matlab environment the real and imaginary parts of the saline water dielectric constant using the double-Debye model. Applicable range of input parameters:

- $0 \leq T (^{\circ}\text{C}) \leq 30$
- $0 \leq f (\text{GHz}) \leq 1000$
- $0 \leq S (\text{psu}) \leq 40$

Guidelines:

- Set water temperature and salinity to 20 °C and 32.54 psu, respectively
- Show the resulting plots on a bi-logarithmic scale in the whole frequency range
- Compare results with the dielectric behavior of pure water



EXERCISE 2

Relative Dielectric Constant of Saline Water:

$$\varepsilon = \varepsilon_{w\infty} + \frac{\varepsilon_{w0} - \varepsilon_{w1}}{(1 + j2\pi f\tau_{w1})} + \frac{\varepsilon_{w1} - \varepsilon_{w\infty}}{(1 + j2\pi f\tau_{w2})} - j \frac{17.9751 \sigma_i}{f}$$

$$\left\{ \begin{array}{l} \varepsilon_{w0} = 87.85306 e^{(-0.00456992 \cdot T - a_1 \cdot S - a_2 \cdot S^2 - a_3 \cdot S \cdot T)} \\ \varepsilon_{w1} = a_4 e^{(-a_5 \cdot T - a_6 \cdot S - a_7 \cdot S \cdot T)} \\ \varepsilon_{w\infty} = a_{16} + a_{17} \cdot T + a_{18} \cdot S \\ \tau_{w1}(\text{ns}) = (a_8 + a_9 \cdot S) e^{\frac{a_{10}}{T + a_{11}}} \\ \tau_{w2}(\text{ns}) = (a_{12} + a_{13} \cdot S) e^{\frac{a_{14}}{T + a_{15}}} \\ \sigma_i = \sigma(T, 35) \cdot P(S) \cdot Q(T, S) \\ \sigma(T, 35) = 2.903602 + 8.607 \cdot 10^{-2} \cdot T + 4.738817 \cdot 10^{-4} \cdot T^2 - 2.991 \cdot 10^{-6} \cdot T^3 + 4.3041 \cdot 10^{-9} \cdot T^4 \\ P(S) = S \frac{37.5109 + 5.45216 \cdot S + 0.014409 \cdot S^2}{1004.75 + 182.283 \cdot S + S^2} \\ Q(T, S) = 1 + \frac{\left(\frac{6.9431 + 3.2841 \cdot S - 0.099486 \cdot S^2}{84.85 + 69.024 \cdot S + S^2} \right) \cdot (T - 15)}{T + 49.843 - 0.2276 \cdot S + 0.00198 \cdot S^2} \end{array} \right.$$

$$\begin{array}{l} a_1 = 0.46606917\text{E} - 02 \\ a_2 = -0.26087876\text{E} - 04 \\ a_3 = -0.63926782\text{E} - 05 \\ a_4 = 0.63000075\text{E} + 01 \\ a_5 = 0.26242021\text{E} - 02 \\ a_6 = -0.42984155\text{E} - 02 \\ a_7 = 0.34414691\text{E} - 04 \\ a_8 = 0.17667420\text{E} - 03 \\ a_9 = -0.20491560\text{E} - 06 \\ a_{10} = 0.58366888\text{E} + 03 \\ a_{11} = 0.12684992\text{E} + 03 \\ a_{12} = 0.69227972\text{E} - 04 \\ a_{13} = 0.38957681\text{E} - 06 \\ a_{14} = 0.30742330\text{E} + 03 \\ a_{15} = 0.12634992\text{E} + 03 \\ a_{16} = 0.37245044\text{E} + 01 \\ a_{17} = 0.92609781\text{E} - 02 \\ a_{18} = -0.26093754\text{E} - 01 \end{array}$$

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EXERCISE 2

Relative Dielectric Constant of Saline Water:

```
function [epsr epsi] = RelDielConst_SalineWater(t,f,S)

%Model parameters: conductivity
A = [2.903602 8.607e-2 4.738817e-4 -2.991e-6 4.3041e-9];
sig35 = A(1) + A(2)*t + A(3)*t^2 + A(4)*t^3 + A(5)*t^4;
A = [37.5109 5.45216 0.014409 1004.75 182.283];
P = S * ((A(1) + A(2)*S + A(3)*S^2) / (A(4) + A(5)*S + S^2));
A = [6.9431 3.2841 -0.099486 84.85 69.024];
alpha0 = (A(1) + A(2)*S + A(3)*S^2) / (A(4) + A(5)*S + S^2);
A = [49.843 -0.2276 0.00198];
alpha1 = A(1) + A(2)*S + A(3)*S^2;
Q = 1 + ((alpha0*(t-15))/(t+alpha1));
sigma = sig35*P*Q;
%Other model paramaters
a=[0.46606917e-2 -0.26087876e-4 -0.63926782e-5 0.63000075e1 0.26242021e-2 -0.42984155e-2 ...
0.34414691e-4 0.17667420e-3 -0.20491560e-6 0.58366888e3 0.12634992e3 0.69227972e-4 ...
0.38957681e-6 0.30742330e3 0.12634992e3 0.37245044e1 0.92609781e-2 -0.26093754e-1];
epsS = 87.85306*exp(-0.00456992*t - a(1)*S - a(2)*S^2 - a(3)*S*t);
eps0ne = a(4)*exp(-a(5)*t-a(6)*S-a(7)*S*t);
tau1 = (a(8)+a(9)*S)*exp(a(10)/(t+a(11)));
tau2 = (a(12)+a(13)*S)*exp(a(14)/(t+a(15)));
epsInf = a(16) + a(17)*t + a(18)*S;
%Complex permittivity evaluation
eps = ((epsS-eps0ne)./(1+1i*2*pi.*f.*tau1)) + ((eps0ne-epsInf)./(1+1i*2*pi.*f.*tau2)) + epsInf - 1i*((17.9751*sigma)./f);
%Extraction of real and imaginary components of relative permittivity
epsr = real(eps);
epsi = -imag(eps);

end
```



EXERCISE 2

Relative Dielectric Constant of Saline Water:

```
f = linspace(1,1000,20000);  
t2 = 20;  
S = 32.54;
```

```
[dc_sw loss_fact_sw] = RelDielConst_SalineWater(t2,f,S);
```

```
figure(),loglog(f,dc_sw,'b'), grid on, hold on,loglog(f,loss_fact_sw,'r'),  
axis([1 1000 1 100]), xlabel('Frequency (GHz)'), ylabel('Saline water  
dielectric constant @ 20 °C'), legend('Relative permittivity','Loss factor')
```



EXERCISE 3

Relative Dielectric Constant of Heterogeneous Mixture:

Compute in Matlab environment, according to the TVB model, the real and imaginary parts of the equivalent dielectric constant of a two-phase heterogeneous mixture composed by air as continuous medium hosting randomly oriented spherical lossy inclusions.

Inputs:

- Dielectric properties of the host medium (air, ε_h)
- Dielectric properties (human skin @30GHz, $\varepsilon_i = 5 - j11.5$) and shape of sparse particles

Outputs:

- Show a linear-scale plot for mixture permittivity and loss factor
- Plot results versus inclusion volume fraction
- Discuss the effects of inclusions' shape



EXERCISE 3

Showcase: Tinga-Voss-Blossey mixture dielectric model

- Thin circular disc inclusions: $\epsilon_m = \epsilon_h + \frac{v_i}{3} (\epsilon_i - \epsilon_h) \left[\frac{2\epsilon_i(1 - v_i) + \epsilon_h(1 + 2v_i)}{v_i\epsilon_h + (1 - v_i)\epsilon_i} \right]$
- $a_1 = b_1, a_2 = b_2,$
 $c_1 \ll a_1, c_2 \ll a_2$
 $A_{a_1} = A_{a_2} = A_{b_1} = A_{b_2} = 0$
 $A_{c_1} = A_{c_2} = 1$
- Spherical inclusions: $\epsilon_m = \epsilon_h + \frac{3v_i\epsilon_h(\epsilon_i - \epsilon_h)}{(2\epsilon_h + \epsilon_i) - v_i(\epsilon_i - \epsilon_h)}$
- $A_{a_1} = A_{a_2} = A_{b_1} = A_{b_2} = A_{c_1} = A_{c_2} = \frac{1}{3}$
- Long narrow needle inclusions: $\epsilon_m = \epsilon_h + \frac{v_i}{3} (\epsilon_i - \epsilon_h) \left[\frac{\epsilon_h(5 + v_i) + (1 - v_i)\epsilon_i}{\epsilon_h(1 + v_i) + \epsilon_i(1 - v_i)} \right]$
- $A_{a_1} = A_{a_2} = A_{b_1} = A_{b_2} = 0.5$
 $A_{c_1} = A_{c_2} = 0$



EXERCISE 3

Showcase: Tinga-Voss-Blossey mixture dielectric model

```
function [eps_m] = TVBmodel_HeterogeneousMix(eps_i, eps_h, shape, vi)

if shape == 1 % thin circular disc inclusions
eps_m = eps_h + vi./3.*(eps_i - eps_h).*(2.*eps_i.*(1-vi)+eps_h.*(1+2.*vi))./(vi.*eps_h +(1-vi).*eps_i);
end

if shape ==2 % spherical inclusions
eps_m = eps_h + 3*vi*eps_h*(eps_i -eps_h)./((2*eps_h+eps_i)-vi*(eps_i-eps_h));
end

if shape == 3 % needle inclusions
eps_m = eps_h + vi./3.*(eps_i-eps_h).*(eps_h.*(5+vi)+(1-vi).*eps_i)./(eps_h.*(1+ vi)+eps_i.*(1-vi));
end

end
```



EXERCISE 3

Showcase: Tinga-Voss-Blossey mixture dielectric model

```
eps_h = 1; eps_i = 5 - 1i*11.5;
shape = 2;
vi = linspace(0,1,1000);
[eps_m] = TVBmodel_HeterogeneousMix(eps_i, eps_h, shape, vi);

eps_m_1 = real(eps_m); eps_m_2 = -imag(eps_m);

figure(),plot(vi,eps_m_1,'k'), grid on,xlabel('Inclusion volume fraction'), ylabel('Mixture equivalent permittivity')
figure(),plot(vi,eps_m_2,'k'), grid on,xlabel('Inclusion volume fraction'), ylabel('Mixture loss factor')

shape = 1;
[eps_m_D] = TVBmodel_HeterogeneousMix(eps_i, eps_h, shape, vi);
eps_m_D1 = real(eps_m_D); eps_m_D2 = -imag(eps_m_D);

shape = 3;
[eps_m_N] = TVBmodel_HeterogeneousMix(eps_i, eps_h, shape, vi);
eps_m_N1 = real(eps_m_N); eps_m_N2 = -imag(eps_m_N);

figure(),plot(vi,eps_m_1,'k'), grid on, hold on, plot(vi,eps_m_D1,'b'), plot(vi,eps_m_N1,'r'), xlabel('Inclusion volume fraction'), ylabel('Mixture equivalent permittivity'), legend('Spherical','Disc','Needle')
figure(),plot(vi,eps_m_2,'k--'), grid on, hold on, plot(vi,eps_m_D2,'b--'), plot(vi,eps_m_N2,'r--'), xlabel('Inclusion volume fraction'), ylabel('Mixture loss factor'), legend('Spherical','Disc','Needle')
```

